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Ghost DBI-essence in fractal geometry

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Abstract. Focusing on a fractal geometric ghost dark energy, we reconstruct the Dirac-Born-Infeld(DBI)-essence type scalar field and find exact solutions of the potential and warped brane tension. We also discuss statefinders for the selected dark energy description to make it distinguishable among others.

PACS. 04.50.Kd Modified theories of gravity – 95.35.+d Dark Matter – 98.80.-k Cosmology

1 Introduction

The mysteries of our universe always remain very attractive issue in literature. Recent data observed and collected by Supernova Search Team[1], Boomerang Collaboration[2], Supernovae Cosmology Project Collaboration[3,4], Wmap Collaboration[5,6] and Planck collaboration[7] have shown that the Universe has an accelerated expansion. The speedy expansion of our Universe is due to an antigravity force which is drawing galaxies apart from each other, dubbed as exotic dark content. Since the advent of Einstein's general relativity theory several significant issues like the cosmological missing matter problem in curved spacetime have been investigated and many of them have non-specific solutions and are still in doubt. The observations mentioned above have indicated that the Universe is ruled by galactic dark energy having large negative pressure and occupying 68.3 percent[7] density of the Universe. The remaining part is occupied by 26.8 percent dark matter and 4.9 percent other cosmic ordinary matters[7].

The cosmological parameter is the most suitable theoretical candidate of accelerated expansionary behavior, but it yields some problematic cases like the cosmic-coincidence puzzle[8] and fine-tuning[9]. The scalar field desctiptions[10–12,43,14], extra dimensions[15,16], modified theories of gravitation[17,18] and reconstruction in modified gravity[19] are other possible theoretical suggestions to get complete information about the speedy expansion of our Universe.

Scalar fields can describe effectively the galactic dark energy effect and they naturally arise in the M/String-theory and super-symmetric theories. Therefore, scalar field definitions of the dark effect are expected to reveal the dynamical mechanism of accelerated expansion. Fundamental gravitation theories suggest several scalar field formulations, but they cannot give a unique potential definiton. On the other hand, it can be established a connection between dark energy formulations (ghost[20], holographic[21], newagegrapgic[22], pilgrim[23], Chaplygin gas[24], Polytropic gas[25]) and scalar field models to obtain an exact definition for the scalar field potential. Here, we use the ghost energy description effectively in a fractal geometry to investigate dynamics of DBI-essence type scalar field.

In this study, we reestablish the dynamics of DBI-essence scalar field model by considering the fractal version of ghost energy. We briefly introduce the fractal theory in the next section, and we construct a fractal connection between the DBI-essence and ghost energy in the third section. Next, we calculate the deceleration and statefinder diagnostic parameters exactly in the fourth section. In the final section, we share our conclusions.

2 Ghost dark effect scenario in fractal theory

It is further assumed that the exotic dark contents are embedded in a flat Friedmann-Robertson-Walker(FRW) type background which is compatible with the recent cosmological data[5,6,26,27]

$$ds^2 = a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] - dt^2, \tag{1}$$

where $a(t)$ denotes the scale term which determines the expansion rate.

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The four-dimensional time-like fractal theory is described by the following action[28,29]:

$$S = S_G + S_m, \tag{2}$$

where

$$S_G = \frac{1}{2\kappa^2}\int d\chi\sqrt{-g}[R - \upsilon\partial_\mu\eta\partial^\mu\eta], \qquad S_m = \int d\chi\sqrt{-g}\pounds_m. \tag{3}$$

Here $\kappa^2 = 8\pi\tilde{G}$ where $\tilde{G}$ shows the gravitational parameter. Additionally, g, R, υ, η and $\pounds_m$ denote the determinant of metric tensor, Ricci scalar, fractal parameter, fractal function and the lagrangian density of matter part, respectively. It is significant that $d\chi(x)$ defines the Lebesgue-Stieltjes measure extending d^4x which is known as the standard measure and $[\chi] = -D\sigma$ is the dimension of χ with a positive σ constant.

