

Gedankenexperiment for looking at fluctuations in the time component of the metric tensor δg_{tt} for initial expansion of the universe and influence on HUP via inflaton physics., with Bohm theory of QM

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Abstract. We examine through the lens of dynamical systems a 'one dimensional' time mapping of emergent VEV from Pre Planckian space time conditions. We use Licata's and Fiscaletti's Bohmian Quantum mechanics to obtain an energy E as minus the time derivative of a phase transition, as given by Bohmian Q.M. for curved space time, in the Pre Planckian state, where we then use a modified version of the HUP, to then isolate the inflaton. The emergence of the inflaton scalar field, in time evolution is tied into physics of the first order phase transition to QM, and the regular HUP, used by Heisenberg. I.e. a first order generation of initial entropy via Bohmian QM applied in Pre Planckian space-time.

1. Introduction. Bringing up the Quantum Potential used in this problem

We begin with the results from [1], by Iacata and Fiscoletti, where in pages 69-71 of [1] they set up a quantum gravity correspondence between space-time and matter based on [2, 3,4,5,6]

By Eq. (3.6) of reference [1] we have that if Q is the quantum potential and we go to Pre Planck times, with a modification of the Klein Gordon equation as given below

$$g^{uv} \nabla_u S \cdot \nabla_v S = (m^2 c^2) \cdot \exp(Q) \xrightarrow{\text{Pre-Planckian}} g^{00} (\nabla_0 S)^2 = (m^2 c^2) \cdot \exp(Q)$$

$$Q = \frac{\hbar^2}{m^2 c^2} \cdot \frac{\left(\nabla^2 - \frac{1}{c^2} \cdot \partial_t^2 \right) \cdot |\psi|}{|\psi|} \xrightarrow{\text{Pre-Planckian}} \frac{\hbar^2}{m^2 c^2} \cdot \frac{\left(\frac{\partial^2}{\partial r^2} - \frac{1}{c^2} \cdot \partial_t^2 \right) \cdot |\psi|}{|\psi|} \quad (1)$$

The energy expression we will reference comes from [1], page 44 is

$$E(t) = -\partial_t S \quad (2)$$

We will be using Eq. (2) in its variance in order to obtain an energy expression. Before we do so we will, say that Planck mass has 2.177 times 10^{-8} Kilograms, Planck length is 1.616 times 10^{-35} meters, and Plan time as 5.391 times 10^{-44} seconds. Now, due to Planck units, we can and will make the following simplifications, namely

The five universal constants that Planck units, by definition, normalize to 1 are:

- the speed of light in a vacuum, c ,
- the gravitational constant, G ,
- the reduced Planck constant, \hbar ,
- the Coulomb constant, $1/4\pi\epsilon_0$
- the Boltzmann constant, k_B

We will find this extremely useful in order to avoid having the calculations which follow completely messed up. i.e. in doing so we look at the mass, for Planck mass, which is going to be set to 1. i.e. this will have immediate consequences in the equations we will work with next. i.e. Planck mass will be set equal to 1. We will in our own derivations figure in the mass of a graviton, rest, about 10^{-62} kilograms, as by given by these units as

$$m(\text{graviton,rest}) \sim 10^{-54} \quad (3)$$

These units, as well as Planck time set as = 1 , and Planck length, as set = 1 will be used extensively in our manuscript.

2. Use of the Wheeler De Witt equation in the representation of Eq. (1)

This section makes use of [7] by Dalarsson and Dalaarsson, page 271 and then obtains for $r < r_0$

$$|\psi| \propto \frac{N_0 A_1}{2} \cdot \frac{\exp \left[\left(\left(\frac{r}{r_0} \right)^2 - 1 \right)^{3/2} / 3A_1 \right]}{\left[\left(\frac{r}{r_0} \right)^4 - \left(\frac{r}{r_0} \right)^2 \right]} \text{ iff } r < r_0 \quad (4)$$

$$A_1 = (2\hbar G \Lambda / 9\pi c^5) \xrightarrow{\text{Planck-units}} (2\Lambda / 9\pi)$$

Here, we will be taking into account, the issues in [9] as to symmetry breaking, by a change in the HUP.

