Quantitative Labor Theory of Value
---Marxian General Equilibrium Characterized by Transformation Model

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Abstraction: This research attempts to place the formal Sraffian model with linear production sets into a general equilibrium framework and to derive a quantitative transformation theorem about Marxian theory of labor value and production price. Marxian reproduction solution established a dynamic general economic equilibrium, which can be characterized by input-(total) output ratio, namely, the reduced Organic Composite of Capital divided by the total productivity rate. The labor value thus the value rate of profit (ROP) can be determined from the production price by the use of the input-output matrix analysis. The increased value ROP and the decreased price ROP of USA around 2006/2007 revealed that there was an OCC reduction. Under the framework of the dynamic Marxian general equilibrium, it is possible to undergo an optimal planning about an economic system by the regulation of the government input, entrepreneur taxation, and minimal wage rate.

Key words: value theory function, transformation problem, Marx production function, Marx eigenvalue, price eigenvector
JEL: E11, O47

1. Marxian productivity economics[1]

From the Marxian value theory equations system derived formerly,

$LabourValueTheoryEquantion:$

$Q = C + V + M = C + Y = B_0 e^{ft} C^{\beta} V^{1-\beta},$

$Q: TotalValue, C: ConstantCapital, V: VariableCapital, M: SurplusValue;$

$g \equiv \frac{C}{V}: \underline{O}rganic\ \underline{C}apital\ \underline{C}omposite;\ \beta \equiv \frac{g}{g+1}: reducedOCC;$

$1-\beta \equiv d_R, Roundabout\ ProductionDegree;\ f := (1-\beta)\dot{g}^* = \dot{\beta}^*, Productivity\ GrowthRate$

$CostFunction: Cv \equiv C + V = c_0 C^{\beta} V^{1-\beta}, g+1 = c_0 g^{\beta}$

[1] Erman ZENG: http://www.vixra.org/econ/1510.0333; see also: 曾尔曼，《马克思生产力经济学导引》，厦门大学出版社,(Erman ZENG: Introducing Marxian Productivity Economics, Xiamen Univ. Press), p.123, 2016

$$Marx\ Production Function:$$
$$Y = M + V = a_0 e^{Ft} C^{\alpha} V^{1-\alpha},$$
$$F := (1-\alpha)\dot{g}^*,\ Production Development Coefficient$$
$$\alpha \equiv \frac{g}{g+\gamma}, 1-\alpha \equiv d_L : Labor Division Degree,\ \gamma \equiv \frac{P'}{p'} = 1 + \frac{1}{p'}$$

the Marxian general equilibrium is obtained as a macro dynamic process:

$$Y/C = b_0 e^{Ft} g^{\alpha-1} \Rightarrow$$
$$\dot{Y} - \dot{C} = (1-\alpha)\dot{g}^* + (\alpha-1)\dot{g} = (1-\alpha)(\dot{g}^* - \dot{g});$$
$$Q/C = B_0 e^{ft} g^{\beta-1} \Rightarrow$$
$$\dot{Q} - \dot{C} \doteq (1-\beta)\dot{g} + (\beta-1)\dot{g}^* = (1-\beta)(\dot{g}^* - \dot{g});$$
$$\therefore \dot{g} = \dot{g}^* \Rightarrow \dot{Y}^* = \dot{C}^* = \dot{Q}^*$$

and also, the Marxian growth models: (a) optimal growth: $\dot{Y}^* = \dot{C}^* = \dot{Q}^*$; (b) steady state growth: $\dot{Y}^* = \dot{C}^* = \dot{Q}^* = \dot{V}^* = \dot{M}^* \Leftrightarrow \dot{y}^* = \dot{k}^* = \dot{w}^*, \dot{n}^* = \dot{g}^* = p = F = f = 0$; (c) turnpike growth: $\dot{Y}^* = \dot{C}^* = \dot{Q}^* = \frac{\dot{V}^*}{\beta}$. Besides, from

$$Surplus Value Theory Equantion:$$
$$M = b_0 e^{pt} C^{\beta} V^{1-\beta},$$
$$p := \frac{1-\alpha}{\alpha}\beta\dot{g}^*,\ Profit Growth Rate;$$
$$\alpha p = \beta F = \alpha\gamma f,\ p > F > f;$$
$$Suplus Value Rate: m' = M/V = b_0 e^{pt} g^{\beta}$$
$$\text{Profit Rate}: p' = M/Cv = \frac{m'}{g+1} = m'(1-\beta) = m' d_R = b_0 e^{pt}\frac{g^{\beta}}{g+1} = c_0^{-1} b_0 e^{pt}$$
$$\text{Productivity}: P' = Q/Cv = B_0 c_0^{-1} e^{ft} = p'+1 = (1-\alpha)(m'+1) = (m'+1)d_L$$

we see that the rate of profit (p') in terms of value might not tend to fall as long as either the rate of surplus value (m') in terms of value or the degree of the roundabout production (1-β) increases, so does the productivity (P') rate. So far, we are talking about the value system all the time, however, it is the production price system existed in reality; we cannot assure that the price ROP (r') behave the same way as the value ROP (p'):

$$r' = \frac{P_3 M}{P_1 C + P_2 V} = \frac{P_3}{P_2}\frac{m'}{gP+1} = p'\frac{P_3}{P_2}\frac{g+1}{g'+1},\ g' \equiv gP = \frac{P_1 C}{P_2 V}$$

