Manoj Kr. Chaudhary, V. K. Singh, Rajesh Singh
Department of Statistics, Banaras Hindu University
Varanasi-221005, INDIA

Florentin Smarandache
Department of Mathematics, University of New Mexico, Gallup, USA

Estimating the Population Mean in Stratified Population using Auxiliary Information under Non-Response

Published in:
Rajesh Singh, F. Smarandache (Editors)
STUDIES IN SAMPLING TECHNIQUES AND TIME SERIES ANALYSIS
Zip Publishing, Columbus, USA, 2011
ISBN 978-1-59973-159-9
pp. 24 - 39

Abstract

The present chapter deals with the study of general family of factor-type estimators for estimating population mean of stratified population in the presence of non-response whenever information on an auxiliary variable are available. The proposed family includes separate ratio, product, dual to ratio and usual sample mean estimators as its particular cases and exhibits some nice properties as regards to locate the optimum estimator belonging to the family. Choice of appropriate estimator in the family in order to get a desired level of accuracy even if non-response is high, is also discussed. The empirical study has been carried out in support of the results.

Keywords: Factor-type estimators, Stratified population, Non-response, Optimum estimator, Empirical study.

1. Introduction

In sampling theory the use of suitable auxiliary information results in considerable reduction in variance of the estimator. For this reason, many authors used the auxiliary information at the estimation stage. Cochran (1940) was the first who used the auxiliary information at the estimation stage in estimating the population parameters. He proposed the ratio estimator to estimate the population mean or total of a character under study. Hansen *et. al.* (1953) suggested the difference estimator which was subsequently modified to give the linear regression estimator for the population mean or its total. Murthy (1964) have studied the product estimator to estimate the population mean or total when the character under study and the auxiliary character are negatively correlated. These estimators can be used more efficiently than the mean per unit estimator.

There are several authors who have suggested estimators using some known population parameters of an auxiliary variable. Upadhyaya and Singh (1999) have suggested the class of estimators in simple random sampling. Kadilar and Cingi (2003) and Shabbir and Gupta (2005) extended these estimators for the stratified random sampling. Singh et. al. (2008) suggested class of estimators using power transformation based on the estimators developed by Kadilar and Cingi (2003). Kadilar and Cingi (2005) and Shabbir and Gupta (2006) have suggested new ratio estimators in stratified sampling to improve the efficiency of the estimators. Koyuncu and Kadilar (2008) have proposed families of estimators for estimating population mean in stratified random sampling by considering the estimators proposed in Searls (1964) and Khoshnevisan et. al. (2007). Singh and Vishwakarma (2008) have suggested a family of estimators using transformation in the stratified random sampling. Recently, Koyuncu and Kadilar (2009) have proposed a general family of estimators, which uses the information of two auxiliary variables in the stratified random sampling to estimate the population mean of the variable under study.

The works which have been mentioned above are based on the assumption that both the study and auxiliary variables are free from any kind of non-sampling error. But, in practice, however the problem of non-response often arises in sample surveys. In such situations while single survey variable is under investigation, the problem of estimating population mean using sub-sampling scheme was first considered by Hansen and Hurwitz (1946). If we have incomplete information on study variable X_0 and complete information on auxiliary variable X_1, in other words if the study variable is affected by non-response error but the auxiliary variable is free from non-response. Then utilizing the Hansen-Hurwitz (1946) technique of sub-sampling of the non-respondents, the conventional ratio and product estimators in the presence of non-response are respectively given by

$$T_{0R}^{*} = \left(T_{0HH} / \bar{x}_1\right)\bar{X}_1 \tag{1.1}$$

and $$T_{0P}^{*} = \left(T_{0HH}.\bar{x}_1\right)/\bar{X}_1 . \qquad (1.2)$$

The purpose of the present chapter is to suggest separate-type estimators in stratified population for estimating population mean using the concept of sub-sampling of non-respondents in the presence of non-response in study variable in the population. In this context, the information on an auxiliary characteristic closely related to the study variable, has been utilized assuming that it is free from non-response.

In order to suggest separate-type estimators, we have made use of Factor-Type Estimators (FTE) proposed by Singh and Shukla (1987). FTE define a class of estimators involving usual sample mean estimator, usual ratio and product estimators and some other estimators existing in literature. This class of estimators exhibits some nice properties which have been discussed in subsequent sections.

