

Gedankenexperiment for looking at 5th force arguments for contribution for a Pre Planckian inflaton. With consequences.

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Abstract. Using work by Fishbach and Talmadge, we use 5th force arguments and the common Virial theorem, plus a modified Wheeler De Witt wave function to set up an energy fluctuation equation. From there, we also use a modified Heisenberg Uncertainty principle to isolate the role of an inflaton in Pre Planckian space – time. Its consequences are remarked upon **with λ in the range of a purported 5th force and α scaled as between 10^{-1} to 10^{-3} .**

1. Looking at the virial theorem

We begin via first re stating the Gasiorowitz version of the Virial theorem [1] which is rendered as

$$2\langle K.E. \rangle = \left\langle \frac{p^2}{m} \right\rangle = \langle r \cdot \nabla V(r) \rangle \quad (1)$$

I.e. we will attempt to connect this, with the 5th force potential as given by [2,3] as

$$V(r) = -\frac{G_\infty m_i m_j}{r} \cdot [1 \mp \alpha \cdot e^{-r/\lambda}] \quad (2)$$

Here, in the early universe, we are assuming, using [3]

$$\begin{aligned} m_i m_j &\xrightarrow{\text{Pre-Planckian}} m_{\text{graviton}}^2 \cdot 10^{2\mathfrak{S}}; 0 < \mathfrak{S} < 30 \\ G_\infty &\xrightarrow{\text{Pre-Planckian}} G \\ \alpha &\xrightarrow{\text{Pre-Planckian}} G \cdot (10^{-1} - 10^{-3}) \\ V(r) &= -\frac{G \cdot m_{\text{graviton}}^2 \cdot 10^{2\mathfrak{S}}}{r} \cdot [1 \mp (10^{-1} - 10^{-3}) \cdot e^{-r/\lambda}] \end{aligned} \quad (3)$$

The above explicitly uses the Fifth force charges $Q_i = f_M \cdot m_i$

Next, we will look at a wave function with the real part as given by [4], in the Wheeler De Witt approximation [4]

$$\begin{aligned} \Psi\left(\frac{r}{r_0}\right) &= \frac{N_0}{2A_1 \cdot \left[\left(\frac{r}{r_0}\right)^4 - \left(\frac{r}{r_0}\right)^2\right]^{1/2}} \cdot \exp\left[\frac{A_1}{3} \cdot \left[\left(\frac{r}{r_0}\right)^2 - 1\right]^{3/2}\right] \\ A_1 &= \frac{9\pi c^5}{2\hbar G \Lambda} \end{aligned} \quad (4)$$

While we are at this, we will also use [5] Here, we will be taking into account, the issues in [9] as to symmetry breaking, by a change in the HUP.

Then, set from [8], i.e. Begin with the starting point of [5,6,7, 8,9]

$$\Delta l \cdot \Delta p \geq \frac{\hbar}{2} \quad (5)$$

We will be using the approximation given by Unruh [6,7] , of a generalization we will write as

$$\begin{aligned} (\Delta l)_{ij} &= \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \\ (\Delta p)_{ij} &= \Delta T_{ij} \cdot \delta t \cdot \Delta A \end{aligned} \quad (6)$$

If we use the following, from the Roberson-Walker metric [9].

$$\begin{aligned} g_{tt} &= 1 \\ g_{rr} &= \frac{-a^2(t)}{1-k \cdot r^2} \\ g_{\theta\theta} &= -a^2(t) \cdot r^2 \\ g_{\phi\phi} &= -a^2(t) \cdot \sin^2 \theta \cdot d\phi^2 \end{aligned} \quad (7)$$

Following Unruh [6,7] , write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110}, r \equiv l_p \sim 10^{-35} \text{ meters} \quad (8)$$

Then, if $\Delta T_{tt} \sim \Delta \rho$

$$\begin{aligned} V^{(4)} &= \delta t \cdot \Delta A \cdot r \\ \delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} &\geq \frac{\hbar}{2} \\ \Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} &\geq \frac{\hbar}{V^{(4)}} \end{aligned} \quad (9)$$

