Kinetic Theory of Dark Energy

Author: Nikola Perkovic*

perce90gm@gmail.com

University of Novi Sad, Faculty of Sciences, Institute of Physics and Mathematics

Abstract: This paper will present a new theory for what we call "Dark Energy" by explaining it as a form of kinetic energy. It will be presented within General Relativity where we will use new terms and explain them in detail. Some of the new terms that will be presented also fall under Quantum Mechanics. It will be explained in detail how and why dark energy comes to be and why it is now dominant in the Universe which will explain the observational evidence that has been attained on this subject and has so far been rather puzzling to scientists when it comes to the very nature of "Dark Energy".

Introduction

1. Expanding flat spacetime

The Universe will be represented as homogenous and isotropic. Isotropy means that the metric must be diagonal since it will be show that space is allowed to be curved. Therefore we will use spherical coordinates to describe the metric.

The metric is given by the following line element:

$$(1)\ ds^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

where we measure (θ) from the north pole and at the south pole it will equal (π).

In order to simplify the calculations, we abbreviate the term between the brackets as:

$$(2)\ d\omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$$

because it is a measure of angle, which can be thought of as "on the sky" from the observers point of view. It is important to mention that the observers are at the center of the spherical coordinate system.

Due to the isotropy of the Universe the angle between two galaxies, for the observers, is the true angle from the observers' vantage point and the expansion of the Universe does not change this angle.

Finally, we represent flat space as:

$$(3)\ ds^2 = dr^2 + r^2 d\omega^2$$

Robertson and Walker proved that the only alternative metric that obeys both isotropy and homogeneity is:

$$(4)\ ds^2 = dr^2 + f_K(r)^2 d\omega^2$$

where ($f_K(r)$) is the curvature function given by:

$$(4)\ f_K(r) = \begin{cases} K^{-1/2}\ for\ K > 0 \\ r\ for\ K = 0 \\ K^{-1/2} \sin h\left(K^{1/2} r\right)\ for\ K < 0 \end{cases}$$

which means that the circumference of a sphere around the observers with a radius (r) is, for ($K \neq 0$), not anymore equal to ($C = 2\pi r$) but smaller for ($K > 0$) and larger for ($K < 0$).

The surface area of that sphere would no longer be ($S = \left({4\pi}/{3}\right) r^3$) but smaller for ($K > 0$) and larger for ($K < 0$). If ($r$) is ($r \ll |K|^{-1/2}$) the deviation from ($C = 2\pi r$) and ($S = \left({4\pi}/{3}\right) r^3$) is very small, but as (r) approaches ($|K|^{-1/2}$) the deviation can become rather large.

The metric in the equation (1) can also be written as:

$$(5)\ ds^2 = \frac{dr^2}{1 - Kr^2} + r^2 d\omega^2$$

If we determine an alternative radius (r) as:

$$(6)\ r \equiv f_K(r)$$

This metric is different only in the way we chose our coordinate (r); other than that there is no physical difference with the equation (47).

1.1. Friedmann equations

We can now build our model by taking for each point in time a RW space. We allow the scale factor and the curvature of the RW space to vary with time. This gives the generic metric:

$$(7)\ ds^2 = -dt^2 + a(t)^2[dx^2 + f_K(x)^2 x^2 d\omega^2]$$

the function ($a(t)$) is the scale factor that depends on time and it will describe the expansion of it, that influences spatial expansion, hence the expansion of the whole Universe. We use (x) instead of (r) because the radial coordinate, in this form, no longer has meaning as a true distance.

We now insert equation (7) into the Einstein equations and after calculus, we obtain two equations:

$$(8)\ \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2} + \frac{\lambda}{3}$$

$$(9)\ \left(\frac{\ddot{a}}{a}\right)^2 = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\lambda}{3}$$

These equations are known as Friedmann equations and the first equation is from the "00 component" and the second from the "ii component" of the Einstein equations.

