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A Projection Model of Neutrosophic Numbers for Multiple Attribute Decision Making of Clay-Brick Selection

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Abstract. Brick plays a significant role in building construction. So we should use the effective mathematical decision making tool to select quality clay-bricks for building construction. The purpose of this paper is to present a projection model of neutrosophic numbers and its decision-making method for the selecting problems of clay-bricks with neutrosophic number information. The projection method of neutrosophic numbers is one useful tool that can deal with decision-making problems with indeterminacy data. By the projection measure between each alternative and the ideal alternative, all the alternatives can be ranked to select the best one. Finally, an actual example on clay-brick selection in construction field demonstrates the application and effectiveness of the projection method.

Keywords: Neutrosophic number, projection method, clay-brick selection, decision making.

1 Introduction

As we know, in realistic decision making situations, some information cannot be described only by unique crisp numbers, and then may imply indeterminacy. In order to deal with this situation, Smarandache [1-3] introduced neutrosophic numbers. To apply them in real situations, Ye [4, 5] proposed the method of de-neutrosophication and possibility degree ranking order of neutrosophic numbers and the bidirectional projection method respectively, and then applied them to multiple attribute group decision-making problems under neutrosophic number environments. Then, Ye [6] developed a fault diagnosis method of steam turbine using the exponential similarity measure of neutrosophic numbers. Further Kong et al. [7] presented the misfire fault diagnosis method of gasoline engine by using the cosine similarity measure of neutrosophic numbers.

Clay-brick selection problem in construction field is a multiple attribute decision-making problem. Hence, Mondal and Pramanik [8] presented a quality clay-brick selection approach based on multiple attribute decision making with single valued neutrosophic grey relational analysis. However, so far neutrosophic numbers are not applied to decision making problems in construction field. To do it, this paper introduces a projection-based model of neutrosophic numbers and applies it to the multiple attribute decision-making problem of clay-brick selection in construction field under neutrosophic number environment.

The rest of the paper is organized as the following. Section 2 reviews basic concepts of neutrosophic numbers. Section 3 introduces a projection measure of neutrosophic numbers. Section 4 presents a multiple attribute decision-making method based on the projection model under neutrosophic number environment. In section 5, an actual example is provided for the decision-making problem of clay-brick selection to illustrate the application of the proposed method. Section 6 presents conclusions and future research direction.

2 Basic concept of neutrosophic numbers

A neutrosophic number, proposed by Smarandache [1-3], consists of the determinate part and the indeterminate part, which is denoted by $N = d + uI$, where d and u are real numbers and I is indeterminacy, such that $I^n = I$ for $n > 0$, $0 \times I = 0$, and uI/kI = undefined for any real number k.

For example, assume that there is a neutrosophic number $N = 2 + 2I$. If $I \in [0, 0.2]$, it is equivalent to $N \in [2, 2.4]$ for sure $N \geq 2$, this means that its determinate part is 2 and its indeterminate part is $2I$ with the indeterminacy $I \in [0, 0.2]$ and the possibility for the number ''N'' is within the interval [2, 2.4]. In general, a neutrosophic number may be considered as a changeable interval.

Let $N = d + uI$ be a neutrosophic number. If $d, u \geq 0$, then N is called positive neutrosophic numbers. In the following, all neutrosophic numbers are considered as positive neutrosophic numbers, which are called neutrosophic numbers for short, unless they are stated. Based on the cosine measure and projection model [5, 7], we introduce the following definitions.

Let $N_1 = d_1 + u_1I$ and $N_2 = d_2 + u_2I$ be two neutrosophic numbers, then there are the following operational relations of neutrosophic numbers [1-3]:

(1) $N_1 + N_2 = d_1 + d_2 + (u_1 + u_2)I$;

(2) $N_1 - N_2 = d_1 - d_2 + (u_1 - u_2)I$;;

(3) $N_1 \times N_2 = d_1d_2 + (d_1u_2 + u_1d_2 + u_1u_2)I$;

(4) $N_1^2 = (d_1 + u_1I)^2 = d_1^2 + (2d_1u_1 + u_1^2)I$;