This Eq.(9) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time [5]

$$T_{ii} = \text{diag}(\rho, -p, -p, -p) \quad (10)$$

Then [5]

$$\Delta T_{tt} \sim \Delta \rho \sim \frac{\Delta E}{V^{(3)}} \quad (11)$$

Then, Eq. (9) and Eq. (10) and Eq. (11) together yield

$$\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \quad (12)$$

Unless $\delta g_{tt} \sim O(1)$

And we will be using the Giovannini approximation of [5,9]

$$g_{tt} = \phi \cdot a_{\min}^2 \quad (13)$$

Finally, we will use the results from Padmanabhan [10] as to a scalar inflation field, and a given cosmological potential

$$\begin{aligned} a(t) &= a_{\text{initial}} t^\gamma \\ V(\phi) &= V_0 \exp \left[-\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi \right] \\ \phi(t) &= \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left[\sqrt{\frac{8GV_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right] \end{aligned} \quad (14)$$

In short, Eq. (14), Eq. (13), Eq. (12), Eq. (4), Eq. (3) and Eq. (1) will all be combined, inter related and used as the focus of a way to represent the inflaton in Pre Planck time physics, this with [11]

$$m(\text{graviton, rest}) \sim 10^{-54} \quad (15)$$

While doing this, with a low mass for a 'massive graviton' we will be looking at [12,13, 14] as far as restraints, as well as comparing our results to that given by [15,16]

Here, we are assuming one half the value of Planck time, i.e. about $\Delta t \sim \frac{t_{\text{Planck}}}{2} = .5$

And dr/dt set = 1 $r_0 \sim l_{\text{Planck}} = 1$, and a temperature $T^4 \sim T_{\text{Re-Heat}}^4 \sim \Delta t^{-2} m_{\text{Planck}}^2 \equiv \Delta t^{-2}$

$$\begin{aligned}
\Delta E \Delta t &\sim 1 / \phi(\text{initial}) \cdot a_{\text{initial}}^2 \\
\Rightarrow \phi(\text{initial}) &\sim 1 / \Delta E \Delta t \cdot a_{\text{initial}}^2 \\
&\&\Delta E \sim \partial_t S \sim m(\text{val}) \cdot \exp \left[\frac{1}{2(m(\text{val}))^2} \cdot \frac{(\partial_r^2 - \partial_t^2)|\psi|}{|\psi|} \right] \\
&\sim \left[N(\text{gravitons}) \sim 10^{54} \cdot 10^{\aleph} \right] \cdot \exp \left[\frac{1}{2(10^{54} \cdot 10^{\aleph})^2} \cdot \frac{(\partial_r^2 - \partial_t^2)|\psi|}{|\psi|} \right]
\end{aligned} \tag{16}$$

If the above energy is also, by thermal arguments equal to scaling by [17, 18]

$$\rho \sim \Delta E \sim a_{\text{min}}^{-4} \tag{17}$$

Also

The energy expression we will reference comes from [16], page 44 is

$$E(t) = -\partial_t S \tag{18}$$

2. Comparison of the formulas, with a derived value compared to the ideas/ results of [15, 16]

We will be using Eq. (16) in its variance in order to obtain an energy expression. Before we do so we will, say that Planck mass has 2.177 times 10^{-8} Kilograms, Planck length is 1.616 times 10^{-35} meters, and Plan time as 5.391 times 10^{-44} seconds. Now, due to Planck units, we can and will make the following simplifications, namely

The five universal constants that Planck units, by definition, normalize to 1 are:

- the speed of light in a vacuum, c ,
- the gravitational constant, G ,
- the reduced Planck constant, \hbar ,
- the Coulomb constant, $1/4\pi\epsilon_0$
- the Boltzmann constant, k_B

We will find this extremely useful in order to avoid having the calculations which follow completely messed up. i.e. in doing so we look at the mass, for Planck mass, which is going to be set to 1. i.e. this will have immediate consequences in the equations we will work with next. i.e. Planck mass will be set

equal to 1. We will in our own derivations figure in the mass of a graviton, rest, about 10^{-62} kilograms, as by given by these units as

$$m(\text{graviton,rest}) \sim 10^{-54} \quad (19)$$

These units, as well as Planck time set as = 1 , and Planck length, as set = 1 will be used extensively in our manuscript.