The two equations can be combined to make the adiabatic equation:

$$(10)\ \frac{d}{dt}(\rho a^3 c^2) + p\frac{d}{dt}(a^3) = 0$$

We will define a new term here named "spatial kinetic factor" (Д) as:

$$(11)\ Д = Kc^2 + \frac{1}{\rho_{crit}}$$

This factor is important to define some other terms that will be presented and it represents the spatial factor of the kinetic energy that will be defined in the conclusion of this paper. There will also be a “temporal kinetic factor” ($\langle \widehat{Д}_{\twoheadrightarrow} \rangle$).

1.2. Scaling of relativistic and non-relativistic matter

Cold matter is matter for which the pressure ($p \ll \rho c^2$) leads us to reduce the equation (10) to:

$$(12)\ \frac{d}{dt}(\rho a^3) = 0$$

meaning that the equation of state for such cold matter is:

$$(13)\ \rho \propto \frac{1}{a^2}$$

If we look at the other limiting case, of ultra-relativistic matter, we have the maximum possible relativistic isotropic pressure:

$$(14)\ p = \frac{\rho c^2}{3}$$

for radiation (ultra-hot matter).

Equation (10) now reduces to:

$$(15)\ \frac{d}{dt}(\rho a^3 c^2) + \frac{\rho c^2}{3}\frac{d}{dt}(a^3) = 0$$

hence:

$$(16)\ \rho \propto \frac{1}{a^4}$$

for radiation.

1.3. Critical density

Critical velocity turns into a critical density; the best way to define this is to start from the first Friedmann equation and rewrite it as:

$$(17)\ H^2 = \frac{8\pi G}{3}(\rho + \rho_\lambda) - \frac{Kc^2}{a^2}$$

with the Hubble constant ($H = \dot{a}/a$), and we have (λ) as (ρ_λ) according to:

$$(18)\ \rho_\lambda = \frac{\lambda}{8\pi G}$$

The density (ρ) can be written as contributions from matter, meaning baryons, cold dark matter and radiation:

$$(19)\ \rho = \rho_m + \rho_r$$

where baryonic and cold dark matter are:

$$(20)\ \rho_m = \rho_b + \rho_{cdm}$$

for matter density (ρ_m).

We write that radiation consists of photons and neutrinos:

$$(21)\ \rho_r = \rho_\gamma + \rho_v$$

The first Friedmann equation becomes:

$$(22)\ H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \rho_\lambda) - \frac{Kc^2}{a^2}$$

If we define (ρ_{crit}) as:

$$(23)\ \rho_{crit} = \frac{3H^2}{8\pi G}$$

then we see that if the total density ($\rho_m + \rho_r + \rho_\lambda$) equals the critical density, then ($K = 0$), which means that the Universe is flat. By the equivalence of curvature and expansion rate, it would also mean that the Universe expands critically. Therefore the critical density is the density at which the Universe expands critically, given the value for (H).

1.4. Dimensionless Friedmann equation

We define (H_0) as the Hubble constant at the present time and ($\rho_{crit;0}$) as the critical density at the present time, forming the first Friedmann equation as:

$$(24)\ H^2 = {H_0}^2\left(\frac{\rho_m}{\rho_{crit;0}} + \frac{\rho_r}{\rho_{crit;0}} + \frac{\rho_\lambda}{\rho_{crit;0}}\right) - \frac{Kc^2}{a^2}$$

Allowing us to introduce the following dimensionless densities:

$$(25)\ \Omega_m(a) = \frac{\rho_m(a)}{\rho_{crit}(a)}$$

$$(26)\ \Omega_r(a) = \frac{\rho_r(a)}{\rho_{crit}(a)}$$

$$(27)\ \Omega_\lambda(a) = \frac{\rho_\lambda(a)}{\rho_{crit}(a)}$$

The values of these quantities are denoted as ($\Omega_{m;0}$), ($\Omega_{r;0}$) and ($\Omega_{\lambda;0}$).

At this point we introduce ($\Omega_K(a)$), respectively ($\Omega_{K;0}$).