2. Transformation Model[2]

If there is no inflation, the total values of input commodities represented by the production prices remain unchanged:

$$dN = d(P_1C + P_2V) = CdP_1 + P_1dC + P_2dV + VdP_2 = CdP_1 + VdP_2$$
$$= Cv[\beta dP_1 + (1-\beta)dP_2] = Cv[\beta\delta P_1 + (1-\beta)\delta P_2]$$
$$\cong Cv[\beta\ln(1+\delta P_1) + (1-\beta)\ln(1+\delta P_2)] = Cv[\beta\ln P_1 + (1-\beta)\ln P_2]$$
$$= Cv\ln(P_1^{\beta}P_2^{1-\beta}) = 0$$
$$\Rightarrow P_1^{\beta}P_2^{1-\beta} = 1,$$
$$P^{\beta} = 1/P_2,\ P \equiv P_1/P_2$$
$$\beta\dot{P}_1 + (1-\beta)\dot{P}_2 = 0,\ or:\ \beta\delta P_1 + (1-\beta)\delta P_2 = 0$$

$$M' = P_3M = b(P_1C)^{\beta}(P_2V)^{1-\beta} = MP_1^{\beta}P_2^{1-\beta},$$
$$\Rightarrow P_3 = P_1^{\beta}P_2^{1-\beta} = 1;$$
$$Q' = P_1C + P_2V + P_3M = B(P_1C)^{\beta}(P_2V)^{1-\beta} = QP_1^{\beta}P_2^{1-\beta}$$
$$\Rightarrow Q' = Q = C + V + M$$
$$\Rightarrow P_1C + P_2V = C + V;$$
$$M' = Q' - (P_1C + P_2V) = P_3M = M = p'(C+V) = r'(P_1C + P_2V),\ P_3 = 1$$
$$\Rightarrow p' = r'$$

therefore, total production prices equal to total values of labor and commodities; total profits equal to total surplus values; the price rate of profit (r’) equals the value rate of profit (p’). When there is some inflation:

$$P_1^{\beta}P_2^{1-\beta} = I = P^{\beta}P_2 = P_3 \neq 1,\ P \equiv \frac{P_1}{P_2}$$

$$r' = \frac{P_3M}{P_1C + P_2V} = \frac{P_3}{P_2}\frac{m'}{gP+1} = p'P^{\beta}\frac{g+1}{g'+1},\ g' \equiv gP = \frac{P_1C}{P_2V}$$

$$r \equiv \frac{dr'}{r'dt} = p + \beta\dot{P} + \beta\dot{g} - \beta'\dot{g}' = p + (\beta-\beta')\dot{g}' = \dot{m}' + \beta\dot{P} - \beta'\dot{g}',\ \beta' \equiv \frac{g'}{g'+1}$$

$$r < 0 \Leftrightarrow p < (\beta'-\beta)\dot{g}';$$

$$\beta'-\beta = \frac{g'}{g'+1} - \frac{g}{g+1} = \beta\frac{P-1}{g'+1} = P_1\left(\frac{1}{P_2} - \frac{1}{P_1}\right)\beta(1-\beta') < 0$$

$$\Leftrightarrow p = \frac{1-\alpha}{\alpha}\beta\dot{g}^* < P_1\left(\frac{1}{P_2} - \frac{1}{P_1}\right)\beta(1-\beta')\dot{g}'$$

$$\Leftrightarrow \dot{g}^* < \frac{\alpha}{1-\alpha}P_1\left(\frac{1}{P_2} - \frac{1}{P_1}\right)(1-\beta')\dot{g}' = \frac{\alpha}{1-\alpha}(P-1)(1-\beta')(\dot{g} + \dot{P})$$

Marx is right about the falling tendency of the price rate of profit, but not the value rate of profit.

[2] Samuelson,PA.: Understanding the Marxian notion of exploitation: a summary of the so-called transformation problem between Marxian values and competitive prices, *Jour. Econ. Liter.* 1971, 9, 399-431

At the same time, the labor value can be determined by the transformation model; the companying eigenvectors of a 3x3 matrix are the coefficients of price-value ratio of an economic system, which is divided into three production sectors, namely, first the production of the means of production (Department I), second the production of articles of consumption (Department II), and third the production of capital goods (Department III):

$$C_1 P_1 + V_1 P_2 + M_1 P_3 = Q_1 P_1$$

$$C_2 P_1 + V_2 P_2 + M_2 P_3 = Q_2 P_2$$

$$C_3 P_1 + V_3 P_2 + M_3 P_3 = Q_3 P_3$$

$$\begin{pmatrix} \frac{C_1}{Q_1} & \frac{V_1}{Q_1} & \frac{M_1}{Q_1} \\ \frac{C_2}{Q_2} & \frac{V_2}{Q_2} & \frac{M_2}{Q_2} \\ \frac{C_3}{Q_3} & \frac{V_3}{Q_3} & \frac{M_3}{Q_3} \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} \equiv \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \lambda \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}$$

so, the mean labor value of the three production departments can be calculated as following:

$$\begin{pmatrix} \frac{C'_1}{Q'_1} & \frac{V'_1}{Q'_1} & \frac{M'_1}{Q'_1} \\ \frac{C'_2}{Q'_2} & \frac{V'_2}{Q'_2} & \frac{M'_2}{Q'_2} \\ \frac{C'_3}{Q'_3} & \frac{V'_3}{Q'_3} & \frac{M'_3}{Q'_3} \end{pmatrix} \begin{pmatrix} \frac{1}{P_1} \\ \frac{1}{P_2} \\ \frac{1}{P_3} \end{pmatrix} \equiv \begin{pmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{pmatrix} \begin{pmatrix} \frac{1}{P_1} \\ \frac{1}{P_2} \\ \frac{1}{P_3} \end{pmatrix} = \lambda \begin{pmatrix} \frac{1}{P_1} \\ \frac{1}{P_2} \\ \frac{1}{P_3} \end{pmatrix},$$

$$w_i = \frac{V'_i}{P_2 L_i} = \frac{w'_i}{P_2}$$

3. Marxian General Equilibrium

In Marxian models, competition among firms may lead to an “equilibrium” characterized by equal profit rates or exploitation/surplus rates in all sectors. But here is no such guarantee that a dynamic economic process of price adjustment will converge to a uniform rate of surplus value or equal-profit-rate[3]. However, the Marxian general equilibrium state could be characterized by the input/(total) output ratio with the reduced organic composite of capital β divided by the productivity P’:

[3] Nikaido，H：Refutation of the dynamic equalization of profit rates in Marx’s scheme of reproduction”1978, Univ. South. Cal.；

$$Q = C + V + M = (C+V)(1+p') = C(1+\frac{1}{g})(1+p') = C\frac{1+p'}{\beta} = \frac{P'}{\beta}C$$

$$\Rightarrow C = \zeta Q, \zeta \equiv \frac{\beta}{P'} = \frac{\beta}{1+p'} = \frac{C}{Q} = \frac{C}{C+Y} = (\frac{Y}{C}+1)^{-1}$$

$$\frac{C}{Q} = \frac{Q-Y}{Q} = 1 - \frac{1-\beta}{1-\alpha} = \frac{\beta-\alpha}{1-\alpha} < \frac{C}{Y} = \frac{Q-Y}{Y} = \frac{1-\alpha}{1-\beta} - 1 = \frac{\beta-\alpha}{1-\beta}$$

$$\dot{\zeta} = \dot{C} - \dot{Q} = \dot{\beta} - \dot{P}' = (1-\beta)\dot{g} - f = \dot{\beta} - \dot{\beta}* = (1-\beta)(\dot{g} - \dot{g}*)$$

$$\left|\sum_j \frac{C'_{ij}}{P_i}\right| = \frac{\beta_i}{1+p'_i}\left|\frac{Q'_i}{P_i}\right| = \lambda\left|\frac{Q'_i}{P_i}\right| \Leftrightarrow \sum_j \left|\frac{C'_{ij}}{Q'_i}\right|\left|\frac{1}{P_i}\right| = \frac{\beta_i}{1+p'_i}\left|\frac{1}{P_i}\right| = \lambda\left|\frac{1}{P_i}\right|$$

Here we see ζ is much more suitable than the surplus value rate m' or the profit rate p' to characterize the economic equilibrium state since we can take advantage of the intermediate input coefficient matrix to calculate the eigenvalue ζ directly, besides when approaching the Marxian general equilibrium state, the value of ζ changed little.

$$\frac{1}{m'} = \frac{V}{M} = \frac{Y}{M} - 1 = \frac{\beta}{\alpha} - 1 = \frac{1}{p'(g+1)} = \frac{1-\beta}{p'};$$

$$\frac{C}{Y} = \frac{C}{V+M} = \frac{g}{1+m'} = \frac{g}{1+p'(1+g)} = \frac{\alpha}{p'}$$

$$Q = C + V + M = (C+V)(1+p') = V(g+1+m') = Vm'\gamma \Rightarrow$$

$$1)\ \frac{1}{1+p'} = \frac{1}{P'} = \frac{C+V}{Q} = \frac{Cv}{Q} > \frac{C}{Q}, -\dot{P}' = -f = -\dot{\beta}* = \dot{C}v - \dot{Q} = (\beta-1)\dot{g}*;$$

$$2)\ \frac{1}{m'} = \frac{V\gamma}{Q} = \frac{Y-M}{M} = \frac{\beta}{\alpha} - 1 = \frac{\beta-\alpha}{\alpha}, -\dot{m}' = -(p+\beta\dot{g})$$

By using the Matrix analysis technique upon the Input-Output Table, the eigenvalue of the intermediate input coefficients matrix could be obtained, and the reduced OCC β thus the degree of roundabout production(1-β)[4], as well as the value rate of profit p'. Similarly,

$$\frac{C}{Y} = \frac{\alpha}{p'} \rightarrow \left|\sum_j \frac{C'_{ij}}{P_i}\right| = \frac{\alpha_i}{p'_i}\left|\frac{Q'_i - C'_i}{P_i}\right| = \frac{\alpha_i}{p'_i}\left|\frac{V'_i + M'_i}{P_i}\right| = \xi\left|\frac{Y'_i}{P_i}\right|$$

$$\sum_j \left|\frac{C'_{ij}}{Y'_i}\right|\left|\frac{1}{P_i}\right| = \frac{\alpha_i}{p'_i}\left|\frac{1}{P_i}\right| = \xi\left|\frac{1}{P_i}\right|$$

from the eigenvalue of the intermediate input coefficients matrix, the elasticity of the capital production α thus the degree of labor division[5] could also be obtained, and so on as well as OCC and ROP:

[4] Young AA. Increasing Returns and Economic Progress [J]. The Economic Journal, 1928, 38: 527-42.

