INFORMATION AND CONDITIONAL PROBABILITY TO GO BEYOND HIDDEN VARIABLES

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(Dated: June 8, 2016)

We study the relation between the possibility of describing quantum correlation with hidden variables and the existence of the Bloch sphere. We derive some proposition concerning a quantum expected value under an assumption about the existence of the Bloch sphere in N spin-1/2 systems. However, the hidden variables theory violates the proposition with a magnitude that grows exponentially with the number of particles. Therefore, we have to give up either the existence of the Bloch sphere or the hidden variables theory. We show that the introduction of curved information and the continuity equation of probability is in agreement with classical quantum mechanics. So we give up the hidden variable theory as local theory and we accept the Bloch sphere as global theory connected with the information space.

PACS numbers: 03.65.Ud, 03.65.Ca

I. INTRODUCTION

The quantum theory (cf. [1–6]) gives approximate and at times remarkably accurate numerical predictions. Much experimental data approximately fits to the quantum predictions for the past some 100 years. We do not doubt the correctness of the quantum theory. The quantum theory also says new science with respect to information theory. The science is called the quantum information theory [6]. Therefore, the quantum theory gives us very useful another theory in order to create new information science and to explain the handling of raw experimental data in our physical world.

As for the foundations of the quantum theory, Legget type non-local variables theory [7] is experimentally investigated [8–10]. The experiments report that the quantum theory does not accept Leggett-type non-local variables interpretation. As for the applications of the quantum theory, implementation of a quantum algorithm to solve Deutsch's problem [11] on a nuclear magnetic resonance quantum computer is reported firstly [12]. Implementation of the Deutsch-Jozsa algorithm on an ion-trap quantum computer is also reported [13]. There are several attempts to use single-photon two-qubit states for quantum computing. Oliveira *et al.* implement Deutsch's algorithm with polarization and transverse spatial modes of the electromagnetic field as qubits [14]. Single-photon Bell states are prepared and measured [15]. Also the decoherence-free implementation of Deutsch's algorithm is reported by using such single-photon and by using two logical qubits [16]. More recently, a one way based experimental implementation of Deutsch's algorithm is reported [17].

We study the relation between a significant specific hidden variables theory and the existence of the Bloch sphere. The results of measurements are either $+1$ or -1. We derive some proposition concerning a quantum expected value under an assumption about the existence of the Bloch sphere in N spin-1/2 systems. However, the hidden variables theory violates the proposition with a magnitude that grows exponentially with the number of particles. Therefore, we have to give up either the existence of the Bloch sphere or the hidden variables theory. We solve the previous dilemma we introduce the information space and the continuity equation to show how quantum mechanics is consequence of the information locate in all the space so is impossible to represent by local hidden variables. In conclusion we agree on the existence of the Bloch sphere.

II. FROM PROBABILITY CONTINUITY EQUATION AND INFORMATION SPACE TO SHRODINGER EQUATION

1) Continuity equation

Given the continuity equation of the probability (global conservation of the probability) we have

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \tag{1}$$

When the probability density $\rho = R^2$ we have

$$\frac{\partial R^2}{\partial t} + \nabla \cdot (R^2 v) = 2R\frac{\partial R}{\partial t} + \nabla \cdot (R^2 v)$$
$$= 2R\frac{\partial R}{\partial t} + 2R\nabla R v + R^2 \nabla v = 0 \quad (2)$$

For $R \neq 0$ we have

$$\frac{\partial R}{\partial t} + \nabla R v + \frac{1}{2} R \nabla v$$
$$= \frac{\partial R}{\partial t} + \frac{1}{2m}(2\nabla R m v + R \nabla m v) \quad (3)$$

Now in classical mechanics we have for the action S the relation

$$\nabla S = mv = p \quad (4)$$

So

$$\frac{\partial R}{\partial t} + \frac{1}{2m}(R\nabla^2 S + 2\nabla R \nabla S) = 0 \quad (5)$$

where S is the action and for the wave the phase of the wave.

