Ontological-Phase Topological Field Theory

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We thank Newton for inspiring strict adherence to *hypotheses non-fingo*[1], and claim reasonable *a posteriori* surety in positing the need for an Ontological-Phase Topological Field Theory (OPTFT) as the final step in describing the remaining requirements for bulk UQC. Let's surmise with little doubt that a radical new theory needs to be correlated with the looming 3rd regime of Unified Field Mechanics (UFM). If the author knows one thing for sure, it is that gravity is not quantized! The physics community is so invested in quantizing the gravitational force that it could still be years away from this inevitable conclusion. There is still a serious conundrum to be dealt with however; discovery of the complex Manifold of Uncertainty (MOU), the associated 'semi-quantum limit' and the fact of a duality between Newton's and Einstein's gravity, may allow some sort of wave-particle-like duality with a quantal-like virtual graviton in the semi-quantum limit. Why mention the gravitational field? Relativistic information processing (RIP) introduces gravitational effects in the 'parallel transport' aspects of topological switching in branes. There are A and B type topological string theories, and a related Topological M-Theory with mirror symmetry, that are somewhat interesting especially since they allow sufficient dimensionality with Calabi-Yau mirror symmetric dual 3-tori perceived as essential elements for developing a UFM. But a distinction between these theories and the ontology of an energyless topological switching of information (Shannon related) through topological charge in brane dynamics, perhaps defined in a manner making correspondence to a higher dimensional (HD) de-Broglie-Bohm super-quantum potential synonymous with a 'Force of coherence' of the unified field is of interest. Thus the term 'OPTFT' has been chosen to address this issue as best as the *Zeitgeist* is able to conceive at the time of writing…

It is possible to make 'intelligent guesses and conjectures – Atiyah [2].

1 Abductive *a Priori a Posteriori* Tautology

Not all who wander are lost. - J. R. R. Tolkien

Reality leaves a lot to the imagination. - John Lennon

And those who were seen dancing were thought to be insane by those who could not hear the music – Friedrich Nietzche.

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[1] In B. Motte's 1729 English translation of Newton's essay 'General Scholium', 2nd (1713) edition of *Principia*, the phrase appeared as "*I do not frame hypotheses*". This translation was objected to by Koyre in 1965, who pointed out that '*fingo*' means 'feign', not 'frame' [1].

This has been among the most challenging works for the author, the conception of which wasn't even in the list of topics when the recent volume on Universal Quantum Computing was first conceived in 2014; and not knowing sufficient Group Theory limits current enfolding. I didn't suspect there would be much to say about Anyon – quasi-particle – quantum Hall TQC [3] because it was perceived as an LD 'Toy Model' of the HD UFM UQC architecture proposed to take its place. My expertise at the time on TQFT and TQC was sparse such that quite a can of worms was opened into my world view in bringing myself sufficiently up to speed with study and tad of tutoring given graciously by a world-renowned topologist.

The necessity of r-qubits (relativistic qubits) had already been embraced since first hearing of them at Physcomp96 [4]; and again in the course of getting up to speed, discovered that a corner of the QC R&D community finally began a discussion of their utility for modeling relativistic quantum computing (RIP) with a version of r-qubits [5-7].

I felt that attempting to develop a relativistic-TQFT was not a correct nomenclatural framework for both mathematical and physical reasons. Most acutely that the universe is not fundamentally quantum (anymore) and that gravitation, unlike the other three known phenomenological fields, is not quantized. The hearty belief in a quantum gravity persists only because of a herd mentality confounded by the current belief that fundamental reality is indeed quantum. Also, as you will see, we go far beyond RIP.

Most likely, the imminent age of discovery will be described topologically. Field theory has evolved from classical field theory to the current 2nd regime modes of QFT, RQFT and TQFT. It is proposed that the 3rd regime of reality, Unified Field Mechanics (UFM) will be described by an Ontological-Phase Topological Field Theory (OPTFT). In terms of the nature of reality, quantum information processing and the measurement problem, there has been a recent introduction of relativistic parameters including relativistic r-qubits and not just an Amplituhedron but more saliently a dual-Amplituhedron replacing spacetime, all bringing into question the historically fundamental basis of and need to be restricted to 'locality and unitarity' as in the Copenhagen interpretation of QM.

We briefly review this dilemma in terms of Bell's inequalities, the no-cloning theorem and discuss correspondence to the epistemic view of the Copenhagen Interpretation versus the ontic consideration of objective realism and as merged by W. Zurek's epi-ontic blend of quantum redundancy in quantum Darwinism [8-10]. Finally, we delve into the UFM ontological-phase topology requiring a new set of topological transformations beyond the Galilean, Lorentz-Poincairé.

A radical paradigm shift is needed to incorporate the new 3rd regime of Unified Field Mechanics (UFM), which appears to be inherently topological, suggesting extensions of current theory are required. If I was M. Atiya's clone, I would write a seminal introduction to an extended topological field theory as he did in 1986 [11]. UFM does not imply a 5th force, is not quantized, but entails an ontological mediation of information by a 'force of coherence' transferring information (by a form of topological charge) in a Shannon sense in the geometric topology of brane dynamics. This process, as we continue to mention, is an energyless process called 'topological switching' utilizing 'topological charge' [12-14].

2. The Phasor (Phase Vector) Complex Probability Amplitude

As the first step in trying to figure out how to develop a new concept of Ontological-phase we wish to adapt the phasor or phase vector concept as a precursor for describing ontological topological phase. In general, a phasor is a complex number for a sinusoidal (π rotation) function with Amplitude A, angular frequency ω and initial phase θ, which are all time invariant. The complex constant is the phasor [15].

Euler's formula allows sinusoids to be represented as the sum of two complex-valued functions:

$$A\cdot\cos\left(\omega t+\theta\right)=A\cdot\frac{e^{i(\omega t+\theta)}+e^{-i(\omega t+\theta)}}{2}, \tag{1}$$

or as the real part of one of the functions:

$$A \cdot \cos(\omega t + \theta) = \mathrm{Re}\left\{A \cdot e^{i(\omega t+\theta)}\right\} = \mathrm{Re}\left\{Ae^{i\theta} \cdot e^{i\omega t}\right\}. \quad (2)$$

The function $A \cdot e^{i(\omega t+\theta)}$ is the analytic representation of $A \cdot \cos(\omega t + \theta)$. Multiplication of the phasor $Ae^{i\theta}e^{i\omega t}$ by a complex constant, $Be^{i\phi}$, produces another phasor that *changes the amplitude and phase* of the underlying sinusoid**:**

$$\begin{aligned} \mathrm{Re}\left\{\left(Ae^{i\theta} \cdot Be^{i\phi}\right) \cdot e^{i\omega t}\right\} &= \mathrm{Re}\left\{\left(ABe^{i(\theta+\phi)}\right) \cdot e^{i\omega t}\right\} \\ &= AB\cos\left(\omega t + (\theta + \phi)\right). \end{aligned} \quad (3)$$

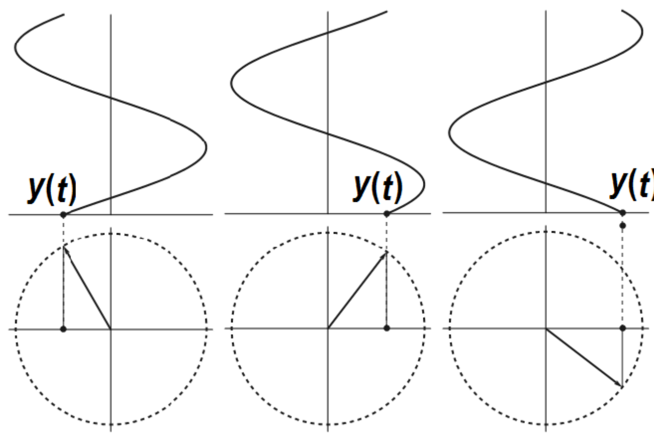

Fig. 1. Top sine waves - Phase transform in the complex plane. Bottom, can also be thought of as 2D rotation of the reference circle, and 1D sliding point on the line segment, helping us ponder the 2D nature of anyon braid topology. Thus elements of the figure can be considered in 1D, 2D and 3D.

When function $A \cdot e^{i(\omega t+\theta)}$ is depicted in the complex plane (Fig. 1), the vector formed by the imaginary and real parts rotates around the origin. *A* is the magnitude, *i* is the imaginary unit $i^2 = -1$, one cycle is completed every $2\pi / \omega$ seconds, and θ is the angle formed with the real axis at $t = n \cdot 2\pi / \omega$, for integer values of *n* [16].

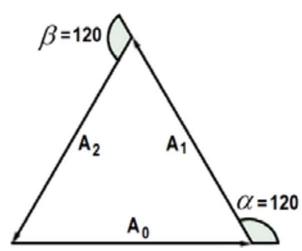

Fig. 2 Phasor diagram of three waves in perfect destructive interference.

This type of addition occurs when sinusoids interfere with each other constructively or destructively. Three identical sinusoids with a specific phase difference between them may perfectly cancel. To illustrate, we take three vectors of equal length placed head to tail so that the last head matches up with the first tail forming an equilateral triangle with the angle between each phasor being 120° (2π/3 radians), or one third of a wavelength $\lambda / 3$. Thus the phase difference between each wave is 120°,

$$\cos(\omega t) + \cos(\omega t + 2\pi / 3) + \cos(\omega t - 2\pi / 3) = 0. \tag{4}$$

In the Fig.2 example of three waves, the phase difference between the first and the last wave is 240°, In the limit of many waves, the phasors must form a circle for destructive interference, so that the first phasor is nearly parallel with the last. This means that for many sources, destructive interference happens when the first and last wave differ by 360°, a full wavelength, λ [16].

2.1 *Complex Phase Factor*

For any complex number written in polar form, such as $re^{i\theta}$, the phase factor is the complex exponential factor, $e^{i\theta}$. As such, the term 'phase factor' is related more generally to the term phasor, which may have any magnitude (i.e., not necessarily part of the circle group). The phase factor is a unit complex number of absolute value 1 as commonly used in quantum mechanics.

