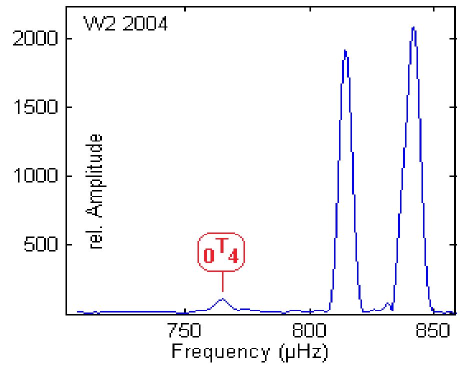
SG Gravimeters measure Toroidal Modes of the Earth

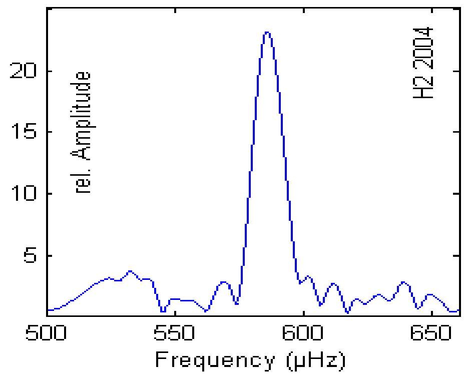
Herbert Weidner [a]

Abstract: In contrast to previous assumptions, gravimeters can measure not only radial movements of the earth's surface, but also horizontal movement. This is first shown on the mode ${}_0T_4$.

Introduction



After earthquakes, the Earth vibrates at different eigenfrequencies. The amplitudes are very small and require extremely sensitive detection devices. Superconducting gravimeters deliver particularly trouble-free recordings, but they may not register all normal modes. According to theoretical considerations, they should only respond to spheroidal modes in which the Earth's surface moves in the radial direction. They should be "blind" for tangential movements and sometimes it is claimed that "*The motion in a toroidal mode has no radial component and there is no compression or dilation so they are not recorded on gravimeters*"[1]. But as soon as a careful preparation of the data avoids the generation of intermodulation, the spectral lines of toroidal modes clearly exceed the noise level of superconducting gravimeters.



Some of the existing measurement problems are probably caused by the method commonly used. In geophysics, the spectral analysis is the standard method to measure the natural frequencies of the earth. This corresponds to a very insensitive receiver which is hardly suitable to detect weak signals. In radio astronomy, nobody would use such a broadband technique in order to study the properties of distant gas clouds.

The Fourier transform alone has no selectivity and handles all frequencies equally. This invites interference from adjacent spectral lines and – even worse – it lowers the signal-to-noise ratio. In sensitive receivers, the frequency of the wanted signal is *always* reduced in order to refine the frequency resolution. At a lower frequency, it is easier to construct narrow band filters in order to reduce the noise. A proven rule is to select an intermediate frequency which is about twenty times higher than the highest expected frequency of the modulation. The final decomposition of the signal into two orthogonal components is suitable to measure the center frequency and to reconstruct the amplitude and the modulation of the signal.

The Preparation of the data

In a first step, the raw data (the most common sampling time is one second) of all available SG stations between 2004-12-26 and 2004-12-31 were made machine readable. One week after the earthquake, the amplitudes of most natural oscillations of the earth disappear in the background noise. The influence of atmosphere pressure variation on the gravity data was omitted because the air mass above the instruments changes much slower than the oscillation time of the analyzed normal modes.

In order not to worsen the low SNR of the SG records, the very intense tides must be effectively

a) 13. June 2016, email: herbertweidner@gmx.de

reduced by a comb filter in the *first* processing step. The huge dynamic range of the data (more than 100 dB) plus a safety margin of about 40 dB inside the filter programs must not override the math co-processor of the computer. Otherwise there is a risk that the large dynamic range leads to an inaccurate representation of weak signals with too few significant figures, equivalent to a low measurement resolution. This form of non-linearity always produces unwanted intermodulation products which can not be distinguished from actual spectral lines. The application of a comb filter is very simple: all samples are shifted by several seconds and subtracted from the original record. The ratio of this time difference to the period(s) of the oscillations produces the desired effect: The very intense amplitudes of the slow tides ($T \approx$ several hours) are largely compensated, while the amplitude of the much faster oscillations is doubled. A low-pass filter ($f < 8$ mHz), followed by a second "*shift and subtract*" procedure generates the final data string. A multiple application does not bring further improvement.