2) Condition probability from statistical parameters (average value, standard deviation and others)

We assume that to found a particle in a particular state is a probabilistic phenomena for which we have join probability that the particle can be in a particular state. Now the novelty is to assume that the probability is function of other external elements as parameters. The average of the position for the particle is a parameter the movement of the particle in a particular environment for example inside of a tube or in other boundary condition (see boundary condition in Shrodinger solution) can change the probability for a particular state. Any far or near change of the environment change the probability of the state (Bell theorem and entanglement) In conclusion the join probability of a state of different variables is conditioned by a set of parameters that statically or physically can define the environment where the particle move. We denote all this parameters as the information relate to the environment where the particle is locate. The set of external parameters are the information space that can have curvature as in the Berry phase phenomena that show that in the Shrodinger solution any loop can change the original phase. In differential geometry any loop in a space with curvature change the original phase of the vectors. Now we built the information space which geodesic tensor is the Fisher entropy or Fisher information by which we can compute the covariant derivatives and the curvature.

Given the system of the conditional probabilities

$$\rho = \rho(x_1, \dots, x_q | \theta_1, \dots, \theta_p) \quad (6)$$

We have the p dimensional information reference

$$e_\alpha = \begin{bmatrix} \int_\Omega \frac{\partial \log \rho}{\partial \theta_1} dx \\ \int_\Omega \frac{\partial \log \rho}{\partial \theta_2} dx \\ \dots \\ \int_\Omega \frac{\partial \log \rho}{\partial \theta_p} dx \end{bmatrix}. \quad (7)$$

Given the vector

$$V = V^\alpha e_\alpha \quad (8)$$

The derivative is

$$\frac{\partial V}{\partial \theta^\beta} = \frac{\partial V^\alpha}{\partial \theta^\beta} e_\alpha + V^\alpha \frac{\partial e_\alpha}{\partial \theta^\beta}$$
$$= \frac{\partial V^\alpha}{\partial \theta^\beta} e_\alpha + V^\gamma \frac{\partial e_\gamma}{\partial \theta^\beta}$$
$$= \frac{\partial V^\alpha}{\partial \theta^\beta} e_\alpha + V^\gamma \int_\Omega \frac{\partial^2 \log \rho}{\partial \theta^\beta \partial \theta^\gamma} dx \quad (9)$$

where

$$\int_\Omega \frac{\partial^2 \log \rho}{\partial \theta^\beta \partial \theta^\gamma} dx = \Gamma^\alpha_{\gamma\beta^{e}\alpha} \quad (10)$$

is the Fisher information matrix connected with the Christoffel symbols $\Gamma^\alpha_{\gamma\beta}$. For

$$\frac{\partial^2 \log \rho}{\partial \theta^k \partial \theta^\gamma} V^\gamma = \frac{\frac{\partial^2 \log \rho}{\partial \theta^k \partial \theta^\gamma}}{\frac{\partial \log \rho}{\partial \theta^k}\frac{\partial \log \rho}{\partial \theta^\gamma}} \frac{\partial \log \rho}{\partial \theta^k} \quad (11)$$

Here,

$$V^\gamma = \frac{1}{\frac{\partial \log \rho}{\partial \theta^\gamma}} \quad (12)$$

For the Fisher information we have

$$\frac{E\left[\frac{\partial^2 \log \rho_j}{\partial \theta^p \partial \theta^\gamma}\right]}{E\left[\frac{\partial \log \rho_j}{\partial \theta^p}\frac{\partial \log \rho}{\partial \theta^\gamma}\right]} = 1 \quad (13)$$

where E is the average operator so we can write in the first approximation

$$D_k = \frac{\partial}{\partial \theta^k} - \frac{\partial \log \rho}{\partial \theta^k} \quad (14)$$

and

$$P_k = p_k - \frac{\partial \log \rho}{\partial \theta^k} \quad (15)$$

where p_k is the momentum. So we have

$$\delta S = \delta \int \rho[\frac{\partial S}{\partial t} + \frac{1}{2m} p_i p_j + V) dt d^n x$$
$$+\delta \frac{1}{2m} \int \rho \frac{\partial \log \rho}{\partial x_i}\frac{\partial \log \rho}{\partial x_j})] dt d^n x = 0 \quad (16)$$