If we consider the first Friedmann equation at the present time:

$$(28)\ H^2 = {H_0}^2\left(\Omega_{m;0} + \Omega_{r;0} + \Omega_{\lambda;0}\right) - Kc^2$$

We can evaluate the curvature:

$$(29)\ Kc^2 = {H_0}^2\left(\Omega_{m;0} + \Omega_{r;0} + \Omega_{\lambda;0} - 1\right)$$

We define the curvature density ($\Omega_{K;0}$) as:

$$(30)\ \Omega_{K;0} \equiv -\frac{Kc^2}{{H_0}^2} = 1 - \Omega_{m;0} - \Omega_{r;0} - \Omega_{\lambda;0}$$

concluding that all (Ω_s) add up to (1). We can define ($\Omega_K(a)$) in terms of a “curvature density”:

$$(31)\ \Omega_K(a) = \frac{\rho_K(a)}{\rho_{crit}(a)}$$

The (Ω) symbol can be used to rewrite the Friedmann equations. The matter density goes as ($1/a^3$), the radiation as ($1/a^4$) and the (Ω_λ) stays constant. The (Ω_K) is, according to equation (77), ($1/a^2$). Now we can write:

$$(32)\ H^2 = {H_0}^2\left(\frac{\Omega_{m;0}}{a^3} + \frac{\Omega_{r;0}}{a^4} + \Omega_{\lambda;0} + \frac{\Omega_{K;0}}{a^2}\right) = {H_0}^2E^2(a)$$

at present time ($a = 1$).

2. The Standard Model

The current understanding of the Universe tells us that it is flat ($\Omega_{K;0} \simeq 0$), but that it contains matter, radiation and that it has a non-zero cosmological constant. We are currently dominated by (λ) by a factor of three, which means a phase of exponential growth.

But before that, around ($z \gtrsim 0.5$), the Universe was dominated by cold matter and before that, around ($z \gtrsim 3200$), the Universe was dominated by radiation.

The late Universe, ($z = few$) until ($z = 0$), in which both matter and (λ) are important, but radiation is not important, can also be integrated analytically. During this period, most of the structure formation in the Universe occurred.

We take ($0 < \Omega_{m;0} < 1$) and ($\Omega_{\lambda;0} = 1 - \Omega_{m;0}$) and set ($\Omega_r = 0$) and ($\Omega_K = 0$). Then we have:

$$(33)\ \frac{1}{a}\frac{da}{dt} = H_0\sqrt{\frac{\Omega_{m;0}}{a^3} + \Omega_{\lambda;0}}$$

which can be integrated as:

$$(34)\ t = \frac{1}{H_0}\int_0^a \frac{da'}{a'\sqrt{\frac{\Omega_{m;0}}{a'^3} + \Omega_{\lambda;0}}} = \frac{1}{H_0}\int_0^a \frac{\sqrt{a'}da'}{\sqrt{\Omega_{m;0} + \Omega_{\lambda;0}\cdot a'^3}}$$

by submitting ($x = a^{3/2}$) we integrate to:

$$(35)\ t = \frac{2}{3H_0\sqrt{1-\Omega_{m;0}}}\operatorname{arcsin}h\left(\sqrt{\frac{1-\Omega_{m;0}}{\Omega_{m;0}}}a^{3/2}\right)$$

if (a) has a small value, the formula above approaches the equation for the matter dominated era, which is:

$$(36)\ a(t) \simeq \left(\frac{3}{2}H_0\sqrt{\Omega_{m;0}}t\right)^{2/3}$$

and that for ($a \gg 1$) this formula describes an exponentially expanding Universe.

The equation (35) is accurate for all redshifts up to around ($z \simeq 1000$), meaning that it can be used for the estimation of age of the Universe by inserting ($a = 1$) into the equation (82) hence we obtain ($\Omega_{m;0} = 0.273$) an age of ($1376Gyr$).

Temporal Motion

Unlike spatial motion, temporal motion requires no specific direction. Instead of a trajectory it needs expansion and it needs a velocity. Time expands in all direction and it influences spatial expansion, hence it "inflates space", which forms the space-time continuum. Temporal motion ($\twoheadrightarrow$) produces kinetic energy which is why it is observable.