The variable θ is usually referred to as the phase. Multiplying the equation for a plane wave $A^{ei(k\cdot r-\omega t)}$ by a phase factor shifts the phase of the wave by θ:

$$e^{i\theta} A e^{i(k\cdot r-\omega t)} = A e^{i(k\cdot r-\omega t+\theta)}. \tag{5}$$

In quantum mechanics, a phase factor is a complex coefficient $e^{i\theta}$ that multiplies a ket $|\psi\rangle$ or bra $\langle\phi|$. *It does not, in itself, have any physical meaning in the standard formulation of QM*, since the introduction of a phase factor does not change the expectation values of a Hermitian operator. That is, the values of $\langle\phi|A|\phi\rangle$ and $\langle\phi|e^{-i\theta}Ae^{i\theta}|\phi\rangle$ are the same [17].

However, *differences* in phase factors between two interacting quantum states can be measurable under certain conditions such as in Berry phase, which has important consequences. The argument for a complex number $z = x + iy$, denoted arg z, is defined equivalently as:

- Geometrically, in the complex plane, as the angle φ from the positive real axis to the vector representing *z*. The numeric value given by the angle in radians is positive if measured counterclockwise. This is more precise if $z(t)$ is defined on a covering space, not the complex plane.
- Algebraically, the argument is defined as any real quantity φ such that $z = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$ for some positive real *r* (Euler's formula). The quantity *r* is the *modulus* of *z*, as $|z|$: $r = \sqrt{x^2 + y^2}$.

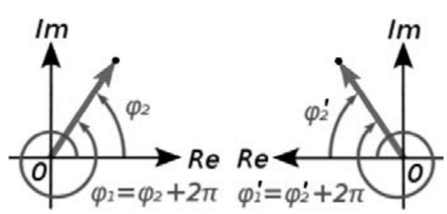

Fig. 3. Left-Right phase argument.

Use of the terms *amplitude* for the modulus and *phase* for the argument are sometimes used equivalently. Under both definitions, it can be seen that the argument of any (non-zero) complex number has many possible values: firstly, as a geometrical angle, whole circle rotations do not change the point, so angles differing by an integer multiple of 2π radians are the same. Similarly, from the periodicity of sin and cos, the second definition also has this property.

An N-particle system can be represented in non-relativistic quantum mechanics by a wavefunction, $\psi\left(x_1, x_2, \ldots x_n\right)$, where each x_i is a point in 3D-space. A classical phase space contains a real-valued function in 6N dimensions (each particle contributes 3-spatial coordinates and 3-momenta. Quantum phase space involves a complex-valued function on a 3N dimensional space. Position and momenta are represented by operators that do not commute, and ψ lives in the mathematical structure of a Hilbert space. Aside from these differences, the analogy holds.

In physics, this sort of addition occurs when sinusoids interfere with each other, constructively or destructively. The static vector concept provides useful insight into questions like: What phase difference would be required between three identical sinusoids for perfect cancellation? (Figure 2) In this case, simply imagine taking three vectors of equal length and placing them head to tail such that the last head matches up with the first tail. Clearly, the shape which satisfies these conditions is an equilateral triangle, so the angle between each phasor to the next is 120° ($2\pi / 3$ radians), or one third of a wavelength $\lambda / 3$. So the phase difference between each wave must also be 120°. In other words, what this shows is: $\cos\left(\omega t\right) + \cos\left(\omega t + 2\pi / 3\right) + \cos\left(\omega t - 2\pi / 3\right) = 0.$

2.2 *Geometric Phase - Berry Phase*

A Berry phase difference acquired over the course of a cycle, when a system is subjected to cyclic adiabatic processes resulting from the geometrical properties of the parameter space of the Hamiltonian [18].
This phenomenon was first discovered in 1956, [19] and rediscovered in 1984 [20]. It can be seen in the Aharonov-Bohm effect and in the conical intersection.

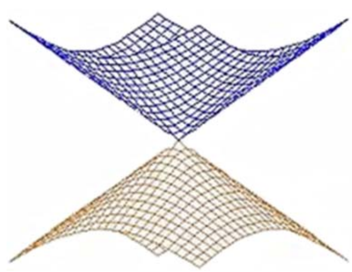

Fig. 4. Conical intersection of two potential energy surfaces.

A conical intersection of two potential energy surfaces is the set of geometrical points where the two potential energy surfaces are degenerate (intersect) and the non-adiabatic couplings between these two states are non-vanishing. For the Aharonov–Bohm effect, the adiabatic parameter is the magnetic field enclosed by two interference paths, and is cyclic because the two paths form a loop. For a conical intersection, the adiabatic parameters are molecular coordinates. In addition to quantum mechanics it can occur whenever there are at least two parameters describing a wave in the vicinity of a singularity or topological hole.

In a quantum system at the n^{th} eigenstate, if adiabatic (adapts to gradually changing external conditions; but for rapidly varying conditions there is insufficient time, so the spatial probability density remains unchanged) evolution of the Hamiltonian evolves the system such that it remains in the n^{th} eigenstate, while also obtaining a phase factor. The phase obtained has a contribution from the state's time evolution and another from the variation of the eigenstate with the changing Hamiltonian.

The second term corresponds to the Berry phase which for non-cyclical variations of the Hamiltonian can be made to vanish by a different choice of the phase associated with the eigenstates of the Hamiltonian at each point in the evolution. However, if the variation is cyclical, the Berry phase cannot be cancelled, it is invariant and becomes an observable property of the system. From the Schrödinger equation the Berry phase γ is:

$$\gamma[C] = i\oint_C \langle n,t|\left(\nabla_R|n,t\rangle\right)dR \tag{6}$$

where R parametrizes the cyclic adiabatic process. It follows a closed path C in the appropriate parameter space. Geometric phase along the closed path C can also be calculated by integrating the Berry curvature over surface enclosed by C [21].

One of the simplest examples of geometric phase is the Foucault pendulum [22]. The pendulum precess when it is taken around a general path *C*. For transport along the equator, the pendulum does not precess. But if *C* is made up of geodesic segments, precession arises from the angles where the segments of the geodesics meet; the total precession is equal to the net deficit angle, which equals the solid angle enclosed by *C* modulo 2π. We can approximate any loop by a sequence of geodesic segments, from which the most general result is that the net precession is equal to the enclosed solid angle. Since there are no inertial forces on the pendulum precess, precession, relative to the direction of motion along the path, is entirely due to the turning of the path. Thus the orientation of the pendulum undergoes parallel transport [22].

2.3 *The Toric Code*

The toric code introduced by Alexei Kitaev, is named from its periodic boundary conditions giving it the shape of a torus allowing the model to have translational invariance useful in TQC. Putative experimental realization requires open boundary conditions, allowing the system to be embedded on a 2D surface. Toric code and its generalized surface codes provides a basis for 2D anyonic computation by braiding defects. The unique nature of topological codes, like Kitaev's toric code, is that stabilizer violations can be interpreted as quasiparticles [23].

Kitaev defines the Toric Code on a periodic 2D lattice, usually the square lattice, with a spin-1/2 degree of freedom located on each edge. Stabilizer operators are defined on the spins around each vertex *v* and plaquette *p* of the lattice:

$$A_v = \prod_{i \in v} \sigma_i^x,\ B_p = \prod_{i \in p} \sigma_i^{\approx}. \tag{7}$$

Where $i \in v$ denotes edges touching the vertex *v*, and $i \in p$ denotes the edges surrounding the plaquette *p*. The stabilizer space of the code is where all stabilizers act trivially,

$$A_v|\psi\rangle = |\psi\rangle,\ \forall v,\ B_p|\psi\rangle = |\psi\rangle,\ \forall p, \tag{8}$$

for any state $|\psi\rangle$. For the toric code, this is a 4D space, so it can store two qubits. The occurrence of errors moves the state out of the stabilizer space, resulting in vertices and plaquettes for which the above condition does not hold. The positions of these violations is the 'syndrome of the code', and is used for error correction. The unique nature of topological toric codes, is that stabilizer violations can be interpreted as quasiparticles. Specifically, if the code is in a state $|\phi\rangle$ such that, $A_v|\phi\rangle = -|\phi\rangle$, a quasiparticle called an *e* anyon exists on the vertex *v* [23,24].

Another method introduces a distance truncature at the antipode of each set of points. In Fig. 5, the square is a flat Euclidean torus with null curvature everywhere [25].

From a geometrical point of view in the Figs. below, the points A,B,C,D must be identified to an antipode of point P on the torus. For the Euclidean square torus, straight lines are geodesics of the torus. The gravitational action of a mass located at the antipodal point (A,B,C,D) on the point P is zero, which is the same for a mass located in (H,K) or (M,N) [25]. See fig. 6 (Right). The corresponding geodesic path lengths are basically different (Fig. 5) as shown in (9):

$$\begin{aligned} &\mathrm{PA} = \mathrm{PB} = \mathrm{PC} = \mathrm{PD} = \frac{\sqrt{2}}{2}\ \mathrm{L} \\ &\mathrm{PM} = \mathrm{PN} = \mathrm{PH} = \mathrm{PK} = \frac{L}{2}. \end{aligned} \tag{9}$$

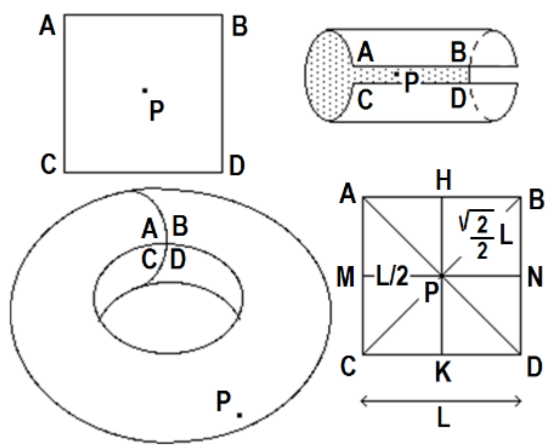

Fig. 5. a) Square Euclidean torus.