2. Sampling Strategy and Estimation Procedure

Let us consider a population consisting of N units divided into k strata. Let the size of i^{th} stratum is N_i, ($i = 1,2,...,k$) and we decide to select a sample of size n from the entire population in such a way that n_i units are selected from the i^{th} stratum. Thus, we have $\sum_{i=1}^{k} n_i = n$. Let the non-response occurs in each stratum. Then using Hansen and Hurwitz (1946) procedure we select a sample of size m_i units out of n_{i2} non-respondent units in the i^{th} stratum with the help of simple random sampling without replacement (SRSWOR) scheme such that $n_{i2} = L_i m_i$, $L_i \geq 1$ and the information are observed on all the m_i units by interview method.

The Hansen-Hurwitz estimator of population mean $\bar{X}_{0i}$ of study variable X_0 for the i^{th} stratum will be

$$T_{0i}^{*} = \frac{n_{i1}\bar{x}_{0i1} + n_{i2}\bar{x}_{0mi}}{n_i}, \qquad \left(i = 1,2,...,k\right) \qquad (2.1)$$

where $\bar{x}_{0i1}$ and $\bar{x}_{0mi}$ are the sample means based on n_{i1} respondent units and m_i non-respondent units respectively in the i^{th} stratum for the study variable.

Obviously T_{0i}^* is an unbiased estimator of $\bar{X}_{0i}$. Combining the estimators over all the strata we get the estimator of population mean $\bar{X}_0$ of study variable X_0, given by

$$T_{0st}^* = \sum_{i=1}^{k} p_i T_{0i}^* \tag{2.2}$$

where $p_i = \dfrac{N_i}{N}$.

which is an unbiased estimator of $\bar{X}_0$. Now, we define the estimator of population mean $\bar{X}_1$ of auxiliary variable X_1 as

$$T_{1st} = \sum_{i=1}^{k} p_i \bar{x}_{1i} \tag{2.3}$$

where $\bar{x}_{1i}$ is the sample mean based on n_i units in the i^{th} stratum for the auxiliary variable. It can easily be seen that T_{1st} is an unbiased estimator of $\bar{X}_1$ because $\bar{x}_{1i}$ gives unbiased estimates of the population mean $\bar{X}_{1i}$ of auxiliary variable for the i^{th} stratum.

3. Suggested Family of Estimators

Let us now consider the situation in which the study variable is subjected to non-response and the auxiliary variable is free from non-response. Motivated by Singh and Shukla (1987), we define the separate-type family of estimators of population mean $\bar{X}_0$ using factor-type estimators as

$$T_{FS}(\alpha) = \sum_{i=1}^{k} p_i T_{Fi}^*(\alpha) \tag{3.1}$$

where $$T_{Fi}^{*}(\alpha)=T_{0i}^{*}\left[\frac{(A+C)\overline{X}_{1i}+fB\overline{x}_{1i}}{(A+fB)\overline{X}_{1i}+C\overline{x}_{1i}}\right] \tag{3.2}$$

and $f=\frac{n}{N}$, $A=(\alpha-1)(\alpha-2)$, $B=(\alpha-1)(\alpha-4)$, $C=(\alpha-2)(\alpha-3)(\alpha-4)$; $\alpha>0$.

3.1 Particular Cases of $T_{FS}(\alpha)$

Case-1: If $\alpha=1$ then $A=B=0$, $C=-6$

so that $T_{Fi}^{*}(1)=T_{0i}^{*}\frac{\overline{X}_{1i}}{\overline{x}_{1i}}$

and hence $$T_{FS}(1)=\sum_{i=1}^{k}p_iT_{0i}^{*}\frac{\overline{X}_{1i}}{\overline{\overline{x}}_{1i}}. \tag{3.3}$$

Thus, $T_{FS}(1)$ is the usual separate ratio estimator under non-response.

Case-2: If $\alpha=2$ then $A=0=C$, $B=-2$

so that $T_{Fi}^{*}(2)=T_{0i}^{*}\frac{\overline{x}_{1i}}{\overline{X}_{1i}}$

and hence $$T_{FS}(2)=\sum_{i=1}^{k}p_iT_{0i}^{*}\frac{\overline{x}_{1i}}{\overline{X}_{1i}} \tag{3.4}$$

which is the usual separate product estimator under non-response.