The fractal gravitation theory is Lorentz invariant, power-counting renormalizable and also free from the ultraviolet divergence[30]. Recently, Calcagni[28,29] has studied the quantum theory of cosmology in a fractal geometric model. Making use of a time-like case $\eta = t^{-\alpha}$ where the fractal paremeter $\alpha = 4(1-\sigma)$ describes the dimension, Calcagni[29] recovered that

$$H^2 - \alpha H t^{-1} + \frac{\upsilon\alpha^2}{6t^{2(\alpha+1)}} = \frac{1}{3M_p^2}(\rho_D + \rho_m), \tag{4}$$

where $M_p = (8\pi\tilde{G})^{-\frac{1}{2}}$, $H = \frac{\dot{a}}{a}$, ρ_D and ρ_m show the Planck mass, Hubble parameter and the densities of dark quantities, respectively. We further consider a pressureless dark matter condition, i.e. $p_m = 0$. Next, it is known also that $\alpha = 0$ implies the infra-red divide while $\alpha = 2$ describes the ultra-violet territory[29].

Additionally, the fractal continuity relation[29] is given as

$$\dot{\rho} - (\frac{\alpha}{t} - 3H)(\rho + p) = 0, \tag{5}$$

where p and ρ represent the pressure and total energy density, respectively. Nonetheless, the fractal form of gravitational constraint[29] is

$$\dot{H} + 3H^2 + \left(2 + \frac{3\upsilon}{t^{2\alpha}}\right)\frac{\alpha H}{t} - \frac{\alpha(\alpha+1)}{t^2} - \frac{\upsilon\alpha(2\alpha+1)}{t^{2\alpha+2}} = 0. \tag{6}$$

Note that, in the infra-red sector, equation (4) can be reduced to the corresponding relation of general relativity. Therefore, the gravitational constraint in the ultra-violet territory becomes

$$\dot{H} + 3H^2 + \left(2 + \frac{3\upsilon}{t^4}\right)\frac{2H}{t} - \frac{6}{t^2} - \frac{10\upsilon}{t^6} = 0. \tag{7}$$

After solving this equation, it is obtained that[29]

$$a^3(t) = \frac{1}{t^6}\Theta(\frac{11}{4};\frac{13}{4};\frac{3\upsilon}{2t^4}), \tag{8}$$

$$H(t) = -\frac{2}{t} - \frac{22\upsilon}{13t^5}\frac{\Theta(\frac{15}{4};\frac{17}{4};\frac{3\upsilon}{2t^4})}{\Theta(\frac{11}{4};\frac{13}{4};\frac{3\upsilon}{2t^4})}, \tag{9}$$

were Θ is the first kind confluent hypergeometric function:

$$\Theta(a;b;x) \equiv \frac{\Gamma(b)}{\Gamma(a)}\sum_{n=0}^{+\infty}\frac{\Gamma(a+n)}{\Gamma(b+n)}\frac{x^n}{n!}. \tag{10}$$

Defining some new dimensionless densities,

$$\Omega_D = \frac{\rho_D}{3H^2M_p^2}, \qquad \Omega_m = \frac{\rho_m}{3H^2M_p^2}, \qquad \Omega_f = \frac{\alpha}{H^2}\left(\frac{H}{t} - \frac{\upsilon\alpha}{6t^{2(\alpha+1)}}\right), \tag{11}$$

the new version of fractal Friedmann equation may be given in another form as

$$\Omega_D + \Omega_m + \Omega_f = 1. \tag{12}$$

The Veneziano ghost model of the exotic dark energy effect is one of the most noteworthy formulations[20,31–33]. It was introduced mainly to study the $U(1)A$ puzzle in quantum chromodynamics[34–36], but the model is decoupled from the physical territory[37,38]. The ghost dark energy definition seems to be a meaningless model in the quantum mechanical field theory in the Minkowski universe, but it also emcees a very significant non-trivial physical efficacy in an expanding spacetime and defines a vacuum energy density $\rho_D \sim (10^{-3}eV)^4$[39]. As a result of this, the ghost energy gets rid of the well-known fine-tuning puzzle[31,32].