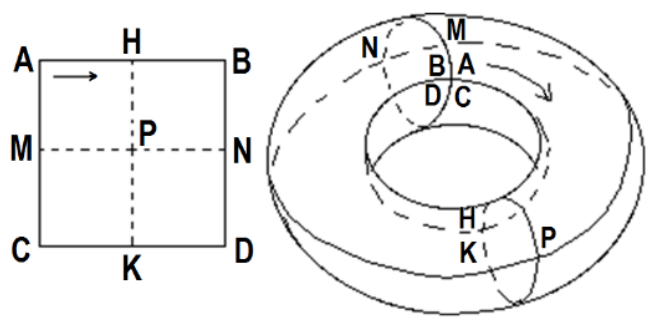

Fig. 6. The P torus point owns three antipodal points (A,B,C,D), (M,N) and (H,K).

Note that on a torus there are an infinite number of geodesics joining two given points, one being the shortest. When computing a corresponding gravitational interaction, both lengths must be considered, $d = \alpha R d' = R(2\alpha\pi - \alpha)$ [25].

3. Transitioning from TQFT to OPTFT

Topological quantum field theories (TQFT) were originally created to avoid the infinities plaguing quantum field theory [11,26]. Atiyah [11] initially to an axiomatic approach to TQFT, which has been realized in low dimensions and the primary method for modeling anyonic QC. The motivation for topological field theories stems from modern physical theories being defined by invariance under certain group actions like gauge groups in particle physics, diffeomorphism groups in general relativity, or unitary operator groups in quantum mechanics. In topological field theory, the concern is topological invariants, which are objects computed from a topological space (smooth manifold) without any metric [27]. Topological invariance is invariance under the diffeomorphism group of the manifold. Important milestones were Thom's theory of cobordism [28], de Rham cohomology, and knot theory. Through theories such as the Chern-Weil theory linking differential geometry and algebraic topology, abstract formalisms found powerful geometric applications which were applied to physics beginning in the 70's [29] and flourished through the work of Witten and Atiyah [30].

Fundamental strings map out 2D surfaces. The $N = (1,1)$ sigma model quantum field theory is defined on each surface. It consists of maps from the surface to a supermanifold interpreted physically as spacetime and each map is interpreted as the embedding of the string in spacetime. Only certain spacetimes admit topological strings. Classically one must choose a spacetime that allows an additional pair of supersymmetries, so in fact the theory is an $N = (2,2)$ sigma model. This is the case for a Kähler manifold where the H-flux is identically equal to zero [30].

Ordinary strings on special backgrounds are never topological. To make these strings topological, one needs to modify the sigma model by a procedure called a *topological twist* invented by Witten in 1988 [31]. The central observation is that these theories have two U(1) symmetries known as R-symmetries, where the Lorentz symmetry may be modified by mixing rotations and R-symmetries. One may use either of the two R-symmetries, leading to two different theories, called the A model and the B model. After this twist the action of the theory is BRST exact, and as a result the theory has no dynamics, instead all observables depend on the topology of a configuration [26].

Twisting is not possible for anomalies. In the Kähler case where $H = 0$ the twist leading to the A-model is always possible, but that leading to the B-model is only possible when the first Chern class of the spacetime vanishes, implying that the spacetime is Calabi-Yau. More generally $N = (2,2)$ theories have two complex structures and the B model exists when the first Chern classes of associated bundles sum to zero, whereas the A model exists when the difference of the Chern classes is zero. In the Kähler case the two complex structures are the same and so the difference is always zero, which is why the A model always exists [31].

3.1 *The A and B-Models of Topological Field Theory*

The topological A-model comes with a target space which is a real-6D generalized Kähler spacetime describing two objects. There are fundamental strings, which wrap two real-dimensional holomorphic curves. Amplitudes for the scattering of these strings depend only on the Kähler form of the spacetime, and not on the complex structure [30].

The B-model also contains fundamental strings, but their scattering amplitudes depend entirely upon the complex structure and are independent of the Kähler structure. In particular, they are insensitive to worldsheet instanton effects and so can often be calculated exactly. Mirror symmetry then relates them to A-model amplitudes, allowing one to compute Gromov–Witten invariants. The B-model also comes with D(-1), D1, D3 and D5-branes, which wrap holomorphic 0, 2, 4 and 6-submanifolds respectively. The 6-submanifold is a connected component of the spacetime. The theory on a D5-brane is known as holomorphic Chern-Simons theory [29].

3.2. *Dualities Between Topological String Theories (TSTs)*

A number of dualities relate the above theories. The A-model and B-model on two mirror manifolds are related by mirror symmetry, which has been described as a T-duality on a 3-torus. The A-model and B-model on the same manifold are thought to be related by S-duality, implying the existence of several new branes, called NS branes by analogy with the NS5-brane, which wrap the same cycles as the original branes but in the opposite theory. Also a combination of the A-model and a sum of the B-model and its conjugate are related to topological M-theory by a kind of *dimensional reduction*. Here the degrees of freedom of the A-model and the B-models appear to not be simultaneously observable, but have a relation similar to that between position and momentum in quantum mechanics [26,30].

3.3. *The Holomorphic Anomaly*

The sum of the B-model and its conjugate appears in the above duality because it is the theory whose low energy effective action is expected to be described by Hitchin's formalism. This is because the B-model suffers from a holomorphic anomaly, which states that the dependence on complex quantities, while classically holomorphic, receives non-holomorphic quantum corrections. In Quantum Background Independent String Theory, Witten argued that this structure is analogous to a structure that one finds geometrically quantizing the space of complex structures. Once this space has been quantized, only half of the dimensions simultaneously commute and so the number of degrees of freedom has been halved. This halving depends on an arbitrary choice, called a polarization. The conjugate model contains the missing degrees of freedom, and so by tensoring the B-model and its conjugate one reobtains all of the missing degrees of freedom and also eliminates the dependence on the arbitrary choice of polarization [23,24,26,30].

4 Topological Vacuum Bubbles by Anyon Braiding

According to a basic rule of fermionic and bosonic many-body physics, known as the linked cluster theorem, physical observables are not affected by vacuum bubbles, which represent virtual particles created from vacuum and self-annihilating without interacting with real particles. Here we show that this conventional knowledge must be revised for anyons, quasiparticles that obey fractional exchange statistics intermediate between fermions and bosons. We find that a certain class of vacuum bubbles of Abelian anyons does affect physical observables. They represent virtually excited anyons that wind around real anyonic excitations. These topological bubbles result in a temperature-dependent phase shift of Fabry-Perot interference patterns in the fractional quantum Hall regime accessible in current experiments, thus providing a tool for direct and unambiguous observation of elusive fractional statistics [32].

When two identical particles adiabatically exchange positions $\mathrm{r}_{i=1,2,}$ their final state ψ, to dynamical phase, relates to the initial state through an exchange statistics phase θ^*,

$$\psi(\mathrm{r}_2,\mathrm{r}_1)=e^{i\theta*}\psi(\mathrm{r}_1,\mathrm{r}_2), \tag{10}$$

with $\theta^*=0(\pi)$ [33].

In many-body quantum theory [33], Feynman diagrams are used to compute the expectation value of observables. This approach invokes vacuum bubble diagrams, which describe virtual particles excited from vacuum and self-annihilating without interacting with real particles. According to the linked cluster theorem [33], each diagram having vacuum bubbles comes with a partner diagram of the same magnitude but of opposite sign that it is exactly cancelled by. Consequently, vacuum bubbles do not contribute to physical observables.

This common wisdom must be revised for anyons because a certain class of vacuum bubbles of Abelian anyons does affect observables. These virtual particles, called topological vacuum bubbles, wind around a real anyonic excitation, gaining the braiding phase $\pm 2\pi v$ [32].

Han's team proposes an experimental procedure for detecting them and $\theta^*=\pi v$, where πv is the anyon phase and θ the interference phase shift [32]. For an interference a_1a_2 between processes a_1 and a_2 for propagation of a real particle, in a_1, a virtual particle-hole pair is excited then self-annihilates after the virtual particle winds around the real particle, forming a vacuum bubble, which is not excited in a_2. The winding results in a braiding phase $2\pi v$ and an Aharonov–Bohm phase $2\pi\left(\Phi/\Phi_0^*\right)$ from the magnetic flux Φ enclosed by the winding path, contributing to the interference signal as $e\exp i\left(2\pi\left(\Phi/\Phi_0^*\right)+2\pi v\right)$; $\Phi_0^*=h/e^*$ as the anyon flux quantum [32].

The limiting cases of bosons (v = 0) and fermions (v = 1) imply that this bubble diagram appears together with, and is cancelled by, a partner diagram. The partner diagram has a bubble not encircling the real particle and involves only $2\pi\left(\Phi/\Phi_0^*\right)$. The two diagrams (and complex conjugates) yield

$$\text{Interference signal} \propto \operatorname{Re}\left[e^{i\left(2\pi\left(\Phi/\Phi_0^*\right)+2\pi v\right)} - e^{2\pi i\left(\Phi/\Phi_0^*\right)}\right]$$
$$= -\sin\left(\pi v\right)\sin\left(2\pi\left(\Phi/\Phi_0^*\right)+\pi v\right). \tag{11}$$

For bosons and fermions, the two diagrams fully cancel each other with $\sin\left(\pi v\right)=0$ in agreement with the linked cluster theorem; thus, the signal disappears. By contrast, for anyons they cancel only partially, producing non-vanishing interference in an observable, and are topological as the braiding phase is involved [32].

The astute reader will begin to notice, that the anyon braid topology begins to overlap with the UFM OPTFT. The question will be whether the cryogenic TQC will be built as a 'proof of concept' or a 'leap-frog' will occur to the table top room temperature UFM model. If the utility of the Aharonov-Bohm effect remains a key element of 'Topological vacuum bubbles by anyon braiding' interferometry; it is easy to add Aharonov-Bohm effect parameters to the OPTF dynamics.

5 Topological Switching – Key to Ontological-Phase

The 2-state formalism currently forms the basis of QC. Qubits, are 2-state systems. Any QC operation is a unitary operation that rotates the state vector on the Bloch sphere. To move from Hilbert space to ontological-phase space we must begin to define what we mean by topological switching [12-14. We begin with a number of ways of looking at the ambiguous Necker cube [34].

