New coordinate solution in Cosmology

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ABSTRACT

In the general relativity theory, we discover new solution in cosmology by Einstein's gravity field equation. We investigate a light by the new coordinate observer's time.

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1.Introduction

We solve new solution in the cosmology by gravity field equation.

The spherical coordinate is

$$d\tau^2 = W(r,t)dt^2 - \frac{1}{c^2}[U(t,r)dr^2 + V(t,r)\{d\theta^2 + \sin^2\theta d\phi^2\}] \quad (1)$$

In this time, Einstein's gravity equation is

$$R_{tt} = \frac{1}{2}\frac{\ddot{U}U - \dot{U}^2}{U^2} + \frac{1}{2}\frac{-W''U + W'U'}{U^2} + \frac{1}{4}\frac{W'^2}{UW} + \frac{\dot{U}^2}{4U^2} - \frac{\dot{U}\dot{W}}{4UW} - \frac{1}{4}\frac{U'W'}{U^2}$$
$$+ \frac{\ddot{V}V - \dot{V}^2}{V^2} + \frac{1}{2}\frac{\dot{V}^2}{V^2} - \frac{\dot{W}\dot{V}}{2WV} - \frac{W'V'}{2UV} \quad (2)$$

$$R_{rr} = \frac{1}{2}\frac{W''W - W'^2}{W^2} - \frac{\ddot{U}W - \dot{U}\dot{W}}{2W^2} + \frac{W'^2}{4W^2} + \frac{\dot{U}^2}{4UW} - \frac{\dot{U}\dot{W}}{4W^2} - \frac{U'W'}{4WU}$$
$$+ \frac{V''V - V'^2}{V^2} + \frac{1}{2}\frac{V'^2}{V^2} - \frac{U'V'}{2UV} - \frac{\dot{U}\dot{V}}{2WV} \quad (3)$$

$$R_{\theta\theta} = \frac{1}{2}\frac{-\ddot{V}W + \dot{V}\dot{W}}{W^2} - \frac{\dot{W}\dot{V}}{4W^2} + \frac{W'V'}{4UW} + \frac{V''U - V'U'}{2U^2} - \frac{\dot{U}\dot{V}}{4UW} + \frac{U'V'}{4U^2} - 1 \quad (4)$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta} \quad (5)$$

$$R_{tr} = \frac{\dot{V}'}{V} - \frac{\dot{V}V'}{2V^2} - \frac{\dot{U}V'}{2UV} - \frac{W'\dot{V}}{2WV} \quad (6)$$

In this time, $A' = \frac{\partial A}{\partial r}, \dot{A} = \frac{1}{c}\frac{\partial A}{\partial t}$

2. New solution in cosmology

We think

$$W(r,t) = g(r),\ U(t,r) = 1,\ V(t,r) = C_1c^2t^2 \quad (7)$$

In vacuum, Eq(6) is

$$R_{tr} = \frac{\dot{V}'}{V} - \frac{\dot{V}V'}{2V^2} - \frac{\dot{U}V'}{2UV} - \frac{W'\dot{V}}{2WV} = -\frac{g'}{2g}\frac{2}{t} = 0,\ g' = 0 \to g = 1$$

In vacuum, Eq(2)-(4) is

$$R_{tt} = \frac{1}{2}\frac{-W''U}{U^2} + \frac{1}{4}\frac{U'W'}{U^2} + \frac{1}{4}\frac{W'^2}{UW} + \frac{\ddot{V}}{V^2} - \frac{1}{2}\frac{\dot{V}^2}{V^2}$$
$$= \frac{2}{t^2} - \frac{1}{2}(\frac{2}{t})^2 = 0 \quad (8)$$

$$R_{rr} = -\frac{\dot{U}\dot{V}}{2WV} = 0 \quad (9)$$

$$R_{\theta\theta} = \frac{1}{2}\frac{-\ddot{V}W}{W^2} - 1 = -\frac{1}{2}(2C_1) - 1 = 0 \rightarrow C_1 = -1 \tag{10}$$

Therefore, new solution is in vacuum in cosmology

$$ds^2 = -c^2 d\tau^2 = -c^2 dt^2 + dr^2 - c^2 t^2 (d\theta^2 + \sin^2\theta d\phi^2) \tag{11}$$

In the solution, the density ρ and the pressure p are zero. Energy-momentum tensor $T_{\mu\upsilon} = 0$ .In cosmology, if the solution satisfy this condition, we think the solution is the light coordinate solution or the static universe solution.

If the observer on the coordinate solution see a light

$$ds^2 = -c^2 d\tau^2 = -c^2 dt^2 + dr^2 - c^2 t^2 (d\theta^2 + \sin^2\theta d\phi^2) = 0 \tag{12}$$

$$c^2 dt^2 = dr^2 - c^2 t^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$ct = \int \sqrt{dr^2 - c^2 t^2 (d\theta^2 + \sin^2\theta d\phi^2)} \tag{13}$$

Therefore, the time of the observer on the coordinate solution is

$$t = \frac{1}{c}\int \sqrt{dr^2 - c^2 t^2 (d\theta^2 + \sin^2\theta d\phi^2)} \tag{14}$$

3. Conclusion

Therefore, new spherical solution in the cosmology is

$$d\tau^2 = dt^2 - \frac{1}{c^2} dr^2 + t^2 (d\theta^2 + \sin^2\theta d\phi^2) \tag{15}$$

Eq(15) is new coordinate solution.

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