

Second mathematical Model of Ball Lightning

Abstract

Based on the Maxwell's equations and on the understanding of the electrical conductivity of the body of ball lightning, a mathematical model of ball lightning is built; the structure of the electromagnetic field and of electric current in it is shown. Next it is shown (as a consequence of this model) that in a ball lightning the flow of electromagnetic energy can circulate and thus the energy obtained by a ball lightning when it occurs can be saved. Sustainability, luminescence, charge, time being, the mechanism of formation of ball lightning are briefly discussed.

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1. Introduction

The hypotheses that were made about the nature of ball lightning are unacceptable because they are contrary to the law of energy conservation. This occurs because the luminescence of ball lightning is usually attributed to the energy released in any molecular or chemical transformation, and so it is suggested source of energy, due to which the ball lightning glows is located in it.

Kapitsa P.L. 1955 [1]

This assertion (as far as the author knows) is true also today. It is reinforced by the fact that the currently estimated typical ball lightning contains tens of kilojoules [2], released during its explosion.

It is generally accepted that ball lightning is somehow connected with the electromagnetic phenomena, but there is no rigorous description of these processes.

A mathematical model of a globe lightning based on the Maxwell equations, which enabled us to explain many properties of the globe lightning, is proposed in [8]. However, this model turned out to be quite intricate as to the used mathematical description. Another model of the ball lightning which is substantiated to a greater extent and makes it possible to obtain less intricate mathematical description is outlined below. Moreover, this model agrees with the model of a spherical capacitor [9].

When constructing the mathematical model, it will be assumed that the globe lightning is plasma, i.e. gas consisting of charged particles – electrons, and positive charged ions, i.e. the globe lightning plasma is fully ionized. In addition, it is assumed that the number of positive charges equal to the number of negative charges, and, hence, the total charge of the globe lightning is equal to zero. For the plasma, we usually consider charge and current densities averaged over an elementary volume. Electric and magnetic fields created by the average “charge” density and the “average” current density in the plasma obey the Maxwell equations [1]. The effect of particles collision in the plasma is usually described by the function of particle distribution in the plasma. These effects will be accounted for the Maxwell equations assuming that the plasma possesses some electric resistance or conductivity.

For reader convenience, this paper repeats many fragments from paper [8].

2. The solution of Maxwell equations in spherical coordinates

Fig. 1 shows a system of spherical coordinates (ρ, θ, φ) and the Table 1 gives the expressions for rotor and divergence of vector \mathbf{E} in these coordinates. Here

E - intensity of electric field,

H - intensity of magnetic field,

J - currents density,

μ - absolute permeability,

ε - absolute dielectric permittivity.

The Maxwell equations in the spherical coordinates in the GHS system without any non-compensated charges are presented in Table 2.

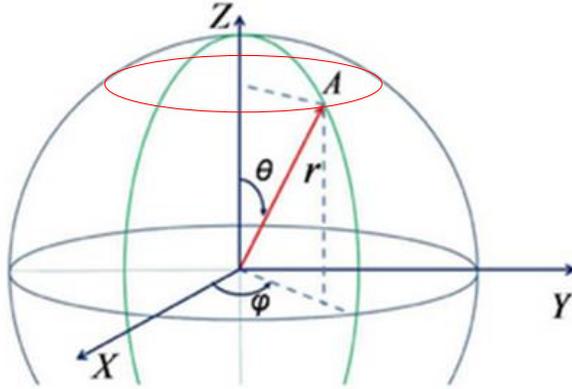


Fig. 1.

Table 1.

1	2	3
1	$\text{rot}_\rho(E)$	$\frac{E_\varphi}{\rho g(\theta)} + \frac{\partial E_\varphi}{\rho \partial \theta} - \frac{\partial E_\theta}{\rho \sin(\theta) \partial \varphi}$
2	$\text{rot}_\theta(E)$	$\frac{\partial E_\rho}{\rho \sin(\theta) \partial \varphi} - \frac{E_\varphi}{\rho} - \frac{\partial E_\varphi}{\partial \rho}$
3	$\text{rot}_\varphi(E)$	$\frac{E_\theta}{\rho} + \frac{\partial E_\theta}{\partial \rho} - \frac{\partial E_\rho}{\rho \partial \varphi}$
4	$\text{div}(E)$	$\frac{E_\rho}{\rho} + \frac{\partial E_\rho}{\partial \rho} + \frac{E_\theta}{\rho g(\theta)} + \frac{\partial E_\theta}{\rho \partial \theta} + \frac{\partial E_\varphi}{\rho \sin(\theta) \partial \varphi}$

Table 2.