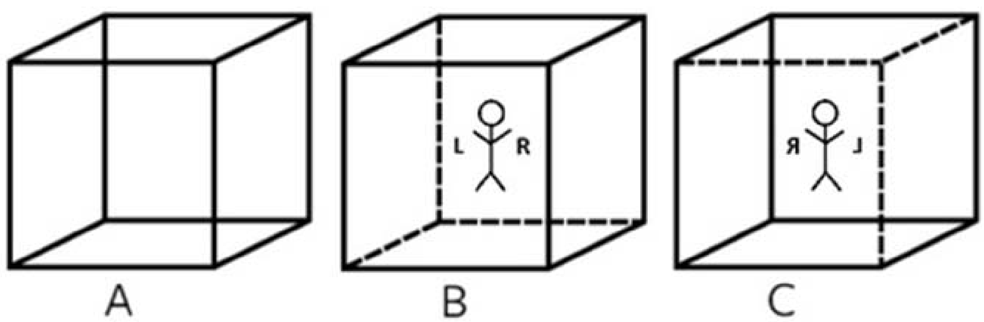

Fig. 7. Ambiguous Necker cube, left, mirror image, center and perceived shift between the two states in 4D.

The oscillation or inversion of the central vertices of the ambiguous or Necker cube, is key to conceptually understanding the process of 'energyless' or ontological transfer of information between branes.

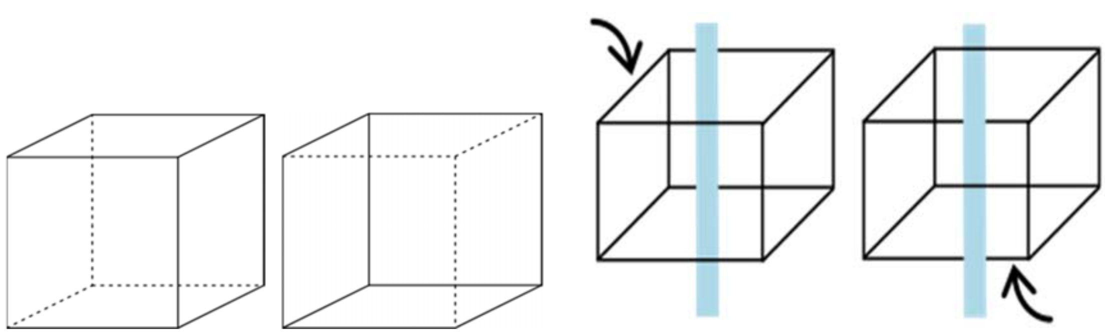

Fig. 8. Two states of the Necker cube. A physically real description is needed.

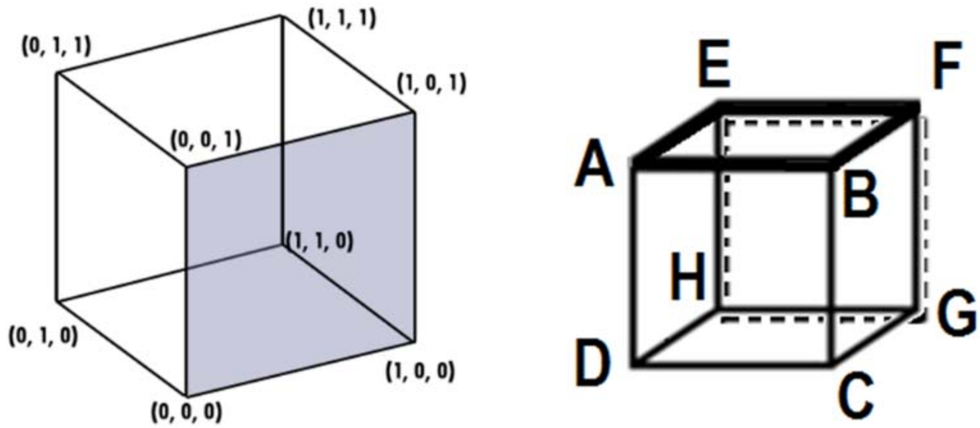

Fig. 9. A first step towards physicality might be distinguishing the vertices.

Quaternions have the ability to represent rotations of 3D space. If we represent 3-space, $\mathbb{R}^3$ as the set of pure quaternions of the form $\mathbb{Z} = ai + bj + ck$ with *a, b, c* real numbers, then *g* is a unit quaternion mapping $\rho : \mathbb{R}^3 \to \mathbb{R}^3$ defined by the equation $\rho(\mathbb{Z}) = g\mathbb{Z}g^{-1}$ that describes a 3-space rotation by angle θ around axis μ when

$$g = \cos(\theta / 2) + \sin(\theta / 2)\mu. \tag{12}$$

In this manner, μ is a unit length quaternion giving a direction to a vector in 3-space, a rotation is specified by an angle θ about an axis *U*, which in the case below is in the positive direction [35].

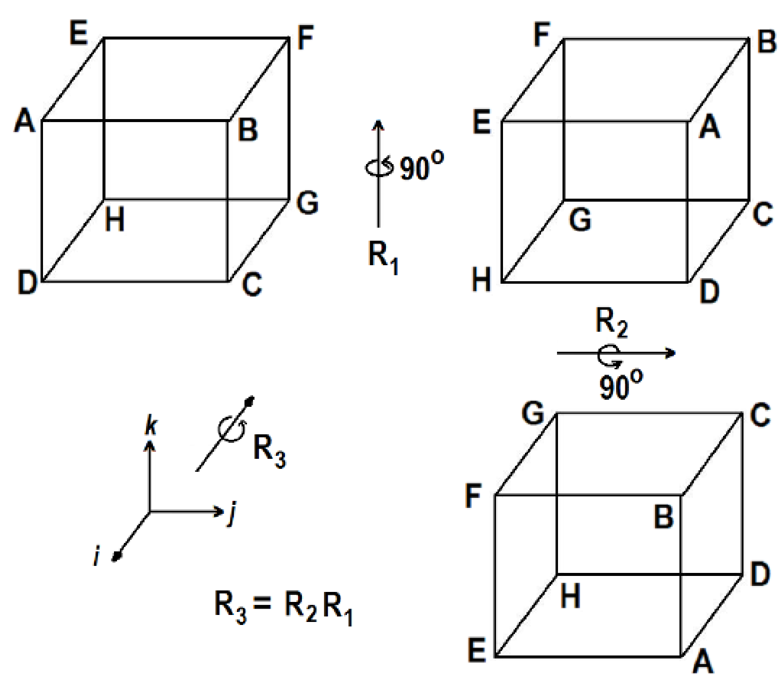

Fig. 10. Denoting two 90º rotations R_1 and R_2, we write $R_3 = R_2R_1$ for the rotation obtained by 1st performing R_1 and then R_2. R_3 fixes the corners B and H; Thus R_3 is a 120º rotation about the diagonal axis.

Thus, following Kauffman [35],

$$e^{j(\pi/4)}e^{k(\pi/4)} = \left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}+k\frac{\sqrt{2}}{2}\right)$$
$$=\frac{1}{2}(1+j)(1+k)=\frac{1}{2}(1+j+k+jk).$$
$$=\frac{1}{2}(1+i+j+k)=\frac{1}{2}+\frac{\sqrt{3}}{2}\left(\frac{i+j+k}{\sqrt{3}}\right).$$

$$\therefore e^{j(\pi/4)}e^{k(\pi/4)} = e^{\left[(i+j+k)/\sqrt{3}\right][2\pi/3]/2}$$
$$\uparrow \qquad \uparrow$$
$$\text{diagonal axis} \quad 120^{\text{o}}.$$

These quaternion rotations can be considered phase changes under certain conditions; but they do not correspond to the ontological phase we are looking for because Euclidean geometry has no natural inherent perspective. It appears we need a duo-morphic projection perhaps involving Berry phase because the ambiguous vertices of the Necker cube are not distinguished in Kauffman's quaternion rotation system [35].

To clarify how projective a transformation loses orientable information, rotating a triangle in a plane is used as an illustration [36].

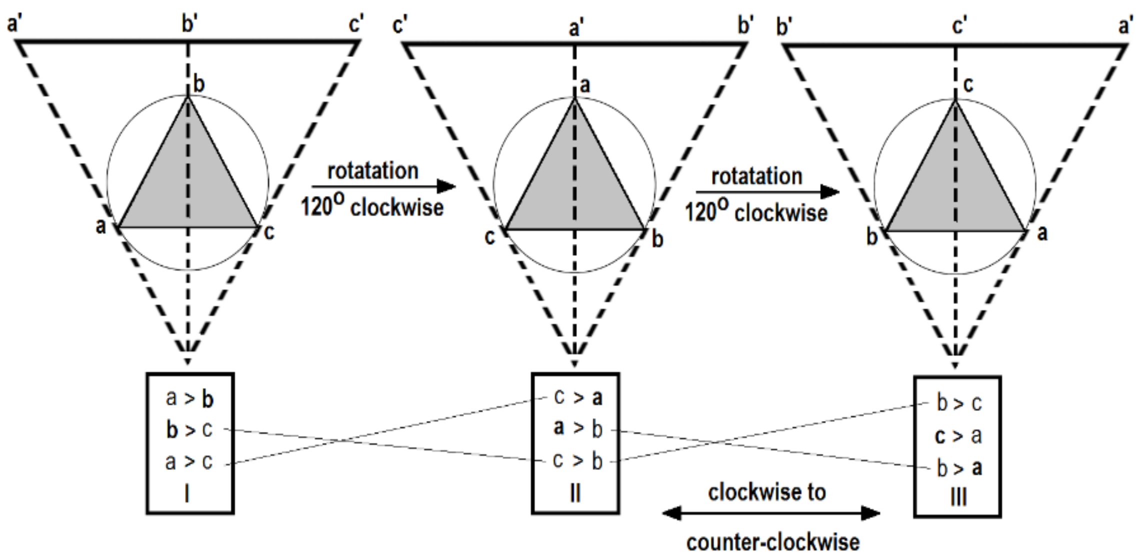

Fig. 11. Removing ambiguity from a projected rotation, with > denoting order of sequence occurrence – to the left on the projective line. Bold letters are the front range of projective mapping. Fig. redrawn from [36].