Then, set from [8], ie. Begin with the starting point of [9,10, 11]

$$\Delta l \cdot \Delta p \geq \frac{\hbar}{2} \quad (5)$$

We will be using the approximation given by Unruh [10, 11] , of a generalization we will write as

$$(\Delta l)_{ij} = \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2} \quad (6)$$

$$(\Delta p)_{ij} = \Delta T_{ij} \cdot \delta t \cdot \Delta A$$

If we use the following, from the Roberson-Walker metric [12].

$$g_{tt} = 1$$

$$g_{rr} = \frac{-a^2(t)}{1 - k \cdot r^2} \quad (7)$$

$$g_{\theta\theta} = -a^2(t) \cdot r^2$$

$$g_{\phi\phi} = -a^2(t) \cdot \sin^2 \theta \cdot d\phi^2$$

Following Unruh [9, 10] , write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110}, r \equiv l_p \sim 10^{-35} \text{ meters} \quad (8)$$

Then, if $\Delta T_{tt} \sim \Delta \rho$

$$\begin{aligned} V^{(4)} &= \delta t \cdot \Delta A \cdot r \\ \delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} &\geq \frac{\hbar}{2} \\ \Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} &\geq \frac{\hbar}{V^{(4)}} \end{aligned} \quad (9)$$

This Eq.(9) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time [11, 12]

$$T_{ii} = \text{diag}(\rho, -p, -p, -p) \quad (10)$$

Then [8]

$$\Delta T_{tt} \sim \Delta \rho \sim \frac{\Delta E}{V^{(3)}} \quad (11)$$

Then, Eq. (9) and Eq. (10) and Eq. (11) together yield

$$\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \quad (12)$$

$$\text{Unless } \delta g_{tt} \sim O(1)$$

What we will refer to is also, besides Eq. (12), that in Pre Planckian space-time we have due to [8] cancellation which we give as follows. To begin this process, we will break it down into the following

coordinates. In the $rr, \theta\theta$ and $\phi\phi$ coordinates, we will use the Fluid approximation, $T_{ii} = \text{diag}(\rho, -p, -p, -p)$ [8] with the results as given in [8] to be

$$\begin{aligned}
\delta g_{rr} T_{rr} &\geq - \left| \frac{\hbar \cdot a^2(t) \cdot r^2}{V^{(4)}} \right| \Bigg|_{a \rightarrow 0} \rightarrow 0 \\
\delta g_{\theta\theta} T_{\theta\theta} &\geq - \left| \frac{\hbar \cdot a^2(t)}{V^{(4)} (1 - k \cdot r^2)} \right| \Bigg|_{a \rightarrow 0} \rightarrow 0 \\
\delta g_{\phi\phi} T_{\phi\phi} &\geq - \left| \frac{\hbar \cdot a^2(t) \cdot \sin^2 \theta \cdot d\phi^2}{V^{(4)}} \right| \Bigg|_{a \rightarrow 0} \rightarrow 0
\end{aligned} \tag{13}$$

Furthermore from Giovanni, we will use the additional approximation of, from [8] and [13,14,15,16]

$$g_{tt} \approx \phi(\text{initial}) \cdot a_{\text{initial}}^2 \tag{14}$$

In particular, reference [16] gives a good reason as to why the initial scale factor is not set to zero, and we follow this convention, while asserting its commonality with work done by Corda as to the inflaton, [17] where the final part of our work will be to, give a parallel development as to a Pre Planckian space-time version of the Inflaton which will be the tail end of what will an algebraically complicated result.

In doing so, we will also, with additional work, if the infaton is so identified, permit investigation into the issues bought up in [18] and [19] in future research.

