A New Solution to Einstein's Infinite Mass Challenge Based on Maximum Frequency*

Espen Gaarder Haug
Norwegian University of Life Sciences

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Abstract

In 1905, Einstein presented his famous relativistic mass energy equation $\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$. When v approaches c, the expression containing the moving mass approaches infinity. Einstein interpreted this in the following way: since one needs an infinite amount of energy to accelerate even a small mass to the speed of light, it would appear that no mass can ever reach the speed of light. In this paper, we present a new solution to the infinite mass challenge based on combining special relativity with insights from Max Planck and maximum frequency. By doing this we show that there is an exact limit on the speed v in Einstein's formula. This limit is only dependent on the Planck length and the reduced Compton wavelength of the mass in question. This also gives us a limit on the maximum relativistic Doppler shift that is derived and discussed.

Key words: Relativistic mass, maximum frequency, maximum speed, Planck length, Planck mass, relativistic Doppler shift.

1 A Maximum Frequency and a Maximum Velocity for Mass?

Einstein [1, 2] gave the following relativistic energy mass formula:

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{1}$$

Further, Einstein commented on his own formula

> *This expression approaches infinity as the velocity v approaches the velocity of light c. The velocity must therefore always remain less than c, however great may be the energies used to produce the acceleration*[1]

Einstein's argument is that the mass will become infinite as v approaches c and this means that we would need an infinite amount of energy to accelerate even an electron to the speed of light. Here we will combine Einstein's special relativity with insight from Max Planck [3, 4] and show that there is another possible solution to the infinite mass challenge. The relativistic mass relationship can be derived from a two-sided Einstein relativistic Doppler shift. Assume a mass is sending out two energy beams. These beams have a frequency of f_{ab} and f_{ba} as observed from the mass sending them out, and further $f_{ab} = f_{ba}$ as observed from this frame. From another frame moving at speed v relative to this mass, based on the relativistic Doppler shift, one will observe these frequencies to be

$$\hat{f}_a = f_a \frac{(1-\frac{v}{c})}{\sqrt{1-\frac{v^2}{c^2}}} \tag{2}$$

and

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[1] This quote is taken from page 53 in the 1931 edition of Einstein's book *Relativity: The Special and General Theory.* English translation version of Einstein's book by Robert W. Lawson.

$$\hat{f}_b = f_b \frac{(1+\frac{v}{c})}{\sqrt{1-\frac{v^2}{c^2}}} \tag{3}$$

An important question involves whether or not there is a maximum frequency of so-called electromagnetic waves and also of the relativistic Doppler shift. We will here assume that the maximum two-sided frequency is given by

$$\hat{f}_{max} = \hat{f}_a + \hat{f}_b = 2\frac{c}{l_p} \tag{4}$$

We assume the frequency is limited by the Planck length l_p, which is to say that we assume no wavelength can be shorter than the Planck length. We can use this limit on the relativistic Doppler shift and find the maximum velocity of a moving mass. We will assume a mass can send out a two-sided frequency with wavelength equal to its reduced Compton wavelength, as observed from the frame the mass is at rest in. From a frame moving at speed v relative to the moving mass (and based on the relativistic Doppler shift), the two-sided frequency must be

$$\begin{aligned}
\hat{f}_{max} &= \hat{f}_a + \hat{f}_b \\
\hat{f}_{max} &= f_a \frac{(1-\frac{v}{c})}{\sqrt{1-\frac{v^2}{c^2}}} + f_b \frac{(1+\frac{v}{c})}{\sqrt{1-\frac{v^2}{c^2}}} \\
2\frac{c}{l_p} &= \frac{c}{\bar{\lambda}} \frac{(1-\frac{v}{c})}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{c}{\bar{\lambda}} \frac{(1+\frac{v}{c})}{\sqrt{1-\frac{v^2}{c^2}}} \\
2\frac{c}{l_p} &= 2\frac{c}{\bar{\lambda}} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\
\sqrt{1-\frac{v^2}{c^2}} &= \frac{l_p}{\bar{\lambda}} \\
v &= c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}
\end{aligned} \tag{5}$$

