TWO NEW DARK MATTER DENSITY PROFILES FOR M31 HALO GOT FROM ROTATION CURVE

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1. ABSTRACT

In this work has been calculated two new DM density profiles inside halo region of M31 galaxy and it has been demonstrated that both ones are mathematically equivalents.

The first is called direct DM density because it is got directly from velocity as power regression of radius in rotation curve. In other words velocity of rotation curve depend on radius as a power function. In fact galactic rotation curve inside M31 halo has a power regression of velocity depending on radius with a correlation coefficient bigger than 0,95 inside halo region.

The second one, called Bernoulli profile has been introduced by author in previous papers, [8] Abarca,M.2016, and others papers quoted in bibliography, where it has been used to study DM in several galaxies. It is called Bernoulli because it is got from a Bernoulli differential equation.

Hypothesis which is the basis to get Bernoulli profile stated that DM is generated locally by the own gravitational field according a power law. DM density = A· E^B where A& B are coefficients and E is gravitational intensity of field. In addition A& B are similar for different galaxies on condition that galaxies are similar and giants, not dwarfs.

To find reasons that author has to do so daring statement, reader can consult [1] Abarca,M.2014. *Dark matter model by quantum vacuum*. [8] Abarca,M.2016. *Dark matter density on big galaxies depend on gravitational field as Universal law* and other papers quoted in bibliography.

Briefly will be explained method followed to develop this paper. Firstly are presented rotation curve and table with data points inside M31 halo. These data come from [5] Sofue,Y.2015. In addition it is got a power regression for rotation curve points in halo region whose function is $v = a \cdot r^b$ getting a correlation coefficient bigger than 0,95.

In fourth chapter it is developed a mathematical method to get a new DM density depending on radius called direct DM density because it is got directly from power regression function got in chapter three.

In fifth chapter it has been demonstrated that a power regression function for rotation curve is mathematically equivalent that DM density depend on gravitational field, as a power function i.e. DM density = A· E^B where A& B are cleared up depending on a & b (parameters of power regression of rotation curve).

In sixth chapter it has been got that for radius bigger than 40 kpc then ratio baryonic density versus DM density is under 1% so it is reasonable to consider negligible baryonic density in order to develop theory introduced in this work.

The seventh is a short chapter where is compared direct DM density got with NFW density profile fitted by Sofue in his paper. [5] Sofue, Y.2015. Relative differences between both density profiles are under 25% inside main part of halo dominion. In addition it is shown that NFW profile is bigger than direct DM through all dominion.

In eighth chapter is got a Bernoulli differential equation for gravitational field. Hypothesis to state this equation is that DM density is a power function of gravitational field i.e. DM density = A· E^B. Solution for E allows to get Bernoulli density profile.

In ninth chapter is demonstrated that Bernoulli profile is mathematically equivalent to direct DM profile.

In tenth chapter has been got masses for M31 by direct DM and NFW profiles at different radius. Through whole dominion masses got by NFW are bigger than direct DM profile although relative difference between both profiles are under 25 %. In conclusion chapter will be pointed reason, which in author opinion, explain that NFW profile gives bigger values than direct DM profile.

2. INTRODUCTION

As reader knows M31 is the twin galaxy of Milky Way in Local Group of galaxies. Its disk radius is approximately 35 kpc and according [5] Sofue, Y. 2015. Its baryonic mass is $M_{BARYONIC} = 1{,}61 \cdot 10^{11}\ M_{SUN}$
In previous paper [10] Abarca,M.2016. *A New Dark Matter Density Profile for M31 Galaxy to Demonstrate that Dark Matter is Generated by Gravitational Field,* author has studied DM inside M31 halo through Bernoulli DM profile. However in such paper DM density used it was NFW profile provided by [5] Sofue, Y. 2015 whereas in current paper DM density profile has been got directly from a power regression function on rotation curve in halo region.

This new DM profile has been called direct DM density because this profile is fitted directly from data measures inside halo region. In this work radius dominion begin at 40 kpc because at this distance baryonic density is negligible as it will be shown in chapter six. Therefore the only one kind of matter in halo region it is supposed to be non baryonic dark matter.

It is known that there is baryonic dark matter such us giant planets, cold gas clouds, brown dwarfs but this kind of DM is more probable to be placed inside galactic disk. Reader can consult: [11] Nieuwenhuizen,T.M. 2010. [12] Nieuwenhuizen,T.M. 2012. [13] Nieuwenhuizen,T.M. 2010 [14] Wyrzykowski,L.2010. [15] M.R.S. Hawkins 2015.In fact there are an important amount of researchers in this way because baryonic DM and non baryonic DM are open problems still.

As it is known, NFW profile is fitted over bulge, disk and galactic halo and taking in consideration that there is an unknown amount of baryonic DM in bulge and galactic disk it is needed concluded that NFW profile is more imprecise than direct DM profile in order to study non baryonic DM in halo because direct DM density has been fitted exclusively with data of DM non baryonic in halo region.

