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Numeralogy (III) or Properties of Numbers

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1) Odd Sequence:

1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, ...

How many of them are primes?

2) even Sequence:

1, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, ...

Conjecture: No number in this sequence is an even power.

3) Prime Sequence:

2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, ...

How many of them are primes?

Conjecture: A finite number.

4)S-sequence:

General definition: Let $S = \{s_1, s_2, s_3, \ldots, s_n, \ldots\}$ be an infinite sequence of integers. Then the corresponding S-sequence is $\{s_1, s_1s_2, \ldots, s_1s_2 \cdots s_n, \ldots\}$ where the numbers are concatenated together.

Question 1: How many termsof the S-sequence are found in the original set S?

Question 2: How many terms of the S-sequence satisfy the properties of other given sequence?

For example, the odd sequence above is built from the set $S = \{1, 3, 5, 7, 9, \ldots\}$ and every element of the S-sequence is found in S. The even sequence is built from the set $S = \{2, 4, 6, 8, 10, \ldots\}$ and every element of the corresponding S-sequence is also in S. However, the question is much harder for the prime sequence.

Study the case when S is the Fibonacci numbers $\{1, 1, 2, 3, 5, 8, 13, 21, \ldots\}$. The corresponding F-sequenceis then $\{1, 11, 112, 1123, 11235, 112358, 11235813, \ldots\}$. In particular, how many primes are in the F-sequence?

5) Uniform sequences:

General definition: Let $n \neq 0$ be an integer and $d_1, d_2, \ldots, d_r$ distinct digits in base $B > r$. Then, multiples of n, written using only the comlete set of digits $d_1, d_2, \ldots, d_r$ in base B, increasingly ordered, is called the uniform sequence.

Some particular examples involve one digit only.

a) Multiples of 7 written in base 10 using only the digit 1.

111111, 111111111111, 111111111111111111, 111111111111111111111111, ...

b) Multiples of 7 written in base 10 using only digit 2.

222222, 222222222222, 222222222222222222, 222222222222222222222222, ...

c) Multiples of 79365 written in base 10 using only the digit 5.

55555, 555555555555, 555555555555555555, 555555555555555555555555, ...

In many cases, the uniform sequence is empty.

d) It is possible to create multiples of 79365 in base 10 using only the digit 6.

Remark: If there exists at least one such multiple of n written with the digits $d_1, d_2, \ldots, d_r$ in base B, then there exists an infinite number of multiples of n. If m is the initial multiple, then they all have the form, $m, mm, mmm, \ldots$

With a computer program it is easy to select all multiples of a given number written with a set of digits, up to a maximum number of digits.

Exercise: Find the general term expression for multiples of 7 using only the digits $\{1,3,5\}$ in base 10.

6) Operation Sequence:

General definition: Let E be an ordered set of elements, $E = \{e_1, e_2, \ldots\}$ and Θ a set of binary operations well-defined on E. Then

$$a_1 \in \{e_1, e_2, \ldots\}$$

$$a_{n+1} = \min\{e_1\theta_1 e_2\theta_2 \ldots \theta_n e_{n+1}\} > a_n, \text{ for } n \geq 1.$$

where all θ_i are oprations belonging to Θ.

Some examples:

a) When E is the set of natural numbers and $\Theta = \{+, -, *, /\}$, the four standard arithmetic operations.

Then

$a_1 = 1$

$a_{n+1} = \min\{1\theta_1\theta_2 \ldots \theta_n(n+1)\} > a_n$, for $n \geq 1$.

where $\theta_i \in \Theta$.