[5] Adam Smith : The Wealth of Nations (Bantam Classics),2003

$$\frac{C}{M}=\frac{\beta}{p'}\to\left|\sum_j\frac{C'_{ij}}{P_i}\right|=\frac{\beta_i}{p'_i}\left|\frac{M'_i}{P_i}\right|\Leftrightarrow\sum_j\left|\frac{C'_{ij}}{M'_i}\right|\left|\frac{1}{P_i}\right|=\frac{\beta_i}{p'_i}\left|\frac{1}{P_i}\right|=\mu\left|\frac{1}{P_i}\right|$$

$$\frac{C}{V}=g=\frac{\beta}{1-\beta}\to\left|\sum_j\frac{C'_{ij}}{P_i}\right|=\frac{\beta_i}{1-\beta_i}\left|\frac{V'_i}{P_i}\right|\Leftrightarrow\sum_j\left|\frac{C'_{ij}}{V'_i}\right|\left|\frac{1}{P_i}\right|=\frac{\beta_i}{1-\beta_i}\left|\frac{1}{P_i}\right|=\upsilon\left|\frac{1}{P_i}\right|$$

$$Q=C+V+M=(C+V)(1+p')=V(1+g)(1+p')=V\frac{1+p'}{1-\beta}\Rightarrow$$

$$\frac{V'_i}{P_{2i}}=\frac{1-\beta_i}{1+p_i'}\frac{Q'_i}{P_i}=\left(\frac{1}{1+p_i'}-\zeta\right)\frac{Q'_i}{P_i};$$

$$\sum_j\frac{C'_{ij}}{P_j}+\frac{V'_i}{P_{2i}}=\frac{1}{1+p_i'}\frac{Q'_i}{P_i}=\frac{1}{p_i'}\frac{M'_i}{P_{3i}};$$

$$\sum_j\frac{C'_{ij}}{P_j}+\frac{V'_i}{P_{2i}}+\frac{M'_i}{P_{3i}}=\frac{Q'_i}{P_i}=\frac{M'_i}{P_{3i}}\frac{1+p_i'}{p_i'}.$$

According to the Germany 2000 input-output matrix[6] (7-sectoral aggregation, Table 1), the eigenvalue ζ equals 0.510, the companying (value-price transformation) eigenvector is: [0. 345, 0. 638, 0. 474, 0. 391, 0. 189, 0. 196, 0. 149]^T.

Table 1.(Germany)	1	2	3	4	5	6	7
1. Agri.	0. 028	0	0. 045	0	0	0. 002	0. 002
2. ManuExpo	0. 09	0. 282	0. 05	0. 022	0. 003	0. 008	0. 011
3. OthManu	0. 142	0. 232	0. 324	0. 287	0. 03	0. 055	0. 065
4. Cnst.	0. 007	0. 003	0. 006	0. 017	0. 006	0. 028	0. 016
5. BizSer	0. 142	0. 121	0. 14	0. 107	0. 332	0. 134	0. 096
6. CnsmSer	0. 036	0. 053	0. 051	0. 108	0. 072	0. 152	0. 049
7. Social Se	0. 031	0. 006	0. 011	0. 007	0. 007	0. 013	0. 024

According to the UK 2000 input-output data[7] (change from 123x123 to 3x3, Table 2), the eigenvalue of ζ equals to 0.537, the companying (value-price transformation) eigenvector is: [0. 8906 0. 3504 0. 2898]^T, the value rate of profit (ROP) p'=8.55%, the price ROP r'=20.7%, β=ζ*(1+p')=0.537*1.0855=0.583, α=λ*p'=1.1899*0.0855=0.102:

[6] Flaschel,P: 《Topics in Classical Micro- and Macroeconomics: Elements of a Critique of Neoricardian Theory》, p.64, Springer, 2010

[7] https://data.gov.uk/dataset/input-output_supply_and_use_tables

Table 2. (UK)	Ic	IIv	IIIm			
Ic	10289. 57	2095. 598	1683. 799	0. 016655	0. 103059	0. 001421
IIv	291327. 8	5920. 842	137581. 3	0. 471564	0. 291179	0. 116074
IIIm	77415. 61	3516. 56	455519. 9	0. 125311	0. 17294	0. 38431
V	140424	3177	381289	0. 61353	0. 567178	0. 501805
M	98333	5624	209218	0. 22730	0. 156241	0. 321684
O	617790	20334	1185292	0. 15917	0. 276581	0. 176512

According to the PRC 2000 input-output direct consumption coefficient data[8](6x6 to 3x3, Table 3):

Table 3 (PRC)	Agr	Ind	Con	T	Bus	Other
Agric	0.1525828	0.0577784	0.0038685	0.0011993	0.0537537	0.0073763
Indus	0.2047609	0.5685721	0.5426612	0.3459779	0.2759972	0.229536
Const	0.002155	0.0009704	0.0005986	0.0196918	0.0043322	0.0282213
T&T	0.0139507	0.0243611	0.0694626	0.0390754	0.0314647	0.0656174
Busi	0.0190034	0.0405975	0.0650467	0.019845	0.0863221	0.0394385
Other	0.0292091	0.0258501	0.0501379	0.0589961	0.1138938	0.1140228

the eigenvalue of ζ equals to 0.658, changed to a 3x3 table:

0.7696	0.5042	0.6025
0.1206	0.4024	0.2815
0.1098	0.0933	0.1160

the companying (value-price transformation) eigenvector is: $(0.954, 0.261, 0.146)^T$, the value rate of profit p'=2.20% the price rate of profit r'=12.9%， β=ζ*(1+p')= 0.673.