Case-3: If $\alpha=3$ then $A=2$, $B=-2$, $C=0$

so that $T_{Fi}^{*}(3)=T_{0i}^{*}\frac{\overline{X}_{1i}-f\overline{x}_{1i}}{(1-f)\overline{X}_{1i}}$

and hence $$T_{FS}(3)=\sum_{i=1}^{k}p_iT_{Fi}^{*}(3) \tag{3.5}$$

which is the separate dual to ratio-type estimator under non-response. The dual to ratio type estimator was proposed by Srivenkataramana (1980).

Case-4: If $\alpha = 4$ then $A = 6$, $B = 0$, $C = 0$

so that $T^*_{Fi}(4) = T^*_{0i}$

and hence $T_{FS}(4) = \sum_{i=1}^{k} p_i T^*_{0i} = T^*_{0st}$ (3.6)

which is usual mean estimator defined in stratified population under non-response.

3.2 Properties of $T_{FS}(\alpha)$

Using large sample approximation, the bias of the estimator $T_{FS}(\alpha)$, up to the first order of approximation was obtained following Singh and Shukla (1987) as

$$B[T_{FS}(\alpha)] = E[T_{FS}(\alpha) - \overline{X}_0]$$

$$= \phi(\alpha) \sum_{i=1}^{k} p_i \overline{X}_{0i} \left(\frac{1}{n_i} - \frac{1}{N_i} \right) \left[\frac{C}{A + fB + C} C_{1i}^2 - \rho_{01i} C_{0i} C_{1i} \right] \tag{3.7}$$

where $\phi(\alpha) = \frac{C - fB}{A + fB + C}$, $C_{0i} = \frac{S_{0i}}{\overline{X}_{0i}}$, $C_{1i} = \frac{S_{1i}}{\overline{X}_{1i}}$, S_{0i}^2 and S_{1i}^2 are the population mean squares of study and auxiliary variables respectively in the i^{th} stratum . ρ_{01i} is the population correlation coefficient between X_0 and X_1 in the i^{th} stratum. The Mean Square Error (MSE) up to the first order of approximation was derived as

$$M[T_{FS}(\alpha)] = E[T_{FS}(\alpha) - \overline{X}_0]^2$$

$$= \sum_{i=1}^{k} p_i^2 MSE[T^*_{Fi}(\alpha)]$$

$$= \sum_{i=1}^{k} p_i^2 \overline{X}_{0i}^2 \left[\frac{V(T_{0i}^*)}{\overline{X}_{0i}^2} + \phi^2(\alpha) \; \frac{V(\overline{x}_{1i})}{\overline{X}_{1i}^2} - 2\phi(\alpha) \frac{Cov(T_{0i}^*, \overline{x}_{1i})}{\overline{X}_{0i} \overline{X}_{1i}} \right].$$

Since $\quad V(T_{0i}^*) = \left(\frac{1}{n_i} - \frac{1}{N_i} \right) S_{0i}^2 + \frac{L_i - 1}{n_i} W_{i2} S_{0i2}^2, \; V(\overline{x}_{1i}) = \left(\frac{1}{n_i} - \frac{1}{N_i} \right) S_{1i}^2$

and $\quad Cov(T_{0i}^*, \overline{x}_{1i}) = \left(\frac{1}{n_i} - \frac{1}{N_i} \right) \rho_{01i} S_{0i} S_{1i}$ [due to Singh (1998)].

where S_{0i2}^2 is the population mean square of the non-response group in the i^{th} stratum and W_{i2} is the non-response rate of the i^{th} stratum in the population.