We define that temporal motion of a natural vacuum on quantum level equals:

$$(37)\ \delta \twoheadrightarrow = \delta \int d\text{Д}\, L(a(t), \dot{a}(t))$$

where the (dД) is the spatial kinetic factor, the function ($a(t)$) is the scale factor that depends on time and it will describe expansion and ($\dot{a}(t)$) is the velocity. We also define that:

$$(38)\ \dot{a}(t) = c$$

Where (c) is "the speed of light". This is the reason that time dilatation is caused by velocity and why (c) is the speed necessary to achieve maximal time dilatation. What (c) actually is, is the speed of temporal motion. Any velocity will cause time dilatation to some extent as every mass of a celestial body will cause gravitational time dilatation to some extent. Since both mass and velocity cause time dilatation they will mutually dilate, causing the law that nothing with a mass can reach the speed of light. It is difficult to observe time dilatation for small velocities.

We also define that

$$(39)\ d\text{Д} = \frac{d}{dt}\text{Д}$$

which allows us to form the final equation for natural vacuum:

$$(40)\ \delta \twoheadrightarrow = \delta \int d\text{Д}\, L(a(t), c)$$

This allows us to form an equation for the temporal kinetic factor.

Since temporal motion is more difficult to explain than any spatial motion, the simplest way would be to use frames. We would split the Universe in trillions of frames, however we would only use very few where there is a significant difference compared to the previous one.

These frames mark different eras in the Universe. The first frame represents Cosmological inflation, where ($\twoheadrightarrow = 1$), initiating the birth of the Universe known as the Big Bang.