Having said that, it is time to calculate first the energy, and then the total mass, where the mass in what we are assuming is akin to a multiple of the graviton mass, with the graviton ‘rest mass’ set as about to 10^{-62} kilograms, or maybe less, by [20], and we will do our best not to contravene the basic physics given by Abbot et.al, in [21]

3. Calculating of energy, and then from there a minimum fluctuation of energy

Our initial assumption is starting off with $dr/dt = c = 1$, i.e. we disregard the very initial slight difference in graviton speed for heavy gravity as given $< c = 1$ due to the considerations for massive gravity given in [21], and approximate due to the smallness of the ‘massive graviton’ as given by [20] a top off of propagation speed of about $c=1$. Having done this, using Eq. (12) and then wanting to isolate (E) as due to Eq. (1) and Eq. (2), we then after algebra obtain, for $-20 < \aleph < 20$ and using Planck unit normalization. Here, we are assuming one half the value of Planck time, i.e. about $\Delta t \sim \frac{t_{\text{Planck}}}{2} = .5$

And dr/dt set =1 $r_0 \sim l_{\text{Planck}} = 1$, and a temperature $T^4 \sim T_{\text{Re-Heat}}^4 \sim \Delta t^{-2} m_{\text{Planck}}^2 \equiv \Delta t^{-2}$

$$\begin{aligned}
\Delta E \Delta t &\sim 1 / \phi(\text{initial}) \cdot a_{\text{initial}}^2 \\
\Rightarrow \phi(\text{initial}) &\sim 1 / \Delta E \Delta t \cdot a_{\text{initial}}^2 \\
&\&\Delta E \sim \partial_t S \sim m(\text{val}) \cdot \exp \left[\frac{1}{2(m(\text{val}))^2} \cdot \frac{(\partial_r^2 - \partial_t^2)|\psi|}{|\psi|} \right] \\
&\sim \left[N(\text{gravitons}) \sim 10^{54} \cdot 10^{\aleph} \right] \cdot \exp \left[\frac{1}{2(10^{54} \cdot 10^{\aleph})^2} \cdot \frac{(\partial_r^2 - \partial_t^2)|\psi|}{|\psi|} \right]
\end{aligned} \tag{15}$$

If the above energy is also, by thermal arguments equal to scaling by [23]

$$\rho \sim \Delta E \sim a_{\text{min}}^{-4} \tag{16}$$

We obtain then a scaling argument as to a scalar potential as expressed in Eq. (16) and more over a first estimate as to the entropy, which is in this case equal to by Ng [24]

$$S_{\text{Initial-Entropy}} \sim N(\text{gravitons}) \sim 10^{54} \cdot 10^{\aleph} \tag{17}$$

We will next, finally Out in the explicit value for Eq. (15) while using Eq. (17) as well as coming up with an estimate of entropy, by explicit calculations.

If as an example, we have negative pressure, with $T_{rr}, T_{\theta\theta}$ and $T_{\phi\phi} < 0$, and $p = -\rho$, then the only choice we have, then is to set $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$, **since there is no way that $p = -\rho$ is zero valued.** This observation and others will color our concluding statement and explicit calculation philosophy.

4. Conclusion. Explicit representation of Pre Planckian inflaton value, entropy and other issues.

We will in the conclusion give explicit calculation values and do it with respect to Planckian physics units scaling as identified at the start of the paper. Plus suggestions as to numerical simulation work as follow up, and possible interpretations of the above. First we set up a work in progress, and then next we will elucidate what we have done with the inflaton calculation explicitly in this formulation.

