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Two-Warehouse Fuzzy Inventory Model with K-Release Rule

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ABSTRACT: Fuzzy set theory is primarily concerned with how to quantitatively deal with imprecision and uncertainty, and offers the decision maker another tool in addition to the classical deterministic and probabilistic mathematical tools that are used in modeling real-world problems. The present study investigates a fuzzy economic order quantity model for two storage facility. The demand, holding cost, ordering cost, storage capacity of the own - warehouse are taken as a trapezoidal fuzzy numbers. Graded Mean Representation is used to defuzzify the total cost function and the results obtained by this method are compared with the help of a numerical example. Sensitivity analysis is also carried out to explore the effect of changes in the values of some of the system parameters. The proposed methodology is applicable to other inventory models under uncertainty.

Keywords: Inventory, Two – warehouse system, Fuzzy Variable, Trapezoidal Fuzzy Number, Graded mean representation method and K – release rule.

1. INTRODUCTION

In most of the inventory models that had been proposed in the early literature, the associated costs are assumed to be precise, although the real-world inventory costs usually exist with imprecise components. In this case, customer demand as one of the key parameters and source of uncertainty have been most often treated by a probability distribution. However, the probability-based approaches may not be sufficient enough to reflect all uncertainties that may arise in a real-world inventory system. Modelers may face some difficulties while trying to build a valid model of an inventory system, in which the related costs cannot be determined precisely. For example, costs may be dependent on some foreign monetary unit. In such a case, due to a change in the exchange rates, the costs are often not known precisely.

Fuzzy set theory, originally introduced by Zadeh [1], provides a framework for considering parameters that are vaguely or unclearly defined or whose values are imprecise or determined based on subjective beliefs of individuals. Petrovic et al. [2] presented newsboy problem assuming that demand and backorder cost are fuzzy numbers. Kaufamann and Gupta [3] introduced to fuzzy arithmetic: theory and application. The application of fuzzy theory to inventory problem has been proposed by Kacprzyk and Staniewski [4]. Roy and Maiti [5] presented a fuzzy inventory model with constraint. Roy and Maiti [6] developed a fuzzy EOQ model with demand-dependent unit cost under limited storage capacity. Ishii and Konno [7] introduced fuzziness of shortage cost explicitly into classical inventory problem. Chen and Hsieh [8] established a fuzzy economic production model to treat the inventory problem with all the parameters and variables, which are fuzzy numbers. Hsieh [9] presented a fuzzy production inventory model. Yao and Chiang [10] presented an inventory model without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. Dutta et al. [11] developed a single-period inventory model with fuzzy random variable demand. In that study, they have applied graded mean integration representation method to find the optimum order quantity. Chen and Chang [12] presented an optimization of fuzzy production inventory model. In this study, they have used 'Function Principle' as arithmetical operations of fuzzy total production inventory cost and also used the 'Graded Mean Integration Representation method' to defuzzify the fuzzy total production and inventory cost. Mahata and Goswami [13] presented a fuzzy inventory model for deteriorating items with the help of fuzzy numbers and so on.

Most of the classical inventory models discussed in the literature deals with the situation of a single warehouse. Because of capacity limitation a single warehouse would not be always sufficient. Additional warehouse are necessary to store excess items. Therefore due to the limited capacity of the existing warehouse (Rented warehouse, RW) is acquired to keep excess items. In practice, large stock attracts the management due to either an attractive price discount for bulk purchase or the acquisition cost being higher than the holding cost in RW. The actual service to the customer is done at OW only. Usually the holding cost is greater in RW than in OW. So in order to reduce the holding cost. The stock of rented warehouse is transferred to the own warehouse. Hartley [14] was discussed a model under the assumption that the cost of transporting a unit from RW to OW is not significantly high. It was as the case with two levels of storage. Sarma [15] extended the model with two levels of storage given by Hartley, by considering the transportation cost of a unit from rented warehouse to own warehouse. Maurdeswar and Sathe [16] discussed this model by relaxing the condition on production rate (finite production rate). Dave [17] considered it for finite and infinite replenishment, assuming the cost of transportation depending on the quantity to be transported. Pakkala and Achary [18] developed a model for deteriorating items with two warehouses. They extended it with bulk release rule, after words, Gowsami and Chaudhari [19] formulated models for time dependent demand. Kar et al. [20] suggested a two level inventory model for linear trend in demand. Yang [21] considered a two-warehouse inventory models for deteriorating items with shortages under inflation. Singh et al. [22] presented two-warehouse inventory model without shortage for exponential demand rate and an optimum release rule. Jaggi and Verma [23] developed a deterministic order level inventory model with two storage facilities. It has been observed in supermarkets that the demand rate is usually influenced by the amount of stock level, that is, the demand rate may go up or down with the on-hand stock level. Singh et al. [24] developed a deterministic two-warehouse inventory model for deteriorating items with stock-dependent demand and shortages. Neeraj et al. [25] developed three echelon supply chain inventory model with two storage facility. Neeraj et al. [26] presented a two-warehouse inventory model with K-release rule and learning effect. Neeraj et al. [27] considered effect of salvage value on a two-warehouse inventory model. Recently, Kumar and Kumar [28] developed an inventory model with stock dependent demand rate for deterioration items.