Ghost model's energy density is related with the Hubble parameter[40]

$$\rho_D = \rho_G = \gamma H, \tag{13}$$

where γ is a constant. So, the fractal conservation equations read as

$$\dot{\rho_m} + \left(3H - \frac{\alpha}{t}\right)\rho_m = 0, \tag{14}$$

$$\dot{\rho_G} + \left(3H - \frac{\alpha}{t}\right)(1+\omega_G)\rho_G = 0. \tag{15}$$

Here $\omega_G = p_G\rho_G^{-1}$ is the ghost equation-of-state parameter. Time derivative of the definition (13) and the fractal Friedmann equation (4) give

$$\dot{\rho}_G = \frac{\gamma}{2H - \frac{\alpha}{t}}\left[\frac{\upsilon\alpha^2(\alpha+1)}{3t^{2\alpha+3}} - \frac{\alpha H}{t^2} - \frac{\rho_G}{3M_p^2}(3H - \frac{\alpha}{t})(1+\omega_G+\varrho)\right], \tag{16}$$

where

$$\varrho = \rho_m\rho_G^{-1}. \tag{17}$$

Making use of the above result with the fractal continuity relation and dimensionless density parameters one can find

$$\omega_G = -1 + \frac{\frac{\upsilon\alpha^2(\alpha+1)}{3H^2t^{2\alpha+3}} - \frac{\alpha}{Ht^2} - \left(3H - \frac{\alpha}{t}\right)(1 - \Omega_G - \Omega_f)}{\left(3H - \frac{\alpha}{t}\right)\left(\Omega_G - 2 - \frac{\alpha}{t}\right)}. \tag{18}$$

It can be seen easily here that we have $\omega_G = \frac{1}{\Omega_G - 2}$ in the infra-red fractal sector. At early times, i.e. $t \to 0$, we find $\Omega_G \ll 1$ and $\omega_G = -\frac{1}{2}$. On the other hand, at later times, i.e. $t \to \infty$, we have $\Omega_G \to 1$ and $\omega_G = -1$ which means the fractal ghost energy has a cosmological constant behavior. Besides, it is calculated in the ultra-violet era that

$$\omega_G = -1 + \frac{\frac{4\upsilon}{H^2t^7} - \frac{2}{Ht^2} - \left(3H - \frac{2}{t}\right)(1 - \Omega_G - \Omega_f^{UV})}{\left(3H - \frac{2}{t}\right)\left(\Omega_G - 2 - \frac{2}{t}\right)}, \tag{19}$$

where

$$\Omega_f^{UV} = \frac{2}{H^2}\left(\frac{H}{t} - \frac{\upsilon}{3t^6}\right). \tag{20}$$

3 Ghost reconstruction of DBI-essence

The corresponding action of DBI-essence scalar field model is given as

$$S = -\int d^4x\sqrt{-g}\left[T(\phi)\left(\sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}} - 1\right) + V(\phi)\right], \tag{21}$$

where $V(\phi)$ denotes self-interacting potential while $T(\phi)$ represents warped brane tension[41]. Next, the corresponding energy and pressure densities of the DBI-essence are given by the following definitions[41,42]:

$$\rho_{DBI} = (\lambda - 1)T(\phi) + V(\phi), \tag{22}$$

$$p_{DBI} = \left[\lambda - \frac{1}{\lambda}\right]T(\phi) - V(\phi), \tag{23}$$

where

$$\lambda = \left[1 - \frac{\dot{\phi}^2}{T(\phi)}\right]^{-\frac{1}{2}}. \tag{24}$$

The above equation indicates that we always have $T(\phi) > \dot{\phi}^2$ and $\lambda > 1$. Besides, equations (22) and (23) give that $\rho_{DBI} + p_{DBI} = \left(\lambda - \frac{1}{\lambda}\right) T(\phi)$ have positive values[42].