Let us now begin the process of looking at the new development, with λ the range of a purported 5th force and α scaled as between 10^{-1} to 10^{-3} .

$$K.E \approx \Delta E \sim \frac{G \cdot m_{\text{graviton}}^2 \cdot 10^{23}}{\lambda} \cdot \left[\left\langle \Psi \left| \frac{1}{r} \right| \Psi \right\rangle \mp \frac{\alpha}{\lambda} \left(\left\langle \Psi \left| e^{-r/\lambda} \right| \Psi \right\rangle + \left\langle \Psi \left| \frac{\lambda \cdot e^{-r/\lambda}}{r} \right| \Psi \right\rangle \right) \right] \quad (20)$$

Here, if we are looking at Pre-Planckian space-time we may be able to use a convenient trick to analyze Eq. (20) above. The wave function,

$$\Psi = \Psi(\text{WKB}) \times e^{i\phi}; \phi = \text{phase} \quad (21)$$

and we then will look at the Mean value theorem for integrals, with [19]

Mean value theorem for integrals (what you need to know)

For c between b and a , the and $f(x)$ a continuous function between a and b , as a mid-point

$$f(c) \cdot (b - a) = \int_a^b f(r) \cdot dr \quad (22)$$

If we apply Eq. (22) to Eq. (21), and Eq. (20) and make the following substitutions, this is what we get

$$\begin{aligned} b &= l_p \\ a &= l_p / 100 \\ c &= l_p / 2 \\ l_p &= \text{Planck - length} \end{aligned} \quad (23)$$

$$\begin{aligned}
\Delta E &\sim \frac{G \cdot m_{\text{graviton}}^2 \cdot 10^{2\cdot 3}}{\lambda} \cdot \left[\left\langle \Psi \left| \frac{1}{r} \right| \Psi \right\rangle \mp \frac{\alpha}{\lambda} \left(\left\langle \Psi \left| e^{-r/\lambda} \right| \Psi \right\rangle + \left\langle \Psi \left| \frac{\lambda \cdot e^{-r/\lambda}}{r} \right| \Psi \right\rangle \right) \right] \\
&\approx \frac{G \cdot m_{\text{graviton}}^2 \cdot 10^{2\cdot 3}}{\lambda \cdot c} \cdot \frac{N_0^2 \cdot (b-a)}{2A_1 \cdot \left[\left(\frac{c}{r_0} \right)^4 - \left(\frac{c}{r_0} \right)^2 \right]} \cdot \exp \left[\frac{2A_1}{3} \cdot \left[\left(\frac{c}{r_0} \right)^2 - 1 \right]^{3/2} \right] \\
&\mp \frac{G \cdot m_{\text{graviton}}^2 \cdot 10^{2\cdot 3}}{\lambda^2} \cdot \frac{e^{-c/\lambda} \cdot \alpha \cdot N_0^2 \cdot (b-a)}{2A_1 \cdot \left[\left(\frac{c}{r_0} \right)^4 - \left(\frac{c}{r_0} \right)^2 \right]} \cdot \exp \left[\frac{2A_1}{3} \cdot \left[\left(\frac{c}{r_0} \right)^2 - 1 \right]^{3/2} \right] \\
&\mp \frac{G \cdot m_{\text{graviton}}^2 \cdot 10^{2\cdot 3}}{\lambda c} \cdot \frac{e^{-c/\lambda} \cdot \alpha \cdot N_0^2 \cdot (b-a)}{2A_1 \cdot \left[\left(\frac{c}{r_0} \right)^4 - \left(\frac{c}{r_0} \right)^2 \right]} \cdot \exp \left[\frac{2A_1}{3} \cdot \left[\left(\frac{c}{r_0} \right)^2 - 1 \right]^{3/2} \right]
\end{aligned} \tag{24}$$