Here, in this paper the cost of transporting a unit is considered to be significant and the effect of releasing the stocks of RW in n shipments with a bulk size of K units per shipment, instead of withdrawing an arbitrary quantity, is assumed. Here, K is to be decided optimally and is call this as K-release rule. This problem is to decide the optimal values of Q and C, which minimize the sum of ordering, holding and transportation costs of the system. Here, we assumed that the storage capacity of the own – warehouse, the holding cost in both warehouses and ordering cost is fuzzy in nature. The associated total cost minimization is illustrated by numerical example and sensitivity analysis is carried out by using *MATHEMATICA–5.2* for the feasibility and applicability of our model.

2. ASSUMPTIONS AND NOTATIONS:

The following assumptions are used to analyze this inventory model:

1. D is the constant demand rate.
2. W is the storage capacity of the OW.

3. A is the fixed set – up cost per order.
4. C(Q) is the cost function.
5. Q is the highest inventory level.
6. H is the holding cost in OW.
7. F is the holding cost in RW.
8. $\tilde{D}$ is the fuzzy demand rate.
9. $\tilde{A}$ is the fuzzy set – up cost per order.
10. $\tilde{H}$ is the fuzzy holding cost in OW.
11. $\tilde{F}$ is the fuzzy holding cost in RW.
12. $\tilde{C}(Q)$ is the fuzzy cost function.
13. $\tilde{W}$ is the fuzzy storage capacity of the OW.
14. The holding cost per unit in OW is higher than in RW.
15. The storage capacity of OW as W and that of RW is unlimited.
16. The transportation cost of K units from RW to OW is C_t at a time, which is constant over time.
17. The items of RW are transferred to OW in 'n' shipments of which K (K ≤ W) units are transported in each shipment.
18. Replenishment rate is infinite.
19. Lead-time is zero.
20. Consumption takes place only in OW.

3. FUZZY SETS, MEMBERSHIP FUNCTION, DEFUZZIFYING APPROACH AND ARITHMETICAL OPERATIONS

3.1. Fuzzy Sets

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. Let X={x} denote a space of objects. Then a fuzzy set A in X is a set of ordered pairs:

$$A = \{ x, \mu_A(x) \},\ x \in X$$

Where, $\mu_A(x)$ is termed " the grade of the membership of x in A". For simplicity, $\mu_A(x)$ is a number in the interval [0, 1], with the grades of unity and zero respectively, full membership and non-membership in the fuzzy set. An object (point) P contained in a set (class) Q is an element of $Q\ (P \subset Q)$.

3.2. Membership Function

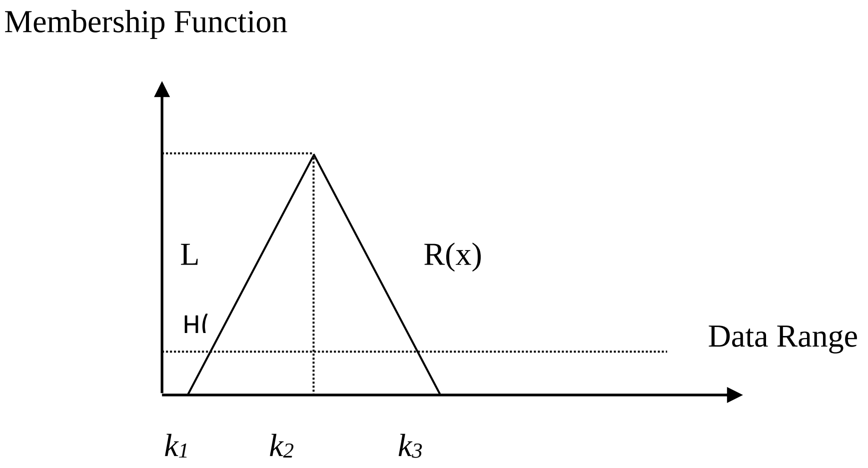

Fig. 1 Membership function for triangle number

At the outset it would be prudent introduce the concept of membership function. There are different shapes of membership function in the inventory control such as the triangle and trapezoid. The shapes of the triangle membership function and the trapezoid membership function are shown in Fig. 1 and 2.