Questions:

a) Given N as the set of numbers and $\Theta = \{+, -, *, /\}$ as the set of operations, is there a general formula for the sequence?

b) If the finite sequence is defined with the finite set of numbers $\{1, 2, 3, \ldots, 99\}$ and the set of operations the same as above, where

$a_1 = 1$

$$a_{n+1} = \min\{1\theta_1 2\theta_2 \ldots \theta_{98} 99\} > a_n, \text{ for } n \geq 1.$$

Same questions as in (a).

c) Let N be the set of numbers and $\Theta = \{+, -, *, /, **, (\sqrt{})\}$, where $x^{**}y$ is x to the power y and $x(\sqrt{})y$ is the x-th root of y. Define the sequence by

$a_1 = 1$

$$a_{n+1} = \min\{1\theta_1 2\theta_2 \ldots \theta_n (n+1)\} > a_n, \text{ for } n \geq 1.$$

The same questions can be asked, althought they are harder and perhaps more intresting.

d) Using the same set of operations, the algebraic operation finite sequence can be defined:

$a_1 = 1$

$a_{n+1} = \min\{1\theta_1 2\theta_2 \ldots \theta_{98} 99\} > a_n$, for $n \geq 1$.

And pose the same questions as in (b).

More generally, the binary operations can be replaced by k_i-ary operations, where all k_i are integers.

$a_1 \in \{e_1, e_2, \ldots\}$

$a_{n+1} = \min\{1\theta_1 2\theta_2 \ldots \theta_1 k_1 \theta_2 (k_1 + 1)\theta_2 \ldots \theta_2 (k_1 + k_2 - 1) \ldots (n + 2 - k_r \theta_r \ldots \theta_r (n+1)\} > a_n$

where $n \geq 1$.

Where each θ_i is a k_i-ary relation and $k_1 + (k_2 - 1) + \ldots + (k_r - 1) = n + 1$. Note that the last element of the k_i relation is the first element of the k_{i+1} realtion.

Remark: The questions are much easier when $\Theta = \{+, -\}$. Study the operation type sequences in this easier case.

e) Operators sequences at random:

Same definition and questions as the previous sequences, except that the minimum condition is removed.

$a_{n+1} = \{e_1\theta_1e_2\theta_2\ldots\theta_ne_{n+1}\} > a_n$, for $n \geq 1$.

Therefore, a_{n+1} will be chosen at random, with the only restriction being that it be greater than a_n.

Study these sequences using a computer program with a random number generator to choose a_{n+1}.

References

[1] F.Smarandache, "Properties of the Numbers", University of Craiova Archives, 1975. [Also see the Arizona State University Special Collections, Tempe, Arizona, USA].

P-Q Relationships and Sequences

Let $A = \{a_n\}, n \geq 1$ be a sequence of numbers and q, p integers ≥ 1.

We say that the terms $a_{k+1}, a_{k+2}, \ldots, a_{k+p}, a_{k+p+1}, a_{k+p+2}, \ldots, a_{k+p+q}$ satisfy a $p-q$ relationship if

$$a_{k+1}\Diamond a_{k+2}\Diamond\ldots\Diamond a_{k+p} = a_{k+p+1}\Diamond a_{k+p+2}\Diamond\ldots\Diamond a_{k+p+q}$$

where $\Diamond$ may be any arithmetic operation, although it is generally a binary relation on A. If this relationship is satisfied for any $k \geq 1$, then $\{a_n\}, n \geq 1$ is said to be a $p-q-\Diamond$ sequence. For operations such as addition, where $\Diamond = +$, the sequence is called a $p-q$-additive sequence.

As a specific case, we can easily see that the Fibonacci/Lucas sequence ($a_n + a_{n+1} = a_{n+2}$, for $n \geq 1$), is a $3-1$-additive sequence.

Definition. *Given any integer $n \geq 1$, the value of the Smarandache function $S(n)$ is the smallest integer m such that n divides $m!$.*

If we consider the sequence of numbers that are the values of the Smarandache function for the integers $n \geq 1$,

$1, 2, 3, 4, 5, 3, 7, 4, 6, 5, 11, 4, 13, 7, 5, 6, 17, \ldots$

they can be incorporated into questions involving the $p-q-\Diamond$ relationships.