The rotation sequences in Fig. 11 are I, II, III for clockwise and I, III, II for counter-clockwise. According to Shaw the direction of rotation reverses if the back and front ranges are interchanged. This is denoted by the connecting lines in the boxes below the rotation triangles. Bold letters mark the front range; this system is able to preserve orientation information under projected rotation.

The 3D wire-frame Necker cube can be projected onto a 2D surface, collapsing the cubes six faces into a complex of one to seven coplanar polygons depending on orientation of the cube.

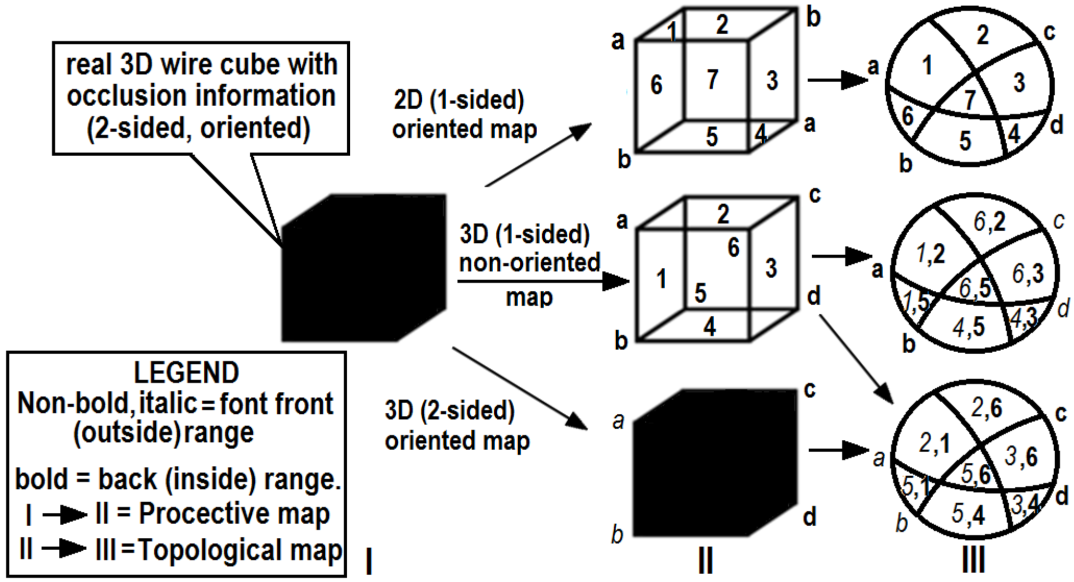

Fig. 12 Contrasting nonoriented - oriented projective geometries. Redrawn from [37].

Figure 12 illustrates three different forms of projection.

- I $\rightarrow$ II-Top $\rightarrow$ III-Top: no occlusion information
- I $\rightarrow$ II-Middle $\rightarrow$ III-Middle and III-Bottom: occlusion information is specified ambiguously
- I $\rightarrow$ II-Bottom $\rightarrow$ III-Bottom: occlusion information is specified unambiguously.

The Necker cube, like the Möbius strip is an ambiguous figure because of the problem of projective mapping. In ordinary projective space, the Möbius strip and Necker cube, are one-sided (Fig. 12). The spherical model of this geometry represents the fact that the projections of a point on the back of the sphere and of a point on its front both have the same image in the Euclidean (projective) plane. All of the projected points, regardless of the hemisphere to which they belong, cover the projective plane in the usual way without any designation of where they originated. The loss of orientation is due to this failure of the projective mapping to preserve the distinction between the front and back range, collapsing both into positive values of the dimension of depth *w*. This loss of orientation is represented by the fact that relationships (e.g., the arrows) invert when the projective angle passes through the points at infinity [36].

To keep the front and back ranges distinguished, traditional computational geometries use the line at infinity as a reference; but this move is not a real solution to the orientation problem in projective geometry because it is tantamount to a return to Euclidean geometry which has no inherent natural perspective.

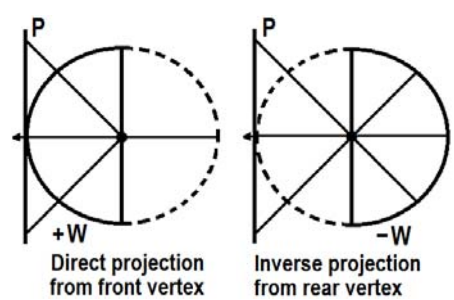

Fig. 13 Duo-morphic oriented projections (+W, -W) yield a double covering of the projective plane, P.

To distinguish front and back ambiguous vertices of the Necker cube is a problem of orientation. Oriented projective geometry introduces a methodology for distinguishing the ambiguous vertices of the Necker cube [36]. Shaw [37] assigns a dual range, +W and -W to represent front and rear ranges of a sphere.

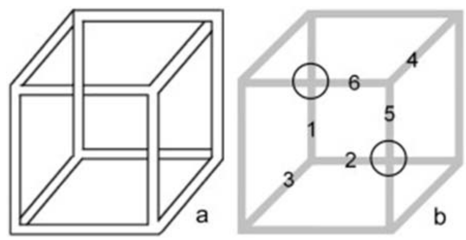

Fig. 14. Ambiguity needs a method of labeling for clarity.

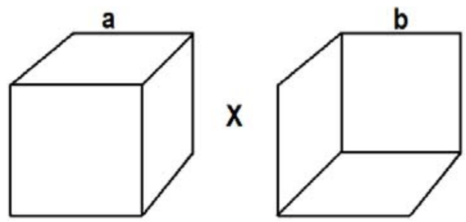

Fig. 15. Visual test of stereoscopic construction of a Necker cube. Focus on the 'X' between the L-R Necker perspectives.

Figure 15 separates the ambiguous Necker cube into its component perspectives. Although what we are about to illustrate is usually considered a mental construct, we use it here to illustrate what we mean by ontological phase and an ontological phase transformation. Focus on the 'X' halfway between the 2D L-R Necker perspectives; relax one's eyes and allow them to lose focus and cross. Soon, a 3rd image appears between the two printed L-R images fusing the original perspective into one apparent 3D image, confirmed by noticing the labels 'a' and 'b' are now superposed. This stereoscopic condition is the scenario we want to utilize to define ontological-phase.

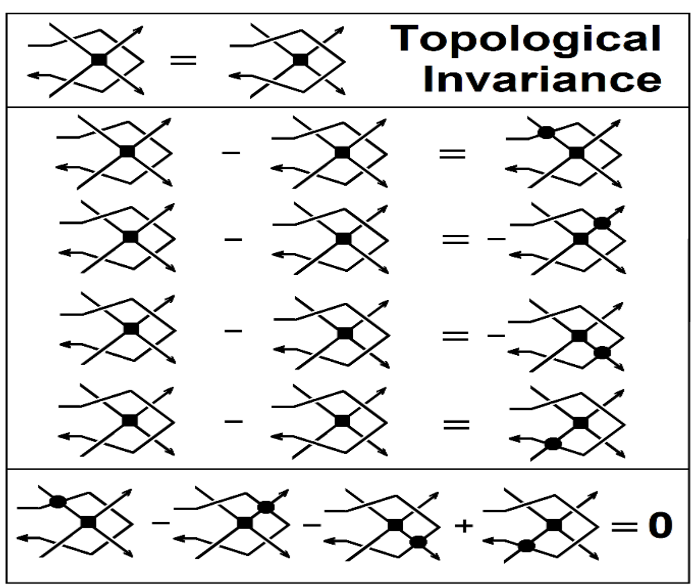

Fig.16. Topological Invariance must be included in any phase labeling. Figure redrawn from [25].

Masahide & Satoh generalize the class of roll-spun knots for 2-knot theory and show how to calculate the quandle cocycle invariant for any roll-spun knot [38]. For the case $X = S_4 = \mathbb{Z}\left[t,t^{-1}\right]\big/\left(2,t^2+t+1\right)$, the element $w = 1\cdot t^{-1}\cdot 0\cdot\left(t+1\right)^{-1}$ satisfies $\varphi_w = \mathrm{id}_{S4}$; such that

$$\begin{array}{ccccc} 0 & \xrightarrow{\varphi_1} t+1 & \xrightarrow{\varphi_t^{-1}} 1 & \xrightarrow{\varphi_0} t & \xrightarrow{\varphi_{t+1}^{-1}} 0 \\ 1 & \mapsto 1 & \mapsto 0 & \mapsto 0 & \mapsto 1 \\ t & \mapsto 0 & \mapsto t+1 & \mapsto 1 & \mapsto t \\ t+1 & \mapsto t & \mapsto t & \mapsto t+1 & \mapsto t+1. \end{array} \tag{13}$$

Since $\mathrm{ind}(w) = 0,$ it holds that $w \in G_0\left(S_4\right)$. Figure 17 shows that $w^2 = 1$ in $\mathbf{G}_0(S_4)$, and that w is the generator of $G_0\left(S_4\right) \cong \mathbb{Z}_2$.

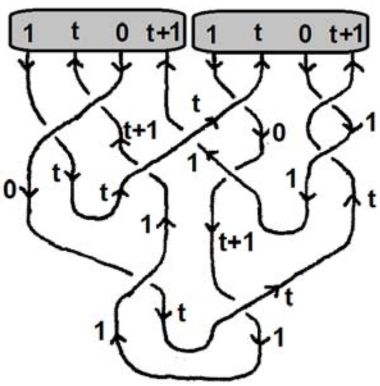

Fig. 17. Deform-spun knot tangle diagram. Redrawn from [38].

The spun knot is explored as a possible component topological move for ontological-phase transitions. When parallel transport creates a deficit angle in brane raising and lowering dynamics, in addition to Reidemeister moves, rotations, reflections and any other topological moves, spun knot components may add another type of phase transition with lattice charge.