Therefore, we have

$$M[T_{FS}(\alpha)] = \sum_{i=1}^{k} \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 \left[S_{0i}^2 + \varphi(\alpha)^2 R_{01i}^2 S_{1i}^2 - 2\varphi(\alpha) R_{01i} \rho_{01i} S_{0i} S_{1i} \right]$$

$$+ \sum_{i=1}^{k} \frac{L_i - 1}{n_i} W_{i2} p_i^2 S_{0i2}^2 \qquad (3.8)$$

where $\quad R_{01i} = \frac{\overline{X}_{0i}}{\overline{X}_{1i}}.$

3.3 Optimum Choice of α

In order to obtain minimum MSE of $T_{FS}(\alpha)$, we differentiate the MSE with respect to α and equate the derivative to zero

$$\sum_{i=1}^{k} \left(\frac{1}{n_i} - \frac{1}{N_i} \right) p_i^2 \left[2\phi'(\alpha)\phi(\alpha) R_{01i}^2 S_{1i}^2 - 2\phi'(\alpha) R_{01i} \rho_{01i} S_{0i} S_{1i} \right] = 0, \qquad (3.9)$$

where $\phi'(\alpha)$ stands for first derivative of $\phi(\alpha)$.From the above expression, we have

$$\phi(\alpha)=\frac{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}\rho_{01i}S_{0i}S_{1i}}{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}^2 S_{1i}^2}=V \text{ (say).} \tag{3.10}$$

It is easy to observe that $\phi(\alpha)$ is a cubic equation in the parameter α. Therefore, the equation (3.10) will have at the most three real roots at which the MSE of the estimator $T_{FS}(\alpha)$ attains its minimum.

Let the equation (3.10) yields solutions as α_0, α_1 and α_2 such that $M[T_{FS}(\alpha)]$ is same. A criterion of making a choice between α_0, α_1 and α_2 is that "compute the bias of the estimator at $\alpha=\alpha_0$, α_1 and α_2 and select α_{opt} at which bias is the least". This is a novel property of the FTE.

3.4 Reducing MSE through Appropriate Choice of α

By using FTE for defining the separate-type estimators in this chapter, we have an advantage in terms of the reduction of the value of MSE of the estimator to a desired extent by an appropriate choice of the parameter α even if the non-response rate is high in the population. The procedure is described below:

Since MSE's of the proposed strategies are functions of the unknown parameter α as well as functions of non-response rates W_{i2}, it is obvious that if α is taken to be constant, MSE's increase with increasing non-response rate, if other characteristics of the population remain unchanged, along with the ratio to be sub sampled in the non-response class, that is, L_i. It is also true that more the non-response rate, greater would be the size of the non-response group in the sample and, therefore, in order to lowering down the MSE of the estimator, the size of sub sampled units should be increased so as to keep the value of L_i in the vicinity of 1; but this would, in term, cost more because more effort and money would be required to obtain information on sub sampled units through personal interview method. Thus, increasing the size of the sub sampled units in order to reduce the MSE is not a feasible solution if non-response rate is supposed to be large enough.

The classical estimators such as T_{0HH}, T_{0R}^{*}, T_{0P}^{*}, discussed earlier in literature in presence of non-response are not helpful in the reduction of MSE to a desired level. In all these estimators, the only controlling factor for lowering down the MSE is L_i, if one desires so.

By utilizing FTE in order to propose separate- type estimators in the present work, we are able to control the precision of the estimator to a desired level only by making an appropriate choice of α.

Let the non-response rate and mean-square of the non-response group in the i^{th} stratum at a time be $W_{i2} = \frac{N_{i2}}{N_i}$ and S_{0i2}^2 respectively. Then, for a choice of $\alpha = \alpha_0$, the MSE of the estimator would be

$$M\left[T_{FS}(\alpha)/W_{i2}\right] = \sum_{i=1}^{k}\left(\frac{1}{n_i} - \frac{1}{N_i}\right)p_i^2\left[S_{0i}^2 + \phi(\alpha_0)^2 R_{01i}^2 S_{1i}^2 - 2\phi(\alpha_0) R_{01i}\rho_{01i}S_{0i}S_{1i}\right]$$

$$+\sum_{i=1}^{k}\frac{L_i - 1}{n_i}W_{i2}p_i^2 S_{0i2}^2 \qquad (3.11)$$

Let us now suppose that the non-response rate increased over time and it is $W_{i2}^{'} = \frac{N_{i2}^{'}}{N_i}$ such that $N_{i2}^{'} > N_{i2}$. Obviously, with change in non-response rate, only the parameter S_{0i2}^2 will change. Let it becomes $S_{0i2}^{'2}$. Then we have

