Exact Trigonometric Periodic Solutions to Inverted Quadratic Mathews-Lakshmanan Oscillator Equations by Means of Linearizing Transformation

M. D. Monsia[1] , J. Akande[1], D. K. K. Adjaï[1], L. H. Koudahoun[1], Y. J. F. Kpomahou[2]

1. Department of Physics, University of Abomey-Calavi, Abomey-Calavi, 01.B.P. 526, Cotonou, BENIN.

2. Department of Industrial and Technical Sciences , ENSET-Lokossa, University of Lokossa, Lokossa, BENIN.

Abstract

The present letter adds to the paper '' A Class of Position-Dependent Mass Liénard Differential Equations via a General Nonlocal Transformation''. The purpose is to emphasize the fact that the mathematical theory of position-dependent mass nonlinear oscillator differential equations previously developed [1] provides exact analytical trigonometric periodic solutions to inverted quadratic Mathews-Lakshmanan oscillator equations.

Theory

Let us consider the general class of exactly solvable quadratic Liénard-type nonlinear differential equations [1-3]

$$\ddot{x} - \gamma\varphi'(x)\dot{x}^2 + \omega^2 x \exp(2\gamma\varphi(x)) = 0 \tag{1}$$

It was mentioned that for $\varphi(x) = \ln(f(x))$, equation (1) reduces to [2,3]

$$\ddot{x} - \gamma\frac{f'(x)}{f(x)}\dot{x}^2 + \omega^2 x f(x)^{2\gamma} = 0 \tag{2}$$

so that letting $f(x) = \frac{1}{\sqrt{1-\lambda x^2}}$, and $\gamma = 1$, gives immediately the inverted position-dependent mass Mathews-Lakshmanan nonlinear oscillator equation [2]

$$\ddot{x} - \frac{\lambda x}{1-\lambda x^2}\dot{x}^2 + \frac{\omega^2 x}{1-\lambda x^2} = 0 \tag{3}$$

By application of the linearizing transformation [1-3]

$$y(\tau) = x(t) \;\; , \;\; d\tau = \frac{dt}{\sqrt{1-\lambda x^2}} \tag{4}$$

equation (3) becomes the well-known linear harmonic oscillator equation

$$y''(\tau) + \omega^2 y(\tau) = 0 \tag{5}$$

where prime denotes here differentiation with respect to τ . In such a situation, if the solution to equation (5) reads

[1] Corresponding author.
E-mail address: monsiadelphin@yahoo.fr

$$y(\tau) = A_0 \sin(\omega\tau + \theta_0) \tag{6}$$

where A_0 and θ_0 are arbitrary constants, then the exact analytical trigonometric periodic solution to the inverted position-dependent mass Mathews-Lakshmanan oscillator (3) is expressed as a sinusoidal function

$$x(t) = A_0 \sin[\omega\phi(t) + \theta_0] \tag{7}$$

where the function $\tau = \phi(t)$, satisfies

$$\frac{d\tau}{dt} = \frac{1}{\sqrt{1 - \lambda A_0^2 \sin^2(\omega\tau + \theta_0)}} \tag{8}$$

On the other hand, in [3] it was shown that the equation of motion of a particle moving on a rotating parabola, which can be considered as an inverted version of Mathews-Lakshmanan oscillator equation, possesses a trigonometric function, that is to say, a sinusoidal function as exact analytical periodic solution. So, it is worth to note that any quadratic Liénard type nonlinear differential equation that belongs to the general class of exactly integrable position-dependent mass oscillator equations (1) admits a trigonometric function as exact analytical periodic solution.

References

[1] M. D. Monsia, J. Akande, D. K. K. Adjaï, L. H. Koudahoun, Y. J. F. Kpomahou, A class of position-dependent mass Liénard differential equations via a general nonlocal transformation, viXra:1608.0226v1.(2016).

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[3] M. D. Monsia, J. Akande, D. K. K. Adjaï, L. H. Koudahoun, Y. J. F. Kpomahou, Exact Analytical Periodic Solutions with Sinusoidal Form to a Class of Position-Dependent Mass Liénard-Type Oscillator Equations, viXra: 1608.0368v1.(2016).