$$\delta \int \rho[\frac{\partial S}{\partial t} + \frac{1}{2m} p_i p_j + V) dt d^n x$$
$$\frac{\delta \rho(\frac{\partial S}{\partial t} + \frac{1}{2m} p_i p_j + V)}{\partial \rho}$$
$$-\frac{\partial}{\partial x_\mu} \frac{\delta \rho(\frac{\partial S}{\partial t} + \frac{1}{2m} p_i p_j + V)}{\partial \frac{\partial \rho}{\partial x_\eta}}$$
$$= \frac{\partial S}{\partial t} + \frac{1}{2m} p_i p_j + V \quad (17)$$

and

$$\delta \int \rho \frac{\partial \log \rho}{\partial x_i} \frac{\partial \log \rho}{\partial x_j})]dtd^n x$$
$$= \frac{\partial (\rho \frac{\partial \log \rho}{\partial x_i} \frac{\partial \log \rho}{\partial x_j})}{\partial \rho}$$
$$- \frac{\partial}{\partial x_\mu} \frac{\partial (\rho \frac{\partial \log \rho}{\partial x_i} \frac{\partial \log \rho}{\partial x_j})}{\partial \frac{\partial \rho}{\partial x_\eta}}$$
$$= \frac{\partial \log \rho}{\partial x_i} \frac{\partial \log \rho}{\partial x_j} - \frac{1}{\rho} \frac{\partial (\frac{\partial \rho}{\partial x_i} \frac{\partial \rho}{\partial x_j})}{\partial \frac{\partial \rho}{\partial x_\mu}}$$
$$= \frac{1}{\rho^2} \frac{\partial \rho}{\partial x_i} \frac{\partial \rho}{\partial x_j} - \frac{1}{\rho} \frac{\partial}{\partial x_\mu} \frac{\partial (\frac{\partial \rho}{\partial x_i} \frac{\partial \rho}{\partial x_j})}{\partial \frac{\partial \rho}{\partial x_\eta}}$$
$$= \frac{1}{\rho^2} \frac{\partial \rho}{\partial x_i} \frac{\partial \rho}{\partial x_j} - \frac{2}{\rho} \frac{\partial^2 \rho}{\partial x_i \partial x_j} = Q \tag{18}$$

So

$$\frac{\partial S}{\partial t} + \frac{1}{2m} p_i p_j + V + \frac{1}{2m} (\frac{1}{\rho^2} \frac{\partial \rho}{\partial x_i} \frac{\partial \rho}{\partial x_j} - \frac{2}{\rho} \frac{\partial^2 \rho}{\partial x_i \partial x_j})$$
$$= \frac{\partial S}{\partial t} + \frac{1}{2m} p_i p_j + V + Q = 0 \tag{19}$$

Now for $R^2 = \rho$ and the Plank constant is equal to 1 we have

$$Q = -\frac{1}{2m} \frac{\nabla^2 R}{R} \tag{20}$$

So we have

$$\frac{\partial S}{\partial t} + \frac{|\nabla S|^2}{2m} - \frac{1}{2m} \frac{\nabla^2 R}{R} + V = 0 \tag{21}$$

where Q is the Bohm quantum potential that is a consequence for the extreme condition of Fisher information (minimum or maximum condition for the Fisher information). We know that the quantum potential as real part and the continuous equation as the imaginary part from the Boltzmann entropic geometry we can generate the Schr?dinger equation. We can also use the Schrodinger equation and came back to the Fisher information and to the pure conditional probability interpretation of the quantum mechanics.