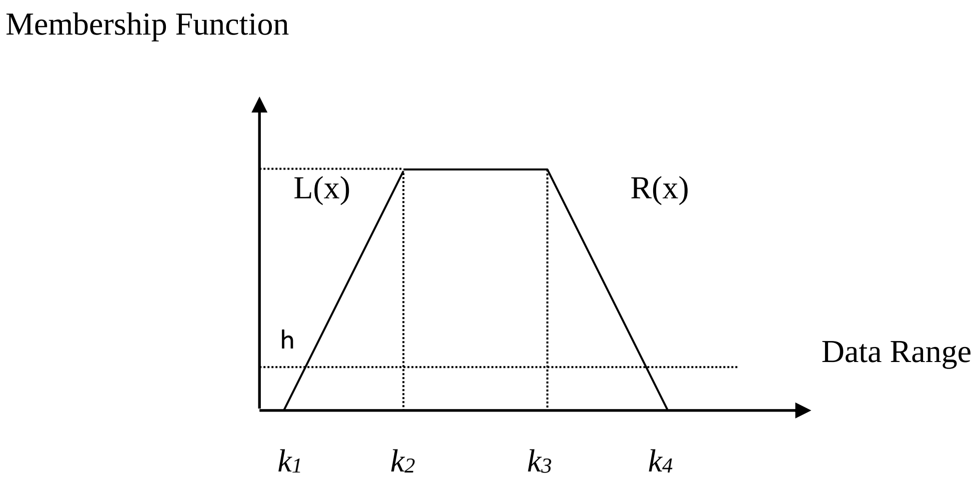

Fig. 2 Membership function for trapezoid number

Ã is assumed as a fuzzy number. If Ã is a triangle number, Ã can be represented as Ã = $[k_1, k_2, k_3]$ subject to the constraint $0 < k_1 \le k_2 \le k_3$. While Ã is a trapezoid fuzzy number, Ã = $[k_1, k_2, k_3, k_4]$ subject to the constraint that $0 < k_1 \le k_2 \le k_3 \le k_4$. Membership function of the triangle and trapezoid fuzzy numbers can be defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < k_1,\ x > k_3 \\ L(x) = \dfrac{x - k_1}{k_2 - k_1} & k_1 \le x < k_2 \\ R(x) = \dfrac{k_3 - x}{k_3 - k_2} & k_2 \le x < k_3 \end{cases}$$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < k_1, x > k_4 \\ L(x) = \dfrac{x - k_1}{k_2 - k_1} & k_1 \le x < k_2 \\ 1 & k_2 \le x < k_3 \\ R(x) = \dfrac{k_4 - x}{k_4 - k_3} & k_3 \le x \le k_4 \end{cases}$$

where $\mu_{\tilde{A}}(x)$ is a membership function.

3.3. Graded Mean Integration Representation Method

In this study, generalized fuzzy number Ã was denoted in Fig. 6.1 as Ã = $(c, a, b, d, \omega_A)_{LR}$. When ω_A = 1, we simplify the notation as $\tilde{A} = (c, a, b, d)_{LR}$. Chen and Hsieh (1999) introduced the graded mean integration representation method of generalized fuzzy number based on the integral value of graded mean h–level of generalized fuzzy number. Its meaning is as follows:

Let L^{-1} and R^{-1} are inverse function of L and R respectively, then the graded mean h–level value of generalized fuzzy number $\tilde{A} = (c, a, b, d, W_A)_{LR}$ is $h\ \left(L^{-1}(h) + R^{-1}(h)\right)$ / 2 as Fig. 3.

Then the graded mean integration representation of Ã is

$$P(\tilde{A}) = \int_o^{W_A} \frac{h\left(L^{-1}(A) + R^{-1}(h)\right)}{2} dh \Big/ \int_o^{W_A} h dh\ ,$$

where $0 < h \le W_A$ and $0 < W_A \le 1$.