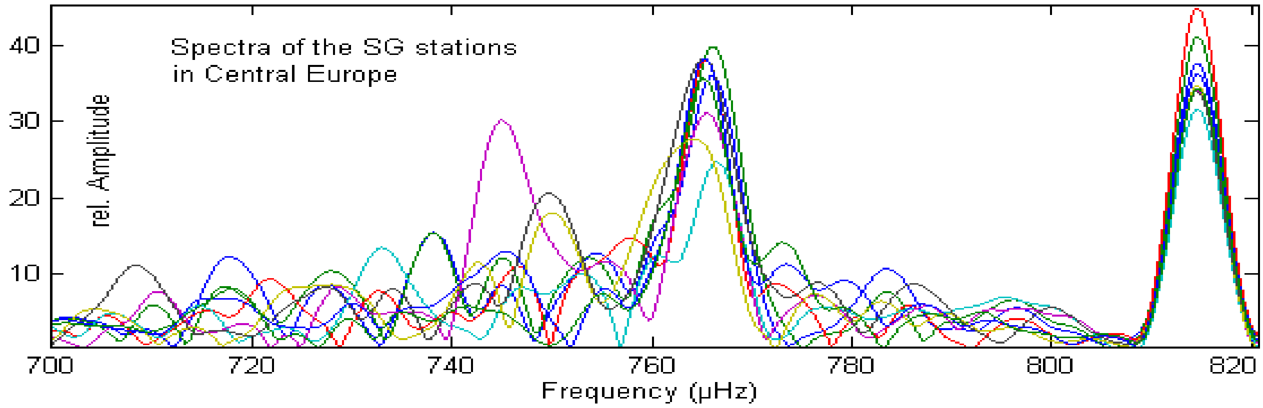
After the comb filter has reduced the dynamic range of the data, the bandwidth around the region of interest is limited to ± 40 µHz and the mean frequency is shifted to the intermediate frequency 130 µHz, using the mixing method[2]. This low value allows to extend the sampling rate of the records to 100 seconds without losing information. Then the bandwidth is further reduced to the value indicated below. In order not to degrade the noisy data and to avoid any distortion of the spectra, nearly all filtering is done with the time consuming convolution with *windowed Sinc* functions.

Frequency and Quality factor of ${}_0T_4$

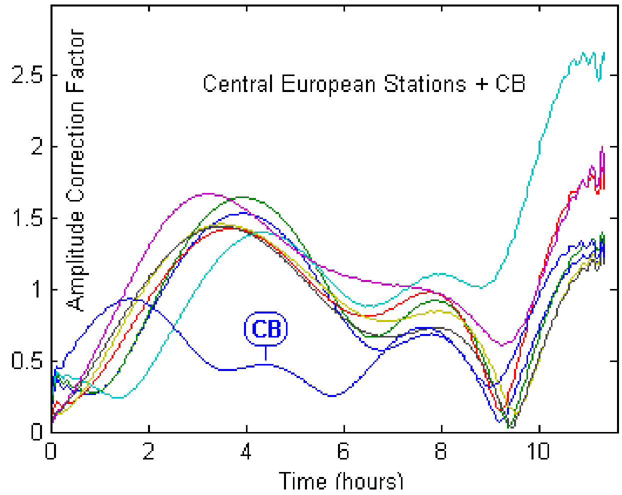
The table shows the results ${}_0T_4$ after narrow band filtering. The much stronger ${}_0S_0$ spectral line near 814 µHz is far outside the bandwidth. Even without proper choice of the filter parameters, neither the signals nor the noise in the region around ${}_0T_4$ will be affected.

	Bandwidth = 26 µHz		Bandwidth = 12 µHz	
SG Station	Frequency (µHz)	*Q*	Frequency (µHz)	*Q*
CB	765.83	215	760.58 767.35	-1340 (see below) 519
H1	765.48	684	766.12	344
H2	765.44	643	766.17	404
M1	765.07	451	765.47	596
MB	765.78	3176	766.5	496
MC	765.31	362	765.54	437
ME	766.7	-754	766.68	-2615 noisy
NY	766.32	425	766.05	321 noisy
S1	764.14	-998	755.02 763.23	3791 -4374
S2	764.96	-894	755.03 ~765.3 split?	-1562
ST	764.76	594	764.36	535
VI	764.86	608	765.49	408
W1	765.04	404	765.28	483
W2	765.06	378	765.39	430
Average	765.34 ± 0.18		765.59 ± 0.21	459 ± 25

The spectral line of ${}_0T_4$ is not identifiable in the records of the Stations WU, TC, MA and ES. The Q values in the third column vary considerably, so it does not make sense to calculate the average. The reason is the wide bandwidth of 26 µHz, inviting a lot of noise. After restricting the bandwidth to 12 µHz, the Q values of the Central European stations can be calculated reliably (green fields). The spectra of the stations CB, S1 and S2 are discussed separately below.