1	2
1.	$\text{rot}_\rho H - \varepsilon \frac{\partial E_\rho}{\partial t} - J_\rho = 0$
2.	$\text{rot}_\theta H - \varepsilon \frac{\partial E_\theta}{\partial t} - J_\theta = 0$
3.	$\text{rot}_\varphi H - \varepsilon \frac{\partial E_\varphi}{\partial t} - J_\varphi = 0$
4.	$\text{rot}_\rho E - \mu \frac{\partial H_\rho}{\partial t} = 0$

5.	$\text{rot}_\theta E - \mu \frac{\partial H_\theta}{\partial t} = 0$
6.	$\text{rot}_\phi E - \mu \frac{\partial H_\phi}{\partial t} = 0$
7.	$\text{div}(E) = 0$
8.	$\text{div}(H) = 0$

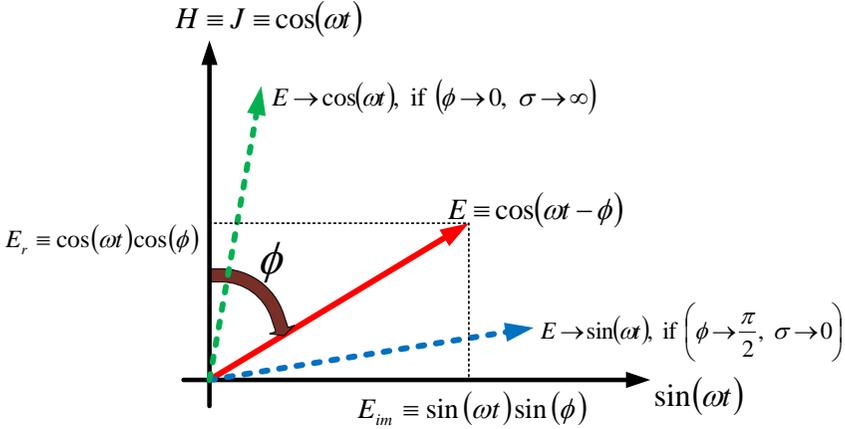


Fig. 2.

A monochromatic solution to these equations will be sought for below. For this purpose, let us write the functions E , H , J in the time domain in the following form:

$$H = H_o \cos(\omega t),$$

$$E = E_o (\sin(\omega t) \sin(\phi) + \cos(\omega t) \cos(\phi)),$$

$$J = E_o \sigma \cos(\omega t) \cos(\phi),$$

where ϕ is the phase angle between the electric and the magnetic strength - see Fig. 2. Considering this assumption, the solution to the Maxwell equations will be sought for in the form of functions E , H , J presented in Table 3, where the functions of type $E_{\phi\phi}(\rho)$ are to be determined. It should be noted here that these functions are independent of the argument ϕ .

Table 3.

1	2
	$E_\rho = E_{\rho\rho}(\rho)\cos(\theta)(\sin(\phi)\sin(\omega t) + \sigma\cos(\phi)\cos(\omega t))$
	$E_\theta = E_{\theta\rho}(\rho)\sin(\theta)(\sin(\phi)\sin(\omega t) + \sigma\cos(\phi)\cos(\omega t))$
	$E_\varphi = E_{\varphi\rho}(\rho)\sin(\theta)(\sin(\phi)\sin(\omega t) + \sigma\cos(\phi)\cos(\omega t))$
	$H_\rho = H_{\rho\rho}(\rho)\cos(\theta)\cos(\omega t)$
	$H_\theta = H_{\theta\rho}(\rho)\sin(\theta)\cos(\omega t)$
	$H_\varphi = H_{\varphi\rho}(\rho)\sin(\theta)\cos(\omega t)$