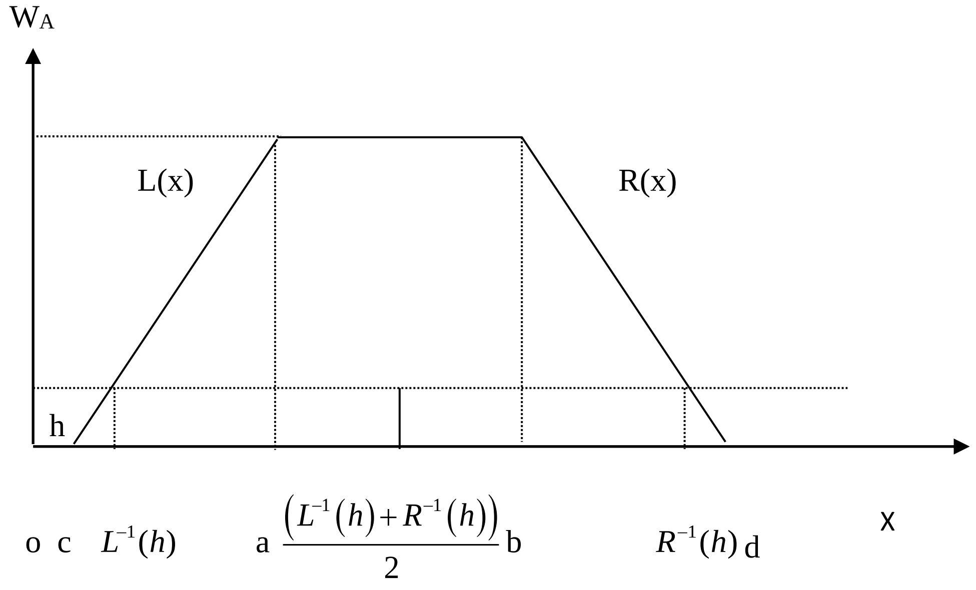

Fig. 3 The graded mean h-level of generalized fuzzy number A = (c, a, b, d, W_A)$_{LR}$

In the present, the generalized trapezoidal fuzzy number has been used as the type of all fuzzy parameters in our proposed inventory models. The very popular generalized trapezoidal fuzzy number B is a special case of generalized fuzzy number and can be denoted as $\tilde{B} = (c, a, b, d; W_B)$ its' corresponding graded mean integration representation is

$$P(\tilde{B}) = \int_0^{W_B} \frac{h(c + d + (a - c - d + b)h / W_B)dh}{2} \Bigg/ \int_0^{W_B} hdh = \frac{c + 2a + 2b + d}{6}$$

where a, b, c, d are any real numbers.

3.4. Properties of Second Function Principle

Chen (1985) proposed second function principal to be as the fuzzy arithmetical operations between generalized trapezoidal fuzzy numbers. Because it does not change the type of membership function of generalized fuzzy number after arithmetical operations. It reduces the trouble and tediousness of operations. Furthermore, Chen already proved the properties of fuzzy arithmetical operations under second function principle. Here some properties of the fuzzy arithmetical operations have been described as follows:

Suppose $\tilde{A}_1 = (c_1, a_1, b_1, d_1)$ and $\tilde{A}_2 = (c_2, a_2, b_2, d_2)$ are two generalized trapezoidal fuzzy numbers. Then

1. The addition of $\tilde{A}_1$ and $\tilde{A}_2$ is $\tilde{A}_1 \oplus \tilde{A}_2 = (c_1 + c_2, a_1 + a_2, b_1 + b_2, d_1 + d_2)$
2. The multiplication of $\tilde{A}_1$ and $\tilde{A}_2$ is $\tilde{A}_1 \otimes \tilde{A}_2 = (c_1c_2, a_1a_2, b_1b_2, d_1d_2)$
3. $-\tilde{A}_2 = (-d_2, -b_2, -a_2, -c_2)$ Then the subtraction of $\tilde{A}_1$ and $\tilde{A}_2$ is $\tilde{A}_1 \Theta \tilde{A}_2$ = $(c_1 - d_2, a_1 - b_2, b_1 - a_2, d_1 - c_2)$
4. $1/\tilde{A}_2$ = $A^{-1}{}_2 = \left(\frac{1}{d_2}, \frac{1}{b_2}, \frac{1}{a_2}, \frac{1}{c_2}\right)$ where c_2, a_2, b_2 and d_2 are all positive real numbers. If $c_1, a_1, b_1, d_1, c_2, a_2, b_2$ and d_2 are all non zero positive real numbers, then the division of $\tilde{A}_1$ and $\tilde{A}_2$ is $\tilde{A}_1 \oslash \tilde{A}_2$ = $\left(\frac{c_1}{d_2}, \frac{a_1}{b_2}, \frac{b_1}{a_2}, \frac{d_1}{c_2}\right)$.

4. MODEL DEVELOPMENT

Initially the company ordered Q units of the item, out of which W units is kept in OW and Z units are kept in RW, where Z = (Q - W). Initially, demand is satisfied using the stocks of OW until the stock level drops to (W-K) units. At this stage, K units from RW are transported to OW to meet further demand and this process is repeated ‘n’ times until the stocks of RW are exhausted. The remaining (W-K) units in OW are used again at this stage. The inventory situation in RW and OW are shown in the figure 1.