Based on this, the maximum speed any particle can take is a function of the Planck length and the reduced Compton wavelength of the particle in question. Next let's look at what happens with a moving mass at this speed. Assume we have an electron and it accelerates to a velocity of $v = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}_e^2}}$, where $\bar{\lambda}_e$ is the reduced Compton wavelength of the electron. This gives a relativistic mass of

$$\frac{m_e}{\sqrt{1-\frac{v_{max}^2}{c^2}}} = \frac{m_e}{\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}_e^2}}\right)^2}{c^2}}} = \frac{m_e}{\frac{l_p}{\bar{\lambda}_e}} = \frac{\bar{\lambda}_e}{l_p} m_e = m_p \approx 2.1765 \times 10^{-8} \tag{6}$$

where m_p is the Planck mass. Thus when a fundamental particle reaches this maximum speed of $v_{max} < c$, it will become a Planck mass. Similar results have been derived based on a new fundamental physics theory rooted in atomism; see [5, 6] as well as [7]. Haug suggests that there are good reasons to assume that the Planck mass will dissolve into energy within a Planck second[2]. However, we are not relying directly on atomism here, but merely on the assumption that there must be a maximum frequency limited by the Planck length and we will get the same maximum speed as [6].

Using the known reduced Compton wavelength of some subatomic particles, we can find their maximum speed. An electron has a reduced Compton wavelength of $\bar{\lambda}_e \approx 3.86159 \times 10^{-13}$ m and can never be accelerated to a velocity faster than

$$v = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}_e^2}} = c \times 0.99999999999999999999999999999999999999999999999908202$$

In the above calculation, we have assumed a Planck length of 1.616199×10^{-35}. However, the Planck length: $l_p = \sqrt{\frac{G\hbar}{c^3}}$ is dependent on big G in addition to $\hbar$ and c. Therefore, our assumed theoretical speed limit for the electron is also dependent on big G. As there is considerable uncertainty about the exact value for big G, there is also some uncertainty about the theoretical value for the maximum speed

[2] As measured with Einstein- Poincaré-synchronized clocks.

limit of the electron. In 2007, a research team measured big G to 6.693×10^{-11}, while in 2014 another team measured big G to 6.67191×10^{-11}, see [8, 9], for example. Assuming $\hbar = 1.054571800 \times 10^{-34}$ and 6.67×10^{-11} to 6.9×10^{-11} as the range for big G, we get a theoretical range for the speed limit of the electron equal to:

$G = 6.7 \times 10^{-11}$ corresponds to $l_p = 1.61936379 \times 10^{-35}$ and

$$v = c \times 0.999999999999999999999999999999999999999999999999907824$$

and

$G = 6.67 \times 10^{-11}$ corresponds to $l_p = 1.61573428 \times 10^{-35}$ and

$$v = c \times 0.999999999999999999999999999999999999999999999999908258$$

These calculations require very high precision and were calculated in Mathematica.[3] A proton will hypothetically turn into pure energy at the speed of[4]

$$v = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}_P^2}} = c \times 0.9999999999999999999999999999999999999999999640656$$

For comparison, at the Large Hadron Collider (LHC) in 2008, the team talked about the possibility of accelerating protons to the speed of 99.9999991% of the speed of light [10]. When the LHC went full force in 2015, they increased the maximum speed slightly above this (likely to around 99.99999974% of the speed of light). In any case, the maximum speeds (and energy levels) mentioned in relation to proton accelerations at the LHC are far below what is needed to reach the maximum speed of a proton or an electron as given by the derivations in this paper.

"Surprisingly" the minimum energy needed to accelerate any subatomic particle (fundamental) to its maximum speed is the same. This is only possible because the maximum speed, based on our theory, is inversely related to the particle's rest-mass, this because the maximum mass any subatomic "fundamental" particle can reach is the Planck mass. The minimum energy needed to accelerate any mass to its maximum mass is the Planck mass energy, which is $E = m_p c^2 \approx 1.22 \times 10^{16}$ TeV. The LHC is currently operating at 13 TeV and it is therefore extremely unlikely we will see this theory verified experimentally, as it would require a much more powerful particle accelerator. Still, the main point is that we do not need infinite energy for a mass to reach the speed of light. It will turn into Planck mass just before it reaches the speed of light, and at that time, the Planck mass will likely be highly unstable and will dissolve into pure energy, which is indeed traveling at the speed of light.

In reality, if a proton consists of a series of other subatomic particles, then the speed limit given above for a proton will not be very accurate. Alternatively, we could have looked at the reduced Compton wavelength of the quarks that the standard model claims make up the proton. As the quarks in the proton have different reduced Compton wavelengths, then the proton could have several maximum speeds where parts of the proton mass turns into a Planck mass and then burst into energy.