We are now in a position to construct a fractal relation between the DBI-essence field and ghost dark energy models. Thence, we assume that the DBI-essence model is the underlying theory for the galactic dark effect. Therefore, one can easily see that equation-of-state parameter of the DBI-essence can be given as

$$\omega_{DBI} = \frac{\frac{\lambda^2-1}{\lambda}T(\phi) - V(\phi)}{(\lambda-1)T(\phi) + V(\phi)}. \tag{25}$$

From equations (22), (23) and (24) we have[42]

$$T(\phi) = \frac{\dot{\phi}^2(\rho_{DBI} + p_{DBI})^2}{(\rho_{DBI} + p_{DBI})^2 - \dot{\phi}^4}, \tag{26}$$

$$V(\phi) = \frac{\dot{\phi}^2\rho_{DBI} - p_{DBI}(\rho_{DBI} + p_{DBI})}{\dot{\phi}^2 + \rho_{DBI} + p_{DBI}}. \tag{27}$$

In order to construct a relation between the DBI-essence field and ghost energy, it is identified here that $\rho_{DBI} = \rho_G$ and $\omega_{DBI} = \omega_G$. Hence, we can focus on two special cases[42]: $\lambda = constant$ and $\lambda \neq constant$.

Case (i): If one assumes that $\lambda = constant$, it can be easily seen that

$$\begin{aligned}\dot{\phi}^2 = \frac{\rho_G + p_G}{\lambda} &= (1+\omega_G)\frac{\rho_G}{\lambda} \\ &= \frac{\frac{\upsilon\alpha^2(\alpha+1)}{3H^2t^{2\alpha+3}} - \frac{\alpha}{Ht^2} - \left(3H - \frac{\alpha}{t}\right)(1 - \Omega_G - \Omega_f)}{\lambda(\gamma H)^{-1}\left(3H - \frac{\alpha}{t}\right)\left(\Omega_G - 2 - \frac{\alpha}{t}\right)}.\end{aligned} \tag{28}$$

Therefore, we also obtain that

$$T(\phi) = \frac{\frac{\upsilon\alpha^2(\alpha+1)}{3H^2t^{2\alpha+3}} - \frac{\alpha}{Ht^2} - \left(3H - \frac{\alpha}{t}\right)(1 - \Omega_G - \Omega_f)}{(\lambda^2-1)(\lambda\gamma H)^{-1}\left(3H - \frac{\alpha}{t}\right)\left(\Omega_G - 2 - \frac{\alpha}{t}\right)}, \tag{29}$$

$$V(\phi) = \gamma H - \frac{\frac{\upsilon\alpha^2(\alpha+1)}{3H^2t^{2\alpha+3}} - \frac{\alpha}{Ht^2} - \left(3H - \frac{\alpha}{t}\right)(1 - \Omega_G - \Omega_f)}{(1+\lambda)(\lambda\gamma H)^{-1}\left(3H - \frac{\alpha}{t}\right)\left(\Omega_G - 2 - \frac{\alpha}{t}\right)}. \tag{30}$$

Making use of equation (28) it is found

$$\dot{\phi} = \sqrt{\frac{\frac{\upsilon\alpha^2(\alpha+1)}{3H^2t^{2\alpha+3}} - \frac{\alpha}{Ht^2} - \left(3H - \frac{\alpha}{t}\right)(1 - \Omega_G - \Omega_f)}{\lambda(\gamma H)^{-1}\left(3H - \frac{\alpha}{t}\right)\left(\Omega_G - 2 - \frac{\alpha}{t}\right)}}. \tag{31}$$

Additionally, we introduce a new variable which helps us to rewrite the above relation in a very useful form:

$$x = \ln a. \tag{32}$$

So, considering this new variable, it can be written that

$$\frac{d}{dx} = H\frac{d}{dt}, \tag{33}$$

and we obtain

$$\dot{\phi} = H\phi'. \tag{34}$$

Here, the derivative with respect to x is denoted by a prime. We further consider the fractal cases. Thence, in the infra-red fractal sector, integrating equation (31) yields

$$\phi_{IR}(a) = \phi_{IR}(a_0) + \int_{a_0}^{a}\frac{da}{a}\sqrt{\frac{3M_p^2\Omega_G(1-\Omega_G)}{\lambda(2-\Omega_G)}}, \tag{35}$$

where a_0 denotes present value of the scale term. We also have

$$T_{IR}(\phi)\mid_{(\lambda=constant)}=\frac{3\lambda M_p^2\Omega_G(1-\Omega_G)}{(\lambda^2-1)(2-\Omega_G)}, \tag{36}$$

$$V_{IR}(\phi)\mid_{(\lambda=constant)}=\gamma H+\frac{3\lambda M_p^2\Omega_G(1-\Omega_G)}{(1+\lambda)(\Omega_G-2)}. \tag{37}$$

