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On Another Erdös' Open Problem

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ON ANOTHER ERDÖS' OPEN PROBLEM

Paul Erdös has proposed the following problem:

(1) "Is it true that $\lim_{n\to\infty} \max_{m<n}(m+d(m))-n=\infty$?, where $d(m)$ represents the number of all positive divisors of m."

We clearly have :

Lemma 1. $(\forall)n\in\mathbb{N}\setminus\{0,1,2\}$, $(\exists)!s\in\mathbb{N}^*$, $(\exists)!\alpha_1,...,\alpha_s\in\mathbb{N}$, $\alpha_s\neq 0$, such that $n=p_1^{\alpha_1}\cdots p_s^{\alpha_s}+1$, where $p_1,p_2,...$ constitute the increasing sequence of all positive primes.

Lemma 2. Let $s\in\mathbb{N}^*$. We define the subsequence $n_s(i)=p_1^{\alpha_1}\cdots p_s^{\alpha_s}+1$, where $\alpha_1,...,\alpha_s$ are arbitrary elements of $\mathbb{N}$, such that $\alpha_s\neq 0$ and $\alpha_1+...+\alpha_s\to\infty$ and we order it such that $n_s(1)<n_s(2)<...$ (increasing sequence).

We find an infinite number of subsequences $\{n_s(i)\}$, when s traverses $\mathbb{N}^*$, with the properties:

a) $\lim_{i\to\infty} n_s(i)=\infty$ for all $s\in\mathbb{N}^*$.

b) $\{n_{s_1}(i),\ i\in\mathbb{N}^*\}\cap\{n_{s_2}(j),\ j\in\mathbb{N}^*\}=\Phi$, for $s_1\neq s_2$ (distinct subsequences).

c) $\mathbb{N}\setminus\{0,1,2\}=\bigcup_{s\in\mathbb{N}^*}\{n_s(i),\ i\in\mathbb{N}^*\}$

Then:

Lemma 3. If in (1) we calculate the limit for each subsequence $\{n_s(i)\}$ we obtain:

$$\lim_{n\to\infty}\left(\max_{m<p_1^{\alpha_1}\cdots p_s^{\alpha_s}}(m+d(m))-p_1^{\alpha_1}\cdots p_s^{\alpha_s}-1\right)\geq\lim_{n\to\infty}\left(p_1^{\alpha_1}\cdots p_s^{\alpha_s}+(\alpha_1+1)...(\alpha_s+1)-p_1^{\alpha_1}\cdots p_s^{\alpha_s}-1\right)=$$

$$=\lim_{n\to\infty}\left((\alpha_1+1)...(\alpha_s+1)-1\right)>\lim_{n\to\infty}\left(\alpha_1+...+\alpha_s\right)=\infty$$

From these lemmas it results the following:

Theorem: We have $\overline{\lim_{n\to\infty}}\max_{m<n}(m+d(m))-n=\infty$.

REFERENCES

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