$$M\left[T_{FS}(\alpha)/W_{i2}^{'}\right] = \sum_{i=1}^{k}\left(\frac{1}{n_i} - \frac{1}{N_i}\right)p_i^2\left[S_{0i}^2 + \phi(\alpha_1)^2 R_{01i}^2 S_{1i}^2 - 2\phi(\alpha_1) R_{01i}\rho_{01i}S_{0i}S_{1i}\right]$$

$$+\sum_{i=1}^{k}\frac{L_i - 1}{n_i}W_{i2}^{'}p_i^2 S_{0i2}^{'2} \qquad (3.12)$$

Clearly, if $\alpha_0 = \alpha_1$ and $S_{0i2}^{'2} > S_{0i2}^2$ then $M[T_{FS}(\alpha)|W_{i2}^{'}] > M[T_{FS}(\alpha)|W_{i2}]$. Therefore, we have to select a suitable value α_1, such that even if $W_{i2}^{'} > W_{i2}$ and $S_{0i2}^{'2} > S_{0i2}^2$, expression (3.12) becomes equal to equation (3.11) that is, the MSE of $T_{FS}(\alpha)$ is reduced to a desired level given by (3.11). Equating (3.11) to (3.12) and solving for $\phi(\alpha_1)$, we get

$$\phi(\alpha_1)^2 \sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}^2 S_{1i}^2 - 2\phi(\alpha_1)\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}\rho_{01i}S_{0i}S_{1i}$$

$$-\left[\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2\left\{\phi(\alpha_0)^2 R_{01i}^2 S_{1i}^2 - 2\phi(\alpha_0)R_{01i}\rho_{01i}S_{0i}S_{1i}\right\}\right.$$

$$\left.+\sum_{i=1}^{k}\frac{L_i-1}{n_i}p_i^2\left(W_{i2}S_{0i2}^2 - W_{i2}^{'}S_{0i2}^{'2}\right)\right] = 0, \qquad (3.13)$$

which is quadratic equation in $\phi(\alpha_1)$. On solving the above equation, the roots are obtained as

$$\phi(\alpha_1) = \frac{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}\rho_{01i}S_{0i}S_{1i}}{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}^2 S_{1i}^2} \pm \left[\left\{\frac{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}\rho_{01i}S_{0i}S_{1i}}{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}^2 S_{1i}^2}\right\}^2 + \right.$$

$$\left.\frac{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2\left\{\phi(\alpha_0)^2 R_{01i}^2 S_{1i}^2 - 2\phi(\alpha_0)R_{01i}\rho_{01i}S_{0i}S_{1i}\right\} - \sum_{i=1}^{k}\frac{L_i-1}{n_i}p_i^2\left(W_{i2}^{'}S_{0i2}^{'2} - W_{i2}S_{0i2}^2\right)}{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}^2 S_{1i}^2}\right]^{\frac{1}{2}}$$

(3.14)

The above equation provides the value of α on which one can obtain the precision to a desired level. Sometimes the roots given by the above equation may be imaginary. So, in order that the roots are real, the conditions on the value of α_0 are given by

$$\phi(\alpha_0) > \frac{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}\rho_{01i}S_{0i}S_{1i}}{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}^2 S_{1i}^2} + \left[\frac{\sum_{i=1}^{k}\frac{L_i-1}{n_i}p_i^2\left(W_{i2}^{'}S_{0i2}^{'2}-W_{i2}S_{0i2}^2\right)}{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}^2 S_{1i}^2}\right]^{\frac{1}{2}} \qquad (3.15)$$

and $$\phi(\alpha_0) < \frac{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}\rho_{01i}S_{0i}S_{1i}}{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}^2 S_{1i}^2} - \left[\frac{\sum_{i=1}^{k}\frac{L_i-1}{n_i}p_i^2\left(W_{i2}^{'}S_{0i2}^{'2}-W_{i2}S_{0i2}^2\right)}{\sum_{i=1}^{k}\left(\frac{1}{n_i}-\frac{1}{N_i}\right)p_i^2 R_{01i}^2 S_{1i}^2}\right]^{\frac{1}{2}} \qquad (3.16)$$

4. Empirical Study

In this section, therefore, we have illustrated the results, derived above, on the basis of some empirical data. For this purpose, a data set has been taken into consideration. Here the population is MU284 population available in Sarndal et. al. (1992, page 652, Appendix B). We have considered the population in the year 1985 as study variable and that in the year 1975 as auxiliary variable. There are 284 municipalities which have been divided randomly in to four strata having sizes 73, 70, 97 and 44.