Hence, it is found that

$$\omega_{DBI}^{IR}\mid_{(\lambda=constant)}=-1+\frac{(2\lambda^2-3\lambda-1)T_{IR}}{2\lambda(\lambda-1)T_{IR}+\gamma\lambda H}. \tag{38}$$

In Figures 1 and 2, we plot evolution of the warped brane tension, self-interacting potential and equation-of-state quantity of ghost DBI-essence for the infra-red fractal profile.

Furthermore, making use of the ultra-violet fractal case we get

$$\phi_{UV}(t)=\phi_{UV}(t_0)+\int_{t_0}^{t}dt\sqrt{\frac{\frac{4\upsilon}{H^2t^7}-\frac{2}{Ht^2}-\left(3H-\frac{2}{t}\right)\left(1-\Omega_G-\Omega_f^{UV}\right)}{\lambda(H\gamma)^{-1}\left(3H-\frac{2}{t}\right)\left(\Omega_G-2-\frac{2}{t}\right)}}, \tag{39}$$

$$T_{UV}(\phi)\mid_{(\lambda=constant)}=\frac{\frac{4\upsilon}{H^2t^7}-\frac{2}{Ht^2}-\left(3H-\frac{2}{t}\right)\left(1-\Omega_G-\Omega_f^{UV}\right)}{(\lambda^2-1)(\lambda\gamma H)^{-1}\left(3H-\frac{2}{t}\right)\left(\Omega_G-2-\frac{2}{t}\right)}, \tag{40}$$

$$V_{UV}(\phi)\mid_{(\lambda=constant)}=\gamma H-\frac{\frac{4\upsilon}{H^2t^7}-\frac{2}{Ht^2}-\left(3H-\frac{2}{t}\right)\left(1-\Omega_G-\Omega_f^{UV}\right)}{(1+\lambda)(\lambda\gamma H)^{-1}\left(3H-\frac{2}{t}\right)\left(\Omega_G-2-\frac{2}{t}\right)}, \tag{41}$$

where

$$H(t)=-\frac{2}{t}-\frac{22\upsilon}{13t^5}\frac{\Gamma(\frac{17}{4})\Gamma(\frac{11}{4})\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{15}{4}+m)}{\Gamma(\frac{17}{4}+m)}\frac{\left(\frac{3\upsilon}{2t^4}\right)^m}{m!}}{\Gamma(\frac{15}{4})\Gamma(\frac{13}{4})\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+n)}{\Gamma(\frac{13}{4}+n)}\frac{\left(\frac{3\upsilon}{2t^4}\right)^n}{n!}}. \tag{42}$$

Besides, we also have

$$\omega_{DBI}^{UV}\mid_{(\lambda=constant)}=-1+\frac{\lambda-1}{\gamma\lambda H}T_{UV}. \tag{43}$$

In Figures 3 and 4, we analyze numerically the evolution of warped brane tension, self-interacting potential and equation-of-state parameter of the reconstructed DBI-essence scalar field for the ultra-violet fractal case. The Figure 3 implies that the warped brane tension and potential of fractal DBI-essence increase during evolution of the Universe. Next, the Figure 4 shows that the ultra-violet equation-of-state parameter at late times $t\to\infty$ goes to -0.6 which behaves like the quintessence model[43].