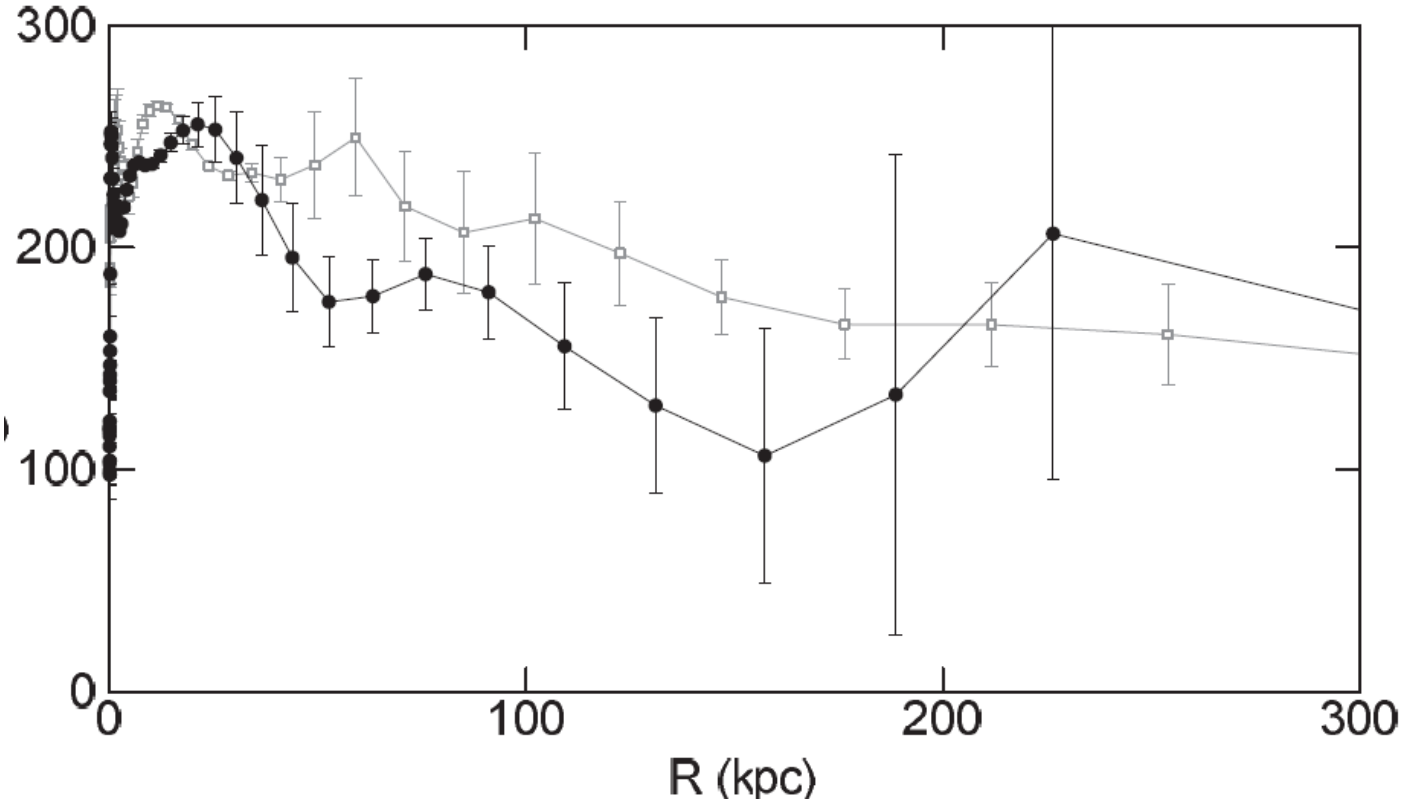
However DM theory introduced in [1] Abarca, M.2014. *Dark matter model by quantum vacuum* and developed in others papers quoted in bibliography refers a kind of weird matter which is generated by the own gravitational field. Therefore it is needed to consider a radius dominion where it is supposed that baryonic matter is negligible.

In fact according [5] Sofue, Y. 2015 data, in chapter six will be got that for radius bigger than 40 kpc baryonic matter density is under 1% versus DM density. This is the reason why radius dominion in this work is from 40 kpc to 300 kpc. In chapter eight it will be got a simple Bernoulli differential equation for gravitational field. However to get a so simple differential equation it is needed that $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ .In other words, it is needed that density of baryonic matter would be negligible versus D.M. density. In addition it is supposed hypothesis $\varphi_{DM}(r) = A \cdot E^B$

In paper [1] Abarca,M.2014, it was postulated that DM density depends on gravitational field. Further papers [2] Abarca,M.2015 and others have studied DM density as power of gravitational field in several galaxies: Milky Way, M33, NGC3198 and others galaxies. Results got support such hypothesis. $\varphi_{DM}(r) = A \cdot E^B$

Summarising, in this paper it is got a new DM density called direct which is calculated directly from a power regression function in halo of rotation curve of M31.As it is supposed that for radius bigger than 40 kpc baryonic matter is negligible then direct DM profile is a precise function to know non baryonic DM density in halo region whereas NFW profile offers more imprecise results for non baryonic DM in halo region.

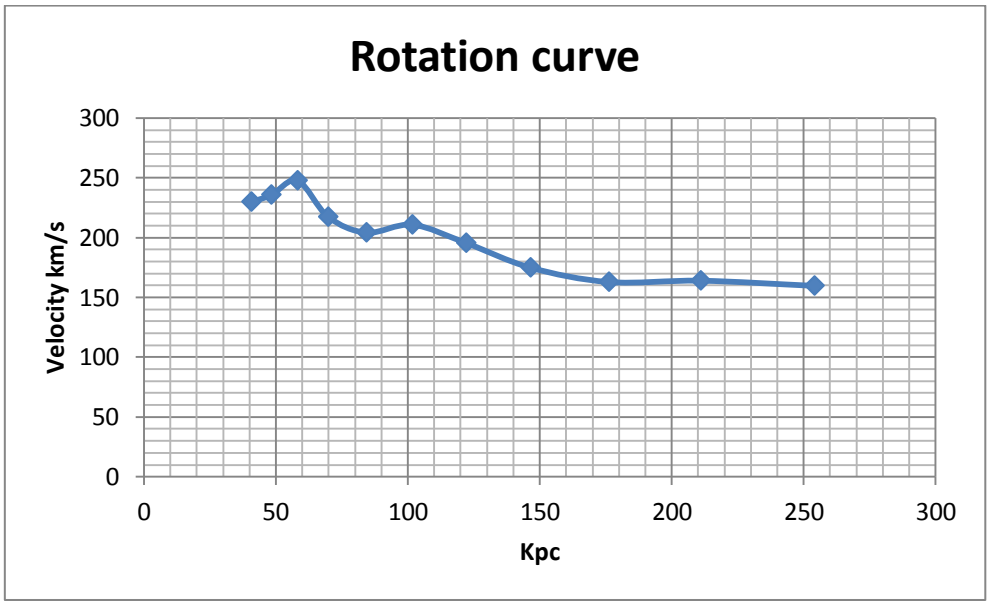
3. OBSERVATIONAL DATA FROM SOFUE. 2015 PAPER



Graphic come from [5] Sofue,Y. 2015.