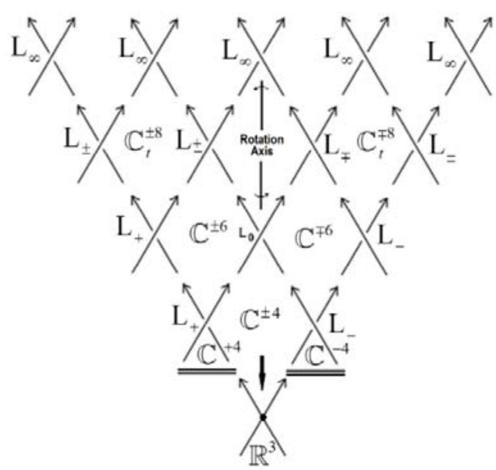

Fig. 18. Rolling spun knots. The infusion of topological charge as a UFM 'force of coherence' driving evolution throughout the multidimensional brane hierarchy can allow multiple types of moves to occur at multiple levels simultaneously.

An important feature of TQFTs is that they do not presume a fixed topology for space or spacetime. In other words, when dealing with an n-dimensional TQFT, one is free to choose any (n - 1)-dimensional manifold to represent space at a given time. Moreover, given two such manifolds, say S and S', one is free to choose any nD manifold M to represent the portion of spacetime between S and S'. Mathematicians call M a `cobordism' from S to S'. We write $M : S \to S'$, because we may think of M as the process of time passing from the moment S to the moment S'.

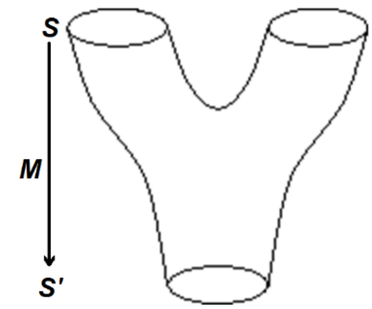

Fig. 18. A basic cobordism.

For example, in Fig. 18 we depict a 2D manifold M going from a 1D manifold S (a pair of circles) to a 1D manifold S' (single circle). Crudely speaking, M represents a process in which two separate spaces collide to form a single one! This may seem *outré*, but currently physicists are quite willing to speculate about processes in which the topology of space changes with the passage of time [39].

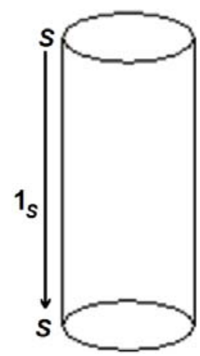

Fig. 19. Identity cobordism.

There are various important operations one can perform on cobordisms, but we only describe two. First, we may `compose' two cobordisms $M : S \to S'$ and $M : S' \to S''$, obtaining a cobordism $M'M : S \to S''$, as illustrated in Fig. 20. The idea here is that the passage of time corresponding to M followed by the passage of time corresponding to M' equals the passage of time corresponding to $M'M$. This is analogous to the familiar idea that waiting t seconds followed by waiting t' seconds is the same as waiting $t + t'$ seconds. The big difference is that in topological quantum field theory we cannot measure time in seconds, because there is no background metric available to let us count the passage of time! We can only keep track of topology change. Just as ordinary addition is associative, composition of cobordisms satisfies the associative law:

$$\left(M''M'\right)M = M''\left(M'M\right). \tag{16}$$

However, composition of cobordisms is not commutative. As we shall see, this is related to the famous noncommutativity of observables in quantum theory [39].

Second, for any (n–1)D manifold S representing space, there is a cobordism $1_S : S \to S$ called the `identity' cobordism, which represents a passage of time without topological change. For example, when S is a circle, the identity cobordism 1_S is a cylinder, as shown in Fig. 19 In general, the identity cobordism 1_S has the property that for any cobordism $M : S' \to S$ we have $1_S M = M$, while for any cobordism $M : S \to S'$ we have $M 1_S = M$ [39].

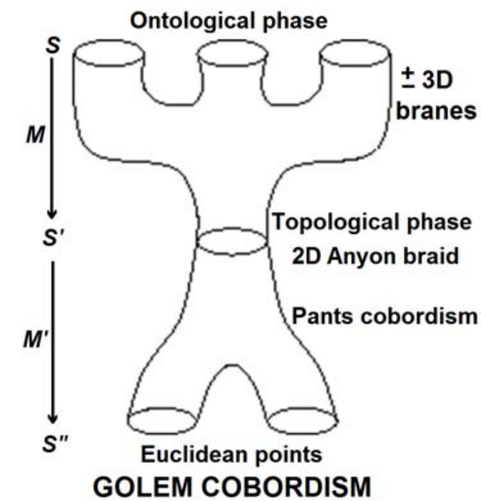

Fig. 20. The Golem, composition of cobordisms designed to handle ontological-phase.

These properties say that an identity cobordism is analogous to waiting 0 seconds: if you wait 0 seconds and then wait t more seconds, or wait t seconds and then wait 0 more seconds, this is the same as waiting t seconds.

These operations just formalize of the notion of `the passage of time' in a context where the topology of spacetime is arbitrary and there is no background metric. Atiyah's axioms relate this notion to quantum theory as follows. First, a TQFT must assign a Hilbert space $Z(S)$ to each (n – 1)D manifold S. Vectors in this Hilbert space represent possible states of the universe given that space is the manifold S. Second, the TQFT must assign a linear operator $Z(M) : Z(S) \to Z(S')$ to each nD cobordism $M : S \to S'$. This operator describes how states change given that the portion of spacetime between S and S' is the manifold M. In other words, if space is initially the manifold S and the state of the universe is ψ, after the passage of time corresponding to M the state of the universe will be $Z(M)\psi$ [39].

In addition, the TQFT must satisfy a list of properties. Let me just mention two. First, the TQFT must preserve composition. That is, given cobordisms $M : S \to S'$ and $M' : S' \to S''$, we must have $Z(M'M) = Z(M')Z(M)$, where the right-hand side denotes the composite of the operators $Z(M)$ and $Z(M')$. Second, it must preserve identities. That is, given any manifold S representing space, we must have $Z(1_S) = 1_{Z(S)}$.
where the right-hand side denotes the identity operator on the Hilbert space $Z(S)$ [39].

Both these axioms are eminently reasonable if one ponders them a bit. The first says that the passage of time corresponding to the cobordism M followed by the passage of time corresponding to M' has the same effect on a state as the combined passage of time corresponding to $M'M$. The second says that a passage of time in which no topology change occurs has no effect at all on the state of the universe. This seems paradoxical at first, since it seems we regularly observe things happening even in the absence of topology change. However, this paradox is easily resolved: a TQFT describes a world quite unlike ours, one without local degrees of freedom. In such a world, nothing local happens, so the state of the universe can only change when the topology of space itself changes [39].

The most interesting thing about the TQFT axioms is their common formal character. Loosely speaking, they all say that a TQFT maps structures in differential topology (the study of manifolds) to corresponding structures in quantum theory. In coming up with these axioms, Atiyah took advantage of a powerful analogy between differential topology and quantum theory, summarized in Table 1 [39].

This analogy between differential topology and quantum theory the sort of clue we should pursue for a deeper understanding of quantum gravity. At first glance, general relativity and quantum theory look very different mathematically: one deals with space and spacetime, the other with Hilbert spaces and operators. Combining them has always seemed a bit like mixing oil and water. But topological quantum field theory suggests that perhaps they are not so different after all! Even better, it suggests a concrete program of synthesizing the two, which many mathematical physicists are currently pursuing. Sometimes this goes by the name of `quantum topology' [2,11].

DIFFERENTIAL TOPOLOGY	QUANTUM THEORY
(n – 1)-dimensional manifold (space)	Hilbert space (states)
cobordism between (n – 1)-dimensional manifolds (spacetime)	operator (process)
composition of cobordisms	composition of operators
identity cobordism	identity operator

Table 1: Analogy between differential topology and quantum theory.

Quantum topology is very technical, as anything involving mathematical physicists inevitably becomes. But if we stand back a moment, it should be perfectly obvious that differential topology and quantum theory must merge if we are to understand background-free quantum field theories. In physics that ignores general relativity, we treat space as a background on which states of the world are displayed. Similarly, we treat spacetime as a background on which the process of change occurs. But these are idealizations which we must overcome in a background-free theory. In fact, the concepts of `space' and `state' are two aspects of a unified whole, and likewise for the concepts of `spacetime' and `process'. It is a challenge, not just for mathematical physicists, but also for philosophers, to understand this more deeply [39].

We begin to explore various types of crossover inks and moves to start cataloguing the variety of moves that maybe applicable to HD M-Theoretic ontological-phase transitions.

Fig. 21. Simple crossover links.

To operate an ontological-phase transition, more and more complex links are required.

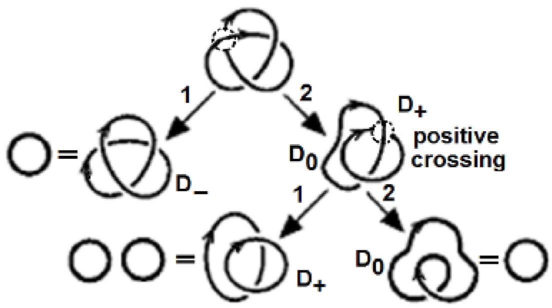

Fig. 22. Crossings for octonion trefoil knots**.**

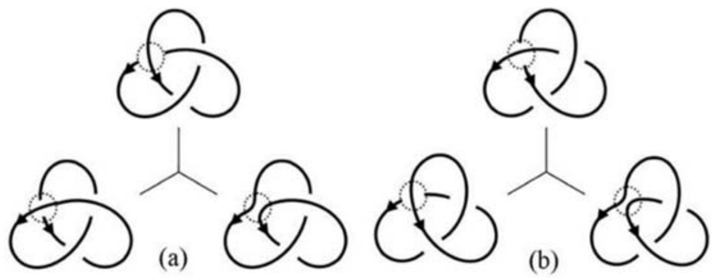

Fig. 23. Reduction schemes for the left- and right-handed trefoil knots. (a) Top: left-handed trefoil knot; bottom: writhe γ_- and a Hopf link H_-, with crossing −1. (b) Top: right-handed trefoil knot; bottom: writhe γ_+ and a Hopf link H_+, with crossing +1. The two knots are mirror images of one another. Figure adapted from [40].