Case (ii): Assuming $\lambda=\dot{\phi}^{-1}\neq constant$[42] gives

$$\dot{\phi}^2=(\rho_G+p_G)^2=(1+\omega_G)^2\rho_G^2=\xi, \tag{44}$$

$$\frac{1}{T(\phi)}=-1+\frac{1}{\xi}, \tag{45}$$

$$V(\phi)=\frac{\xi-\gamma H(\sqrt{\xi}-1)}{1+\sqrt{\xi}}, \tag{46}$$

where

$$\xi(t)=\frac{\frac{\upsilon\alpha^2(\alpha+1)}{3H^2t^{2\alpha+3}}-\frac{\alpha}{Ht^2}-\left(3H-\frac{\alpha}{t}\right)\left(1-\Omega_G-\Omega_f\right)}{(\gamma H)^{-1}\left(3H-\frac{\alpha}{t}\right)\left(\Omega_G-2-\frac{\alpha}{t}\right)}. \tag{47}$$

Using the new variable $x = \ln a$ to integrate equation (44) for $\alpha = 0$ profile yields

$$\phi_{IR}(a) = \phi_{IR}(a_0) + \int_{a_0}^{a} \frac{da}{a}\sqrt{\frac{\gamma(1-\Omega_G)}{2-\Omega_G}}, \tag{48}$$

$$\frac{1}{T_{IR}(\phi)} \mid_{(\lambda\neq constant)} = -1 + \frac{(\Omega_G - 2)}{\gamma H(1-\Omega_G)}, \tag{49}$$

$$V_{IR}(\phi) \mid_{(\lambda\neq constant)} = \frac{\frac{\gamma H(1-\Omega_G)}{2-\Omega_G} - \gamma H\left(\sqrt{\frac{\gamma H(1-\Omega_G)}{2-\Omega_G}} - 1\right)}{1 + \sqrt{\frac{\gamma H(1-\Omega_G)}{2-\Omega_G}}}, \tag{50}$$

in addition to this, for $\alpha = 2$, we also obtain

$$\phi_{UV}(t) = \phi_{UV}(t_0) + \int_{t_0}^{t} \zeta(t)dt, \tag{51}$$

$$\frac{1}{T_{UV}(\phi)} \mid_{(\lambda\neq constant)} = -1 + \frac{1}{\zeta^2}, \tag{52}$$

$$V_{UV}(\phi) \mid_{(\lambda\neq constant)} = \frac{\zeta^2 - \gamma H(\zeta - 1)}{1+\zeta}, \tag{53}$$

where

$$\zeta(t) = \sqrt{\frac{\frac{4\upsilon}{H^2t^7} - \frac{2}{Ht^2} - \left(3H - \frac{2}{t}\right)\left(1 - \Omega_G - \Omega_f^{UV}\right)}{(\gamma H)^{-1}\left(3H - \frac{2}{t}\right)\left(\Omega_G - 2 - \frac{2}{t}\right)}}. \tag{54}$$

Ergo, one can find

$$\omega_{DBI}^{IR} \mid_{(\lambda\neq constant)} = -1 + \frac{1-y}{\frac{y(1-y)}{1+y} + \frac{y+\gamma H(1-\sqrt{y})}{1+\sqrt{y}}}, \tag{55}$$

$$\omega_{DBI}^{UV} \mid_{(\lambda\neq constant)} = -1 + \frac{1+\zeta}{\zeta^2 + \gamma H(1-\zeta) - \zeta^2(1+\zeta)}, \tag{56}$$

where

$$y = \frac{\gamma H(1-\Omega_G)}{2-\Omega_G}. \tag{57}$$

Taking $\gamma = 1$, $\Omega_G = 0.72$[44] and $\lambda = \frac{1}{\phi} \neq constant$[42] in two fractal sectors, the numerical results obtained for warped brane tension, self-interacting potential and equation-of-state parameter of the reconstructed DBI-essence are plotted in Figures 5, 6, 7 and 8.