Grey line belong to M31 rotation curve and black line to Milky Way.

M31 point data	
Radius	Velocity
kpc	km/s
40,7	230
48,25	235,9
58,2	247,8
69,8	217,4
84,3	204,3
101,7	210,9
122,1	195,6
146,5	175
176,2	163
211	164,1
254,1	159,8



From graphic it is clear there is a high correlation between spin radius and velocity.

In chapter six will be shown reason why dominion data begin at 40 kpc in this work. It is accepted that disk radius of M31 is approximately 35 kpc.

3.1 POWER REGRESSION TO ROTATION CURVE

It is seen that experimental measures of rotation curve has a very good fitted curve by power regression.

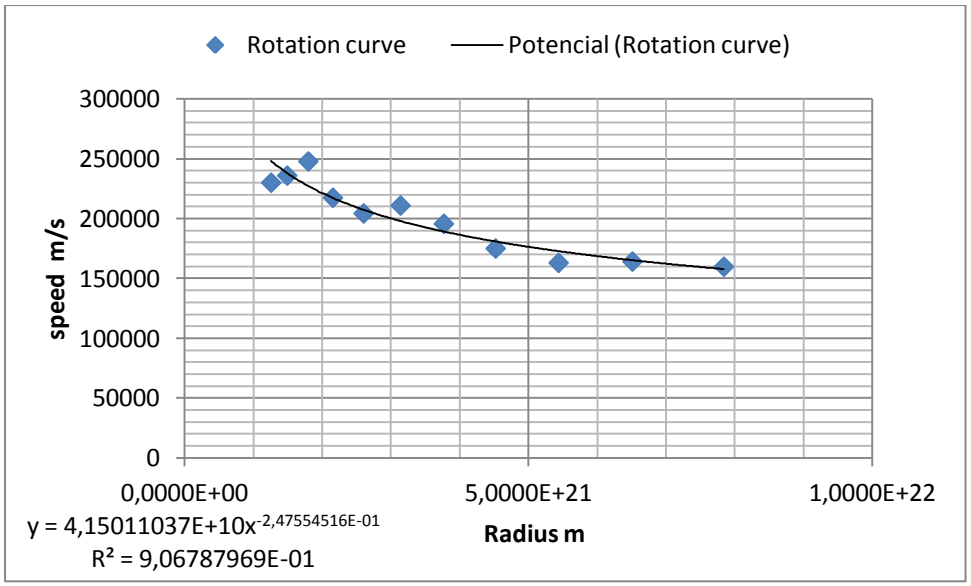
In particular coefficients of $v = a \cdot r^b$ are in table below. Units are into I.S.

Power regression for M31 rot. curve	
V=a*r^b	
a	4,15011040E+10
b	-2,47554520E-01
Correlation coeff.	0,952254

Data fitted are in grey columns below.
In third column is shown results of fitted velocity and fourth column shows relative difference between measures and fitted results.
Correlation coefficient is above 0.95 which is very good correlation.

radius	velocity measures	veloc. Fitted.	Rel. Diff.	Radius
m	m/s		%	kpc
1,2559E+21	230000	2,4827E+05	7,36	40,7
1,4889E+21	235900	2,3803E+05	0,89	48,25
1,7959E+21	247800	2,2723E+05	-9,05	58,2
2,1538E+21	217400	2,1723E+05	-0,08	69,8
2,6012E+21	204300	2,0732E+05	1,46	84,3
3,1382E+21	210900	1,9791E+05	-6,57	101,7
3,7676E+21	195600	1,8915E+05	-3,41	122,1
4,5206E+21	175000	1,8081E+05	3,21	146,5
5,4370E+21	163000	1,7273E+05	5,63	176,2
6,5108E+21	164100	1,6519E+05	0,66	211
7,8408E+21	159800	1,5777E+05	-1,29	254,1

Below is shown a graphic with measures data and power regression function.



In my opinion a correlation coefficient of 0,952254 is a very high correlation if it is considered that M31 is 770 kpc away and errors in measures are not negligibles. Therefore this value support strongly hypothesis that rotation curve of M31 follow a law $v = a \cdot r^b$ where a & b are written above.

4. DIRECT FORMULA FOR DM DENSITY ON M31 HALO GOT FROM ROTATION CURVE

4.1 THEORETICAL DEVELOPPMENT FOR GALACTIC HALOS

Outside disk region, rotation curve it is fitted by power regression with a high correlation coefficient according formula $v = a \cdot r^b$. As $M(<r) = \frac{v^2 \cdot R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of velocity results $M = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G}$

If it is considered outside region of disk where baryonic matter is negligible regarding dark matter it is possible to calculate DM density by a simple derivative. In next chapter will be show that for r > 40 kpc baryonic matter is negligible.

As density of D.M. is $D_{DM} = \frac{dm}{dV}$ where $dm = \frac{a^2 \cdot (2b+1) \cdot r^{2b} dr}{G}$ and $dV = 4\pi r^2 dr$ results

$$D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$$

Writing $L = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot$ results $D_{DM}(r) = L \cdot r^{2b-2}$. In case b = -1/2 DM density is cero which is Keplerian rotation.

4.2 DIRECT DM DENSITY FOR M31 HALO

Parameters a & b from power regression of M31 rotation curve allow calculate easily direct DM density.

Below is such function and table.

Direct DM density for M31 halo 40 < r < 300 kpc
$D_{DM}(r) = L \cdot r^{2b-2}$ kg/m^3
L = 1,03701707086078E $\cdot 10^{30}$
2b -2 = -2,49510904

Below is shown results of DM density inside its dominion. Calculus are into I.S.

Direct DM	Radius	Radius
kg/m^3	m	kpc
2,4570213865E-23	1,234280E+21	40,00
8,9339196948E-24	1,851420E+21	60,00
4,3581911375E-24	2,468560E+21	80,00
2,4974984289E-24	3,085700E+21	100,00
1,5846719874E-24	3,702840E+21	120,00
1,0786979189E-24	4,319980E+21	140,00
7,7304292487E-25	4,937120E+21	160,00
5,7619898451E-25	5,554260E+21	180,00
4,4299881060E-25	6,171400E+21	200,00
3,4923945134E-25	6,788540E+21	220,00
2,8108438328E-25	7,405680E+21	240,00
2,3019796063E-25	8,022820E+21	260,00

5. DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

As independent variable for this function is E, gravitational field, previously will be studied formula for E in the following paragraph.