The speed limit formula we have derived is $v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}}$. When $\bar{\lambda}$ is set to l_p, we have a speed limit of c. This also means the Planck mass is at rest as observed from any reference frame. Haug has suggested that a Planck mass consists of the collision (counter-strike) between two indivisible particles each moving at the speed of light; immediately after collision they will depart and the Planck mass will become energy. However, one does not need to subscribe to this view to see that a maximum frequency leads to a maximum velocity $v_{max} < c$ when the fundamental particle has reached a relativistic Planck mass. With fundamental particle in the context of modern physics[5] we basically think of any particle with a single reduced Compton wavelength.

2 Maximum Doppler Shift

Based on our theory, the maximum possible relativistic Doppler shift must be given by

$$f_2 = f_1\sqrt{\frac{1 + \frac{v_{max}}{c}}{1 - \frac{v_{max}}{c}}} \approx f_1\sqrt{\frac{2 - \frac{lp^2}{\bar{\lambda}^2}\frac{1}{2}}{\frac{lp^2}{\bar{\lambda}^2}\frac{1}{2}}} = f_1\sqrt{\frac{4 - \frac{lp^2}{\bar{\lambda}^2}}{\frac{lp^2}{\bar{\lambda}^2}}} \approx 2f_1\frac{\bar{\lambda}}{l_p} \qquad (7)$$

[3] We used several different set-ups in Mathematica; here is one of them: N[Sqrt[1 − (1616199 ∗ 10^(−41))^2/(3861593 ∗ 10^(−19))^2], 50], where 1616199 ∗ 10^(−41) is the Planck length and 3861593 ∗ 10^(−19) is the reduced Compton wavelength of the electron. An alternative way to write it is: N[Sqrt[1 − (SetPrecision[1.616199 ∗ 10^(−35))^2, 50]/(SetPrecision[3.861593 ∗ 10^(−13))^2, 50]], 50].

[4] Here assuming $l_p = 1.616199 \times 10^{-35}$ and $\bar{\lambda}_P = 2.10309 \times 10^{-16}$.

[5] From atomism point of view there is actually only one fundamental particle, that is an indivisible particle; see [5, 6].

Assume. for example. an electron, which has a frequency (amount of counter-strikes per second, clock frequency) as observed from its rest-frame equal to $f_1 = \frac{c}{\bar{\lambda}_e} \approx 7.7634 \times 10^{20}$. Next assume this electron is accelerated to its maximum velocity relatively to the laboratory frame and that it is traveling towards the laboratory frame. The frequency as observed from the laboratory frame is then given by

$$\begin{aligned} f_2 &\approx f_1 2\frac{\bar{\lambda}_e}{l_p} \\ f_2 &\approx \frac{c}{\bar{\lambda}_e} 2\frac{\bar{\lambda}_e}{l_p} \approx 2\frac{c}{l_p} \end{aligned} \tag{8}$$

That is the maximum relativistic Doppler shift, which again is directly related to the maximum velocity of masses that will "turn" a non-Planck-frequency into the Planck frequency. This is just another way of looking at how a subatomic particle will become a Planck mass when accelerated to its maximum velocity. As the Planck frequency is the very collision point of indivisible particles (the point where the photons changes direction), this also means the Planck frequency is invariant as observed from any reference frame.

Still no one has been able to observe the Planck frequency, or a Planck mass, or the maximum velocity described in this paper "directly". The maximum velocity for any known subatomic particle is far above that achieved at the LHC.

3 Summary and Conclusion

In this paper we have assumed that there is a maximum two-sided Doppler shift frequency of $\hat{f} = 2\frac{c}{l_p}$. Based on this, we have found the maximum speed that fundamental particles can take. At this maximum speed any fundamental particle will have reached a relativistic mass equal to the Planck mass. We expect the Planck mass to be highly unstable and burst into energy within a Planck second. If this is the case, then we do not need an infinite amount of energy to accelerate particles to the speed of light, as they will become Planck masses (standing still for an instant), and then they will burst into pure energy traveling at the speed of light.

To deny that such a maximum velocity just below the speed of light exists, one must assume there is no limitations to the shortness of the wavelength and no limit on the maximum frequency in relativistic Doppler shift. The maximum velocity gives a limit on the maximum relativistic Doppler shift equal to the Planck frequency $2\frac{c}{l_p}$. It also gives a relativistic mass limit equal to the Planck mass, and most importantly, it gives strong evidence towards the concept that the Planck mass is the same as observed from any reference frame. This because the maximum velocity of a Planck mass is zero. However, the Planck mass only exits when it is at rest and in that regard, it only exists for an instant. The Planck mass is the turning point of photons, being the point at which photons collide and change direction.

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