The inventory units in RW can be seen to be equal.

$$A_t = t_{ik}\left[Z + (Z - K) + (Z - 2K) + \ldots\ldots.. + (Z - (n-1)K)\right] = t_{ik}\frac{Z(n+1)}{2} \tag{4.1}$$

Where t_{ik} = K/D, the time taken for the consumption of K units, since Z = (Q - W) and the holding cost in RW is F(i), we have-

$$FA_t = Ft_{ik}\frac{Z(n+1)}{2} = \frac{FK}{D}(Q - W)\frac{(n+1)}{2} = \frac{FK(n+1)(Q-W)}{2D} \tag{4.2}$$

The cost of transporting the units from RW to OW in ‘n’ shipments is given by

$$nC_t = (Z/K)C_t \qquad (4.3)$$

Since $n = Z/K$

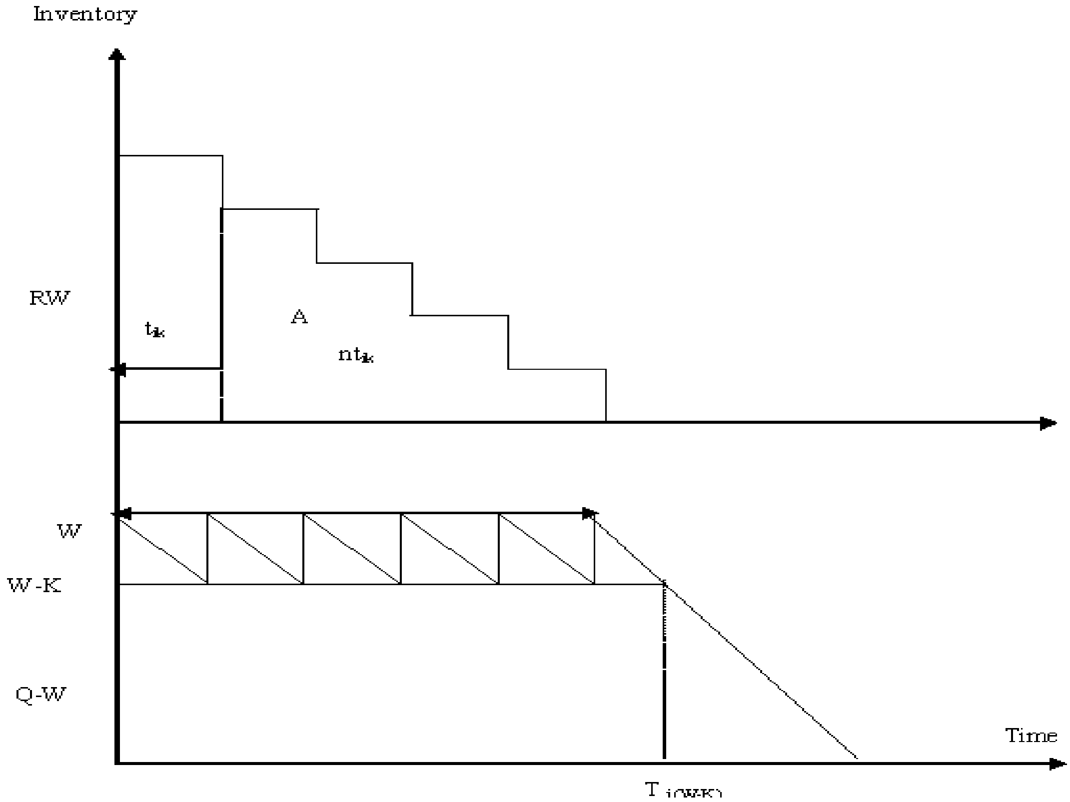

Fig. 1: Graphical representation of two-storage inventory model

When K units are drawn from RW in each shipment, more are carried in OW for a period of t_k and hence account for a holding cost of KH (i) t_{ki} / 2. Since there are ‘n’ such shipments and taking into consideration, the initial K units of OW, the holding cost for these items is $(n+1)HKt_{ik}/2 = (n+1)HK^2/2D$ (4.4)

A quantity of (W- K) units is kept unused in OW for a period of $t_{i(W-K)} = (n+1)t_{ik}$ and an average inventory during usage in OW is (W - K)/2 units for a period $(t - t_{i(W-K)})$. Hence the inventory holding cost in OW for these items is

$$H[K(W-K)(n+1)/D + (W-K)^2/2D]. \qquad (4.5)$$

The fixed ordering cost per order is A. Then the total inventory cost for the system using (4.2) to (4.5) becomes

$$C = A + (n+1)\frac{FK(Q-W)}{2D} + \frac{K^2 H}{2D} + \frac{HK(W-K)}{2D} + nC_t + \frac{(W-K)^2 H}{2D} \quad (4.6)$$

The average inventory cost

C(Q, K) = C / t

But we have t = Q / D, Z = Q – W and n = Z / K = Q – W/K

Total average cost becomes

$$C(Q,K) = \frac{AD}{Q} + \frac{FQ}{2} - W(F-H) + \frac{K}{2}(F-H) - \frac{KW}{2Q}(F-H) + C_t\frac{(Q-W)D}{QK} + \frac{W^2}{2Q}(F-H) \quad (4.7)$$