Provisionally, we will be setting this with $r_0 = l_p$, and we will be putting this into, if $a_{\min}^2 \sim 10^{-110}$

$$\begin{aligned}
\Delta E &\sim \frac{\hbar}{\delta t \cdot \phi_{\text{Pre-Planckian}} \cdot a_{\min}^2} \\
\Leftrightarrow \phi_{\text{Pre-Planckian}} &\sim \frac{\hbar}{\delta t \cdot \Delta E \cdot a_{\min}^2}
\end{aligned} \tag{25}$$

This should be compared with what is in [16] where we write

$$\begin{aligned}
\Delta E \Delta t &\sim 1 / \phi(\text{initial}) \cdot a_{\text{initial}}^2 \\
\Rightarrow \phi(\text{initial}) &\sim 1 / \Delta E \Delta t \cdot a_{\text{initial}}^2 \\
&\&\Delta E \sim \partial_t S \sim m(\text{val}) \cdot \exp \left[\frac{1}{2(m(\text{val}))^2} \cdot \frac{(\partial_r^2 - \partial_t^2) |\psi|}{|\psi|} \right] \\
&\sim \left[N(\text{gravitons}) \sim 10^{54} \cdot 10^8 \right] \cdot \exp \left[\frac{1}{2(10^{54} \cdot 10^8)^2} \cdot \frac{(\partial_r^2 - \partial_t^2) |\psi|}{|\psi|} \right]
\end{aligned} \tag{26}$$

Then, 'We begin with, as given in [15]

$$\begin{aligned}
 f_1(r) &= \left[\left(\frac{r}{r_0} \right)^4 - \left(\frac{r}{r_0} \right)^2 \right] \\
 f_2(r) &= \left[\left(\frac{r}{r_0} \right)^2 - 1 \right]
 \end{aligned} \tag{27}$$

Using the formulation as given in this article, we write using Planckian scaling of units the following. Using Eq. (27) above, explicitly, we obtain the following for the energy

$$\begin{aligned}
 (E)_{\text{initial-Pre-Planck}} &\sim m(\text{val}) \cdot \exp(B_1) \cdot \exp(B_2) \cdot \exp(B_3) \cdot \exp(B_4) \\
 B_j &= \frac{1}{2m(\text{val})^2} \cdot C_j \\
 C_1 &= \left[-\frac{3r^4}{r_0^6 \cdot (f_1(r))^2} + \frac{3}{2} \cdot \frac{\left[\frac{6r^2}{r_0^4} - \frac{1}{r_0^2} \right]}{(f_1(r))^2} \right] \\
 C_2 &= \frac{A_1^{-1}}{2f_1(r)} \cdot \left[\frac{1}{r_0^2} - \left(\frac{12r^2}{r_0^4} - \frac{2}{r_0^2} \right) + \frac{r}{r_0^2 A_1^2} \cdot \left(\frac{4r^3}{r_0^4} - \frac{2r}{r_0^2} \right) \right] \\
 C_3 &= A_1^{-1} \cdot \left[\frac{-r^2}{r_0^4} + \left(\frac{f_2(r)}{r_0^2} \right) + \frac{\left(\frac{12r^2}{r_0^4} - \frac{2}{r_0^2} \right)}{(f_1(r))^2} \right] \\
 C_4 &= A_1^{-2} \cdot \left[-\left(\frac{(f_2(r)) \cdot r}{r_0^3} \right) + \frac{\left(f_2(r) \right) \cdot \left(\frac{12r^4}{r_0^6} - \frac{2r^2}{r_0^4} \right)}{(f_1(r))^2} \right]
 \end{aligned} \tag{28}$$