4 Statefinder Diagnostics

There are many dark energy candidates given in literature and it can be seen that some of them have been analyzed by introducing statefinder diagnostic parameters[45]. These dimensionless parameters give an opportunity to characterize properties of the selected dark energy model in a model independent manner[42]. The set of these parameters is given as[42,45]

$$r = \frac{\dddot{a}}{aH^3}, \tag{58}$$

$$s = \frac{r-1}{3q - \frac{3}{2}}, \tag{59}$$

where

$$q=-\frac{a\ddot{a}}{\dot{a}^2} \tag{60}$$

is the deceleration parameter. It is known that the deceleration parameter q must be negative to define an accelerated behavior for the Universe. The infra-red fractal profile gives the general theory of relativity and there is no fractal contribution in this limit. In addition to this, in the ultra-violet fractal region, we have the following relation

$$\dot{a}(t)=\frac{-1}{3t^3}\left[\frac{\Gamma(\frac{13}{4})}{\Gamma(\frac{11}{4})}\right]^{\frac{5}{3}}\frac{\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+m)}{\Gamma(\frac{13}{4}+m)}\frac{(4m+6)}{m!}\left(\frac{3\eta}{2t^4}\right)^m}{\left[\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+n)}{\Gamma(\frac{13}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}\right]^{\frac{2}{3}}}. \tag{61}$$

Thence, we get

$$r=\frac{\left[\frac{\Gamma(\frac{13}{4})}{\Gamma(\frac{11}{4})}\right]^{\frac{4}{3}}\frac{d^2}{dt^2}\left(\frac{-1}{3t^3}\frac{\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+m)}{\Gamma(\frac{13}{4}+m)}\frac{(4m+6)}{m!}\left(\frac{3\eta}{2t^4}\right)^m}{\left[\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+n)}{\Gamma(\frac{13}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}\right]^{\frac{2}{3}}}.\right)}{\sqrt[3]{\frac{1}{t^6}\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+n)}{\Gamma(\frac{13}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}}\left[-\frac{2}{t}-\frac{22\eta}{13t^5}\frac{\Gamma(\frac{17}{4})\Gamma(\frac{11}{4})\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{15}{4}+n)}{\Gamma(\frac{17}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}}{\Gamma(\frac{15}{4})\Gamma(\frac{13}{4})\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+m)}{\Gamma(\frac{13}{4}+m)}\frac{\left(\frac{3\eta}{2t^4}\right)^m}{m!}}\right]^3}, \tag{62}$$

$$q=-\frac{\frac{d}{dt}\left[\frac{-1}{3t^7}\left[\frac{1}{t^6}\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+n)}{\Gamma(\frac{13}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}\right]^{-\frac{2}{3}}\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+m)}{\Gamma(\frac{13}{4}+m)}\frac{(4m+6)}{m!}\left(\frac{3\eta}{2t^4}\right)^m\right]}{\left[\frac{-1}{3t^7}\left[\frac{1}{t^6}\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+n)}{\Gamma(\frac{13}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}\right]^{-\frac{5}{6}}\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+m)}{\Gamma(\frac{13}{4}+m)}\frac{(4m+6)}{m!}\left(\frac{3\eta}{2t^4}\right)^m\right]^2}, \tag{63}$$

$$s=\frac{\left[\frac{\Gamma(\frac{13}{4})}{\Gamma(\frac{11}{4})}\right]^{\frac{2}{3}}\frac{\frac{d^2}{dt^2}\left(\frac{-1}{3t^7}\left[\frac{1}{t^6}\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+n)}{\Gamma(\frac{13}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}\right]^{-\frac{2}{3}}\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+m)}{\Gamma(\frac{13}{4}+m)}\frac{(4m+6)}{m!}\left(\frac{3\eta}{2t^4}\right)^m\right)}{\sqrt[3]{\frac{1}{t^6}\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+n)}{\Gamma(\frac{13}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}}\left[-\frac{2}{t}-\frac{22\eta}{13t^5}\frac{\Gamma(\frac{17}{4})\Gamma(\frac{11}{4})\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{15}{4}+n)}{\Gamma(\frac{17}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}}{\Gamma(\frac{15}{4})\Gamma(\frac{13}{4})\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+m)}{\Gamma(\frac{13}{4}+m)}\frac{\left(\frac{3\eta}{2t^4}\right)^m}{m!}}\right]^3}-1}{\left[\frac{\Gamma(\frac{13}{4})}{\Gamma(\frac{11}{4})}\right]^{-1}\frac{\frac{d}{dt}\left[\frac{-1}{t^7}\left[\frac{1}{t^6}\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+n)}{\Gamma(\frac{13}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}\right]^{-\frac{2}{3}}\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+m)}{\Gamma(\frac{13}{4}+m)}\frac{(4m+6)}{m!}\left(\frac{3\eta}{2t^4}\right)^m\right]}{\left[\frac{-1}{3t^7}\left[\frac{1}{t^6}\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+n)}{\Gamma(\frac{13}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}\right]^{-\frac{5}{6}}\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+m)}{\Gamma(\frac{13}{4}+m)}\frac{(4m+6)}{m!}\left(\frac{3\eta}{2t^4}\right)^m\right]^2}-\frac{3}{2}}, \tag{64}$$

