FLORENTIN SMARANDACHE

Digital Subsequences

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Let $\{a_n\}$ $n \geq 1$ be a sequence defined by a property (or a relationship involving its terms) P. We then screen this sequence, selecting only the terms whose digits also satisfy the property or relationship.

1) The new sequence is then called a P-digital subsequence.

Examples:

a) Sqare-digital subsequence:

Given the sequence of perfect squares $0, .1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, \ldots$ only those terms whose digits are all perfect squares $\{0, 1, 4, 9\}$ are chosen. The first few terms are $0, 1, 4, 9, 49, 100, 144, 400, 441$.

Disregarding squares of the form $N00\ldots0$, where N is also a perfect square, how many numbers belong to this subsequence?

b) Given the sequence of perfect cubes, $0, 1, 8, 27, 64, 125, \ldots$ only those terms whose digits are all perfect cubes $\{0, 1, 8\}$ are chosen. The first few terms are $0, 1, 8, 1000, 8000$.

Disregarding cubes of the form $N00\ldots0$, where N is also a perfect cube, how many numbers belong to this subsequence?

c) Prime-digital subsequence:

Given the sequence of prime numbers, $2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots$. Only those primes where all digits are prime numbers are chosen. The first few terms are $2, 3, 5, 7, 23, 29, \ldots$.

Conjecture: This subsequence is infinite.

In the same vein, elements of a sequence can be chosen if groups of digits, except the complete number, satisfy a property (or relationship) P. The subsequence is then called a P-partial-digital subsequence.

Examples:

a) Squares-partial-digital subsequence:

$49, 100, 144, 169, 361, 400, 441, \ldots$

In other words, perfect squares whose digits can be partioned into two or more groups that are perfect squares.

For example 169 can be partitioned into 16 and 9.

Disregarding square numbers of the form $N00\ldots0$, where N is also a perfect square, how many numbers belong to this sequence?

b) Cube-partial-digital subsequence:

$1000, 8000, 10648, 27000, \ldots$

i.e. all perfect cubes where the digits can be partioned into two or more groups that are perfect cubes. For example 10648 can be partitioned into 1, 0, 64 and 8.

Disregarding cube numbers of the form $\overline{N\underbrace{00\ldots0}_{2k \text{ times}}}$, where N is also a perfect cube, how many numbers belong to this sequence?

c) Prime-partial-digital subsequence:

23, 37, 53, 73, 113, 137, 173, 193, 197,...

i.e. all prime numbers where the digits can be partioned into two or more groups of digits that are prime numbers. For example, 113 can be partioned into 11 and 3.

Conjecture: This subset of the prime numbers is infinte.

d) Lucas-partial-digital subsequence:

Definition. *A number is a Lucas number of sequence* $L(0) = 2, L(1) = 1$ *and* $L(n+2) = L(n+1) + L(n)$ *for* $n \geq 1$.

The first few elements of this sequence are $2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, \ldots$

A number is an element of the Lucas-partial-digital subsequence if it is a Lucas number and the digits can be partioned into three groups such that the third group, moving left to right, is the sum of the first two groups. For example, 123 satisfies all these properties.

Is 123 the only Lucas number that satisfies the properties of this partition?

Study some P-partial-digital subsequences using the sequences of numbers.

i) Fibonacci numbers. A search was conducted looking for Fibonacci numbers that satisfy the properties of such a partition, but none were found. Are there any such numbers?

ii) Smith numbers, Eulerian numbers, Bernouli numbers, Mock theta numbers and Smarandache type sequences are other candidate sequences.

Remark: Some sequences may not be partitionable in this manner.

If a sequence $\{a_n\}, n \geq 1$ is defined by $a_n = f(n)$, a function of n, then an f-digital sequence is obtained by sceeening the sequence and selecting only those numbers that can be partioned into two groups of digits g_1 and g_2 such that $g_2 = f(g_1)$.

Examples:

a) If $a_n = 2n, n \geq 1$, then the even-digital subsequence is $12, 24, 36, 48, 510, 612, 716, 816, 918, 1020, \ldots$

where 714 can be partitioned into 7 and 14 in that order and

b) Lucky-digital subsequence:

Definition: *Given the set of natural numbers* $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, \ldots$. *First strike out every even numbers, leaving* $1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, \ldots$. *Then strike out* *every third in the remaining list, every fourth number in what remains after that, every fifth number remaining after that and so on. The set of numbers that remains after this infinite sequence is performed are the Lucky numbers.*

$1, 3, 7, 9, 13, 15, 21, 25, 31, 33, 37, 43, 49, 51, 63, \ldots$

A number is said to be a member of the lucky-digital subsequence if the digits can be partitioned into two number mn in that order such that $L_m = n$.

37 and 49 are both elements of this sequence. How many others are there?

Study this type of sequence for other well-known sequences.

References

[1] F.Smarandache, "Properties of the Numbers", University of Craiova Archives, 1975. [See also the Arizona State Special Collections, Tempe, AZ., USA].

Magic Squares

For $n \geq 2$, let A be set of n^2 elements and l an n-ary relation defined on A. As a generalization of the XVIth-XVIIth century magic squares, we present the magic square of order n. This is square array of elements of A arranged so that l applied to all rows and columns yields the same result.

If A is an arithmetic progression and l addition, then many such magic squares are known. The following appeared in Durer's 1514 engraving, "Melancholia"

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Questions:

1) Can you find magic square of order at least three or four where A is a set of prime numbers and l is addition?