Now we combine the continuity equation of the probability with the covariant derivative in the curved information space we have

$$\frac{\partial S}{\partial t} + (\frac{|\nabla S|^2}{2m} - \frac{1}{2m} \frac{\nabla^2 R}{R} + V) \tag{22}$$

$$i(\frac{\partial R}{\partial t}) + \frac{1}{2m}(R\nabla^2 S + 2\nabla R \nabla S)) = 0 \tag{23}$$

where the real part is consequence of the curvature in the information space and the immaginary part is due to the continuous equation for the probability.

Now for $\Psi = Re^{iS}$ the previous real and complex part are the real and immaginary of the classical Schrodinger equation.

$$i\frac{\partial \Psi}{\partial t} = (-\frac{1}{2m}\nabla^2 + V)\Psi \tag{24}$$

with

$$\frac{h}{2\pi} = 1 \tag{25}$$

In conclusion we can make a reverse process used by Shrodinger we can generate the Shrodinger equation by the information space and the continuity equation of the probability. In this way the Hilbert mechanism can be explain only by information , curvature and probability.

III. A HIDDEN VARIABLES THEORY DOES NOT MEET THE EXISTENCE OF THE BLOCH SPHERE

Assume that we have a set of N spins $\frac{1}{2}$. Each of them is a spin-1/2 pure state lying in the x-y plane. Let us assume that one source of N uncorrelated spin-carrying particles emits them in a state, which can be described as a multi spin-1/2 pure uncorrelated state. Let us parameterize the settings of the jth observer with a unit vector $\vec{n}_j$ (its direction along which the spin component is measured) with $j = 1, \ldots, N$. One can introduce the 'hidden variables' correlation function, which is the average of the product of the hidden results of measurement

$$E_{\rm HV}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \langle r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) \rangle_{\rm avg}, \tag{26}$$

where r is the hidden result. We assume the value of r is ± 1 (in $(\hbar/2)^N$ unit), which is obtained if the measurement directions are set at $\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N$. We introduce ergodic averaging as a theoretical model here. We do not pursue the details of the assumption. To pursue the details is an interesting point. It is suitable to the next step of researches.

Also one can introduce a quantum correlation function with the system in such a pure uncorrelated state

$$E_{\rm QM}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = {\rm tr}[\rho \vec{n}_1 \cdot \vec{\sigma} \otimes \vec{n}_2 \cdot \vec{\sigma} \otimes \cdots \otimes \vec{n}_N \cdot \vec{\sigma}] \tag{27}$$

where $\otimes$ denotes the tensor product, $\cdot$ the scalar product in $\mathbf{R}^2$, $\vec{\sigma} = (\sigma_x, \sigma_y)$ is a vector of two Pauli operators, and ρ is the pure uncorrelated state,

$$\rho = \rho_1 \otimes \rho_2 \otimes \cdots \otimes \rho_N \tag{28}$$

with $\rho_j = |\Psi_j\rangle\langle\Psi_j|$ and $|\Psi_j\rangle$ is a spin-1/2 pure state lying in the x-y plane.

One can write the observable (unit) vector $\vec{n}_j$ in a plane coordinate system as follows:

$$\vec{n}_j(\theta_j^{k_j}) = \cos\theta_j^{k_j} \vec{x}_j^{(1)} + \sin\theta_j^{k_j} \vec{x}_j^{(2)}, \tag{29}$$

where $\vec{x}_j^{(1)} = \vec{x}$ and $\vec{x}_j^{(2)} = \vec{y}$ are the Cartesian axes. Here, the angle $\theta_j^{k_j}$ takes two values (two-setting model):

$$\theta_j^1 = 0,\ \theta_j^2 = \frac{\pi}{2}. \tag{30}$$

We derive a necessary condition to be satisfied by the quantum correlation function with the system in a pure uncorrelated state given in (27). In more detail, we derive the value of the product of the quantum correlation function, $E_{\rm QM}$ given in (27), i.e., $\|E_{\rm QM}\|^2$. We use the decomposition (29). We introduce simplified notations as

$$T_{i_1 i_2 \ldots i_N} = {\rm tr}[\rho \vec{x}_1^{(i_1)} \cdot \vec{\sigma} \otimes \vec{x}_2^{(i_2)} \cdot \vec{\sigma} \otimes \cdots \otimes \vec{x}_N^{(i_N)} \cdot \vec{\sigma}] \tag{31}$$

and

$$\vec{c}_j = (c_j^1, c_j^2) = (\cos\theta_j^{k_j}, \sin\theta_j^{k_j}). \tag{32}$$