To begin with in line with what is done in [25] If we understand what the inflaton initially is, and we understand what is done with the physics of generation of entropy, maybe even in the Pre Planckian regime, we may understand the entire point of a possible causal discontinuity, if it exists, or if it does not exist. To paraphrase our point, the degree of entropy as information, along the lines of making sense of the following deliberations, i.e. To do so, in our new argument,

we look at first a simple way to frame the cosmological constant problem as given by Guth [26] as given by

$$\begin{aligned}
\Lambda_{C.Const} g_{uv} &= \langle 0 | T_{uv} | 0 \rangle \xrightarrow{(u,v) \rightarrow (0,0) \text{ Pre-Planckian}} \Lambda_{C.Const} g_{00} = \langle 0 | (T_{00} = \rho) | 0 \rangle \\
\Leftrightarrow \rho &= \frac{\dot{\phi}^2}{2} + V(\phi) + \frac{(\nabla\phi)^2}{2} \xrightarrow{(u,v) \rightarrow (0,0) \text{ Pre-Planckian}} \rho = \frac{\dot{\phi}^2}{2} + V(\phi) \\
\Leftrightarrow \Lambda_{C.Const, \text{Pre-Planckian}} &= \langle 0 | (T_{00} = \rho)_{\text{Pre-Planckian}} | 0 \rangle \cdot g_{00}^{-1}
\end{aligned} \tag{18}$$

This last line, namely $\Lambda_{C.Const, \text{Pre-Planckian}} = \langle 0 | (T_{00} = \rho)_{\text{Pre-Planckian}} | 0 \rangle \cdot g_{00}^{-1}$ is assumed to have the same value as the cosmological constant today, i.e. no quintessence, so what we will be doing is to examine what this says about an inflaton mass, in the spirit of what was said by Corda in [17] In the pre Planckian regime we are having that $(\nabla\phi)^2$ would be of small import, and that there is still though, a small regime of space-time, i.e. a bounce ball of the form given in [27] and [28] and [29] which would have the inflaton only change by time, not space, and then refer to [30] which has an inflaton mass of the form given by , if we use the variable change of $z = \dot{\phi} / H$, and assume that $\dot{\phi}$ is approximately a constant in the interval of time, in the Pre Planckian space-time regime, so that the inflaton mass is given by, if in Pre Planckian space-time

$$d\tau^2 = a^2(\tau) dt^2 \sim (a_{\text{Pre-planckian}} = a_{\text{min}})^2 dt^2 \tag{19}$$

With a_{min} defined in [28], then the equation given in [30] for inflation mass would in the Pre Planckian space-time

$$m \sim -z^{-1} \frac{d^2 z}{d\tau^2} \tag{20}$$

Becomes

$$m \sim -\frac{H}{a_{\text{min}}} \cdot \frac{d^2 H^{-1}}{dt^2} \tag{21}$$

Where, for future work we need to make full sense out of following presentation for the inverse of the Hubble parameter

$$H^{-1} = H_{\text{Planckian-regime}}^{-1} + \frac{H_{\text{Pre-Planckian-regime}}^{-1}}{2} \cdot (t \cdot t_{\text{Planck}} - t^2) \tag{22}$$

Then

The parameter $H_{\text{Pre-Planckian-regime}}^{-1}$ is set for half of the Planck time interval, and the net result is that Eq.(21) becomes scaled as

$$m^2 \sim -\frac{H}{a_{\min}} \cdot \frac{d^2 H^{-1}}{dt^2} \sim \frac{H_{\text{Pre-Planckian-regime}}^{-1}}{a_{\min} \cdot \left[H_{\text{Planckian-regime}}^{-1} + \frac{H_{\text{Pre-Planckian-regime}}^{-1}}{2} \cdot (t \cdot t_{\text{Planck}} - t^2) \right]} \quad (23)$$

Then inflaton based kinetic energy would be , if M_{Planck} is Planck mass

$$\frac{\dot{\phi}^2}{2} \sim \frac{3M_{\text{Planck}}}{\left[H_{\text{Planckian-regime}}^{-1} + \frac{H_{\text{Pre-Planckian-regime}}^{-1}}{2} \cdot (t \cdot t_{\text{Planck}} - t^2) \right]} \quad (24)$$

$$\Leftrightarrow V(\phi)|_{\text{Pre-Planckian-regime}} \sim \lambda \phi^4|_{\text{Pre-Planckian-regime}} \ll \frac{\dot{\phi}^2}{2}$$

Note that we are looking for a numerical simulation to verify and to back up what was stated in reference [32], i.e.