It is demonstrated in [9] that this solution exists with

$$H_{\theta\rho} = \frac{A}{2\rho}(\sin(q_1(\rho - R) + \beta_1) + \sin(q_2(\rho - R) + \beta_2)), \quad (1)$$

$$H_{\varphi\rho} = \frac{-A}{2\rho}(\cos(q_1(\rho - R) + \beta_1) + \cos(q_2(\rho - R) + \beta_2)), \quad (2)$$

$$H_{\rho\rho} = \frac{-A}{\rho^2}\left(\frac{1}{q_1}\cos(q_1(\rho - R) + \beta_1) + \frac{1}{q_2}\cos(q_2(\rho - R) + \beta_2)\right), \quad (3)$$

$$E_{\theta\rho} = \frac{A}{2\rho}(g_1\cos(q_1(\rho - R) + \beta_1) + g_2\cos(q_2(\rho - R) + \beta_2)), \quad (4)$$

$$E_{\varphi\rho} = \frac{A}{2\rho}(g_1\sin(q_1(\rho - R) + \beta_1) + g_2\sin(q_2(\rho - R) + \beta_2)), \quad (5)$$

$$E_{\rho\rho} = \frac{A}{\rho^2}(w_1\sin(q_1(\rho - R) + \beta_1) + w_2\sin(q_2(\rho - R) + \beta_2)), \quad (6)$$

where

$$q_1 = \frac{\omega}{c}\sqrt{\mu\varepsilon}, \quad (7)$$

$$q_2 = \frac{\omega}{c}\sqrt{\mu\varepsilon\sigma \cdot \cos(\phi)}. \quad (8)$$

$$g_1 = \sqrt{\frac{\mu}{\varepsilon}}, \quad (9)$$

$$g_2 = \sqrt{\frac{\mu}{\varepsilon \cdot \sigma \cdot \cos(\phi)}}, \quad (10)$$

$$w_1 = \frac{c}{\omega \varepsilon}, \quad (11)$$

$$w_2 = \frac{c}{\omega \varepsilon \sigma \cdot \cos(\phi)}. \quad (12)$$

A , β_1 , β_2 are the constants.

It is demonstrated in [9] that instead of the pair of vectors \vec{H}_ϕ and \vec{H}_θ we can consider a single sum vector

$$\vec{H}_{\phi\theta} = \vec{H}_\phi + \vec{H}_\theta, \quad (13)$$

which is in the plane tangent to the sphere of radius ρ and has an angle Ψ to the parallel line. The module of this vector and angle Ψ can be determined from the following correlations:

$$|\vec{H}_{\phi\theta}| = \frac{A}{2\rho}, \quad (14)$$

$$\Psi = \frac{\pi}{2} - \frac{\omega}{c}(\rho - R) - \beta, \quad (15)$$

where R is the radius of the sphere, and $\beta = \beta_1 = \beta_2$. From (14) and Table 3 it follows that

$$H_{\phi\theta} = |H_{\phi\theta}| \sin(\theta) \cos(\omega t) = \frac{A}{2\rho} \sin(\theta) \cos(\omega t). \quad (16)$$

Similar correlations do exist for the vectors \vec{E}_ϕ and \vec{E}_θ , namely:

$$|\vec{E}_{\phi\theta}| = \frac{A}{2\rho}, \quad (17)$$

$$\Psi_e = \frac{\omega}{c}(\rho - R) - \beta, \quad (18)$$

or

$$\Psi_e = \frac{\pi}{2} - \Psi. \quad (19)$$

From (17) and Table 3 it follows that

$$E_{\phi\theta} = \frac{A}{2\rho} \sin(\theta) (\sin(\phi) \sin(\omega t) + \sigma \cos(\phi) \cos(\omega t)). \quad (20)$$

Fig. 3 shows vectors \vec{H}_φ , \vec{H}_θ , \vec{E}_φ , \vec{E}_θ , $\vec{H}_{\varphi\theta}$, $\vec{E}_{\varphi\theta}$ going from point T with coordinates (φ, θ) . The angle between the vectors $\vec{H}_{\varphi\theta}$ and $\vec{E}_{\varphi\theta}$ in the plane P is right.