Then, we have

$$\begin{aligned} & \|E_{\rm QM}\|^2 \\ & = \sum_{k_1=1}^{2} \cdots \sum_{k_N=1}^{2} \left(\sum_{i_1,\ldots,i_N=1}^{2} T_{i_1\ldots i_N} c_1^{i_1} \cdots c_N^{i_N} \right)^2 \\ & = \sum_{i_1,\ldots,i_N=1}^{2} T_{i_1\ldots i_N}^2 \le 1, \end{aligned} \tag{33}$$

where we use the orthogonality relation $\sum_{k_j=1}^{2} c_j^{\alpha} c_j^{\beta} = \delta_{\alpha,\beta}$. The value of $\sum_{i_1,\ldots,i_N=1}^{2} T_{i_1\ldots i_N}^2$ is bounded as $\sum_{i_1,\ldots,i_N=1}^{2} T_{i_1\ldots i_N}^2 \le 1$. We have

$$\prod_{j=1}^{N} \sum_{i_j=1}^{2} ({\rm tr}[\rho_j \vec{x}_j^{(i_j)} \cdot \vec{\sigma}])^2 \le 1. \tag{34}$$

From the convex argument, all quantum separable states must satisfy the inequality (33). Therefore, it is a separability inequality. It is important that the separability inequality (33) is saturated iff ρ is a multi spin-1/2 pure uncorrelated state such that, for every j, $|\Psi_j\rangle$ is a spin-1/2 pure state lying in the x-y plane. The reason of the inequality (33) is due to the existence of the Bloch sphere in quantum mechanics

$$\sum_{i_j=1}^{2} ({\rm tr}[\rho_j \vec{x}_j^{(i_j)} \cdot \vec{\sigma}])^2 \le 1. \tag{35}$$

The inequality (35) is saturated iff $\rho_j = |\Psi_j\rangle\langle\Psi_j|$ and $|\Psi_j\rangle$ is a spin-1/2 pure state lying in the x-y plane. The inequality (33) is saturated iff the inequality (35) is saturated for every j. Thus we have the maximal possible value of the scalar product as a quantum proposition concerning the Bloch sphere

$$\|E_{\rm QM}\|^2 = 1 \tag{36}$$

when the system is in such a multi spin-1/2 pure uncorrelated state.

A. Hidden variables & reference frames

A hidden variables correlation function, assuming ergodicity, can be written as a weigthed sum over integer indices. For a function r in a function space $\mathcal{R}$ we see e.g.

$$E_{\rm HV}(\vec{n}_1, \ldots, \vec{n}_N) = \lim_{m\to\infty} \frac{1}{m} \sum_{\ell=1}^{m} r(\vec{n}_1, \ldots, \vec{n}_N, \ell) \tag{37}$$

If a formulation with discrete indexing of hidden variables is a sensible way to describe a correlation, then one may select from $\mathcal{R}$ a function such that

$$E_{\rm HV}(\overline{\overline{n}}) = \lim_{m\to\infty} \frac{1}{m} \sum_{\lambda\in\mathcal{L}_m} r(\overline{\overline{n}}, \lambda) \tag{38}$$

Here, the abbreviation $\overline{\overline{n}} = (\vec{n}_1, \ldots, \vec{n}_N)$ is used. Instead of an index, real variables are employed. In addition we may assume that the function space $\mathcal{R}$ contains a Heaviside type of function,

$$f(x) = \lim_{K\to\infty} \exp\left[-\frac{e^{-Kx}}{K}\right] \in \{0,1\} \tag{39}$$