The parameter $H_{\text{Pre-Planckian-regime}}^{-1}$ is set for half of the Planck time interval. We need to explicitly model it and not call it a pretty concept with no modeling development.

This value for the inflaton mass, would depend upon what we are looking at due to entropy and we will discuss it now, as well as the inflation itself, and explicit energy calculations next.

I.e. filling in the equations explicitly as a way to understand Eq. (23) and Eq. (24) depends in large part upon setting up comprehending the following equations we have set up in Planckian units. i.e. we need to fill in iteratively candidate for the Pre Planckian and Planckian Hubble parameters, and this will numerical iteration and so we will set up a value of the inflaton which will aid us in this endeavor. We begin with

$$f_1(r) = \left[\left(\frac{r}{r_0} \right)^4 - \left(\frac{r}{r} \right)^2 \right] \quad (25)$$

$$f_2(r) = \left[\left(\frac{r}{r_0} \right)^2 - 1 \right]$$

If so then, the energy is given as follows, namely, if we look at say again, as given by B. Hu [31] which we write up as follows: Assuming an energy density as given by , in Pre Planckian space-time is given by , if we have an averaged out mean frequency for particle production given by ω_{average}

$$\begin{aligned} \rho_c &= \frac{1}{V(\text{volume})} \cdot \int \frac{d^3k}{(2\pi)^3} \cdot \left(|\beta_k|^2 + \frac{1}{2} \right) \cdot \omega_k \\ &\sim \left[\frac{1}{V(\text{volume})} \cdot \int \frac{d^3k}{(2\pi)^3} \right] \cdot \left(|\beta_{k_{\text{average}}}|^2 + \frac{1}{2} \right) \cdot \omega_{k_{\text{average}}} \end{aligned} \quad (26)$$

The second line of the above is making the approximation that the insides of the first line, are averaged out to a constant, which is defensible in the situation of a Pre Planckian space-time condition. Secondly, we are assuming in all of this that $|\beta_{k_{\text{average}}}|^2$ is the number of ‘created’ particles in k space, in space-time is in terms of a situation for which we are assuming a very narrow range of k values, so we are when looking at the 2nd line of Eq. (23) referencing an averaged out value for the number of created particles which we then identify as $|\beta_{k_{\text{average}}}|^2$, and have $V(\text{volume}) \leq l_{\text{Planck}}^3$, i.e. with l_{Planck} Planck length.

If so, then we could define having a net energy as given by [31]

$$E_c \sim \left[\int \frac{d^3k}{(2\pi)^3} \right] \cdot \left(|\beta_{k_{\text{average}}}|^2 + \frac{1}{2} \right) \cdot \omega_{k_{\text{average}}} \quad (27)$$

We have several different ways to address what is meant by this energy. Our supposition is that we could make a reference, here, to, if c (speed of light) = 1, to have, here, initially, a transfer of gravitons, as an information carrier, from a prior universe to our present universe so that as a result of a match up in Pre Planckian space-time to Planckian space time we would have Eq. (27) as rendered by, using Hu again, [31]

$$E_c \sim \left[\int \frac{d^3k}{(2\pi)^3} \right] \cdot \left(|\beta_{k_{\text{average}}}|^2 + \frac{1}{2} \right) \cdot \omega_{k_{\text{average}}} \sim \langle n_{\text{gravitons}} \rangle \cdot m_{\text{graviton}} \quad (28)$$

And a graviton count, in the Pre Planckian era we would give as [31]

$$\langle n_{\text{gravitons}} \rangle \sim 1/(\exp(E_c / T_{\text{temp}}) - 1) \quad (29)$$

Here, we would have that $|\beta_{k_{\text{average}}}|^2$ would be the ‘average’ number of particles produced in the Kth mode, and this kth mode would be in Pre Planckian space-time. Then combining Eq. (28) and Eq.(29), if we wish to obtain a ‘Bose’ representation of ‘gravitons’ produced in the immediate aftermath of $|\beta_{k_{\text{average}}}|^2$ as the number of particles produced via a VEV, then we would have, if we have $\hbar = 1$

$$E_c = T_{temp} \cdot \ln\left(1 + \langle n_{graviton} \rangle^{-1}\right) \quad (30)$$