(5) $\frac{N_1}{N_2} = \frac{d_1 + u_1 I}{d_2 + u_2 I} = \frac{d_1}{d_2} + \frac{d_2u_1 - d_1u_2}{d_2(d_2 + u_2)} I$ for $d_2 \neq 0$ and $d_2 \neq -u_2$;

(6) $$\sqrt{N_1} = \sqrt{d_1 + u_1 I} = \begin{cases} \sqrt{d_1} - (\sqrt{d_1} + \sqrt{d_1 + u_1})I \\ \sqrt{d_1} - (\sqrt{d_1} - \sqrt{d_1 + u_1})I \\ -\sqrt{d_1} + (\sqrt{d_1} + \sqrt{d_1 + u_1})I \\ -\sqrt{d_1} + (\sqrt{d_1} - \sqrt{d_1 + u_1})I \end{cases}.$$

Definition 1 [7]. Let $a = (a_1, a_2, \ldots, a_n)$ and $b = (b_1, b_2, \ldots, b_n)$ be two neutrosophic number vectors, where $a_j = [d_{aj} + u_{aj}I^L, d_{aj} + u_{aj}I^U]$ and $b_j = [d_{bj} + u_{bj}I^L, d_{bj} + u_{bj}I^U]$ for $I \in [I^L, I^U]$ and $j = 1, 2, \ldots, n$. Then, the modules of a and b are defined as $\|a\| = \sqrt{\sum_{j=1}^{n}\left(d_{aj} + u_{aj}I^L\right)^2 + \left(d_{aj} + u_{aj}I^U\right)^2}$ and $\|b\| = \sqrt{\sum_{j=1}^{n}\left(d_{bj} + u_{bj}I^L\right)^2 + \left(d_{bj} + u_{bj}I^U\right)^2}$, the inner product between a and b is defined as $a \cdot b = \sum_{j-1}^{n}\left((d_{aj} + u_{aj}I^L)(d_{bj} + u_{bj}I^L) + (d_{aj} + u_{aj}I^U)(d_{bj} + u_{bj}I^U)\right)$. Thus, a cosine measure is defined as

$$\cos(a,b) = \frac{a \cdot b}{\|a\|\|b\|} \quad (1)$$

which is called the cosine of the included angle between a and b.

3 Projection measure of neutrosophic numbers

Definition 2 [5]. Let $a = (a_1, a_2, \ldots, a_n)$ and $b = (b_1, b_2, \ldots, b_n)$ be two neutrosophic number vectors, where $a_j = [d_{aj} + u_{aj}I^L, d_{aj} + u_{aj}I^U]$ and $b_j = [d_{bj} + u_{bj}I^L, d_{bj} + u_{bj}I^U]$ for $I \in [I^L, I^U]$ and $j = 1, 2, \ldots, n$. Then the projection of the vector a on the vector b is defined as

$$\mathrm{Pr}oj_b(a) = \|a\|\cos(a,b) = \frac{a \cdot b}{\|b\|} = \frac{\sum_{j=1}^{n}[(d_{aj} + u_{aj}I^L)(d_{bj} + u_{bj}I^L) + (d_{aj} + u_{aj}I^U)(d_{bj} + u_{bj}I^U)]}{\sqrt{\sum_{j=1}^{n}[(d_{bj} + u_{bj}I^L)^2 + (d_{bj} + u_{bj}I^U)^2]}}. \quad (2)$$

If one considers the importance of each element in neutrosophic number vectors a and b, the weight of each element can be introduced by w_j ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Thus, we introduce the following definition.

Definition 3. Let $a = (a_1, a_2, \ldots, a_n)$ and $b = (b_1, b_2, \ldots, b_n)$ be two neutrosophic number vectors, where $a_j = [d_{aj} + u_{aj}I^L, d_{aj} + u_{aj}I^U]$ and $b_j = [d_{bj} + u_{bj}I^L, d_{bj} + u_{bj}I^U]$ for $I \in [I^L, I^U]$ and $j = 1, 2, \ldots, n$. The weight of the elements is w_j ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Then the projection of the vector a on the vector b is defined as

$$W\,\mathrm{Pr}oj_b(a) = \|a\|_w \cos_w(a,b) = \frac{(a \cdot b)_w}{\|b\|_w} = \frac{\sum_{j=1}^{n} w_j^2[(d_{aj} + u_{aj}I^L)(d_{bj} + u_{bj}I^L) + (d_{aj} + u_{aj}I^U)(d_{bj} + u_{bj}I^U)]}{\sqrt{\sum_{j=1}^{n} w_j^2[(d_{bj} + u_{bj}I^L)^2 + (d_{bj} + u_{bj}I^U)^2]}} \quad (3)$$