A sign function can then be obtained $\propto f(x) - f(-x)$. Those sign functions are in $\mathcal{R}$ and, hence, r can attain this form of sign function. The set $\mathcal{L}_m$ in (38) is a subset of the interval

$$I = \{\lambda \in \mathbb{R} \,|\, -\infty < \ell_I < \lambda < u_I < \infty; \ell_I, u_I \in \mathbb{R}\}$$

and has cardinality m. More explicitly,

$$\mathcal{L}_m = \{\lambda_1, \lambda_2, ..., \lambda_m; \ell_I < \lambda_1 < \lambda_2, \cdots < \lambda_m < u_I\}$$

The elements of $\mathcal{L}_m$ can be shifted with an infinitesimal $\delta\lambda > 0$. This gives, $\lambda' = \lambda + \delta\lambda$, hence,

$$\mathcal{L}'_m = \{\lambda'_1, \lambda'_2, ..., \lambda'_m; \ell_I < \lambda'_1 < \cdots < \lambda'_m < u_I\}$$

and $\ell_I < \lambda_1 < \lambda'_1 < \lambda_2 < \lambda'_2 \cdots < u_I$.

B. Maximum value and product

We are very interested in the maximum value of the square of an expected value in a probability interpretation of quantum measurement theory. Therefore we focus on each measurement result providing a probability. And we study the maximum value when we inspect the summation. In short, we can multiply a measuremenet result by the same measurement result.

Therefore, we wish we would have some sort of Kronecker delta function at our disposal to match proper terms in the sum. In this respect it must be noted that the correlation in (38) can be arbitrary close approximated with the use of $\mathcal{L}'_m$ in the sum. We have, still using the '=' symbol,

$$E_{\rm HV}(\overline{\overline{n}}) = \lim_{m\to\infty} \frac{1}{m} \sum_{\lambda'\in\mathcal{L}'_m} r(\overline{\overline{n}}, \lambda') \tag{40}$$

Assuming, $m \to \infty$, squaring the $E_{\rm HV}$ is in close approximation equal to the product of expressions in (38) and (40)

$$\{E_{\rm HV}(\overline{\overline{n}})\}^2 = \frac{1}{m^2} \sum_{\lambda \in \mathcal{L}_m} \sum_{\lambda' \in \mathcal{L}'_m} r(\overline{\overline{n}}, \lambda) r(\overline{\overline{n}}, \lambda') \tag{41}$$

Because, of small differences, we may write in a Taylor like approximation,

$$r(\overline{\overline{n}}, \lambda') = r(\overline{\overline{n}}, \lambda) + \delta\lambda \frac{\partial r}{\partial \lambda}(\overline{\overline{n}}, \lambda)$$

The r product in (41) can then be re-written as

$$r(\overline{\overline{n}}, \lambda) r(\overline{\overline{n}}, \lambda') = r^2(\overline{\overline{n}}, \lambda) + \delta\lambda\, r(\overline{\overline{n}}, \lambda) \frac{\partial r}{\partial \lambda}(\overline{\overline{n}}, \lambda) \tag{42}$$

If the previous result from (42) is introduced in (41) then the latter can be re-written as

$$\{E_{\rm HV}(\overline{\overline{n}})\}^2 = \frac{1}{m} \sum_{\lambda \in \mathcal{L}_m} \left[r^2(\overline{\overline{n}}, \lambda) + \delta\lambda\, r(\overline{\overline{n}}, \lambda) \frac{\partial r}{\partial \lambda}(\overline{\overline{n}}, \lambda) \right]$$

If for $r \in \mathcal{R}$ a sign form is empoyeld based on (39), then it is easy to see that for proper $\lambda \in \mathcal{L}_m$, the following

$$0 \le \frac{\partial r}{\partial \lambda}(\overline{\overline{n}}, \lambda) \le 1$$

will be true. The form $r(\overline{\overline{n}}, \lambda)\frac{\partial r}{\partial \lambda}(\overline{\overline{n}}, \lambda)$ contains, numerically and in limit, something similar to $x\delta(x)$ because r is a sign $\propto f(x) - f(-x)$. We can conclude that given r is a sign function based on (39), it is plausible to expect for proper $\mathcal{L}_m$ we obtain

$$\begin{aligned} \{E_{\rm HV}(\overline{\overline{n}})\}^2 &\to \tfrac{1}{m} \textstyle\sum_{\lambda \in \mathcal{L}_m} 1 + O(\delta\lambda^2) \\ &\to 1 + O(\delta\lambda^2) \end{aligned} \tag{43}$$

for $m \to \infty$.