According to the USA 1997-2014 input-output data[9] (15x15 to 3x3, Table 4), in the year 2000 the eigenvalue of ζ equals to 0.489：

[8] 刘起运等编著，《投入产出分析》，人大版，2006，p.163

[9] http://www.bea.gov/industry/io_annual.htm

Table 4.		11	21	22	23	31G	42	44RT	48TW	51	FIRE	PROF	6	7	81	G
	λ	Agric	Minin	Utili	Const	Manuf	Whole	Retai	Trans	Infor	Finan	Prof	Educ	Arts	Other	Govern.
2014	**0.485**	0.4364	0.1773	0.1735	0.2956	**0.4964**	0.1413	0.1544	0.3348	0.2270	**0.1185**	0.1586	0.1909	0.2433	0.1786	0.2000
2013	**0.477**	0.4171	0.1636	0.1727	0.3065	**0.4977**	0.1414	0.1605	0.3389	0.2283	**0.1221**	0.1606	0.1946	0.2455	0.1791	0.2102
2012	**0.481**	0.4807	0.1558	0.1500	0.3017	**0.4942**	0.1327	0.1467	0.3235	0.2247	**0.1045**	0.1469	0.1834	0.2355	0.1662	0.2083
2011	**0.478**	0.4452	0.1551	0.1567	0.3110	**0.4967**	0.1417	0.1527	0.3291	0.2192	**0.1160**	0.1526	0.1927	0.2482	0.1723	0.2117
2010	**0.463**	0.4784	0.1548	0.1738	0.3136	**0.4851**	0.1306	0.1546	0.3007	0.2034	**0.1255**	0.1530	0.1926	0.2471	0.1717	0.2092
2009	**0.447**	0.5229	0.1260	0.1628	0.3206	**0.4783**	0.1000	0.1361	0.2798	0.2027	**0.1293**	0.1510	0.1890	0.2444	0.1622	0.2085
2008	**0.492**	-0.4683	-0.1664	-0.2360	-0.3257	**-0.482**	-0.1294	-0.1367	-0.3143	*-0.1738*	**-0.132**	*-0.135*	-0.183	-0.2335	-0.1672	-0.2044
2007	**0.49**	-0.4614	-0.1786	-0.2261	-0.3201	**-0.494**	-0.1262	-0.1378	-0.3116	-0.1845	**-0.131**	-0.143	-0.186	-0.2293	-0.1600	-0.2008
2006	**0.484**	0.4518	0.2100	0.2120	0.3321	**0.4931**	0.1296	0.1350	0.2896	0.2007	**0.1371**	0.1420	0.1913	0.2301	0.1579	0.2020
2005	**0.493**	0.4320	0.2351	0.2509	0.3280	**0.4815**	0.1367	0.1401	0.2848	0.1790	**0.1519**	0.1463	0.2016	0.2350	0.1591	0.1964
2004	**0.473**	-0.4090	-0.2505	-0.2055	-0.3496	**-0.501**	-0.1344	-0.1435	-0.2692	-0.1964	**-0.138**	-0.143	-0.198	-0.2410	-0.1633	-0.2003
2003	**0.466**	0.4398	0.2467	0.2125	0.3430	**0.4957**	0.1275	0.1304	0.2499	0.2176	**0.1209**	0.1381	0.1992	0.2404	0.1582	0.1946
2002	**0.467**	-0.4634	-0.2187	-0.1925	-0.3412	**-0.505**	-0.1324	-0.1277	-0.2439	-0.2236	**-0.109**	-0.134	-0.198	-0.2391	-0.1484	-0.1915
2001	**0.477**	-0.4479	-0.2341	-0.2544	-0.3252	**-0.493**	-0.1165	-0.1196	-0.2293	-0.2476	**-0.110**	-0.1391	-0.197	-0.2406	-0.1553	-0.1909
2000	**0.489**	-0.4301	-0.2612	-0.2337	-0.3268	**-0.492**	-0.1226	-0.1312	-0.2481	-0.2556	**-0.123**	-0.146	-0.195	-0.2373	-0.1354	-0.1854
1999	**0.486**	-0.4651	-0.2341	-0.1781	-0.3439	**-0.510**	-0.1184	-0.1259	-0.2355	-0.2102	**-0.111**	-0.145	-0.194	-0.2532	-0.1409	-0.1811
1998	**0.487**	-0.4431	-0.2400	-0.1565	-0.3518	**-0.523**	-0.1064	-0.1147	-0.2250	-0.2066	**-0.110**	-0.147	-0.202	-0.2792	-0.1440	-0.1819
1997	**0.489**	-0.4310	-0.2342	-0.1379	-0.3607	**-0.523**	-0.1119	-0.1220	-0.2564	-0.2022	**-0.103**	-0.134	-0.1971	-0.2869	-0.1404	-0.1818