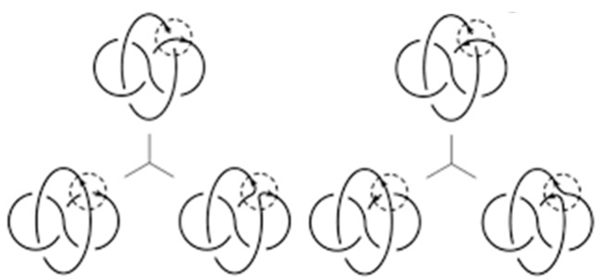

Fig. 24. Reduction schemes for Whitehead links W_+ and W_-. (a) Top: Whitehead link W_+ with crossing +1; bottom: Hopf link H_- and the left-handed trefoil knot T^L. (b) Whitehead link W_- with crossing −1; bottom: Hopf link H_+, and a figure-of-eight knot F^8. Figure adapted from [40].

Thus a true octonion contains three trefoil knots, whereas a split octonion may be specified by mixing a pair of quaternion trefoil lines. To define a tripled Fano plane requires three copies of Furey's particle zoo. It describes a set of 21 = 3 x 7 (left cyclic) modules over a noncommutative ring on eight elements. The ring is given by the upper triangular 2 x 2 matrices over the field with two elements. Similarly, for right cyclic modules [41,42].

The quaternions, *H* are a 4D algebra with basis $1, i, j, k$. To describe the product, it is easy to note:

- 1 is the multiplicative identity,
- i, j, k are square roots of -1,
- we have $ij = k,\ ji = -k$ and all identities obtained from these by cyclic permutations of (i, j, k).

We can summarize the last rule as a diagram

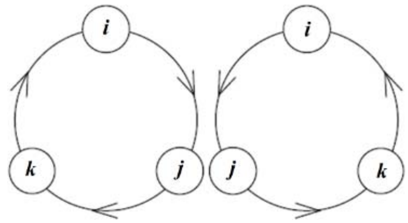

Fig. 25. Clockwise and counterclockwise rule for Quaternion cyclicality.

In multiplying two elements going clockwise around the circle we get the next one: for example, $ij = k$. But when we multiply two going around counterclockwise, we get *minus* the next one: for example, $ji = -k$. We can use the same sort of picture to remember how to multiply octonions:

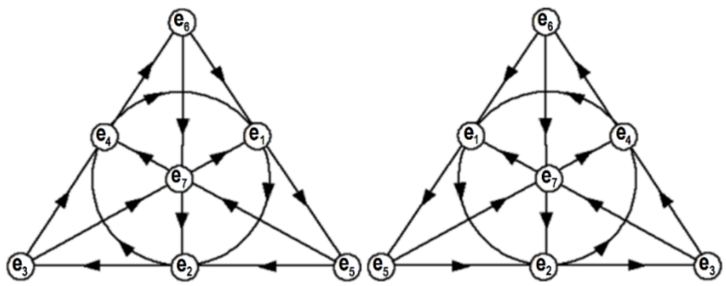

Fig. 26. The Fano plane and its mirror image.

The Fano plane is the finite projective plane of order 2, having the smallest possible number of points and lines, 7 each, with 3 points on every line and 3 lines through every point. The Fano plane has 7 points and 7 lines. The 'lines' are the sides of the triangle, its altitudes, and the circle containing all the midpoints of the sides. Each pair of distinct points lies on a unique line. Each line contains three points, and each of these triples has a cyclic ordering shown by the arrows. If e_i, e_j, e_k are cyclically ordered in this way then $e_i e_j = e_k, \quad e_j e_i = -e_k$.

Together with these rules:

- 1 is the multiplicative identity,
- $e_1, \ldots, e_7$ are square roots of -1,

the Fano plane completely describes the algebra structure of the octonions. Index-doubling corresponds to rotating the picture a third of a turn. Interestingly, The Fano plane is the projective plane over the 2-element field $\mathbb{Z}_2$. In other words, it consists of lines through the origin in the vector space $\mathbb{Z}_2^3$. Since every such line contains a single nonzero element, we can also think of the Fano plane as consisting of the seven nonzero elements of $\mathbb{Z}_2^3$. If we think of the origin in $\mathbb{Z}_2^3$ as corresponding to $1 \in \mathrm{O}$, we get the following picture of the octonions:

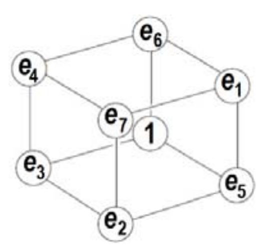

Fig. 27. The octonions for $1 \in \mathrm{O}$.

Note that planes through the origin of this 3D vector space (Fig. 26) give subalgebras of O isomorphic to the quaternions, lines through the origin give subalgebras isomorphic to the complex numbers, and the origin itself gives a subalgebra isomorphic to the real numbers [39].

Now we finally arrive at the fundamental geometric topology for describing ontological-phase topological field theory. When the formalism is next written it will be created by utilizing both topology and complex quaternion/octonions Clifford algebra which is especially suited to handle the manifold embedding [43].

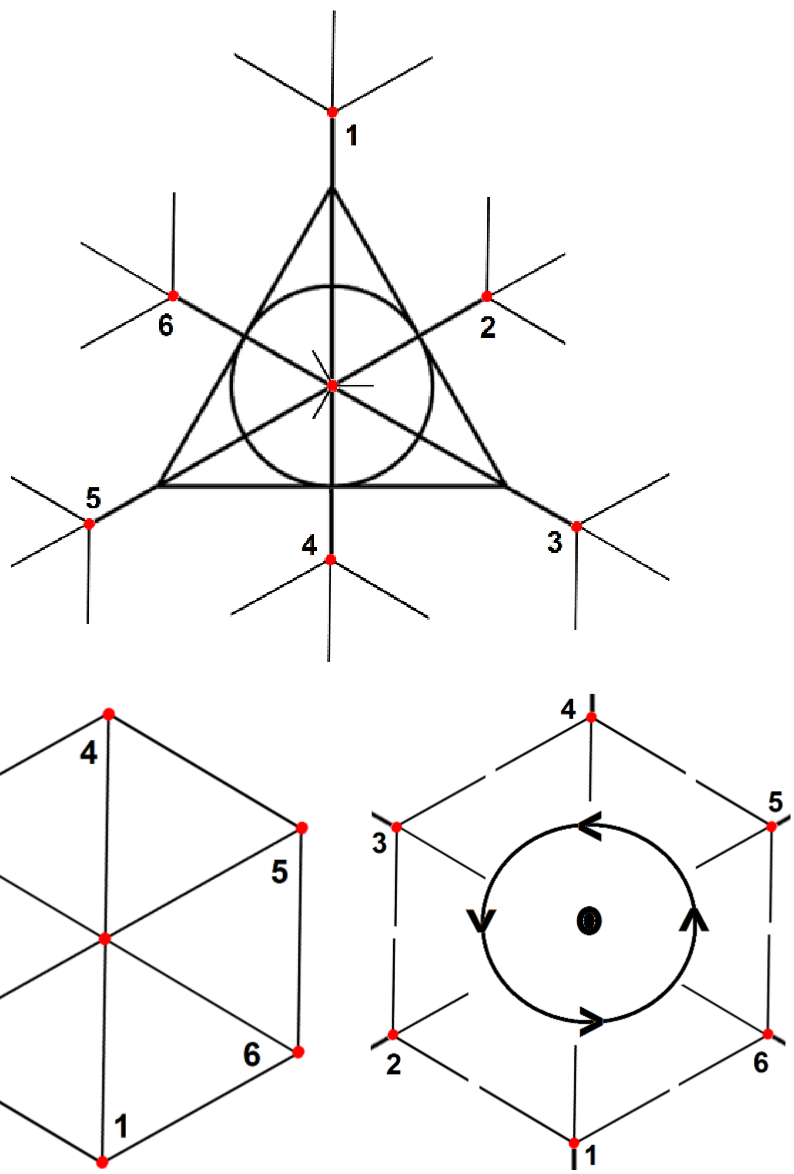

Fig. 28. The 'antennas' (snowflakes) on a Fano plane (top) represent vertices on the circumference of a hexagon or cube (bottom). The center rotates unconnected so position 1 or 2 can create the front/rear vertices of a Necker cube. b) Antennas 1-6 combine to form the outer vertices of a cube/hexagon depending on what dimensional phase the state is in.

The Fano snowflake configuration in Fig, 28 involutes to form a 2D hexagon or vertices of a Euclidean Necker 3-cube. We expect to require a dual set of twin Fano-Snowflakes as would be derived from Fig. 26 to account for all the parameters necessary for 'the mirror image of the mirror image to be causally free of the Euclidean 3-space QED quantum state.