5.1 GRAVITATIONAL FIELD E THROUGH VIRIAL THEOREM

As it is known total gravitational field may be calculated through Virial theorem, formula $E = v^2/R$ whose I.S. unit is m/s^2 is well known. Hereafter, virial gravitational field, E, got through this formula will be called E.

By substitution of $v = a \cdot r^b$ in formula $E = \frac{v^2}{r}$ it is right to get $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ briefly $E = a^2 \cdot r^{2b-1}$

Radius	Radius	E Virial
kpc	m	m/s^2
40,00	1,23428E+21	5,036815266E-11
60,00	1,85142E+21	2,747137450E-11
80,00	2,46856E+21	1,786830491E-11
100,00	3,08570E+21	1,279947977E-11
120,00	3,70284E+21	9,745580687E-12
140,00	4,31998E+21	7,739536828E-12
160,00	4,93712E+21	6,338853095E-12
180,00	5,55426E+21	5,315352710E-12
200,00	6,17140E+21	4,540666972E-12
220,00	6,78854E+21	3,937613385E-12
240,00	7,40568E+21	3,457283979E-12
260,00	8,02282E+21	3,067339974E-12

5.2 DARK MATTER DENSITY AS POWER OF GRAVITATIONAL FIELD

According hypothesis dark matter by quantum vacuum $D_{DM} = A \cdot E^B$. Where A & B are parameters to be calculated. This hypothesis has been widely studied by author in previous papers. [1] Abarca,M. [2] Abarca,M. [8] Abarca,M. [9] Abarca,M. [10] Abarca,M.

As it is known direct DM density $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$ depend on a & b parameters which come from power regression formula for velocity. In previous paragraph has been shown formula for gravitational field $E = \frac{a^2 \cdot r^{2b}}{r} = a^2 \cdot r^{2b-1}$ which depend on a & b as well. Through a simple mathematical treatment it is possible to get A & B to find function of DM density depending on E. Specifically formulas are $A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

M31 galaxy	$D_{DM} = A \cdot E^B$
A	3,766521943774E $\cdot 10^{-6}$
B	1,668847537702

According parameters a & b got in previous chapter, A& B parameters are:

Below is tabulated DM density as power of E and direct DM density, both are identical as it was expected.

		$E = a^2 \cdot r^{2b-1}$	$D_{DM} = A \cdot E^B$	$D_{DM}(r) = L \cdot r^{2b-2}$
Radius	Radius	E Virial	DM power of E	Direct DM
kpc	m	m/s^2	Kg/m^3	kg/m^3
40,00	1,23428E+21	5,036815266E-11	2,45702138653E-23	2,45702138653E-23
60,00	1,85142E+21	2,747137450E-11	8,93391969485E-24	8,93391969485E-24
80,00	2,46856E+21	1,786830491E-11	4,35819113751E-24	4,35819113751E-24
100,00	3,08570E+21	1,279947977E-11	2,49749842886E-24	2,49749842886E-24
120,00	3,70284E+21	9,745580687E-12	1,58467198742E-24	1,58467198742E-24
140,00	4,31998E+21	7,739536828E-12	1,07869791892E-24	1,07869791892E-24
160,00	4,93712E+21	6,338853095E-12	7,73042924866E-25	7,73042924866E-25
180,00	5,55426E+21	5,315352710E-12	5,76198984505E-25	5,76198984505E-25
200,00	6,17140E+21	4,540666972E-12	4,42998810603E-25	4,42998810603E-25
220,00	6,78854E+21	3,937613385E-12	3,49239451343E-25	3,49239451343E-25
240,00	7,40568E+21	3,457283979E-12	2,81084383281E-25	2,81084383281E-25
260,00	8,02282E+21	3,067339974E-12	2,30197960630E-25	2,30197960630E-25

As conclusion, in this chapter has been demonstrated that a power law for velocity

$v = a \cdot r^b$ *is mathematically equivalent a power law for DM density depending on E.* $D_{DM} = A \cdot E^B$

6. RATIO BARYONIC MASS VERSUS DARK MATTER MASS DEPENDING ON RADIUS FOR M31

In this paragraph will be estimated radius which is needed to consider negligible baryonic density regarding DM density in M31 galaxy.

[5] According Sofue, Y. data for M31 disk are

M31 Galaxy	Baryonic Mass at disk	a_d	Σ_0
	$M_d = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$		
	$M_d = 1{,}26 \cdot 10^{11}$ Msun	5,28 kpc	1,5 kg/m^2

Where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ represents superficial density at disk. Total mass disk is given by integration of superficial density from cero to infinite. $M_d = \int_0^\infty 2\pi \cdot r \Sigma(r) \cdot dr = 2\pi \cdot \Sigma_0 \cdot a^2{}_d$

In order to compare baryonic density and DM density it is considered differential baryonic mass and differential DM masses depending on radius.

$dM_{DISK} = 2\pi r\Sigma(r)dr$ where $\Sigma(r) = \Sigma_0 \exp(-r/a_d)$ and

$dM_{DM} = 4\pi r^2 D_{DM}(r)dr$ where $D_{DM}(r) = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$

It is defined ratio function as quotient of both differential quantities $Ratio = \frac{dM_{DISK}}{dM_{DM}} = \frac{\Sigma(r)}{2 \cdot r \cdot D_{DM}(r)}$