the value ROP p'=15.8% the price ROP r'=28.8% ， β=ζ*(1+p')= 0.566. Detailed analyses reveal that, around year 2007/2008, the price ROP and the value ROP underwent a little different trajectory, theoretical derivation explains that USA was in the transition of economic structure transformation:

$$r \equiv \frac{dr'}{r'dt} = p + \beta\dot{P} + \beta\dot{g} - \beta'\dot{g}' = p + (\beta - \beta')\dot{g}' = \dot{m}' + \beta\dot{P} - \beta'\dot{g}', \beta' \equiv \frac{g'}{g'+1}$$

$$if : r < 0, \& \ p = \dot{m}' - \beta\dot{g} > 0 \Leftrightarrow \beta'\dot{g}' - \beta\dot{P} > \dot{m}' > \beta\dot{g} \Leftrightarrow (\beta - \beta')\dot{g}' < 0;$$

$$\because \beta - \beta' = \frac{g}{g+1} - \frac{g'}{g'+1} = \beta\frac{1-P}{g'+1} = P_1(\frac{1}{P_1} - \frac{1}{P_2})\beta(1-\beta') > 0$$

$$\therefore \dot{g}' = \dot{g} + \dot{P} < 0; if : \dot{P} = 0, \dot{g} < 0$$

Table 5	1/Pc	1/Pv	1/Pm	p' (%)	r'	β	l	g	g'
1997	0.8245	0.4009	0.3994	19.2	31.2	0.582	1.639	1.418	2.917
1998	0.8299	0.4194	0.3679	16.9	30.3	0.569	1.617	1.383	2.737
1999	0.8356	0.4108	0.3647	16.7	30.1	0.567	1.627	1.390	2.828
2000	0.8562	0.3724	0.3581	15.8	28.8	0.566	1.680	1.412	3.247
2001	0.8113	0.3914	0.4343	20.2	29.5	0.573	1.683	1.353	2.804
2002	0.8527	0.3799	0.3585	17.2	31.1	0.547	1.690	1.333	2.992
2003	0.8074	0.4018	0.4320	21.5	31.6	0.567	1.675	1.345	2.703
2004	0.8113	0.3914	0.4343	21.5	31.5	0.575	1.680	1.388	2.877
2005	0.8150	0.3713	0.4448	22.2	31.6	0.602	1.678	1.460	3.205
2006	0.7914	0.3841	0.4755	24	31.6	0.6	1.687	1.460	3.008
2007	0.7820	0.3851	0.4900	24.5	31.1	0.61	1.686	1.466	2.978
2008	0.8661	0.3372	0.3689	17.2	30.5	0.577	1.721	1.497	**3.844**
2009	0.8311	0.3723	0.4131	22.4	34.4	0.588	1.680	1.313	2.932
2010	0.7477	0.3976	0.5319	30.2	34.1	0.603	1.719	1.386	2.607
2011	0.8407	0.3383	0.4229	22.2	33.3	0.583	-1.739	1.452	3.608
2012	0.8515	0.3372	0.4016	20.8	33.3	0.581	-1.731	1.451	3.665
2013	0.8373	0.3412	0.4273	22.5	33.4	0.588	-1.730	1.459	3.580
2014	0.8446	0.3418	0.4121	21.3	33.1	0.588	-1.720	1.454	3.593

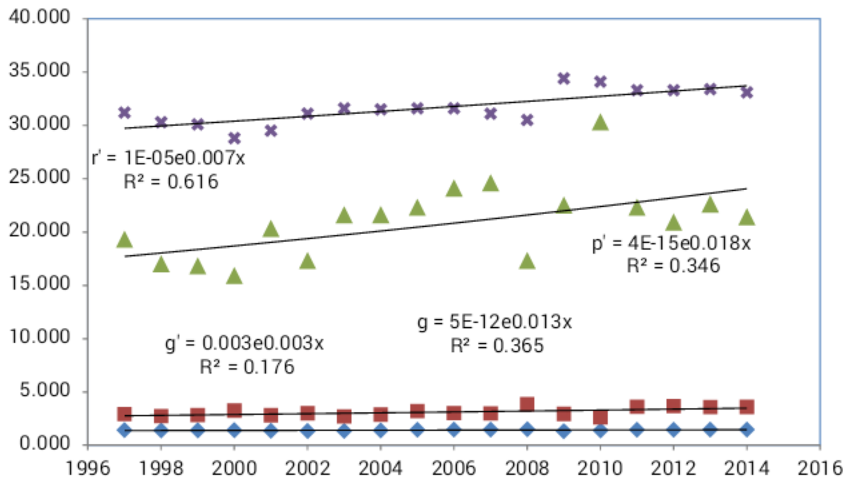

Fig.1 Analysis of USA's price ROP r' vs. value ROP p'

In 2007/2008, there is a changing of the price OCC (organic composite of capital) in the states, the price ROP fell while the value ROP rose, which coincides with the results about the productivity analysis [10]:

$$F = \dot{Y} - \dot{C} - (1-\alpha)\dot{g} = \dot{Y} - \dot{C} - \frac{p'+1}{m'+1}\dot{g}$$

