A Speculative Note on How to Modify Einstein's Field Equation to Hold at the Quantum Scale
Gravity at the Quantum Scale = Strong Force?

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Abstract

In this short note, first we will show a few ways to rewrite Einstein's field equation without basically changing it. Then we speculate a bit more broadly on how to change the equation to make it hold at the quantum scale. More precisely, we modify it for bodies with masses less than the Planck mass.

Key words: Einstein's field equation, Planck mass, subatomic world, strong force, energy density.

1 A few ways to rewrite Einstein's field equation

The Einstein field equation [1] is given by

$$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi G}{c^4}T_{\mu v} \tag{1}$$

The Newton gravitational force [2] between two Planck masses [3], with a radius center to center equal to the Planck length, is given by $F_p = G\frac{m_p m_p}{l_p^2}$, and we have

$$\frac{G}{c^4} = \frac{1}{G\frac{m_p m_p}{l_p^2}} = \frac{1}{F_p} \approx 8.2624 \times 10^{-45} \tag{2}$$

where m_p is the Planck mass and l_p is the Planck lenght. This means we can write Einstein's field equation as

$$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi}{G\frac{m_p m_p}{l_p^2}}T_{\mu v}$$
$$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi}{F_p}T_{\mu v} \tag{3}$$

2 Gravity for objects with masses less than the Planck mass

When we have an object with a mass larger than the Planck mass, we can hypothetically pack it into multiple Planck masses. Assume we are looking at the Newtonian gravitational force between the Sun and the Earth with masses of M_s and M_E respectively, then we can rewrite this as

$$F = G\frac{M_s M_E}{r^2} = G\frac{N_1 m_p N_2 m_p}{r^2} \tag{4}$$

where N_1 is the number of Planck masses we can hypothetically pack the Sun in (mass one) and N_2 is the number of Planck masses we can hypothetically pack the earth in (mass two). When it comes to the gravity between two electrons, for example, we can no longer use the equation above, because the electron mass is much smaller than the Planck mass. However, we can modify the equation to take any two identical masses into account

$$F = G\frac{m_e m_e}{r^2} = G\frac{m_p m_p}{r^2}\frac{l_p^2}{\bar{\lambda}^2} \tag{5}$$

where $\bar{\lambda}$ is the reduced Compton wavelength of two identical subatomic particles, in this case the reduced Compton wavelength of electrons .

3 Speculative modification of Einstein's Field Equation for the quantum realm

We known that the Schwarzschild solution [4, 5] of Einstein's field equation runs into problems at the Schwarzschild radius. One possible interpretation of the issue is that the Schwarzschild solution of Einstein's field equation does not hold for particles with masses less than the Planck mass (or potentially less than the Planck particle). The Planck particle has a mass of $\sqrt{\pi}$ times the Planck mass and Compton wavelength equal to the Schwarzschild radius. All particles with mass less than the Planck particle has a Compton wavelength larger than their Schwarzschild radius. All observed subatomic particles, including the very heavy Higgs particle, have masses much lower then the Planck mass.

Here we speculate that to make Einstein's field equation hold for masses smaller than the Planck mass, then it should be rewritten as

$$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi}{G\frac{m_p m_p}{l_p^2}\frac{l_p^2}{\bar{\lambda}^2}}T_{\mu v} \tag{6}$$

$$R_{\mu v} - \frac{1}{2}g_{\mu v}R = \frac{8\pi}{F_p\frac{l_p^2}{\bar{\lambda}^2}}T_{\mu v} \tag{7}$$

where F is Newton's gravitational force. In the special case of Planck masses, we have $\bar{\lambda} = l_p$ and we then get the standard Einstein field equation. If the modified equation should hold, then we can also rewrite it as

$$\begin{aligned} R_{\mu v} - \frac{1}{2}g_{\mu v}R &= \frac{8\pi}{G\frac{m_p m_p}{l_p^2}\frac{l_p^2}{\bar{\lambda}^2}}T_{\mu v} \\ R_{\mu v} - \frac{1}{2}g_{\mu v}R &= \frac{8\pi G}{\frac{l_p^2}{\bar{\lambda}^2}c^4}T_{\mu v} \end{aligned} \tag{8}$$

The difference between Einstein's field equation and our speculative modification is the $\frac{l_p^2}{\bar{\lambda}^2}$ factor. However, this changes the output relative to the standard Einstein field equation dramatically when working with particles with masses less than a Planck mass, something that either points to the invalidity of this speculative formula, or demonstrates that gravity changes somewhat dramatically in the quantum realm. The value of the factor, $\frac{G}{\frac{l_p^2}{\bar{\lambda}^2}c^4}$ changes by 1.69×10^{38} for two protons relative to the standard Einstein field equation "scaling factor" $\frac{G}{c^4}$. The fact that this is about the same as the difference in the strong force and gravity for two protons (setting the reduced Compton wavelength equal to that of a Proton) is likely a coincident, but it could also be a hint that this speculative formula should be investigated further.

For masses larger or equal to the Planck mass, we have $\bar{\lambda} = l_p$ and then we get the standard Einstein field equation. Whether or not this speculative equation has any validity remains to be seen. We will leave a certain degree of analysis up to experts on Einstein's field equation.

Based on [8, 9] we can also re-write equation 8 as

$$\begin{aligned} R_{\mu v} - \frac{1}{2}g_{\mu v}R &= \frac{8\pi G}{\frac{l_p^2}{\bar{\lambda}^2}c^4}T_{\mu v} \\ R_{\mu v} - \frac{1}{2}g_{\mu v}R &= \frac{8\pi\bar{\lambda}^2}{\hbar c}T_{\mu v} \end{aligned} \tag{9}$$

which is equal to the Einstein field equation for masses equal or larger than the Planck mass.

Appendix

We can also speculate a bit further. It is well known that the Coulomb force [6] between two Planck masses is identical to the gravity between two Planck masses, that is

$$G\frac{m_p m_p}{l_p^2} = k_e\frac{qq}{l_p^2} \tag{10}$$

which means that we can also write Einstein's field equation as

$$
\begin{aligned}
R_{\mu v} - \frac{1}{2} g_{\mu v} R &= \frac{8\pi}{k_e \frac{qq}{l_p^2}} T_{\mu v} \\
R_{\mu v} - \frac{1}{2} g_{\mu v} R &= \frac{8\pi}{F_{k,p}} T_{\mu v} \qquad (11)
\end{aligned}
$$

Haug [7] has recently published a theory of gravitation that seems to unify electromagnetism and gravity. It is not impossible that such a framework can also be extended to the modified Einstein field equation?

References

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