Radius	Radius	Ratio (r)	$\Sigma(r)$	Direct DM
Kpc	m	Ratio	kg/m^2	kg/m^3
30	9,257100E+20	5,480994E-02	5,11103641849E-03	5,0366840424E-23
32	9,874240E+20	4,132947E-02	3,49947979162E-03	4,2875577849E-23
34	1,049138E+21	3,098268E-02	2,39606173959E-03	3,6856697077E-23
36	1,110852E+21	2,310614E-02	1,64056151250E-03	3,1957946476E-23
38	1,172566E+21	1,715255E-02	1,12327743139E-03	2,7924857817E-23
40	1,234280E+21	1,268028E-02	7,69097762116E-04	2,4570213865E-23
42	1,295994E+21	9,339073E-03	5,26594188719E-04	2,1754010061E-23
44	1,357708E+21	6,854954E-03	3,60554214629E-04	1,9370002366E-23
46	1,419422E+21	5,016056E-03	2,46868166174E-04	1,7336491322E-23
48	1,481136E+21	3,660088E-03	1,69028370762E-04	1,5589891544E-23
50	1,542850E+21	2,663744E-03	1,15732176267E-04	1,4080169637E-23
52	1,604564E+21	1,933988E-03	7,92407603715E-05	1,2767558261E-23

For a radius 40 kpc ratio baryonic matter versus DM is only 1,2 % therefore is a good approximation to consider negligible baryonic mass density regarding DM density when radius is bigger than 40 kpc.

This is the reason why in this work dominion for radius begin at 40 kpc.

7. COMPARISON BETWEEN DIRECT DM DENSITY AND NFW DARK MATTER DENSITY

According [5] Sofue,Y., 2015. Parameters of NFW profile for M31 are

Dark matter density function profile NFW
Rs = 34.6 ± 2.1 Kpc
Do = $1.50926 \cdot 10^{-22}$ kg/m^3
Do = $2.23 \pm 0.24 \cdot 10^{-3}$ Msolar/pc^3 = 2.23 mMsolar/pc^3

Where mMsolar/pc^3 = $6{,}768 \cdot 10^{-23}$ Kg /m^3 is a very common unit for DM densities.

$$D_{NFW}(R) = \frac{D_0}{x \cdot (1+x)^2}$$ Where x= radius/ Rs Rs is called length scale and Do is density scale.

Below are tabulated NFW DM density and direct DM density depending on radius both. Last column shows relative differences. Inside main part of dominion relative difference is under 25%.

Direct DM	NFW DM density	Relative diff.	radius
kg/m^3	kg/m^3	%	kpc
2,4570214E-23	2,809191E-23	12,54	40,00
8,9339197E-24	1,164623E-23	23,29	60,00
4,3581911E-24	5,951956E-24	26,78	80,00
2,4974984E-24	3,451667E-24	27,64	100,00
1,5846720E-24	2,180312E-24	27,32	120,00
1,0786979E-24	1,465219E-24	26,38	140,00
7,7304292E-25	1,032080E-24	25,10	160,00
5,7619898E-25	7,543745E-25	23,62	180,00
4,4299881E-25	5,681107E-25	22,02	200,00
3,4923945E-25	4,385100E-25	20,36	220,00
2,8108438E-25	3,455466E-25	18,66	240,00
2,3019796E-25	2,771278E-25	16,93	260,00

NFW DM density dominion include disk and even bulge regions. Inside these region a fraction of DM is baryonic DM such as giant planets and cold cloud of gas. However theory of Dark matter by quantum vacuum,[1] Abarca,M. is a theory for non baryonic dark matter. Therefore we are interested only in halo region where baryonic mass is negligible regarding DM mass.

Taking in consideration that NFW density profile is fitted with a set of DM measures which includes bulge, disk and halo it is right to think that NFW profile does not describe DM density inside halo region as well as theory developed in this paper because direct DM density is a function specifically calculated to fit DM in halo region.

In addition, it is right to think that NFW function fitted should be bigger than direct DM profile because values to fit in disk and bulge has an extra baryonic DM which increase values of function fitted through the whole dominion.

8. BERNOULLI DIFFERENTIAL EQUATION FOR GRAVITATIONAL FIELD IN M31 HALO

It will be considered the region 40 Kpc < Radius < 260 Kpc where density of baryonic matter is negligible versus baryonic density. So for radius bigger than 40 Kpc, it will be considered that derivative of M(r) depend on dark matter density only.

As it is known in this formula $E = G\frac{M(r)}{r^2}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered radius > 40 Kpc then the derivative of M(r) depend on dark matter density only and therefore $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ As $\varphi_{DM}(r) = A{\cdot}E^B(r)$ Where A= 3,76652194377428·10^{-6} and B= 1,668847537702

then $M'(r) = 4\pi r^2 \cdot A{\cdot}E^B$ Now it will differentiated E(r) when r > 40 Kpc

If $E = G\frac{M(r)}{r^2}$ is differentiated it is got $E'(r) = G\frac{M'(r){\cdot}r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ is replaced above it is got $E'(r) = 4\pi G\varphi_{DM}(r) - 2G\frac{M(r)}{r3}$ As $\varphi_{DM}(r) = A{\cdot}E^B(r)$ it is right to get $E'(r) = 4\pi{\cdot}G{\cdot}A{\cdot}E^B(r) - 2\frac{E(r)}{r}$ which is a Bernoulli differential equation.

$E'(r) = K{\cdot}E^B(r) - 2\frac{E(r)}{r}$ being $K = 4\pi{\cdot}G{\cdot}A$ then K = 3,15843170718 ·10^{-15} I.S. as A= 3,76652194377428·10^{-6}

Calling ***y*** to E, the differential equation is written this way $y` = K{\cdot}y^B - \frac{2{\cdot}y}{r}$

Bernoulli family equations $y` = K{\cdot}y^B - \frac{2{\cdot}y}{r}$ may be converted into a differential linear equation with this variable change $u = y^{1-B}$.

General solution is $E(r) = \left(Cr^{2B-2} + \frac{Kr(1-B)}{3-2B}\right)^{\frac{1}{1-B}}$ *with* $B \neq 1$ *and* $B \neq 3/2$ where C is the parameter of initial condition of gravitational field at a specific radius.