[10] 曾尔曼，《马克思生产力经济学导引》，p.169，厦门大学出版社，2016

Table[6] USA	m'	P/T	p'	(p')net	g	F
1997	0.78	5.34	0.32	0.27	1.46	0
1998	0.75	5.24	0.31	0.26	1.43	-0.014
1999	0.75	5.36	0.31	0.26	1.43	-0.002
2000	0.72	5.27	0.29	0.25	1.44	-0.018
2001	0.72	5.43	0.3	0.25	1.39	0.01
2002	0.74	5.27	0.32	0.27	1.35	0.019
2003	0.74	5.27	0.32	0.27	1.36	0
2004	0.77	5.31	0.32	0.27	1.41	0.006
2005	0.79	5.39	0.32	0.27	1.47	-0.004
2006	0.79	5.3	0.32	0.27	1.48	0
2007	0.78	5.34	0.31	0.26	1.5	-0.005
2008	0.77	5.3	0.31	0.26	1.52	-0.012
2009	0.78	5.39	0.33	0.28	1.36	0.036
2010	0.82	5.57	0.34	0.29	1.41	0.01
2000-10:		β=0.467	f=0.0025	α=0.038	F=0.0045	P=0.010

4. Optimal Economic Planning

According to Marxian general equilibrium, which is indeed of macro dynamic, the growth rates of both departments should equal:

$$Q_1 = C_1 + \underline{V_1 + M_1} = C_1 + \underline{Y_1} = C_1 + \underline{C_2} = C$$

$$Q_2 = Y = \underline{C_2} + V_2 + M_2 = \underline{C_2} + Y_2 = \underline{Y_1} + Y_2 = Y$$

$$Q = C + V + M = C + Y = Q_1 + Q_2$$

$$Y / C = b_0 e^{Ft} g^{\alpha - 1} \Rightarrow$$

$$\dot{Y} - \dot{C} = (1-\alpha)\dot{g}^* + (\alpha - 1)\dot{g} = (1-\alpha)(\dot{g}^* - \dot{g});$$

$$Q / C = B_0 e^{ft} g^{\beta - 1} \Rightarrow$$

$$\dot{Q} - \dot{C} \doteq (1-\beta)\dot{g} + (\beta - 1)\dot{g}^* = (1-\beta)(\dot{g}^* - \dot{g});$$

$$\therefore \dot{g} = \dot{g}^* \Rightarrow \dot{Y}^* = \dot{C}^* = \dot{Q}^*$$

Therefore, there could be a policy regulation among the government input, namely constant capital ($C' = C - \delta_k$), and the income ($Y' = Y + \delta_k$) distribution—the wage rate ($\delta_1 / L = \delta_w$), and the surplus value ($\delta_M = \delta_2 + \delta$) including government taxation δ and entrepreneur profit δ_2:

$$\dot{Y}=\frac{V\dot{V}+M\dot{M}}{Y}=\frac{Y-M}{Y}\dot{V}+\frac{P\dot{P}+T\dot{T}}{Y}, M=P+T=T(\kappa+1), \kappa\equiv P/T$$

$$\dot{Y}=(1-\frac{\alpha}{\beta})\dot{V}+\frac{\alpha}{\beta}(\frac{\dot{P}}{1+\kappa^{-1}}+\frac{\dot{T}}{1+\kappa});$$

$$Q\dot{Q}=C\dot{C}+Y\dot{Y}, C'=C-\delta_k, Y'=Y+\delta_k=V'+M'=V'+P'+T',$$

$$\dot{Q}'=\frac{C'}{Q}\dot{C}+\frac{Y'}{Q}\dot{Y}=\dot{C}'=\dot{Y}'\Rightarrow\dot{Q}'=\frac{C_t-\delta_k}{Q_t}\dot{C}+\frac{Y+\delta_k}{Q_t}\dot{Y}=\dot{Q}+\frac{\dot{Y}-\dot{C}}{Q_t}\delta_k=\dot{C}'=\dot{C}-\frac{\delta_k}{C_t}$$

$$\Rightarrow\delta_k=\frac{\dot{C}-\dot{Q}}{\frac{\dot{Y}-\dot{C}}{Q_t}+\frac{1}{C_t}};$$

$$\dot{Y}'=\frac{\Delta Y_t}{Y_t\Delta t}=(1-\frac{\alpha}{\beta})\frac{\Delta V+\delta_1}{V\Delta t}+\frac{\alpha}{\beta}(\frac{\frac{\Delta P_t+\delta_2}{P_t\Delta t}}{1+\kappa_t^{-1}}+\frac{\frac{\Delta T_t+\delta}{T_t\Delta t}}{1+\kappa_t}),$$

$$\delta_1+\delta_2+\delta=\delta_k$$

$$\frac{\delta}{T_t}=\dot{Y}-\dot{T}+\beta\dot{g}*(\frac{1}{\alpha}-1-\frac{1}{\beta})=\dot{Y}-\dot{T}+p\frac{\alpha}{1-\alpha}(\frac{1}{\alpha}-1-\frac{1}{\beta})$$