Fuzzy Model: Due to uncertainly in the environment it is not easy to define all the parameters precisely, accordingly we assume some of these parameters namely $\tilde{D}$, $\tilde{F}$, $\tilde{H}$, $\tilde{A}$ and $\tilde{W}$ may change within some limits. Let $\tilde{D} = [d_1, d_2, d_3, d_4]$, $\tilde{F} = [f_1, f_2, f_3, f_4]$, $\tilde{H} = [h_1, h_2, h_3, h_4]$, $\tilde{A} = [a_1, a_2, a_3, a_4]$, $\tilde{W} = [w_1, w_2, w_3, w_4]$ are as trapezoidal fuzzy numbers. In this case, the total fuzzy cost per unit time is given by

$$\tilde{C}(Q,K) = \left(\left(\tilde{A}\otimes\tilde{D}\right)\oslash Q\right)\oplus\left(\left(\tilde{F}\otimes Q\right)\oslash 2\right)\odot\left(\tilde{W}\otimes\left(\tilde{F}\odot\tilde{H}\right)\right)\oplus\left(\left(K\otimes\left(\tilde{F}\odot\tilde{H}\right)\right)\oslash 2\right)\odot\left(\left(K\otimes\left(\tilde{W}\otimes\left(\tilde{F}\odot\tilde{H}\right)\right)\right)\oslash 2Q\right)$$
$$\oplus\left(\left(\left(C_t\otimes\left(Q\odot\tilde{W}\right)\right)\otimes\tilde{D}\right)\oslash QK\right)\oplus\left(\left(\tilde{W}\otimes\left(\tilde{W}\otimes\left(\tilde{F}\odot\tilde{H}\right)\right)\right)\oslash 2Q\right) \quad (4.8)$$

By second function principal, one has

$$\tilde{C}(Q,K) = \Bigg(\frac{a_1 d_1}{Q} + \frac{f_1 Q}{2} - (f_4 - h_1)w_4 + \frac{K(f_1-h_4)}{2} - \frac{K(f_4-h_1)w_4}{2Q} + \frac{C_t(Q-w_4)d_1}{QK} + \frac{(f_1-h_4)w_1^2}{2Q},$$

$$\frac{a_2 d_2}{Q} + \frac{f_2 Q}{2} - (f_3 - h_2)w_3 + \frac{K(f_2-h_3)}{2} - \frac{K(f_3-h_2)w_3}{2Q} + \frac{C_t(Q-w_3)d_2}{QK} + \frac{(f_2-h_3)w_1^2}{2Q},$$

$$\frac{a_3 d_3}{Q} + \frac{f_3 Q}{2} - (f_2 - h_3)w_2 + \frac{K(f_3-h_2)}{2} - \frac{K(f_2-h_3)w_2}{2Q} + \frac{C_t(Q-w_2)d_3}{QK} + \frac{(f_3-h_2)w_1^2}{2Q},$$

$$\frac{a_4 d_4}{Q} + \frac{f_4 Q}{2} - (f_1 - h_4)w_1 + \frac{K(f_4-h_1)}{2} - \frac{K(f_1-h_1)w_1}{2Q} + \frac{C_t(Q-w_1)d_4}{QK} + \frac{(f_4-h_1)w_1^2}{2Q}\Bigg)$$

Now we defuzzify the total cost per unit time, using graded mean integration representation method, the result is

$$P\left(\tilde{C}(Q,K)\right)=\frac{1}{6}\left[\left(\frac{a_1d_1}{Q}+\frac{f_1Q}{2}-(f_4-h_1)w_4+\frac{K(f_1-h_4)}{2}-\frac{K(f_4-h_1)w_4}{2Q}+\frac{C_t(Q-w_4)d_1}{QK}+\frac{(f_1-h_4)w_1^2}{2Q}\right.\right.$$

$$+2\left(\frac{a_2d_2}{Q}+\frac{f_2Q}{2}-(f_3-h_2)w_3+\frac{K(f_2-h_3)}{2}-\frac{K(f_3-h_2)w_3}{2Q}+\frac{C_t(Q-w_3)d_2}{QK}+\frac{(f_2-h_3)w_1^2}{2Q}\right)$$

$$+2\left(\frac{a_3d_3}{Q}+\frac{f_3Q}{2}-(f_2-h_3)w_2+\frac{K(f_3-h_2)}{2}-\frac{K(f_2-h_3)w_2}{2Q}+\frac{C_t(Q-w_2)d_3}{QK}+\frac{(f_3-h_2)w_1^2}{2Q}\right)$$

$$\left.\left.+\frac{a_4d_4}{Q}+\frac{f_4Q}{2}-(f_1-h_4)w_1+\frac{K(f_4-h_1)}{2}-\frac{K(f_1-h_1)w_1}{2Q}+\frac{C_t(Q-w_1)d_4}{QK}+\frac{(f_4-h_1)w_1^2}{2Q}\right)\right] \tag{4.9}$$