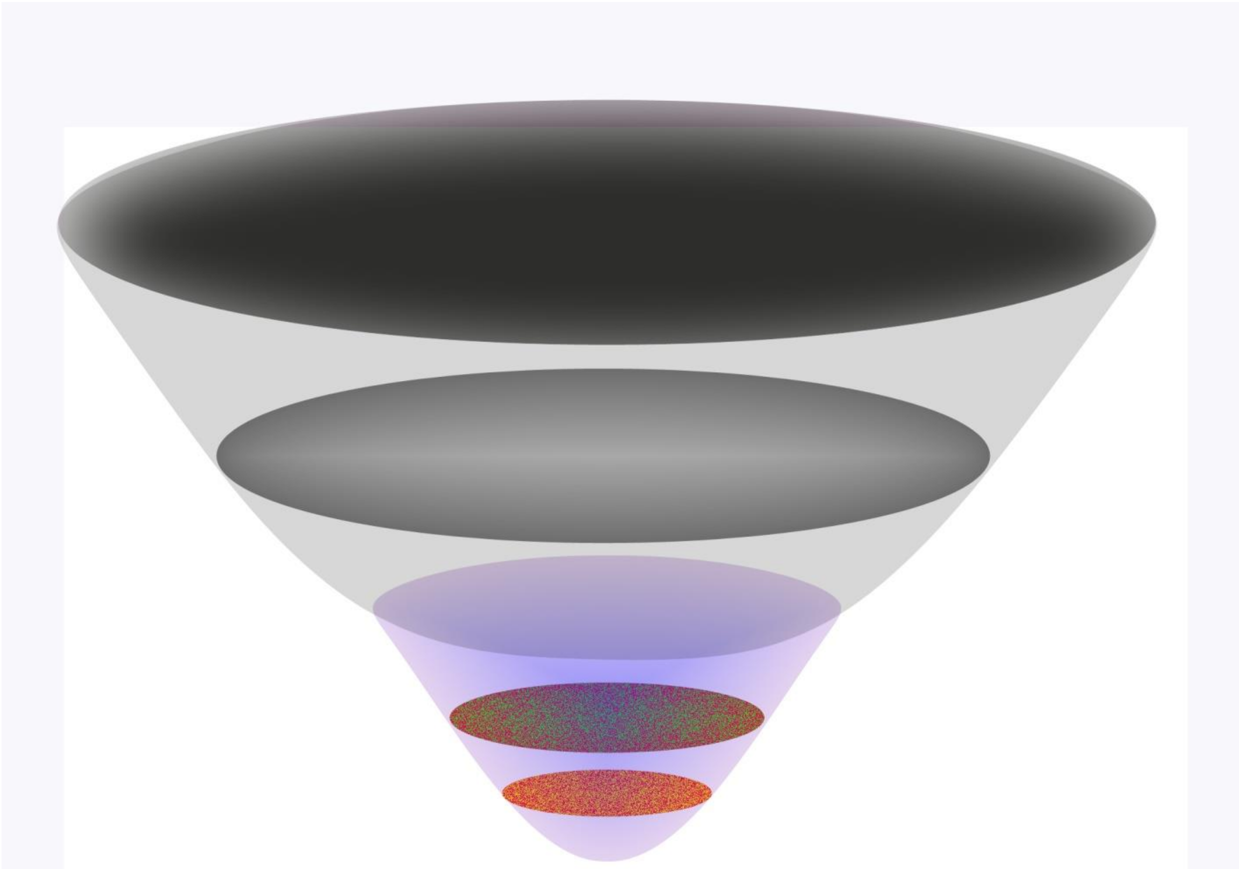

Figure 1: *Some significant frames*

The frame above the first, the i*th* frame ($\twoheadrightarrow = i$), could represent the Universe in the period known as the "radiation dominated era", a period when the Universe was dominated by radiation, around ($z \gtrsim 3200$).

For the early, radiation dominated era we can approximate a solution:

$$(41)\ a(t) \simeq \left(2H_0\sqrt{\Omega_{r;0}}t\right)^{1/2}$$

The early, radiation dominated Universe expanded as:

$$(42)\ a \propto \sqrt{t}$$

Every frame has slightly more temporal-kinetic energy, or "dark energy", than the previous one but since the differences in the trillions of frames are complicated to determine it is therefore simpler and more productive to use only some frames.

Once the presence of temporal-kinetic energy grew enough in the Universe it became dominant. Temporal-kinetic energy strides to accelerate the expansion of the Universe essentially "driving everything away from each other", on the other hand what is known as "dark matter" has a reversed effect of “keeping bodies together” by increasing gravitational influence within galaxies, for example.

This event, where $(\twoheadrightarrow = j)$ thus the j*th* frame, was extremely important since the Universe started expanding more exponentially than it did before. This was the event of the lambda dominance, when the lambda factor became dominant.

We could add other frames $(\twoheadrightarrow = h)$ and $(\twoheadrightarrow = k)$ for different periods, however the final frame that represents the current period is the n*th* one $(\twoheadrightarrow = n)$.