Based on the projection model of interval numbers improved by Xu and Liu [9], the projection model of Eq. (3) is improved as the following form:

$$WP_b(a) = \frac{(a \cdot b)_w}{\|b\|_w^2} = \frac{\sum_{j=1}^{n} w_j^2[(d_{aj} + u_{aj}I^L)(d_{bj} + u_{bj}I^L) + (d_{aj} + u_{aj}I^U)(d_{bj} + u_{bj}I^U)]}{\sum_{j=1}^{n} w_j^2[(d_{bj} + u_{bj}I^L)^2 + (d_{bj} + u_{bj}I^U)^2]} \quad (4)$$

Obviously, the closer the value of $WP_b(a)$ is to 1, the closer the vector a is to the vector b.

4 Decision-making method based on the projection measure

In this section, we present a handling method for multiple attribute decision-making problems by using the proposed projection measure under neutrosophic number environment.

In a multiple attribute decision-making problem, let $S = \{S_1, S_2, ..., S_m\}$ be a set of alternatives and $A = \{A_1, A_2, ..., A_n\}$ be a set of attributes. If the decision maker provides an evaluation value of the attribute A_j (j=1,2,...,n) for the alternative S_i ($i = 1, 2,\ldots, m$) by using a scale from 1 (less fit) to 10 (more fit) with indeterminacy I, which is represented by the form of a neutrosophic number $a_{ij} = d_{ij} + u_{ij}I$ for $I \in [I^L, I^U]$ and constructed as a set of neutrosophic numbers $S_i = \{a_{i1}, a_{i2}, ..., a_{in}\}$ for $i = 1, 2,\ldots, m$ and $j = 1, 2,\ldots, n$. Thus, we can establish the neutrosophic number decision matrix $M = (a_{ij})_{m\times n}$.

If the weights of attributes are considered as the different importance of each attribute A_j (j = 1, 2, …, n), the weight vector of attributes is $W = (w_1, w_2, …, w_n)$ with $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$. Then, the procedure of the decision-making problem is described as follows:

Step 1: Specify the indeterminacy $I \in [I^L, I^U]$ according to decision makers' preference and real requirements, each neutrosophic number $a_{ij} = d_{ij} + u_{ij}I$ in the neutrosophic number decision matrix M can be transformed into an interval numbers $a_{ij} = [d_{ij} + u_{ij}I^L, d_{ij} + u_{ij}I^U]$ for $I \in [I^L, I^U]$ for i = 1, 2, …, m and j = 1, 2, …, n. By $a_j^* = [a_j^{L*}, a_j^{U*}] = [\max_i (d_{ij} + u_{ij}I^L), \max_i (d_{ij} + u_{ij}I^U)]$ (j = 1, 2, …, n), the ideal solution (ideal neutrosophic numbers) can be determined as the ideal alternative $S^* = \{a_1^*, a_2^*, ..., a_n^*\}$.

Step 2: According to Eq. (4), the projection measure between each alternative S_i (i = 1, 2, …, m) and the ideal alternative S^* can be calculated by

$$WP_{S^*}(S_i) = \frac{(S_i \cdot S^*)_w}{\|S^*\|_w^2} = \frac{\sum_{k=1}^{n} w_j^2 [(d_{ij} + u_{ij}I^L)a_j^{L*} + (d_{ij} + u_{ij}I^U)a_j^{U*}]}{\sum_{j=1}^{n} w_j^2 [(a_j^{L*})^2 + (a_j^{U*})^2]} \quad (5)$$

Step 3: The alternatives are ranked in a descending order according to the values of $WP_{S^*}(S_i)$ for i = 1, 2, …, m. The greater value of $WP_{S^*}(S_i)$ means the better alternative S_i.

Step 4: End.

5 Actual example of clay-brick selection

In this section, an actual example on clay-brick selection in construction field adapted from [8] illustrates the application of the projection method.