Essentially, to compare Eq. (28) with Eq. (24) we need to understand if the so called length of the fifth force, in terms of effective interaction, as given by λ plays a significant role. Moreover, it is important to keep in mind the key role [15] plays, as well as looking at the energy is given as follows, namely, if we look at say again, as given by B. Hu [16] which we write up as follows: Assuming an energy density as given by , in Pre Planckian space-time is given by , if we have an averaged out mean frequency for particle production given by $\omega_{k_{average}}$

$$\begin{aligned} \rho_c &= \frac{1}{V(\text{volume})} \cdot \int \frac{d^3k}{(2\pi)^3} \cdot \left(|\beta_k|^2 + \frac{1}{2} \right) \cdot \omega_k \\ &\sim \left[\frac{1}{V(\text{volume})} \cdot \int \frac{d^3k}{(2\pi)^3} \right] \cdot \left(|\beta_{k_{average}}|^2 + \frac{1}{2} \right) \cdot \omega_{k_{average}} \end{aligned} \quad (29)$$

The second line of the above is making the approximation that the insides of the first line, are averaged out to a constant, which is defensible in the situation of a Pre Planckian space-time condition. Secondly, we are assuming in all of this that $|\beta_{k_{average}}|^2$ is the number of 'created' particles in k space, in space-time is in terms of a situation for which we are assuming a very narrow range of k values, so we are when looking at the 2nd line of Eq. (29) referencing an averaged out value for the number of created particles which we then identify as $|\beta_{k_{average}}|^2$, and have $V(\text{volume}) \leq l_{\text{Planck}}^3$, i.e. with l_{Planck} Planck length.

If so, then we could define having a net energy as given by [16]

$$E_c \sim \left[\int \frac{d^3k}{(2\pi)^3} \right] \cdot \left(|\beta_{k_{average}}|^2 + \frac{1}{2} \right) \cdot \omega_{k_{average}} \quad (30)$$

We have several different ways to address what is meant by this energy. Our supposition is that we could make a reference, here, to, if c (speed of light) = 1, to have, here, initially, a transfer of gravitons, as an information carrier, from a prior universe to our present universe so that as a result of a match up in Pre Planckian space-time to Planckian space time we would have Eq. (30) as rendered by, using Hu again, [31]

$$E_c \sim \left[\int \frac{d^3k}{(2\pi)^3} \right] \cdot \left(|\beta_{k_{average}}|^2 + \frac{1}{2} \right) \cdot \omega_{k_{average}} \sim \langle n_{\text{gravitons}} \rangle \cdot m_{\text{graviton}} \quad (31)$$

And a graviton count, in the Pre Planckian era we would give as [16]

$$\langle n_{\text{gravitons}} \rangle \sim 1/(\exp(E_c / T_{\text{temp}}) - 1) \quad (32)$$

3. Conclusion: Examining a tie in with Padmanabhan, of reference [10]

For the sake of convenience, we will be using the [10] conventions as to a Potential energy, and also an inflaton as given by Eq. (14). In doing so, we will consider the situation where the logarithm is expanded to be as given by Beyer [21] on page 299 to have the following behavior

$$\begin{aligned}
 \frac{a_{\min}^{-2}}{\Delta E \delta t} &\approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left[\sqrt{\frac{8GV_0}{\gamma \cdot (3\gamma - 1)}} \cdot \delta t \right] \sim \sqrt{\frac{\gamma}{4\pi G}} \cdot \left(\sqrt{\frac{8GV_0}{\gamma \cdot (3\gamma - 1)}} \delta t - 1 \right) \\
 \Leftrightarrow \frac{a_{\min}^{-2}}{\Delta E \delta t} &\sim \sqrt{\frac{\gamma}{4\pi G}} \cdot \left(\sqrt{\frac{8GV_0}{\gamma \cdot (3\gamma - 1)}} \delta t - 1 \right) \\
 \Leftrightarrow \frac{8GV_0}{\gamma \cdot (3\gamma - 1)} \cdot (\delta t)^2 &\geq 1 + 2 \cdot \varepsilon^+ \\
 \Leftrightarrow \delta E &\sim \frac{a_{\min}^{-2}}{\delta t \cdot \gamma} \left/ \left(\sqrt{\frac{8GV_0}{\gamma \cdot (3\gamma - 1)}} \delta t - 1 \right) \right.
 \end{aligned} \tag{33}$$