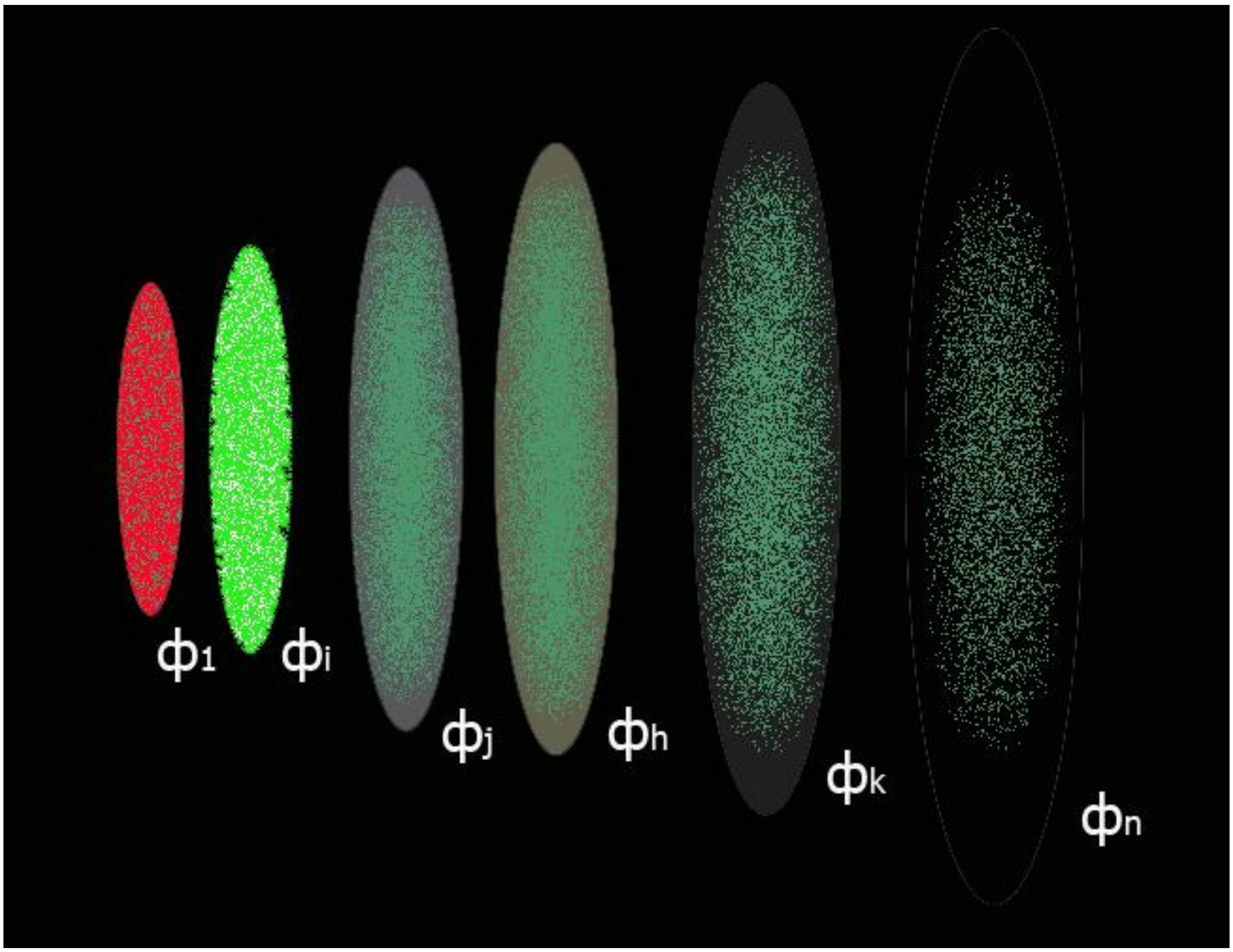

Figure 2: *Frames marking different periods*

This allows us to form the equation:

$$(43)\ \langle \hat{Д}_{\twoheadrightarrow} \rangle = \langle \Psi \left| \sum_{\twoheadrightarrow=1}^{n} \frac{-\hbar^2}{2\rho_\lambda} \nabla_{\twoheadrightarrow}^2 \right| \Psi \rangle = -\frac{\hbar^2}{2\rho_\lambda} \sum_{\twoheadrightarrow=1}^{n} \langle \Psi | \nabla_{\twoheadrightarrow}^2 | \Psi \rangle$$

where ($\nabla^2_{\twoheadrightarrow}$) is the Laplacian of the system and (ρ_λ) is the density of temporal-kinetic energy, or "dark energy".

When the Universe was younger the (λ) factor was significantly lesser than it is now, therefore the rate of acceleration of the expansion was much lesser than it is at the present period.

Conclusion

To conclude the definition of temporal-kinetic energy we form a simple equation:

$$(44)\ E_{tk} = \frac{\langle \widehat{Д}_{\twoheadrightarrow} \rangle}{Д}$$

Temporal-kinetic energy is uniform and smooth across space, its density is very low and it doesn't interact with other energy or matter on any observable scale, which is in accordance with the current observational evidence attained on this subject.

References

[1] S.W. Hawking & G.F.R. Ellis: The Large scale structure of space-time. Cambridge University Press (1973).

[2] R.M. Wald: General Relativity. University of Chicago Press (1984).

[3] G.F.R. Ellis & H.V. Elst: Cosmological Models. University of Cape Town (2008).

[4] A. Riotto: Inflation and the Theory of Cosmological Perturbations. INFN, Sezione di Padova, Padova, Italy.

[5] P. J. E. Peebles & Bharat Ratra: The Cosmological Constant and Dark Energy. Joseph Henry Laboratories, Princeton University, Princeton (2002).

[6] J.R. Primack: Dark matter and structure formation.