C. Quantum and hv values

We study the possibility, $E_{\rm QM} = E_{\rm HV}$. For the ease of the presentation we write, $r(\lambda)$ and suppress the $(\vec{n}_1, \vec{n}_2, \ldots \vec{n}_N)$ notation. Hence, from our previous considerations we then write, under the limit, $m \to \infty$ and employing the result in (43)

$$\begin{aligned} &\|E_{\rm QM}\|^2 \\ &= \sum_{k_1=1}^{2} \cdots \sum_{k_N=1}^{2} \\ &\left(\frac{1}{m} \sum_{\lambda \in \mathcal{L}_m} r(\lambda) \times \frac{1}{m} \sum_{\lambda' \in \mathcal{L}'_m} r(\lambda') \right) \\ &\to \sum_{k_1=1}^{2} \cdots \sum_{k_N=1}^{2} \left(1 + O(\delta\lambda^2)\right) \to 2^N. \end{aligned} \tag{44}$$

We use the following fact

$$(r(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N, \lambda))^2 = +1. \tag{45}$$

for properly selected $\mathcal{L}_m$. Hence one has the following proposition concerning the hidden variables theory

$$\|E_{\rm QM}\|^2 \to 2^N, (m \to \infty). \tag{46}$$

Clearly, we cannot assign the truth value "1" for two propositions (36) (concerning the Bloch sphere) and (46) (concerning the hidden variables theory), simultaneously, when the system is in a multiparticle pure uncorrelated state. Of course we can imagine $\mathcal{L}_m$ where this would be possible but the hv theory would be in need of a sufficient number of special points in the $\mathcal{L}_m$ that makes the r vanish for all $\overline{\overline{n}}$. In general the claim can be made that the selection of hv theories is not as free as one would prefer.

To continue we note, each of the theories refers to a spin-1/2 pure state lying in the x-y plane. Therefore, we are in the contradiction when the system is in such a multiparticle pure uncorrelated state. Thus, we cannot accept a general validity of the proposition of a hidden variables theory, if we assign the truth value "1" for the proposition (36) (concerning the Bloch sphere).

IV. CONCLUSIONS AND DISCUSSIONS

In conclusion, we have studied the relation between a hidden variables theory and the existence of the Bloch sphere and with a new type of conditional probability and Fisher information as metric for information space we show that the hidden variable are not possible. Now we have derived some proposition concerning a quantum expected value under an assumption about the existence of the Bloch sphere in N spin-1/2 systems. However, the hidden variables theory has violated the proposition with a magnitude that grows exponentiall with the number of particles. Therefore, we have had to give up either the existence of the Bloch sphere or the hidden variables theory. The hidden variables theory does not have depictured physical phenomena using the existence of the Bloch sphere with a violation factor that grows exponentially with the number of particles. Now we point out the problem that when we cannot measure an observable we cannot say nothing on this measure as in the non-commutative case. So we have contradictions. In classical interpretation of quantum mechanics does not exist conditional probability and we cannot measure the probability but with the introduction of the information space and Fisher metric we show that conditional probability is possible but limited to statistical parameters as average value or other parameters. So contradiction is eliminate. Now entanglement and Bell theorem can be understand in a new type of set theory that include copula [18] and information [19]. May be we are right that projection operator is not sufficient to understand quantum mechanics so we cannot give Hilbert space axiomatic structure. Now axiomatic Hilbert space is useful but cannot explain completely the meaning of the quantum mechanics. With information space we can give a meaning with the axiomatic Hilbert that is always a useful mathematical instrument to use information and probability together.

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