Thus, in a sphere we may consider only one vector of the electrical field strength $\vec{E}_{\varphi\theta}$ and only one vector of the magnetic field strength $\vec{H}_{\varphi\theta}$. As these vectors lie on sphere, we shall call them spherical vectors. Hence, only spherical $\vec{H}_{\varphi\theta}$ & $\vec{E}_{\varphi\theta}$ and radial \vec{H}_ρ & \vec{E}_ρ strength components exist in the sphere. Fig. 4 shows vectors $\vec{H}_{\varphi\theta}$ and $\vec{E}_{\varphi\theta}$ lying in the plane P and vectors \vec{H}_ρ and \vec{E}_ρ lying along the radius.

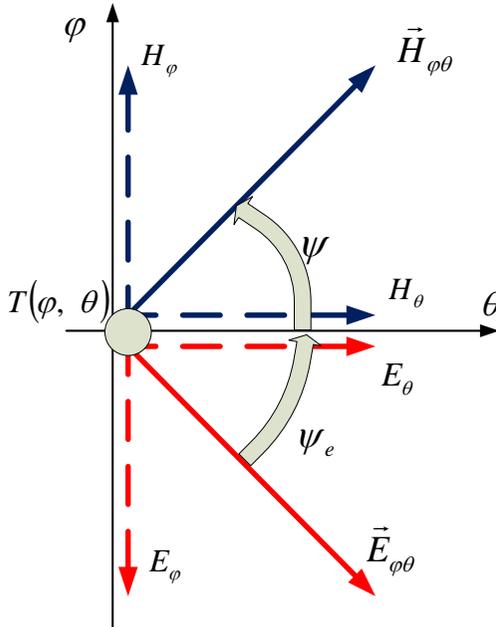


Fig. 3.

Bear in mind that this solution has been obtained under the following assumptions: the sphere is conductive and neutral (does not have any uncompensated charges). Obviously, this solution is not unique. Its existence means only that in a conductive and neutral sphere, an electromagnetic wave can exist, and currents can circulate.

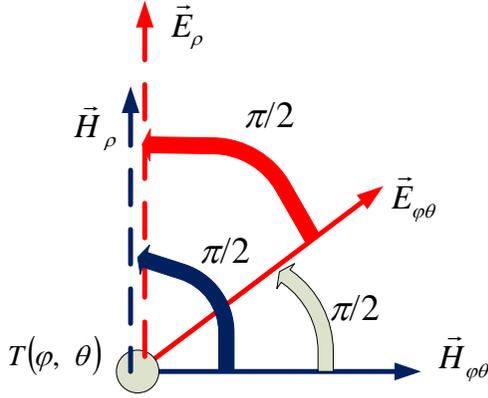


Fig. 4.

3. Energy

From Table 3 follows that a globe lightning contains the following energy components

- Active loss energy W_a – see the second term in the expression for the electric strength:
- Reactive electric energy W_e – see the first term in the expression for the electric strength:
- Reactive magnetic energy W_h – see the expression for the magnetic strength

Let us write these characteristics

$$W_a = (\sigma \cos(\phi) \cos(\omega t))^2 \iint_{\rho, \theta} \left((E_{\rho\rho}(\rho) \cos(\theta))^2 + \sin^2(\theta) ((E_{\theta\rho}(\rho))^2 + (E_{\varphi\rho}(\rho))^2) \right) d\rho d\theta, \quad (21)$$

$$W_e = (\sin(\phi) \sin(\omega t))^2 \iint_{\rho, \theta} \left((E_{\rho\rho}(\rho) \cos(\theta))^2 + \sin^2(\theta) ((E_{\theta\rho}(\rho))^2 + (E_{\varphi\rho}(\rho))^2) \right) d\rho d\theta, \quad (22)$$

$$W_h = (\cos(\omega t))^2 \iint_{\rho, \theta} \left((H_{\rho\rho}(\rho) \cos(\theta))^2 + \sin^2(\theta) ((H_{\theta\rho}(\rho))^2 + (H_{\varphi\rho}(\rho))^2) \right) d\rho d\theta. \quad (23)$$

Obviously, the amplitudes of energies W_e and W_h can be equal when the A , β_1 , β_2 - see (1–6) have certain values. In this case, the energies W_e and W_h transform into each other – see multipliers $(\sin(\omega t))^2$ and $(\cos(\omega t))^2$ in correlations (22, 23). Thus, the energy

conservation law is fulfilled for the globe lighting as a whole in the obtained solution.