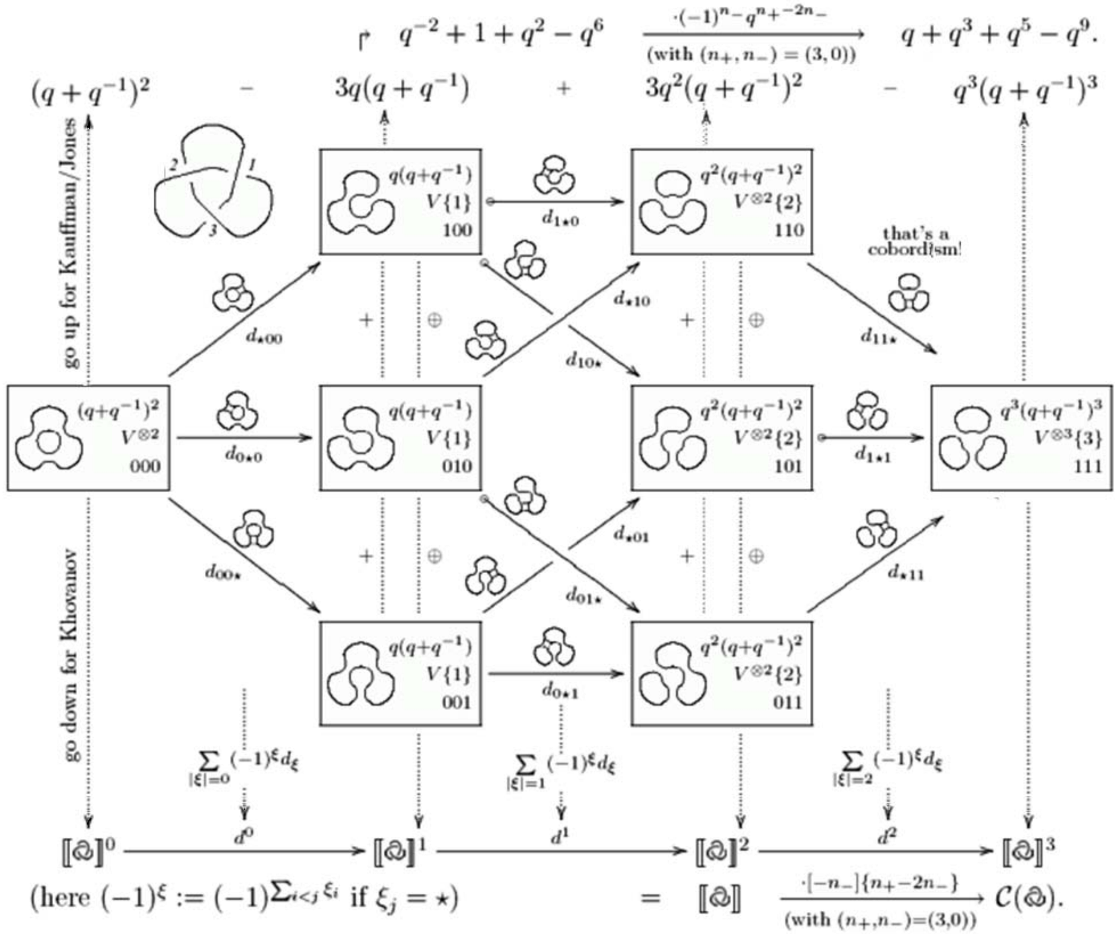

Fig. 29. Construction to improve Khovanov's seminal work on the categorification of the Jones polynomial. Figure adapted form [46].

Some of the complexity for categorizing the Jones polynomial is shown in Fig. 29 as it might apply to modeling ontological-phase.

6. Dual Amplituhedron Geometry and 'Epiontic' Realism

The amplituhedron geometric jewel simplifies particle interaction calculations and challenges the notion that space and time are fundamental components of reality, advancing a long effort to reformulate quantum field theory, the body of laws describing elementary particles and their interactions by calculations with formulas thousands of terms long that can now be described by computing the volume of its amplituhedron, yielding an equivalent one-term expression. The new geometric version of quantum field theory could also facilitate the search for a theory of quantum gravity. Attempts thus far to incorporate gravity into the laws of physics at the quantum scale have run up against nonsensical infinities and deep paradoxes. An amplituhedron type geometry could help by removing two deeply rooted principles of physics: locality and unitarity [47].

Locality is the notion that particles can interact only from adjoining positions in space and time. And unitarity holds that the probabilities of all possible outcomes of a quantum mechanical interaction must add up to one. The concepts are the central pillars of quantum field theory in its original form, but in certain situations involving gravity, both break down, suggesting neither is a fundamental aspect of nature. In keeping with this idea, the new geometric approach to particle interactions removes locality and unitarity from its starting assumptions. The amplituhedron is not built out of space-time and probabilities; these properties merely arise as consequences of the jewel's geometry. The usual picture of space and time, and particles moving around in them, is only a useful construct [47].

Because "we know that ultimately, we need to find a theory that doesn't have" unitarity and locality, Bourjaily said, "*it's a starting point to ultimately describing a quantum theory of gravity*." The 1st part of Bourjaily's statement is correct; however, the 2nd part is not. Most physicists still consider the quantum regime the basement of reality and thus automatically think to progress in unification gravity must be quantized. This is not the regime of integration and therefore obviously why there is no quantum gravity. But transition to the 3rd regime of UFM is confounded 'epiontics'. Reality acquires a semi-quantum (epi) limit on the way to the ontological (ontic) regime of UFM [47,48].

The amplituhedron in HD encodes in its volume "scattering amplitudes," which represent the likelihood that a certain set of particles will turn into certain other particles upon colliding. The twistor theory at the root of it does this kind of simplification. It folds the speed of light into the geometry by mapping point particles to their light cones. The point becomes an intersection of the sphere of light rays that could radiate from it. Then you can do extra stuff like cancelling out the asymmetry of universal expansion by mapping the larger future light cone on to the smaller past light cone [49].

Perhaps often, mathematics corresponds perfectly well to physical reality. But maybe now as we move away from a Hilbert space representation of qubit processing to a truly physical basis, we might surmise 'No wonder it has been difficult to implement bulk QC'. For classical digital computing math itself was sufficient; but as we move to relativistic qubits and topological quantum field theory apparently this is not the case [50].
Jaynes had this to say:

"*… our present formalism is not purely epistemological; it is a … mixture describing in part realities of Nature, in part incomplete human information about Nature … if we cannot separate the subjective and objective aspects of the formalism we cannot know what we are talking about … .*" [50,51].

The term epistemic is used to represent – not real, mind of observer, in contrast to ontic – real; Zurek coined the term epiontic to merge the two philosophies into what he called Quantum Darwinism. Quantum Darwinism describes the proliferation, in the environment, of multiple records of selected states of a quantum system. It explains how the fragility of a state of a single quantum system can lead to the classical robustness of states of their correlated multitude; shows how effective 'wavepacket collapse' arises as a result of proliferation throughout the environment of imprints of the states of quantum system; and provides a framework for the derivation of Born's rule, which relates probability of detecting states to their amplitude. Taken together, these three advances mark considerable progress towards settling the quantum measurement problem [48].

From copying to quantum jumps Quantum Darwinism leads to appearance, in the environment, of multiple copies of the state of the system. However, the no-cloning theorem [52,53] prohibits copying of unknown quantum states. If cloning is outlawed, how can redundancy be possible? Quick answer is that cloning refers to (unknown) quantum states. So, copying of observables evades the theorem. Nevertheless, the tension between the prohibition on cloning and the need for copying is revealing: It leads to breaking of unitary symmetry implied by the superposition principle, accounts for quantum jumps, and suggests origin of the "wavepacket collapse", setting stage for the study of quantum origins of probability [50].

Alexander's horned sphere is a convoluted, intertwined surface with a difficult to define inside and outside that is homeomorphic to a ball, meaning that it can be stretched into a ball without being punctured or broken or vice versa. Embedded in Euclidean 3-space, it can be constructed from a torus (Fig. 30) in the following manner:

1. Remove a radial slice of the torus.
2. Connect a standard punctured torus to each side of the cut, interlinked with the torus on the other side.
3. Repeat steps 1 & 2 on the two tori added in step two *ad infinitum*.

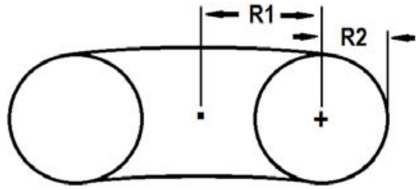

Fig. 30 Torus showing minor and major radii.

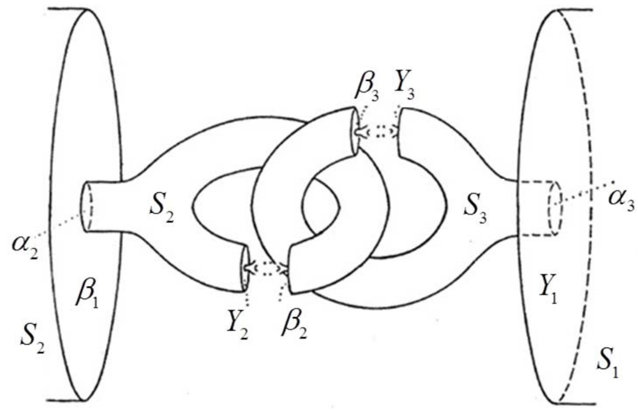

Fig. 31. Alexander's horned sphere with infinite fractal-like embeddings. With a finite number of links, we use it to illustrate the 'chains' of the manifold of uncertainty, that can be opened only by certain topological moves. Figure adapted form [54].

Time to peek out of the Schrödinger box with the eyes of Alexander's horned cat…

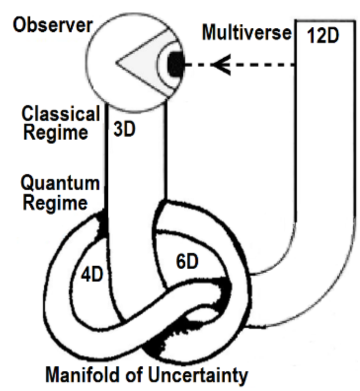

Fig. 30. a) Alexander's horned sphere in the eyes of Schrödinger's Cat. Is reality 'created' by the mind of the observer? Redrawn from [55]. b) Wheeler's Self-Referential Universe, Does the act of observing the universe create it?

States with different topological orders or different patterns of long range entanglements cannot change into each other without a phase transition. In the case of Alexander's horned sphere, we believe this requires an ontological-phase topological transition.

The horned sphere, together with its inside, is a topological 3-ball, the Alexander horned ball, and so is simply connected; i.e., every loop can be shrunk to a point while staying inside. The exterior is *not* simply connected, unlike the exterior of the usual round sphere; a loop linking a torus in the above construction cannot be shrunk to a point without touching the horned sphere. This shows that the Jordan-Schönflies theorem does not hold in three dimensions as Alexander had originally thought. Alexander also proved that the theorem does hold in three dimensions for piecewise linear/smooth embeddings.

This is one of the earliest examples where distinction between the topological category of manifolds, and the categories of differentiable manifolds, and piecewise linear manifolds was noticed.

Now consider Alexander's horned sphere as an embedding into the 3-sphere, considered as the one-point compactification of the 3D Euclidean space R^3. The closure of the non-simply connected domain is called the solid Alexander horned sphere. Although the solid horned sphere is not a manifold, Bing showed that its double (which is the 3-manifold obtained by gluing two copies of the horned sphere together along the corresponding points of their boundaries) is in fact the 3-sphere. One can consider other gluings of the solid horned sphere to a copy of itself, arising from different homeomorphisms of the boundary sphere to itself. This has also been shown to be the 3-sphere. The solid Alexander horned sphere is an example of a crumpled cube; i.e., a closed complementary domain of the embedding of a 2-sphere into the 3-sphere [56].

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