The optimal values of Q and K, which minimizes (4.8), are obtained by solving

$$\frac{\partial P\left(\tilde{C}(Q,K)\right)}{\partial Q}=0 \; and \; \frac{\partial P\left(\tilde{C}(Q,K)\right)}{\partial K}=0 \tag{4.10}$$

we get

$$Q=\left[\frac{\begin{array}{l}2a_1d_1+4a_2d_2+4a_3d_3+2a_4d_4-K\left[(f_4-h_1)w_4+(f_3-h_2)w_3+(f_2-h_3)w_2+(f_1-h_4)w_1\right]\\ -\frac{2C_t}{K}(w_4d_1+w_3d_2+w_2d_3+w_1d_4)+(f_4-h_1)w_4^2+(f_3-h_2)w_3^2+(f_2-h_3)w_2^2+(f_1-h_4)w_1^2\end{array}}{f_1+f_2+f_3+f_4}\right]^{1/2} \tag{4.11}$$

and

$$K=\sqrt{\frac{\frac{C_t}{Q}\left[(Q-w_4)+2(Q-w_3)+2(Q-w_2)+(Q-w_1)\right]}{\begin{array}{l}\frac{(f_1-h_4)}{2}+(f_2-h_3)+(f_3-h_2)+\frac{(f_4-h_1)}{2}\\ -\frac{(f_1-h_4)w_1}{2Q}-\frac{(f_2-h_3)w_2}{Q}-\frac{(f_3-h_2)w_3}{Q}-\frac{(f_4-h_1)w_4}{2Q}\end{array}}} \tag{4.12}$$

5. COST-REDUCTION DUE TO K-RELEASE RULE

The unit cost of transportation with K-release rule is $C_t' = C_t / K$. Suppose the unit cost of transportation is C_t^* without bulk transportation. The bulk transportation will be economical only if $C_t^* > C_t'$. Hence without K-release rule, the cost function becomes-

$$C(Q) = \frac{AD}{Q} + \frac{FQ}{2} + \frac{W^2(F-H)}{2Q} - W(F-H) + \frac{(Q-W)C_t^* D}{Q} \tag{5.1}$$

Fuzzy Model: Due to uncertainly in the environment it is not easy to define all the parameters precisely, accordingly we assume some of these parameters namely $\tilde{D}$, $\tilde{F}$, $\tilde{H}$, $\tilde{A}$ and $\tilde{W}$ may change within some limits.

Let $\tilde{D} = [d_1, d_2, d_3, d_4]$, $\tilde{F} = [f_1, f_2, f_3, f_4]$, $\tilde{H} = [h_1, h_2, h_3, h_4]$, $\tilde{A} = [a_1, a_2, a_3, a_4]$, $\tilde{W} = [w_1, w_2, w_3, w_4]$ are as trapezoidal fuzzy numbers. In this case, the total fuzzy cost per unit time is given by

$$\tilde{C}(Q) = \left(\left(\tilde{A} \otimes \tilde{D}\right) \oslash Q\right) \oplus \left(\left(\tilde{F} \otimes Q\right) \oslash 2\right) \oplus \left(\left(\tilde{W} \otimes \left(\tilde{W} \otimes \left(\tilde{F} \odot \tilde{H}\right)\right)\right) \oslash 2Q\right) \odot \left(\tilde{W} \otimes \left(\tilde{F} \odot \tilde{H}\right)\right)$$
$$\oplus \left(\left(\left(\left(Q \odot \tilde{W}\right) \otimes C_t^*\right) \otimes \tilde{D}\right) \oslash Q\right) \tag{5.2}$$

By second function principal, one has

$$\tilde{C}(Q) = \left(\frac{a_1 d_1}{Q} + \frac{f_1 Q}{2} - (f_4 - h_1) w_4 + \frac{C_t^* (Q - w_4) d_1}{Q} + \frac{(f_1 - h_4) w_1^2}{2Q}, \right.$$

$$\frac{a_2 d_2}{Q} + \frac{f_2 Q}{2} - (f_3 - h_2) w_3 + \frac{C_t^* (Q - w_3) d_2}{Q} + \frac{(f_2 - h_3) w_1^2}{2Q},$$

$$\frac{a_3 d_3}{Q} + \frac{f_3 Q}{2} - (f_2 - h_3) w_2 + \frac{C_t^* (Q - w_2) d_3}{Q} + \frac{(f_3 - h_2) w_1^2}{2Q},$$

$$\left. \frac{a_4 d_4}{Q} + \frac{f_4 Q}{2} - (f_1 - h_4) w_1 + \frac{C_t^* (Q - w_1) d_4}{Q} + \frac{(f_4 - h_1) w_1^2}{2Q} \right)$$