Table 1 shows the values of the parameters of the population under consideration for the four strata which are needed in computational procedure.

Table 1: Parameters of the Population

Stratum (i)	**Stratum Size** (N_i)	**Mean** $(\overline{X}_{0i})$	**Mean** $(\overline{X}_{1i})$	(S_{0i}^2)	(S_{1i}^2)	S_{0i}	S_{1i}	ρ_{01i}	(S_{0i2}^2)
1	73	40.85	39.56	6369.0999	6624.4398	79.8066	81.3907	0.999	618.8844
2	70	27.83	27.57	1051.0725	1147.0111	32.4202	33.8676	0.998	240.9050
3	97	25.79	25.44	2014.9651	2205.4021	44.8884	46.9617	0.999	265.5220
4	44	20.64	20.36	538.4749	485.2655	23.2051	22.0287	0.997	83.6944

The value of $R_{01} = \overline{X}_0 / \overline{X}_1$ comes out to be 1.0192.

We fix the sample size to be 60. Then the allocation of samples to different strata under proportional and Neyman allocations are shown in the following table

Table 2: Allocation of Sample

Stratum (i)	**Size of Samples under**	
	Proportional Allocation	**Neyman Allocation**
1	15	26
2	15	10
3	21	19
4	9	5

On the basis of the equation (3.10), we obtained the optimum values of α :

Under Proportional Allocation

$\phi(\alpha)$= 0.9491, α_{opt} = (31.9975, 2.6128, 1.12) and

Under Neyman Allocation

$\phi(\alpha)$= 0.9527, α_{opt} = (34.1435, 2.6114, 1.1123).

The following table depicts the values of the MSE's of the estimators $T_{FS}(\alpha)$ for α_{opt}, $\alpha = 1$ and 4 under proportional and Neyman allocations. A comparison of MSE of $T_{FS}(\alpha)$ with α_{opt} and $\alpha = 1$ with that at $\alpha = 4$ reveals the fact that the utilization of auxiliary information at the estimation stage certainly improves the efficiency of the estimator as compared to the usual mean estimator T^*_{0st}.

Table 3: MSE Comparison ($L_i = 2$**,** $W_{i2} = 10\%$ for all i)

MSE	Allocation	
	Proportional	**Neyman**
$M[T_{FS}(\alpha)]_{\text{opt}}$	0.6264	0.6015
$M[T_{FS}(1)]$	0.7270	0.6705
$M[T_{FS}(4)] = V[T^*_{0st}]$	35.6069	28.6080

We shall now illustrate how by an appropriate choice of α, the MSE of the estimators $T_{FS}(\alpha)$ can be reduced to a desired level even if the non-response rate is increased.

Let us take $L_i = 2$**,** $W_{i2} = 0.1$, $W'_{i2} = 0.3$ and ${S'_{0i2}}^2 = \frac{4}{3}\left(S^2_{0i2}\right)$ for all i

Under Proportional Allocation

From the condition (3.15) and (3.16), we have conditions for real roots of $\phi(\alpha_1)$ as

$\phi(\alpha_0)$ > 1.1527 and $\phi(\alpha_0)$ < 0.7454.

Therefore, if we take $\phi(\alpha_0)$ = 1.20, then for this choice of $\phi(\alpha_0)$, we get

$$M[T_{FS}(\alpha)|W_{i2}] = 3.0712 \text{ and } M[T_{FS}(\alpha)|W'_{i2}] = 4.6818.$$

Thus, there is about 52 percent increase in the MSE of the estimator if non-response rate is tripled. Now using (3.14), we get $\phi(\alpha_1)$=1.0957 and 0.8025. At this value of $\phi(\alpha_1)$, $M[T_{FS}(\alpha)]$ reduces to 3.0712 even if non-response rate is 30 percent. Thus a possible choice of α may be made in order to reduce the MSE to a desired level.

Under Neyman Allocation

Conditions for real roots of $\phi(\alpha_1)$

$\phi(\alpha_0) > 1.1746$ and $\phi(\alpha_0) < 0.7309$.