At the same time, Table 3 demonstrates that the energy conservation law is not met at each point of the sphere. Hence, there are energy flows between sphere points. This fact will be proved rigorously below.

4. The Energy Flow

4.1. Radial Energy Flux

There is an electromagnetic energy flux along the radius at each point of the sphere, see Fig. 5. The density vector of this flux is equal to

$$\vec{S}_\rho = \vec{E}_{\varphi\theta} \times \vec{H}_{\varphi\theta}. \quad (24)$$

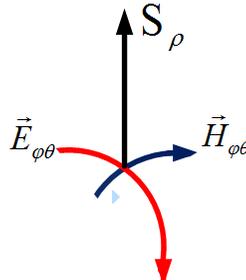


Рис. 5.

As the vectors $\vec{E}_{\varphi\theta}$, $\vec{H}_{\varphi\theta}$ are perpendicular, from (16, 20) it follows that:

$$|\vec{S}_\rho| = |\vec{E}_{\varphi\theta}| |\vec{H}_{\varphi\theta}| = \frac{A^2}{4\rho^2} (\sin(\theta) (\sin(\phi) \sin(\omega t) + \sigma \cos(\phi) \cos(\omega t))) (\sin(\theta) \cos(\omega t))$$

or

$$|\vec{S}_\rho| = \frac{A^2}{4\rho^2} \sin^2(\theta) \cos(\omega t) (\sin(\phi) \sin(\omega t) + \sigma \cos(\phi) \cos(\omega t))$$

or

$$|\vec{S}_\rho| = \frac{A^2}{4\rho^2} \sin^2(\theta) \left(\frac{1}{2} \sin(\phi) \sin(2\omega t) + \sigma \cos(\phi) \cos^2(\omega t) \right) \quad (25)$$

In particular, for $\sigma = 0$ we have: $\sin(\phi) = 1$ and

$$|\vec{S}_\rho| = \frac{A^2}{8\rho^2} \sin^2(\theta) \sin(2\omega t). \quad (26)$$

4.2. Spherical Energy Flux

At each point of the sphere, there are two fluxes of the electromagnetic energy tangent to the sphere, see Fig. 6. The density vector of these fluxes can be written as

$$\vec{S}_1 = \vec{E}_{\varphi\theta} \times \vec{H}_\rho, \quad (27)$$

$$\vec{S}_2 = \vec{H}_{\varphi\theta} \times \vec{E}_\rho. \quad (28)$$

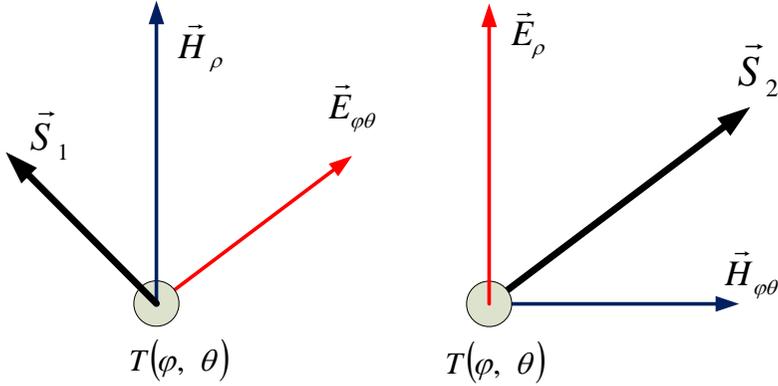


Fig. 6.

