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Conjectures Which Generalize Andrica's Conjecture

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CONJECTURES WHICH GENERALIZE ANDRICA'S CONJECTURE

Five conjectures on paires of consecutive primes are listed below with examples in each case.

1) The equation $p_{n+1}^{x} - p_{n}^{x} = 1,$ (1)
where p_n is the n-th prime, has a unique solution situated in between 0.5 and 1. Checking the first 168 prime numbers (less than 1000), one gets that:

- the maximum occurs of course for n=1, i.e.
$3^x - 2^x = 1$ when x=1.
- the minimum occurs for n=31, i.e.
$127^x - 113^x = 1$ when $x = 0.567148... = a_0$. (2)

Thus, Andrica's Conjecture

$$A_n = \sqrt{p_{n+1}} - \sqrt{p_n} < 1,$$

is generalized to

2) $B_n = p_{n+1}^{a} - p_{n}^{a} < 1, \quad where \quad a < a_0.$ (3)

It is remarkable that the minimum x doesn't occur for
$11^x - 7^x = 1$
as in Andrica's Conjecture the maximum value, but in (2).

Also, the function B_n in (3) is falling asymptotically as A_n in (2). Look at these prime exponential equations solved with a TI-92 Graphing Calculator (approximately: the bigger the prime number gap is, the smaller solution x for the equation (1);
for the same gap between two consecutive primes, the larger the primes, the bigger x):

$3^x - 2^x = 1$, has the solution x = 1.000000.
$5^x - 3^x = 1$, has the solution x ≈ 0.727160.
$7^x - 5^x = 1$, has the solution x ≈ 0.763203.
$11^x - 7^x = 1$, has the solution x ≈ 0.599669.

$13^x - 11^x = 1$, has the solution $x \approx 0.807162$.
$17^x - 13^x = 1$, has the solution $x \approx 0.647855$.
$19^x - 17^x = 1$, has the solution $x \approx 0.826203$.
$29^x - 23^x = 1$, has the solution $x \approx 0.604284$.
$37^x - 31^x = 1$, has the solution $x \approx 0.624992$.
$97^x - 89^x = 1$, has the solution $x \approx 0.638942$.
$127^x - 113^x = 1$, has the solution $x \approx 0.567148$.
$149^x - 139^x = 1$, has the solution $x \approx 0.629722$.
$191^x - 181^x = 1$, has the solution $x \approx 0.643672$.
$223^x - 211^x = 1$, has the solution $x \approx 0.625357$.
$307^x - 293^x = 1$, has the solution $x \approx 0.620871$.
$331^x - 317^x = 1$, has the solution $x \approx 0.624822$.
$497^x - 467^x = 1$, has the solution $x \approx 0.663219$.
$521^x - 509^x = 1$, has the solution $x \approx 0.666917$.
$541^x - 523^x = 1$, has the solution $x \approx 0.616550$.
$751^x - 743^x = 1$, has the solution $x \approx 0.732706$.
$787^x - 773^x = 1$, has the solution $x \approx 0.664972$.
$853^x - 839^x = 1$, has the solution $x \approx 0.668274$.
$877^x - 863^x = 1$, has the solution $x \approx 0.669397$.
$907^x - 887^x = 1$, has the solution $x \approx 0.627848$.
$967^x - 953^x = 1$, has the solution $x \approx 0.673292$.
$997^x - 991^x = 1$, has the solution $x \approx 0.776959$.

If $x > a_0$, the difference of x-powers of consecutive primes is normally grater than 1. Checking more versions:

$3^{0.99}$	-	$2^{0.99}$	≈ 0.981037.
$11^{0.99}$	-	$7^{0.99}$	≈ 3.874270.
$11^{0.60}$	-	$7^{0.60}$	≈ 1.001270.
$11^{0.59}$	-	$7^{0.59}$	≈ 0.963334.
$11^{0.55}$	-	$7^{0.55}$	≈ 0.822980.
$11^{0.50}$	-	$7^{0.50}$	≈ 0.670873 .
$389^{0.99}$	-	$383^{0.99}$	≈ 5.596550.
$11^{0.599}$	-	$7^{0.599}$	≈ 0.997426.
$17^{0.599}$	-	$13^{0.599}$	≈ 0.810218.
$37^{0.599}$	-	$31^{0.599}$	≈ 0.874526.
$127^{0.599}$	-	$113^{0.599}$	≈ 1.230100.

$997^{0.599}$	-	$991^{0.599}$	$\approx 0.225749.$
$127^{0.5}$	-	$113^{0.5}$	$\approx 0.639282.$

3) $C_n = p_{n+1}^{1/k} - p_n^{1/k} < 2/k$, where p_n is the n-th prime, and $k \geq 2$ is an integer.

$11^{1/2}$	-	$7^{1/2}$	$\approx 0.670873.$
$11^{1/4}$	-	$7^{1/4}$	$\approx 0.1945837251.$
$11^{1/5}$	-	$7^{1/5}$	$\approx 0.1396211046.$
$127^{1/5}$	-	$113^{1/5}$	$\approx 0.060837.$
$3^{1/2}$	-	$2^{1/2}$	$\approx 0.317837.$
$3^{1/3}$	-	$2^{1/3}$	$\approx 0.1823285204.$
$5^{1/3}$	-	$3^{1/3}$	$\approx 0.2677263764.$
$7^{1/3}$	-	$5^{1/3}$	$\approx 0.2029552361.$
$11^{1/3}$	-	$7^{1/3}$	$\approx 0.3110489078.$
$13^{1/3}$	-	$11^{1/3}$	$\approx 0.1273545972.$
$17^{1/3}$	-	$13^{1/3}$	$\approx 0.2199469029.$
$37^{1/3}$	-	$31^{1/3}$	$\approx 0.1908411993.$
$127^{1/3}$	-	$113^{1/3}$	$\approx 0.191938.$

4) $D_n = p_{n+1}^a - p_n^a < 1/n,$ (4)

where $a < a_0$ and n big enough, $n = n(a)$, holds for infinitely many consecutive primes.

a) Is this still available for $a < 1$?

b) Is there any rank n_0 depending on a and n such that (4) is verified for all $n \geq n_0$?

A few examples:

$5^{0.8}$	-	$3^{0.8}$	$\approx 1.21567.$
$7^{0.8}$	-	$5^{0.8}$	$\approx 1.11938.$
$11^{0.8}$	-	$7^{0.8}$	$\approx 2.06621.$
$127^{0.8}$	-	$113^{0.8}$	$\approx 4.29973.$
$307^{0.8}$	-	$293^{0.8}$	$\approx 3.57934.$
$997^{0.8}$	-	$991^{0.8}$	$\approx 1.20716.$

5) $P_{n+1}/P_n \leq 5/3,$ (5)

the maximum occurs at n=2.

{The ratio of two consecutive primes is limited, while the

difference $p_{n+1} - p_n$ can be as big as we want!}
However, $1/p_n - 1/p_{n+1} \leq 1/6$, and the maximum occurs at n=1.

Reference:

[1] Sloane, N. J. A., Sequence A001223/M0296 in <An On-Line Version of the Encyclopedia of Integer Sequences>.

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