Here, in this case, $G=1$ in Planckian units whereas δE is either from Eq. (28) or Eq. (24). Given a fluctuation in time say half of Planck time, then the above, would be then, in terms of Eq. (28) or Eq. (24) putting strict limits on the bound of V_0

If we find the bounds due to Eq. (28) and Eq. (24) are giving about the same value as to V_0 , then in terms of inflation, the two schemes outlined are giving equivalent information.

This is something which needs to be investigated. Finally, the issues brought up by [22,23,24] as to first of all fidelity with respect to LIGO and Gravitational waves, ,as well as the foundations of gravity brought up by Dr. Corda need to be vetted. Also the issues brought up in [25,26,27,28,29,30] need to be investigated. Once this is done, and the formulas held to be approximate, it is conceivable that the datum and speculations given by Dr. Corda in [23] will be examinable and hopefully confirmed. In short, we would require an enormous ‘inflaton’ style ϕ valued scalar function, and $a^2(t) \sim 10^{-110}$. I.e. assuming a quantum ‘bounce with $a^2(t) \sim 10^{-110}$, and we hope to confirm it soon.

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References

[1] S. Gasiorowitz, “Quantum Physics, 3rd edition”, Wiley and Sons, 2003, Danvers, Massachusetts, USA

[2] E. Fishbach, C. Talmadge, “The Foftj Fprce: An Introduction to Current research,” pp, AIP press, Springer-Verlag, New York, New York, USA, 1999

[3] E. Fishbach, C. Talmadge, “The Search for Non-Newtonian Gravity”, pp. 369-382, part of the volume, “ 5th Force Neutrino Physics”, Edited by O., Fackler, and J. Tran Than Van, Edition Frontieres, Proceedings of the XXIIIrd Rencontre De Moriond, 1988

[4] M. Dalarsson, and N. Dalarsson, “ Tensors, Relativity and Cosmology”, Elsevier Press, London, UK, 2005

[5] Beckwith, A. (2016) Gedanken Experiment Examining How Kinetic Energy Would Dominate Potential Energy, in Pre-Planckian Space-Time Physics, and Allow Us to Avoid the BICEP 2 Mistake. *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 75-82.
doi: [10.4236/jhepgc.2016.21008](https://doi.org/10.4236/jhepgc.2016.21008).

[6] W. G. Unruh; “Why study quantum theory?”, *Canadian Journal of Physics*, 1986, Vol. 64, No. 2 : pp. 128-130;
(doi: [10.1139/p86-019](https://doi.org/10.1139/p86-019))

[7] W. G. Unruh; “Erratum: Why study quantum gravity?”, *Can. J. Phys.* 64, 128 (1986)

[8] H. Gutfreund, J. Renn, *The Road To Relativity, The history and Meaning of Einstein’s “ The Foundation of General Relativity”, (featuring the Original Manuscript of Einstein’s Masterpiece)* , Princeton University Press, Princeton and Oxford, Princeton, New Jersey, USA, 2015

[9] M. Giovannini, *A Primer on the Physics of the Cosmic Microwave Background* World Press Scientific, Hackensack, New Jersey, USA, 2008

[10] T. Padmanabhan, “An Invitation to Astrophysics,”, World Press Scientific, Singapore, Republic of Singapore, 2006

[11] A. Goldhaber, M. Nieto, “Photon and Graviton Mass Limits”,
Rev.Mod.Phys. 82:939-979,2010, <http://arxiv.org/abs/0809.1003>

[12] Martel, H., P. Shapiro,P., and Weinberg, S., ‘ Likely values of the cosmological constant’
<http://iopscience.iop.org/article/10.1086/305016/fulltext/35789.text.html>