Calling $\alpha = 2B-2$ $\beta = \frac{1}{1-B}$ and $D = \left(\frac{K(1-B)}{3-2B}\right)$ formula may be written as

$E(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ Where specifically values for these parameters are the following ones:

$\alpha = 2B-2$ = 1,156069364

$\beta = \frac{1}{1-B}$ = -1,4951090400

$$D = \left(\frac{K(1-B)}{3-2B} \right) = \; 6{,}25567094167 \cdot 10^{-15}$$

Initial condition for parameter C calculus

Suppose R_0 *and* E_0 are specific initial conditions for radius and gravitational field then $C = \frac{E_0^{\,1/\beta} - D \cdot R_0}{R_0^{\,\alpha}}$

it will be calculated parameter C at 40 kpc.

Radius	radius	E virial	param. C
kpc	m	m/s^2	I.S.
40	$1{,}234280 \cdot 10^{21}$	$5{,}036815 \cdot 10^{-11}$	$-6{,}82969 \cdot 10^{-37}$

Finally it is possible to write formula for gravitational field intensity through Bernoulli method.

Bernoulli Solution for Gravitational field inside halo 40 kpc < Radius < 260 kpc
$E_{BER}(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ C = $-6{,}82969 \cdot 10^{-37}$ D= $6{,}25567094167 \cdot 10^{-15}$ α = 1,156069364 β = -1,49510904

8.1 BERNOULLI PROFILE OF DARK MATTER DENSITY FOR M31 GALAXY

Thanks Bernoulli solution for gravitational field is right to get DM density through power of E formula.

DM Density Bernoulli profile for M31 inside halo 40 kpc < radius < 300 kpc
$E_{BER}(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ C = $-6{,}82969 \cdot 10^{-37}$ D= $6{,}25567094167 \cdot 10^{-15}$ α = 1,156069364 β = -1,49510904
Density $_{\text{D.M. BERNOULLI}}$ (r) = $D_{DM\,B}$(r) = A· E^B(r) Where A= $3{,}76652194377428 \cdot 10^{-6}$ and B= 1,668847537702 unit for density is Kg/ m^3

9. COMPARISON BETWEEN BERNOULLI AND DIRECT DM

9.1 COMPARISON BETWEEN VIRIAL GRAVT. FIELD AND BERNOULLI SOLUTION FOR E

Direct E $E = a^2 \cdot r^{2b-1}$	Bernoulli E	Radius
m/s^2	m/s^2	kpc
5,0368153E-11	5,0368153E-11	40,00
2,7471374E-11	2,7471374E-11	60,00
1,7868305E-11	1,7868305E-11	80,00
1,2799480E-11	1,2799480E-11	100,00
9,7455807E-12	9,7455807E-12	120,00
7,7395368E-12	7,7395368E-12	140,00
6,3388531E-12	6,3388531E-12	160,00
5,3153527E-12	5,3153527E-12	180,00
4,5406670E-12	4,5406670E-12	200,00
3,9376134E-12	3,9376134E-12	220,00
3,4572840E-12	3,4572840E-12	240,00
3,0673400E-12	3,0673400E-12	260,00

There is a perfect agreement between virial gravitational field and Bernoulli field as it was expected.

9.2 COMPARISON BETWEEN DIRECT DM PROFILE AND BERNOULLI DM PROFILE

Bernoulli DM density $D_{DM\,B}(r) = A \cdot E^B(r)$	Direct DM $D_{DM}(r) = L \cdot r^{2b-2}$	Radius
kg/m^3	kg/m^3	kpc
2,457021387E-23	2,457021387E-23	40,00
8,933919695E-24	8,933919695E-24	60,00
4,358191138E-24	4,358191138E-24	80,00
2,497498429E-24	2,497498429E-24	100,00
1,584671987E-24	1,584671987E-24	120,00
1,078697919E-24	1,078697919E-24	140,00
7,730429249E-25	7,730429249E-25	160,00
5,761989845E-25	5,761989845E-25	180,00
4,429988106E-25	4,429988106E-25	200,00
3,492394513E-25	3,492394513E-25	220,00
2,810843833E-25	2,810843833E-25	240,00
2,301979606E-25	2,301979606E-25	260,00

Reader can check agreement between direct DM and Bernoulli DM profile is perfect as it was expected.

It is clear that these table are a numerical proof that Bernoulli DM profile and direct DM profile are mathematically identical.

10. MASSES IN M31

10.1 DYNAMICAL MASS UP TO 40 KPC

It has been chosen 40 kpc, because at this radius baryonic density is negligible regarding DM density in M31.

According power regression law for velocity $v = a \cdot r^b$ rotation velocity is v = $2{,}177 \cdot 10^5$ m/s at 40 kpc.

And $M(<40kpc) = \frac{v^2 \cdot R}{G}$ = $5{,}78 \cdot 10^{11}$ Msun i.e. $M_{DYNAMICAL}$ (< 40 kpc) = $5{,}78 \cdot 10^{11}$ Msun

According Sofue data M31 Baryonic matter = $1{,}6 \cdot 10^{11}$ so DM (<40 kpc) = $4{,}18 \cdot 10^{11}$ Msun

M31 masses up to 40 kpc		
$M_{DYNAMICAL}$ = Total mass	Baryonic mass Bulge + disk	Dark matter mass
$5{,}78 \cdot 10^{11}$ Msun	$1{,}6 \cdot 10^{11}$ Msun	$4{,}18 \cdot 10^{11}$ Msun

10.2 CORONA HALO MASSES THROUGH DYNAMICAL MASSES

As it was shown in chapter four, rotation curve is fitted with a high correlation by this function $v = a \cdot r^b$ whose parameter are in table below.

Power regression for M31 rot. curve	
$V=a \cdot r^b$	
a	4,15011040E+10
b	-2,47554520E-01
Correlation coeff.	0,952254

Also it is known that $M(<r) = \frac{v^2 \cdot R}{G}$ represents total mass enclosed by a sphere with radius r, by substitution of velocity results $M(<r) = \frac{v^2 \cdot R}{G} = \frac{a^2 \cdot r^{2b+1}}{G}$.

As it was demonstrated that baryonic matter is negligible for radius bigger than 40 kpc, it is right to calculate DM corona halo mass subtracting to total mass M(<r) total mass for radius lower 40 kpc.