If $\phi(\alpha_0) = 1.20$ then we have

$M[T_{FS}(\alpha)|W_{i2}] = 2.4885$ and $M[T_{FS}(\alpha)|W_{i2}^{'}] = 4.0072$.

Further, we get from (3.14), $\phi(\alpha_1)$=1.0620 and 0.8435, so that

$M[T_{FS}(\alpha)|W_{i2}^{'}]$=2.4885 for $\phi(\alpha_1)$=1.0620.

5. Conclusion

We have suggested a general family of factor-type estimators for estimating the population mean in stratified random sampling under non-response using an auxiliary variable. The optimum property of the family has been discussed. It has also been discussed about the choice of appropriate estimator of the family in order to get a desired level of accuracy even if non-response is high. The Table 3 reveals that the optimum estimator of the suggested family has greater precision than separate ratio and sample mean estimators. Besides it, the reduction of MSE of the estimators $T_{FS}(\alpha)$ to a desired extent by an appropriate choice of the parameter α even if the non-response rate is high in the population, has also been illustrated.

References

Cochran, W. G. (1940): The estimation of the yields of cereal experiments by sampling for the ratio of grain in total produce. Journ. of The Agr. Sci., 30, 262-275.

Hansen, M. H. and Hurwitz, W. N. (1946): The problem of non-response in sample surveys. Journ. of The Amer. Statis. Assoc., 41, 517-529.

Hansen, M. H., Hurwitz, W. N. and Madow,, W. G. (1953): Sample Survey Methods and Theory, Volume Ii, John Wiley and Sons, Inc., New York.

Kadilar, C. and Cingi, H. (2003): Ratio estimators in stratified random sampling, Biom. Jour. 45 (2), 218–225.

Kadilar, C. and Cingi, H. (2005): A new ratio estimator in stratified sampling, Comm. in Stat.—Theor. and Meth., *34*, 597–602.

Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N., Smarandache, F. (2007): A general family of estimators for estimating population mean using known value of some population parameter(s), Far East Journ. of Theor. Statis., 22, 181–191.

Koyuncu, N. and Kadilar, C. (2008): Ratio and product estimators in stratified random sampling. Journ. of Statis. Plann. and Inf., 3820, 2-7.

Koyuncu, N. and Kadilar, C. (2009): Family of estimators of population mean using two auxiliary variables in stratified random sampling. Comm. in Stat.—Theor. and Meth., 38, 2398–2417.

Murthy, M. N. (1964): Product method of estimation. Sankhya, 26A, 69-74.

Sarndal, C. E., Swensson, B. and Wretman, J. (1992): Model Assisted Survey Sampling, Springer-Verlag, New York, Inc.

Searls, D. T. (1964): The utilization of a known coefficient of variation in the estimation procedure. Journ. of The Amer. Statis. Assoc., 59, 1225-1226.

Shabbir, J. and Gupta, S. (2005): Improved ratio estimators in stratified sampling. Amer. Journ. of Math. and Manag. Sci., 25 (3-4), 293-311.

Shabbir, J. and Gupta, S. (2006): A new estimator of population mean in stratified sampling. Comm. in Stat.—Theor. and Meth., 35, 1201–1209.

Singh, H. P., Tailor, R., Singh, S. and Kim, J. (2008): A modified estimator of population mean using power transformation. Stat. Paper., 49, 37-58.

Singh, H. P. and Vishwakarma, G. K. (2008): A Family of Estimators of Population Mean Using Auxiliary Information in Stratified Sampling**,** Comm. in Stat.—Theor. and Meth., 37, 1038–1050.

Singh, L. B. (1998): Some Classes of Estimators for Finite Population Mean in Presence of Non-response. Unpublished Ph. D. Thesis submitted to Banaras Hindu University, Varanasi, India.

Singh, V. K. and Shukla, D. (1987): One parameter family of factor-type ratio estimators. Metron, 45 (1-2), 273-283.

Srivenkataramana, T. (1980): A dual to ratio estimator in sample surveys, *Biometrika*, 67 (1), 199-204.

Upadhyaya, L. N. and Singh, H. P. (1999): Use of transformed auxiliary variable in estimating the finite population mean, Biom. Journ., 41 (5), 627-636.