Now we defuzzify the total cost per unit time, using graded mean integration representation method, the result is

$$P\left(\tilde{C}(Q)\right)=\frac{1}{6}\left\{\begin{array}{l}\left(\frac{a_1d_1}{Q}+\frac{f_1Q}{2}-(f_4-h_1)w_4+\frac{C_t^*(Q-w_4)d_1}{Q}+\frac{(f_1-h_4)w_1^2}{2Q}\right)\\+2\left(\frac{a_2d_2}{Q}+\frac{f_2Q}{2}-(f_3-h_2)w_3+\frac{C_t^*(Q-w_3)d_2}{Q}+\frac{(f_2-h_3)w_1^2}{2Q}\right)\\+2\left(\frac{a_3d_3}{Q}+\frac{f_3Q}{2}-(f_2-h_3)w_2+\frac{C_t^*(Q-w_2)d_3}{Q}+\frac{(f_3-h_2)w_1^2}{2Q}\right)\\+\left(\frac{a_4d_4}{Q}+\frac{f_4Q}{2}-(f_1-h_4)w_1+\frac{C_t^*(Q-w_1)d_4}{Q}+\frac{(f_4-h_1)w_1^2}{2Q}\right)\end{array}\right\} \tag{5.3}$$

The optimal value of Q, which minimizes (5.1), is obtained by $\frac{dP\left(\tilde{C}(Q)\right)}{dQ}=0$

$$Q=\left[\frac{\begin{array}{l}-a_1d_1-2a_2d_2-2a_3d_3-a_4d_4+C_t\left(w_4d_1+2w_3d_2+2w_2d_3+w_1d_4\right)\\-\frac{(f_4-h_1)}{2}w_4^2-(f_3-h_2)w_3^2-(f_2-h_3)w_2^2-\frac{(f_1-h_4)}{2}w_1^2\end{array}}{-\left(\frac{f_1}{2}+f_2+f_3+\frac{f_4}{2}\right)}\right]^{1/2} \tag{5.4}$$

The proposed K-released rule will be economical if

$$\left[C(Q)-C(Q,K)\right]>0$$

From equation (4.7) and (4.8) we see that-

$$\left[C(Q)-C(Q,K)\right]=\left(1-\frac{W}{Q}\right)\left[D\left(C_t^*-C_t^{'}\right)-\frac{K}{2}(F-H)\right] \tag{5.5}$$

and hence the inequality

$$\left(1-\frac{W}{Q}\right)\left[D\left(C_t^*-C_t^{'}\right)-\frac{K}{2}(F-H)\right]>0$$

$$\Rightarrow\left(C_t^*-C_t^{'}\right)>\frac{K(F-H)}{2D} \tag{5.6}$$

must be satisfied.

Thus for a given situation, if the unit cost of transportation with bulk release rule satisfies the inequality (5.6), K-release rule must be economical.

6. NUMERICAL EXAMPLE

Consider an inventory system with following parametric values:

Crisp Model: demand rate D = 2000, C_t = 0.5, F = 8.5, H = 7.5, W = 100, A = 150. With the help of the above values, we find the optimal values of ordering quantity and total cost with and without K- release which is given as:

With K – release rule: Q = 221.62 & C (Q, K) = 3456.46

And without K – release rule: Q = 216.68 & C (Q, K) = 3585.43

Fuzzy Model: $\tilde{D}$ = [1900, 2000, 2000, 1900], $\tilde{F}$ = [8.075, 8.5, 8.5, 8.075], $\tilde{H}$ = [7.125, 7.5, 7.5, 7.125], $\tilde{A}$ = [142.5, 150, 150, 142.5], $\tilde{W}$ = [95, 100, 100, 95]. The optimal values of ordering quantity and total cost with and without K- release which is given as:

With K – release rule: Q = 225.62 & C (Q, K) = 3458.46

And without K – release rule: Q = 210.68 & C (Q, K) = 3587.43

7. CONCLUSION

Two storage inventory models discussed in this paper and developed under the assumption that the distribution of the items to the customers takes place at OW only. Because of the distance factor, it is natural to consider the transportation cost associated with the transfer of items from RW to OW. Further, the concept of K-release rule is more pragmatic, as holding large inventory in RW is every expensive. With the help of numerical examples, it is clear that the effect of fuzzy cannot be ignored. We can earn more profit by consider the effect of fuzzy on ordering and holding cost in each lot. This model gives the direction to decision makers to take account of fuzzy effect while taking decision and by taking account of this; he/she earn more profit for the organization.

A future extension is to discuss model in more realistic situation by consider impreciseness in different inventory related cost and taking different form of demand pattern likes as time dependent, ram-type demand with inflation and permissible delay.

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