So DM corona halo (40 < r < Ro)= $M(<Ro) - M(<40kpc) = \frac{a^2 \cdot R_o^{2b+1}}{G} - M(<40kpc)$

Where $M(<40kpc)$ = $5{,}78 \cdot 10^{11}$ Msun and Msun = $1{,}99 \cdot 10^{30}$ kg

Below is shown a table with different corona halo masses at different radius.

Velocity	Dynamical mass(<R)	Corona halo 40 kpc < R	Radius
m/s	M sun	Msun	kpc
249335,92	5,7784213E+11	0,00000E+00	40,00
225523,95	7,0911405E+11	1,31272E+11	60,00
210021,39	8,1996729E+11	2,42125E+11	80,00
198734,38	9,1775238E+11	3,39910E+11	100,00
189964,01	1,0062443E+12	4,28402E+11	120,00
182851,43	1,0876878E+12	5,09846E+11	140,00
176905,85	1,1635468E+12	5,85705E+11	160,00
171822,15	1,2348389E+12	6,56997E+11	180,00
167398,54	1,3023054E+12	7,24463E+11	200,00
163495,09	1,3665063E+12	7,88664E+11	220,00
160011,06	1,4278768E+12	8,50035E+11	240,00
156871,66	1,4867632E+12	9,08921E+11	260,00

10.3 DM HALO MASSES THROUGH NFW PROFILE

Below are two different notation for NFW profile.

$$D_{NFW}(R)=\frac{D_0}{x\cdot(1+x)^2} \quad \text{or} \quad D_{NFW}(R)=\frac{4D_S}{x\cdot(1+x)^2} \quad \text{where } x = R / Rs$$

Dark matter density profile NFW for M31 Data come from [5] Sofue, Y.2015.
Rs = 34.6 ± 2.1 Kpc
Do = $2.23 \pm 0.24 \cdot 10^{-3}$ Msolar/pc^3 = 2.23 mMsolar/pc^3 = $1.50926 \cdot 10^{-22}$ kg/m^3
Ds= Do/4= $3.77 \cdot 10^{-23}$ kg/m^3

According NFW DM density profile, total DM enclosed by a sphere with R radius is

$$M(<R)=16\pi R^3{}_S\cdot D_S\cdot\left[\ln(1+x)-\frac{x}{1+x}\right] \quad \text{or} \quad M(<R)=4\pi R^3{}_S\cdot D_o\cdot\left[\ln(1+x)-\frac{x}{1+x}\right]$$

Calling $f(x)=\left[\ln(1+x)-\frac{x}{1+x}\right]=\ln(1+r/Rs)-\frac{r}{r+Rs}$ and $Z_{NFW}=16\pi R^3{}_S\cdot D_S$ then

$$DM_{NFW}(<r)= Z_{NFW}\cdot f(x)$$

According data for M31 galaxy $Z_{NFW} = 11.6\cdot10^{11}$ Msun.

In table below is tabulated total DM inside a sphere with R radius. The third column shows f(x) function.

Radius	NFW DM(<R)	f(x)
kpc	Msun	adimensional
40,00	2,6923E+11	2,320938E-01
60,00	4,3100E+11	3,715543E-01
80,00	5,7944E+11	4,995138E-01
100,00	7,1399E+11	6,155117E-01
120,00	8,3612E+11	7,207908E-01
140,00	9,4750E+11	8,168112E-01
160,00	1,0497E+12	9,048931E-01
180,00	1,1439E+12	9,861523E-01
200,00	1,2314E+12	1,061513E+00
220,00	1,3128E+12	1,131739E+00
240,00	1,3891E+12	1,197463E+00
260,00	1,4607E+12	1,259212E+00

10.4 DYNAMICAL CORONA HALO COMPARED TO NFW CORONA HALO MASSES

As $DM_{NFW}(<r) = Z_{NFW} \cdot f(x)$ represents total DM under radius R, it is right to get DM corona mass halo with this simple formula: DM Corona (R2-R1) = $Z_{NFW} \cdot (f_2 - f_1)$. Where $f_2 = f(R_2)$ and $f_1 = f(R_1)$

In table below in the second column is tabulated Corona DM halo by substraction of dynamical mass at R radius minus dynamical at 40 kpc.

In the third column is tabulated Corona DM halo by subtraction of NFW DM mass at Radius minus same kind of mass at 40 kpc radius.

In fourth column is shown relative difference between both methods to calculate DM corona halo at different radius.

Differences are always under 25 % it is remarkable that NFW method give always bigger masses than dynamical method.

Dynamical or total mass (<R)	Dynamical Corona halo 40 kpc < R	NFW Corona halo 40 kpc < R	Relative difference	Radius
M sun	Msun	Msun	%	kpc
5,7784213E+11	0,00000E+00	0,00000000E+00	0,00	40,00
7,0911405E+11	1,31272E+11	1,61774211E+11	18,85	60,00
8,1996729E+11	2,42125E+11	3,10207255E+11	21,95	80,00
9,1775238E+11	3,39910E+11	4,44764751E+11	23,58	100,00
1,0062443E+12	4,28402E+11	5,66888545E+11	24,43	120,00
1,0876878E+12	5,09846E+11	6,78272190E+11	24,83	140,00
1,1635468E+12	5,85705E+11	7,80447198E+11	24,95	160,00
1,2348389E+12	6,56997E+11	8,74707916E+11	24,89	180,00
1,3023054E+12	7,24463E+11	9,62126666E+11	24,70	200,00
1,3665063E+12	7,88664E+11	1,04358896E+12	24,43	220,00
1,4278768E+12	8,50035E+11	1,11982859E+12	24,09	240,00
1,4867632E+12	9,08921E+11	1,19145736E+12	23,71	260,00

Taking in consideration that NFW density profile is fitted with a set of DM measures which includes bulge, disk and halo it is right to think that NFW profile does not describe DM density inside halo region as well as theory developed in this paper because direct DM density is a function specifically calculated to fit DM in halo region.

10.5 COMPARISON OF DYNAMICAL CORONA HALO VERSUS NFW CORONA HALO IN EXTENDED DOMINION UP TO 385 kpc

According DM generated by gravitational field theory, galactic halo size is radius where gravitational field of neighbour galaxies are balanced.