And not just scale down to energy as directly proportional to 1 over the fourth power of the initial scale factor, as referred to earlier, then if

$$T_{temp} < 1.4167 \times 10^{32} eV \quad (31)$$

This should be compared against the following value for the energy as we have set it up, involving a Wheeler De Witt equation. Also the mass of $m_{graviton} \sim 10^{-62} \text{ grams}$

$$m_{graviton} \sim 10^{-62} \text{ grams} \sim 10^{-29} eV \quad (29)$$

We need to make sense of a given value for the inflaton, and then afterwards, to compare the time derivative, in a future work, of this derived inflaton), In doing so, we will by trial and error, perhaps identify the Pre Planckian Hubble parameter value which is given above. I.e. we claim that to do this we will need numerical simulations. Let us now get a value of the inflation stated by our Bohmian approximation which will then, if confirmed lead to a derivation of the values in Eq. (23) above. Thereby, then, giving more confirmation as to [32].

We shall now, based upon the Bohmian development proceed to obtain a first order confirmation as to an inflaton we claim would lead to confirming [32] as well as set up for numerical time evolution for Eq. (31) above.

Using the formulation as given in this article, we write using Planckian scaling of units the following. Using Eq. (22) above, explicitly, we obtain the following for the energy

$$(E)_{\text{initial-Pre-Planck}} \sim m(\text{val}) \cdot \exp(B_1) \cdot \exp(B_2) \cdot \exp(B_3) \cdot \exp(B_4)$$

$$B_j = \frac{1}{2m(\text{val})^2} \cdot C_j$$

$$C_1 = \left[-\frac{3r^4}{r_0^6 \cdot (f_1(r))^2} + \frac{3}{2} \cdot \frac{\left[\frac{6r^2}{r_0^4} - \frac{1}{r_0^2} \right]}{(f_1(r))^2} \right]$$

$$C_2 = \frac{A_1^{-1}}{2f_1(r)} \cdot \left[\frac{1}{r_0^2} - \left(\frac{12r^2}{r_0^4} - \frac{2}{r_0^2} \right) + \frac{r}{r_0^2 A_1^2} \cdot \left(\frac{4r^3}{r_0^4} - \frac{2r}{r_0^2} \right) \right]$$

$$C_3 = A_1^{-1} \cdot \left[\frac{-r^2}{r_0^4} + \left(\frac{f_2(r)}{r_0^2} \right) + \left(\frac{\frac{12r^2}{r_0^4} - \frac{2}{r_0^2}}{(f_1(r))^2} \right) \right]$$

$$C_4 = A_1^{-2} \cdot \left[-\left(\frac{(f_2(r)) \cdot r}{r_0^3} \right) + \frac{(f_2(r)) \cdot \left(\frac{\frac{12r^4}{r_0^6} - \frac{2r^2}{r_0^4}}{(f_1(r))^2} \right)}{(f_1(r))^2} \right] \quad (30)$$

In Planckian units, all the terms in Eq. (30) will effectively have to be non dimensionalized, and then compared with a numerical scheme, which will when this material is confirmed, be worked out in C++ as to confirm numerically, candidates for the derivative, i.e. the time evolution of Eq. (30) so as to compare it with Eq. (15), and then from there to come up with candidates for the Hubble parameter, in a Pre Planckian time

Once this is done, and the formulas held to be approximate, it is conceivable that the datum and speculations given by Dr. Corda in [17] will be examinable and hopefully confirmed.

In short, we would require an enormous 'inflaton' style ϕ valued scalar function, and $a^2(t) \sim 10^{-110}$. I.e. assuming a quantum 'bounce with $a^2(t) \sim 10^{-110}$, and we hope to confirm it soon.

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