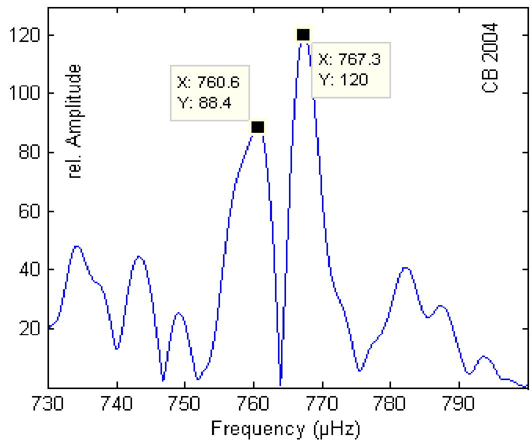


The picture above shows the superposition of the spectra of the Central European stations with extended bandwidth. The amplitudes around 814 µHz are reduced by a factor of twenty.

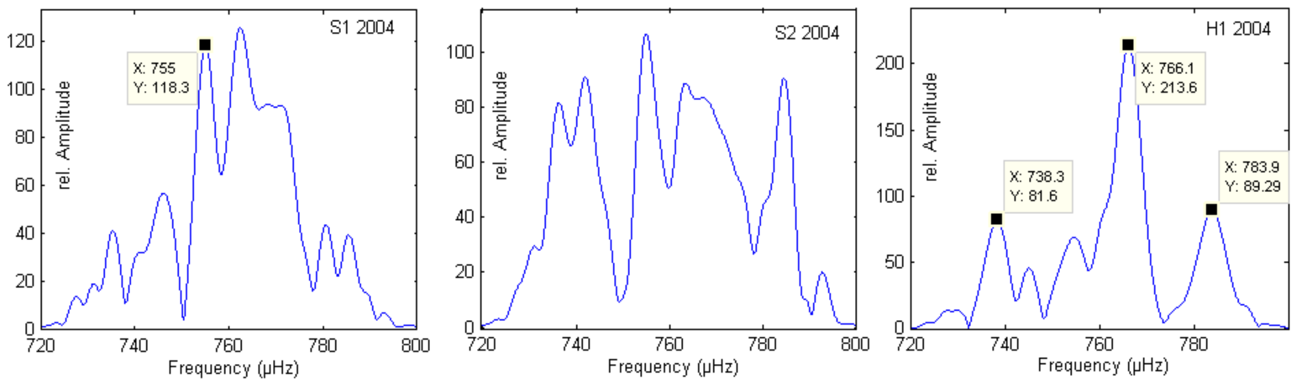


Around the central frequency 765 µHz, one recognizes no symmetrical sidebands, indicating the absence of a sound modulation. Nevertheless, the following figure shows a synchronous time course of the amplitudes, which does not seem to be random. Notably, the decrease in amplitude is not a simple exponential law, it shows a complicated time dependence. A singlet may be described as a decaying oscillation $y_i(t) = A\cos(\omega t + \varphi)\,e^{-\alpha t} \cdot F$ with A as the initial amplitude. With the mean value $Q \approx 460$ we get the attenuation $\alpha = \dfrac{\omega}{2Q}$. In conventional damping, the amplitude correction factor F should always have the value 1. But the measurement shows that F is not constant, it has a complicated time dependence that is virtually identical for all Central European stations. For comparison, F(CB) is also shown with a significantly deviating course. The cause of this amplitude modulation is unknown.

Special cases CB, S1 and S2



All three stations in the southern hemisphere measure not a single resonance frequency of ${}_0T_4$, but several closely spaced spectral lines. In the records taken by station CB, the lower spectral line near 760 µHz is shaped remarkably wide and asymmetrical. Probably there are two closely spaced lines that can not be separated, because the amplitude of the signals drop too quickly. The interference of the oscillations may also explain the temporary increase in the resulting amplitude, which appears in the table as a negative Q.



Much more complex are the spectra of S1 and S2, which differ greatly from one another, although both measuring balls are in close proximity. For better comparison, the typical and relatively simple structured spectrum of a Central European station is shown on the right.

Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[3].

[1] T. Masters, R. Widmer, Free Oscillations: Frequencies and Attenuations, Global Earth Physics, A Handbook of Physical Constants, AGU 1995
[2] H. Weidner, A New method for High-resolution Frequency Measurements, http://viXra.org/abs/1506.0005
[3] The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html