Although M31 and Milky Way are similar espiral giant galaxies, it is known that M31 is a bit massive than Milky Way, therefore halo size of M31 should be a bit bigger than Milky Way halo.

As distance between both galaxies is 770 kpc it is sure that halo size of M31 should be bigger than 385 kpc.

This is the reason why has been extended dominion up to 385 kpc to calculate halo masses of M31.

At this radius dynamical mass of M31 is $1{,}8 \cdot 10^{12}$ Msun.

In table below, first column shows total mass through dynamical method. Second and third column show corona halo mass from 40 kpc up to R by dynamical and NFW method. Fourth column shows its relative differences at different radius.

Dynamical or total mass (<R)	Dynamical Corona halo 40 kpc < R	NFW Corona halo 40 kpc < R	Relative difference	Radius
M sun	Msun	Msun	%	kpc
5,7784213E+11	0,00000E+00	0,00000000E+00	0,00	40,00
7,6650840E+11	1,88666E+11	2,37776815E+11	20,65	70,00
9,1775238E+11	3,39910E+11	4,44764751E+11	23,58	100,00
1,1262438E+12	5,48402E+11	7,30424311E+11	24,92	150,00
1,3023054E+12	7,24463E+11	9,62126666E+11	24,70	200,00
1,4576117E+12	8,79770E+11	1,15618516E+12	23,91	250,00
1,5981580E+12	1,02032E+12	1,32285862E+12	22,87	300,00
1,8126725E+12	1,23483E+12	1,56114411E+12	20,90	385,00

It is important emphasise that NFW profile is a bit bigger than direct DM. This fact will be explained in conclusion chapter.

11. CONCLUSION

This work is focused in halo region of M31 where baryonic density is negligible regarding DM non baryonic. Reason is that the main hypothesis all my papers is that DM non baryonic is generated locally by gravitational field. Therefore it is needed to study radius dominion where it is possible to study gravitational field propagation without interference of baryonic mass density or at least where this density is negligible.

In order to defend properly conclusion this paper is important to emphasise a result got in chapter 3 which is that correlation coefficient of power regression over point measures in rotation curve in halo region is bigger than 0,95.

This high value of correlation between radius and velocity, support strongly that velocity of M31 rotation curve follow a power law regarding radius $v = a \cdot r^b$ whose coefficient a & b were got in chapter 3.

In chapter four was mathematically demonstrated that a power law $v = a \cdot r^b$ in halo region is equivalent a DM density called direct DM in this paper whose formula is $D_{DM} = \frac{a^2 \cdot (2b+1)}{4\pi G} \cdot r^{2b-2}$.

In chapter five was demonstrated mathematically that *a power law for velocity* $v = a \cdot r^b$ *at rotation curve is mathematically equivalent a power law for DM density depending on E.* $D_{DM} = A \cdot E^B$. Where

$A = \frac{a^{\frac{2}{2b-1}} \cdot (2b+1)}{4\pi G}$ & $B = \frac{2b-2}{2b-1}$.

Therefore joining chapters 3,4 and 5 it is concluded that a high correlation coefficient as 0,95 at power regression law for rotation curve $v = a \cdot r^b$ in halo region support strongly that DM density inside halo region is a power of gravitational field $D_{DM} = A \cdot E^B$ whose parameters A & B are written above.

In chapter seven was compared direct DM profile got in this paper with NFW fitted by [5] Sofue, Y.2015. Relative differences fluctuate between 13 % and 28% inside dominion in halo region. Similarly in chapter ten was compared corona masses got through both methods, and its relative difference fluctuate between 20 % and 25 % inside radius dominion. In addition NFW profiles gives values of density a bit bigger than direct DM through all dominion. In next paragraph will be exposed keen ideas which in author opinion might explain the fact that direct DM profile gives values of density a bit lower than values calculated by NFW profile.

As it was pointed at introduction, it is known that there is baryonic dark matter such us giant planets, cold gas clouds, brown dwarfs but this kind of DM is more probable to be placed inside galactic disk and bulge.

Reader can consult these papers about this open problem: [11] Nieuwenhuizen,T.M. 2010. [12] Nieuwenhuizen,T.M. 2012. [13] Nieuwenhuizen,T.M. 2010 [14] Wyrzykowski,L.2010. [15] M.R.S. Hawkins 2015

As it is known, NFW profile is fitted over bulge, disk and galactic halo and taking in consideration that there is an unknown amount of baryonic DM in bulge and galactic disk it is right to conclude that NFW profile gives values of density a bit bigger than direct DM profile through the whole dominion, even in halo region. Because higher values of DM density in bulge and disk produce an increasing of function fitted values in halo region.

In chapter eight has been got Bernoulli DM profile for M31 halo. This new profile has been introduced by author in previous papers quoted in bibliography. This profile is called Bernoulli because gravitational field is got as solution a Bernoulli differential equation. Basis to get Bernoulli profile is that $D_{DM} = A \cdot E^B$ whose coefficients were got in chapter five.

In chapter nine was compared Virial gravitational field $E = a^2 \cdot r^{2b-1}$ (chapter 5) with Bernoulli gravitational field. $E_{BER}(r) = \left(Cr^{\alpha} + Dr\right)^{\beta}$ (chapter 8) results shown that both are mathematically equivalents.

Also were compared Bernoulli DM and direct DM profiles. It was shown that both profiles are mathematically identical because were tabulated and data with ten digits are identical all over radius dominion.

Taking in consideration that Virial gravitational field is a great deal easier than Bernoulli gravitational field and that direct DM density is easier to calculate than Bernoulli DM density it is rightly concluded that is better work with Virial gravitational field and direct DM density.

Nevertheless the importance of formula $D_{DM} = A \cdot E^B$ is that show a functional dependence between DM density and local gravitational field.

Results got about $D_{DM} = A \cdot E^B$ in other galaxies, see [8] Abarca,M.2016, suggest that DM density is similar in different galaxies at a specific E, on condition that galaxies would be similar giant galaxies i.e. its velocity is bigger than 200 km/s in disk region of rotation curves.

In my opinion these facts suggest strongly nature of non baryonic DM. which is very important because shows a way to develop a new quantum gravitation theory.

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