As the multiplied vectors are perpendicular, from (14, 16, 20) and Table 3 we can obtain :

$$|\vec{S}_1| = |\vec{E}_{\varphi\theta} \|\vec{H}_\rho| = \frac{A}{2\rho} \sin(\theta) (\sin(\phi) \sin(\omega t) + \sigma \cos(\phi) \cos(\omega t)) \bullet$$

$$\bullet H_\rho(\rho) \cos(\theta) \cos(\omega t)$$

$$|\vec{S}_2| = |\vec{H}_{\varphi\theta} \|\vec{E}_\rho| = \frac{A}{2\rho} \sin(\theta) \cos(\omega t) \bullet$$

$$\bullet E_\rho(\rho) \cos(\theta) (\sin(\phi) \sin(\omega t) + \sigma \cos(\phi) \cos(\omega t))$$

or

$$|\vec{S}_1| = \frac{A}{2\rho} H_\rho(\rho) \sin(\theta) \cos(\theta) \cos(\omega t) \begin{pmatrix} \sin(\phi) \sin(\omega t) + \\ \sigma \cos(\phi) \cos(\omega t) \end{pmatrix},$$

$$|\vec{S}_2| = \frac{A}{2\rho} E_\rho(\rho) \sin(\theta) \cos(\theta) \cos(\omega t) \begin{pmatrix} \sin(\phi) \sin(\omega t) + \\ \sigma \cos(\phi) \cos(\omega t) \end{pmatrix}.$$

As these fluxes are perpendicular, the module of their sum can be determined by the formula

$$S_3 = |\vec{S}_1 + \vec{S}_2| = \left(\begin{array}{l} \frac{A}{2\rho} \sqrt{(H^2_{\rho\rho}(\rho) + E^2_{\rho\rho}(\rho))} \bullet \\ \bullet \sin(\theta) \cos(\theta) \cos(\omega t) \left(\begin{array}{l} \sin(\phi) \sin(\omega t) + \\ \sigma \cos(\phi) \cos(\omega t) \end{array} \right) \end{array} \right) \quad (29)$$

In particular, for $\sigma = 0$ we have $\sin(\phi) = 1$ and

$$S_3 = |\vec{S}_1 + \vec{S}_2| = \left(\begin{array}{l} \frac{A}{2\rho} \sqrt{(H^2_{\rho\rho}(\rho) + E^2_{\rho\rho}(\rho))} \bullet \\ \bullet \sin(\theta) \cos(\theta) \cos(\omega t) \sin(\omega t) \end{array} \right)$$

or

$$S_3 = |\vec{S}_1 + \vec{S}_2| = \left(\begin{array}{l} \frac{A}{8\rho} \sqrt{(H^2_{\rho\rho}(\rho) + E^2_{\rho\rho}(\rho))} \bullet \\ \bullet \sin(2\theta) \sin(2\omega t) \end{array} \right) \quad (30)$$

Considering (3, 6), for $\sigma = 0$ we have

$$S_3 = \frac{A^2 \sqrt{2}}{8\rho^3 q_1} \sin(2\theta) \sin(2\omega t). \quad (31)$$

4.3. Total Energy Flux

Let us find the electromagnetic energy flux divergence for $\sigma = 0$ from (26, 30):

$$\begin{aligned} \text{div}(S_\rho + S_3) &= \frac{\partial S_\rho}{\partial \rho} + \frac{\partial S_3}{\partial \theta} = \\ &= \frac{\partial}{\partial \rho} \left(\frac{A^2}{8\rho^2} \sin^2(\theta) \sin(2\omega t) \right) + \frac{\partial}{\partial \theta} \left(\frac{A^2 \sqrt{2}}{8\rho^3 q_1} \sin(2\theta) \sin(2\omega t) \right) = \\ &= \frac{-2A^2}{8\rho^3} \sin^2(\theta) \sin(2\omega t) + \frac{2A^2 \sqrt{2}}{8\rho^3 q_1} \cos(2\theta) \sin(2\omega t) = \\ &= \frac{A^2}{4\rho^3} \left(\frac{\sqrt{2}}{q_1} \cos^2(\theta) - \left(\frac{\sqrt{2}}{q_1} + 1 \right) \sin^2(\theta) \right) \sin(2\omega t) \end{aligned} \quad (32)$$

Considering (7), we obtain that $\frac{\sqrt{2}}{q_1} \gg 1$. Then, from (32) one can find

that :

$$\text{div}(S_\rho + S_3) = \frac{A^2 \sqrt{2}}{4\rho^3 q_1} \cos(2\theta) \sin(2\omega t). \quad (33)$$

This divergence of the total electromagnetic energy flux is not zero at many points of the sphere. This means that the energy flux passing through a point is not generally equal to zero. Hence, there is energy exchange between the sphere points. However, the energy conservation law is met for the overall sphere (see above). Thus, in the globe lightning:

- the energy conservation law is met,
- there is an electromagnetic energy flux.