Let us consider a set of four possible alternatives (providers of clay-bricks) $S = \{S_1, S_2, ..., S_m\}$ in construction field, which need to satisfy six attributes (criteria) of clay-bricks: solidity (A_1), color (A_2), size and shape (A_3), and strength of brick (A_4), brick cost (A_5), carrying cost (A_6) [8]. Then, the weighting vector of the six attributes is W = (0.275, 0.175, 0.2, 0.1, 0.05, 0.2).

When the four alternatives with respect to the six attributes are evaluated by the expert corresponding to a scale from 1 (less fit) to 10 (more fit) with indeterminacy I, we can obtain the evaluation values of neutrosophic numbers. For example, the expert give the neutrosophic number of an attribute A_1 for an alternative S_1 as $a_{11} = 7 + 2I$ by using a scale from 1 (less fit) to 10 (more fit) with indeterminacy I, which indicates that the evaluation value of the attribute A_1 for the alternative S_1 is the determinate degree 7 with the indeterminate degree $2I$ with some indeterminacy $I \in [I^L, I^U]$. By the similar evaluation process, we can obtain the following decision matrix:

$$M = (a_{ij})_{4\times 6} = \begin{matrix} & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\ S_1 & 7+2I & 8+I & 7+I & 6+2I & 7 & 5+3I \\ S_2 & 7+I & 7+2I & 8+I & 7+2I & 8+I & 7+2I \\ S_3 & 8+I & 8 & 7+2I & 6+2I & 7+I & 6+2I \\ S_4 & 7 & 9+I & 7+3I & 8+2I & 6+2I & 7+3I \end{matrix}.$$

Assume $I \in [0,1]$, then the above neutrosophic number decision matrix can be transformed into the following de-neutrosophication matrix:

$$M = (a_{ij})_{4\times 6} = \begin{matrix} & A_1 & A_2 & A_3 & A_4 & A_5 & A_6 \\ S_1 & [7,9] & [8,9] & [7,8] & [6,8] & [7,7] & [5,8] \\ S_2 & [7,8] & [7,9] & [8,9] & [7,9] & [8,9] & [7,9] \\ S_3 & [8,9] & [8,8] & [7,9] & [6,8] & [7,8] & [6,8] \\ S_4 & [7,7] & [9,10] & [7,10] & [8,10] & [6,8] & [7,10] \end{matrix}.$$

By $a_j^* = [a_j^{L*}, a_j^{U*}] = [\max_i (d_{ij} + u_{ij}I^L), \max_i (d_{ij} + u_{ij}I^U)]$ (j = 1, 2, …, 6), the ideal solution (ideal neutrosophic numbers) can be determined as the following ideal alternative:

S^* ={[8, 9], [9, 10], [8, 10], [8, 10], [8, 9], [7, 10]}.

According to Eq. (5), the weighted projection measure values between each alternative S_i (i = 1, 2, 3, 4) and the ideal alternative S^* can be obtained as follows:

$WP_{S^*}(S_1)$ = 0.8554, $WP_{S^*}(S_2)$ =0.9026, $WP_{S^*}(S_3)$ = 0.8826, and $WP_{S^*}(S_4)$ = 0.9366.

Since the values of the projection measure are $WP_{S^*}(S_4) > WP_{S^*}(S_2) > WP_{S^*}(S_3) > WP_{S^*}(S_1)$, the ranking order of the four alternatives is $S_4 > S_2 > S_3 > S_1$. Hence, the alternative S_4 is the best choice among all the alternatives.

Compared with the neutrosophic grey relational analysis for clay-brick selection [8], the proposed approach is more convenient and less calculation steps.

6 Conclusion

This paper presented a projection measure of neutrosophic numbers and a projection model-based multiple attribute decision-making method under a neutrosophic number environment. In the decision-making process, through the projection measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined in order to select the best alternative. Finally, an actual example on the selecting problem of clay-bricks demonstrated the application of the proposed method. However, the main advantage of the proposed approach is easy evaluation and calculation in actual applications. In the future work, we shall extend the proposed decision-making method with neutrosophic numbers to the decision-making method with refine neutrosophic numbers.

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Received: February 15, 2016. Accepted: April 30, 2016.