$$\rightarrow\delta=T_t[\dot{Y}-\dot{T}+p(1-\frac{\alpha}{\beta(1-\alpha)})](if:\alpha>0.5,\frac{1}{\alpha}-1-\frac{1}{\beta}<0);$$

$$\dot{Y}'\underline{\underline{\Delta t=1}}(1-\frac{\alpha}{\beta})(\frac{\delta_1}{V_t}+\dot{V})+\frac{\alpha}{\beta}(\frac{\dot{P}+\frac{\delta_2}{P_t}}{1+\kappa_t^{-1}}+\frac{\dot{T}+\frac{\delta}{T_t}}{1+\kappa_t})$$

$$=(1-\frac{\alpha}{\beta})(\dot{V}+\frac{\delta_1}{V_t})+\frac{\alpha}{\beta}(\frac{\dot{P}}{1+\kappa_t^{-1}}+\frac{\dot{T}}{1+\kappa_t})+\frac{\alpha}{\beta}(\frac{\delta_2+\delta}{M_t})$$

$$=\dot{Y}+(1-\frac{\alpha}{\beta})\frac{\delta_1}{V_t}+\frac{\alpha}{\beta}(\frac{\delta_k-\delta_1}{M_t})=\dot{C}'=\dot{C}-\frac{\delta_k}{C_t}$$

$$\rightarrow\delta_1=\frac{\dot{C}-\dot{Y}-\delta_k(\frac{1}{C_t}+\frac{\alpha}{\beta}\frac{1}{M_t})}{(1-\frac{\alpha}{\beta})\frac{1}{V_t}-\frac{\alpha}{\beta}(\frac{1}{M_t})}$$

Analysis of PRChina's input-output data (1997-2012)[11] showed: $\alpha=0.636$, $\beta=0.890$, $p=-0.00679<0$, at year 2010: V_{2010}=1.91^10(k￥), M_{2010}=1.573^10(k￥), T_{2010}=5.99^8, L=7.61^9, k_{2010}=1.626, d_1=1854^8>0, d_w=244，d_2=1458^8，d_k=3657^8，$\dot{C}=0.152>\dot{Q}=0.148>\dot{Y}=0.137\le\dot{T}=0.138, \dot{M}=0.154, \dot{P}=0.165, \dot{V}=0.126$ **:**

[11] 曾尔曼：《马克思生产力经济学导引》，2016,p158，厦门大学出版社.

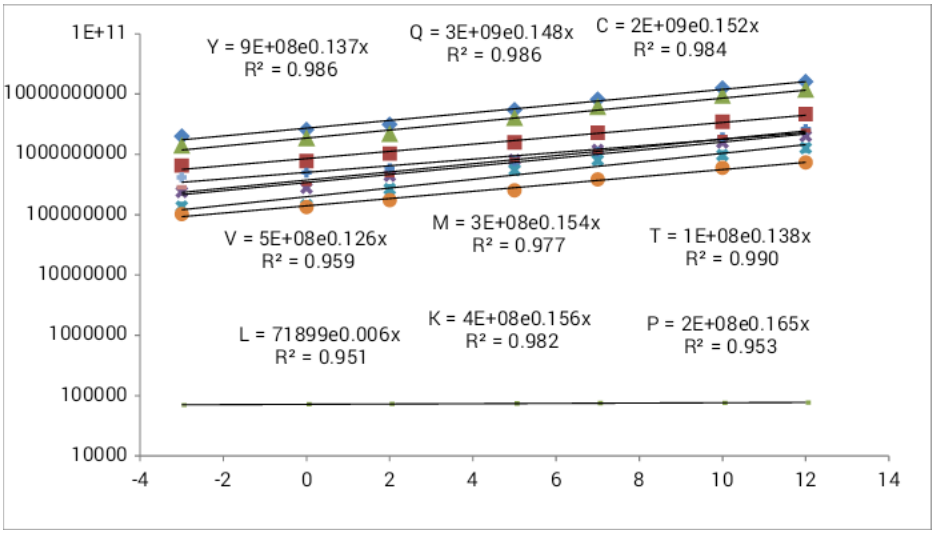

Fig.2 Analysis of PRChina's input-output data (1997-2012)

$d_1>0$, $d_2>0$， $d=d_k-(d_1+d_2)=344\text{^}8(RMB)>0$:

Table 7 PRC	δ2	δ(10κ)	δ1	δk	δw
2012	19983573	4226791	21739075	45949439	283.42
2010	14583165	3440343	18543975	36567483	243.66
2007	11710888	2211914	9937937	23860738	131.94
2005	8517318	1454998	5735870	15708186	76.84
2002	5034615	1002757	2460647	8498019	33.579
2000	2636323	770200	3862052	7268575	53.576
1997	2541417	588307	2303198	5432923	32.988

5. Conclusion

Marxian reproduction solution established an economic equilibrium, which can be characterized by input-(total) output ratio, namely, the reduced Organic Composite of Capital divided by the total productivity. The labor value can be determined from the production price by the use of the input-output matrix analysis. The value ROP and the price ROP analyses of input-output data provide an accurate description on the OCC change, which reflects the industry structure adjustment. Under the framework of the dynamic Marxian general equilibrium, it is possible to undergo an optimization about an economic system by the regulation of the input, taxation, and minimal wage rate, so as to realize the development of the productivity of the society.