[13] C. Will, “ Was Einstein Right? A Centenary Assessment”, pp49-96, of ‘General Relativity and Gravitation, A Centennial Perspective’ Edited by A. Ashtekar, B. Berger, J. Isenberg, M. MacCallum, Cambridge University Press, Cambridge, UK, 2015

[14] C. Will, “The Confrontation between General Relativity and Experiment”,
<http://relativity.livingreviews.org/Articles/lrr-2014-4/download/lrr-2014-4Color.pdf>

[15] A. Beckwith <http://vixra.org/abs/1606.0103>

[16] Iganizio Licata, Davide Fiscaletti, “Quantum Potential: Physics, Geometry and Algebra”, Springer Briefs in Physics, Springer Verlag, Heidelberg, Republic of Germany, 2014

[17] Camara, C.S., de Garcia Maia, M.R., Carvalho, J.C. and Lima, J.A.S. (2004) Nonsingular FRW Cosmology and Non Linear Dynamics. Arxiv astro-ph/0402311 Version 1, Feb 12.

[18] **Andrew Little, “An Introduction to Modern Cosmology, 2nd edition”, Wiley Interscience, Hoboken, New Jersey, NY, 2005**

[19] <http://ocw.mit.edu/resources/res-18-001-calculus-online-textbook-spring-2005/textbook/>

[20] Hu, B. “ Vacuum Viscosity and Entropy Generation in Quantum Gravitational Processes in the Early Universe”, pp. 23-44, in “Cosmology of the Early Universe” with Fang Li Zhi and Ruffini, Remo, World Press Scientific, Singapore, the Republic of Singapore, 1984.

[21] **W. Beyer, “CRC mathematical tables, 28^t edition” CRC press, Boca Racon, Florida, USA, 1987**

[22] B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration) Phys. Rev. Lett. **116**, 061102 – Published 11 February 2016; <https://physics.aps.org/featured-article-pdf/10.1103/PhysRevLett.116.061102>

[23] C. Corda, “**Primordial gravity's breath**”, EJTP 9, 26, 1-10 (2012); <http://arxiv.org/abs/1110.1772>

[24] **C. Corda, Massive gravitational waves from the R**2 theory of gravity: Production and response of interferometers** - _Int. J. Mod. Phys. D 18,2275 (2009)

[25] C. Corda, “Interferometric detection of gravitational waves: the definitive test for General Relativity”, Int. J. Mod. Phys. D18:2275-2282,2009; <http://arxiv.org/abs/0905.2502>

[26] Beckwith, A. (2016) Gedanken Experiment Examining How Kinetic Energy Would Dominate Potential Energy, in Pre-Planckian Space-Time Physics, and Allow Us to Avoid the BICEP 2 Mistake. *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 75-82. doi: [10.4236/jhepgc.2016.21008](https://doi.org/10.4236/jhepgc.2016.21008).

[27] Gao, C. (2012) A Model of Nonsingular Universe. Entropy, 14, 1296-1305.

<http://dx.doi.org/10.3390/e14071296>

[28] Mueller, R. and Lousto, C.O. (1995) Entanglement Entropy in Curved Space-Time with Event Horizons. Physical Review D, 52, 4512-4517. <http://dx.doi.org/10.1103/PhysRevD.52.4512>

[29] Beckwith, A. (2016) Non-Linear Electrodynamics Gedanken Experiment for Modified Zero Point Energy and Planck’s “Constant”, h Bar, in the Beginning of Cosmological Expansion, So $h(\text{Today}) = h(\text{Initial})$. Also How to Link Gravity, Quantum Mechanics, and E and M through Initial Entropy Production in the Early Universe. *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 168-182. doi: [10.4236/jhepgc.2016.22016](https://doi.org/10.4236/jhepgc.2016.22016)

[30] Kolb, E; https://ned.ipac.caltech.edu/level5/Kolb/Kolb3_1.html