5. About Ball Lightning Stability

The question of stability for bodies, in which a flow of electromagnetic energy is circulating, has been treated in [3]. Here we shall consider only such force that acts along the diameter and breaks the ball lightning along diameter plane perpendicular to this diameter. In the first moment it must perform work

$$A = F \frac{dR}{dt}. \quad (34)$$

This work changes the internal energy of the ball lightning, i.e.

$$A = \frac{dW}{dt}. \quad (35)$$

Considering (34, 35) together, we find:

$$F = \frac{dW}{dt} \Big/ \frac{dR}{dt}. \quad (36)$$

If the energy of the global lightning is proportional to the volume, i.e.

$$W = aR^3. \quad (37)$$

where a – is the coefficient of proportionality, then

$$\frac{dW}{dt} = 3aR^2 \frac{dR}{dt}. \quad (38)$$

Thus,

$$F = \frac{dW}{dt} \Big/ \frac{dR}{dt} = 3aR^2 = \frac{3W}{R} \quad (39)$$

Thus, the internal energy of a ball lightning is equivalent to the force creating the stability of ball lightning.

6. About Luminescence of the Ball Lightning

The problem was solved above considering the electric resistance of the globe lightning. Naturally, it is not zero, and when current flows through it, thermal energy is released. This thermal energy is radiated that is the cause of globe lightning illumination.

7. About the Time of Ball Lightning Existence

We can assume that the globe lightning energy is equal to the amplitude of the electric energy, i.e. according to (22),

$$W = \sin^2(\phi) \iint_{\rho, \theta} \left((E_{\rho\rho}(\rho) \cos(\theta))^2 + \sin^2(\theta) (E_{\theta\rho}(\rho))^2 + (E_{\phi\rho}(\rho))^2 \right) d\rho d\theta. \quad (40)$$

The heat loss power is equal to the derivative of the heat loss energy with respect to time. Expression (23) gives the instantaneous energy of heat losses. Therefore,

$$P = \sqrt{2}(\sigma \cos(\phi))^2 \iint_{\rho, \theta} \left((E_{\rho\rho}(\rho) \cos(\theta))^2 + \sin^2(\theta) (E_{\theta\rho}(\rho))^2 + (E_{\phi\rho}(\rho))^2 \right) d\rho d\theta. \quad (41)$$

The existence time of the globe lightning is equal to the time the electrical energy transforms into the heat losses, i.e.

$$\tau = \frac{W}{P}. \quad (42)$$

From (40-42) we can obtain:

$$\tau = \frac{\sin^2(\phi)}{\sqrt{2}(\sigma \cos(\phi))^2} = \frac{tg^2(\phi)}{\sqrt{2}\sigma^2}. \quad (43)$$

8. About a Possible Mechanism of Ball Lightning Formation

The leader of a linear lightning, meeting a certain obstacle, may alter the motion trajectory from linear to circular. This may become the cause of the emergence of the described above electromagnetic fields and currents.

In [4] this process was described as follows:

Another strong bolt of lightning, simultaneous with a bang, illuminated the entire space. I can see how a long and dazzling beam in the color of sun beam approaches to me right in the solar plexus. The end of it is sharp as a razor, but further it becomes thicker and thicker, and reaches something like 0,5 meter. Further I can't see, as I am staring at a downward angle.

Instant thought that it is the end. I see how the tip of the beam approaches. Suddenly it stopped and between the tip and the body began to swell a ball the size of a large grapefruit. There was a thump as if a cork popped from a bottle of champagne. The beam flew into a ball. I see the blindingly bright ball, color of the sun, which

rotates at a breakneck pace, grinding the beam inside. But I do not feel any touch, any beat.

The ball grinds the ray and increases in size. ... The ball does not issue any sounds. At first it was bright and opaque, but then begins to fade, and I see that it is empty. Its shell has changed and it became like a soap bubble. The shell rotates, its diameter remained stable, but the surface was with metallic sheen.

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