To investigate evolutionary nature of the model, we first need to know the time evolution of scale term and Hubble parameter. The deceleration parameter and statefinders versus time are plotted in Figures 9 and 10. The Figure 9 shows that the deceleration parameter shows a cosmic acceleration during history of the Universe which is compatible with the recent observations[1–7]. This result shows that our research fits into the existing body of knowledge.

5 Discussions

It is known that scalar field descriptions[10–12,43,14] can be taken into account as an effective theory to work out the nature of dark energy dominated universe and the ghost model has also been used recently to investigate the galactic dark energy effect[20,31–33]. Besides, the reconstruction of scalar fields based on some well known dark energy models can produce important conclusions and this idea motivated us to reestablish the DBI-essence scalar field model having a non-canonical kinetic term. It is important to mention here that the new model with the redefined potential and warped brane tension is a unique single-scalar description that can explain the evolution of universe. In addition to this, scalar field models have very exciting feature of understanding the phantom crossing and the redefined form of their potentials also give noteworthy physical implications in cosmology. On this purpose, we have mainly established a relation between the DBI-essence and ghost models of galactic dark effect in a fractal universe.

We have also analyzed graphically evolutions of the reconstructed warped brane tensions, potentials and the equation-of-state parameters for both the infra-red and ultra-violet fractal profiles. As we have mentioned before, the infra-red fractal profile, i.e. $\alpha = 0$, gives corresponding relations of General Relativity. Figures 1, 2, 5 and 6 show the evolutionary nature of infra-red quantities for $\lambda = constant$ and $\lambda \neq constant$ cases. However, it is better to be focused on the ultra-violet fractal profile to get meaningful new conclusions. For the ultra-violet fractal regime, i.e. $\alpha = 2$, our calculations develop new knowledge that goes beyond the existing research and literature. For $\lambda = constant$ case, the Figure 3 shows that the potential and warped brane tension of fractal DBI-essence increase during evolution of the Universe. On the other hand, for $\lambda \neq constant$ case, the Figure 7 implies that the potential and warped brane tension of ghost DBI-essence increase during the evolution of fractal universe. It is known that the cosmic inflation and accelerated expansion of our Universe is identified by the equation-of-state parameter. For a constant λ, it can be seen from the Figure 4 that the ultra-violet equation-of-state parameter at late times $t \to \infty$ goes to -0.6 which behaves like the quintessence model. Furthermore, for $\lambda \neq constant$ condition, we have seen from the Figure 8 that the ultra-violet equation-of-state parameter acts like the ΛCDM model, i.e. $\omega_{DBI}^{UV} \to -1$. Besides, we have also discussed the fractal deceleration and statefinder parameters. These parameters have been calculated to make our fractal model distinguishable among many dark energy candidates. Our results give a deceleration parameter definition showing a cosmic acceleration during history of the Universe which is compatible with the recent observations[1–7].

Additionally, the problem considered here can also be extended to the anisotropic case in order to investigate dark energy with generalized equations of state[46]. Making use of an anisotropic spacetime model including three different scale factors and a generalized energy-momentum description showing anisotropic dark energy interacting with dark matter and dark radiation, one can arrive more general equations of state. After calculating the corresponding fractal field equations and using an anisotropic energy-momentum definition, a generalized reconstruction can be performed. Actually, the extended problem may be considered in another work.

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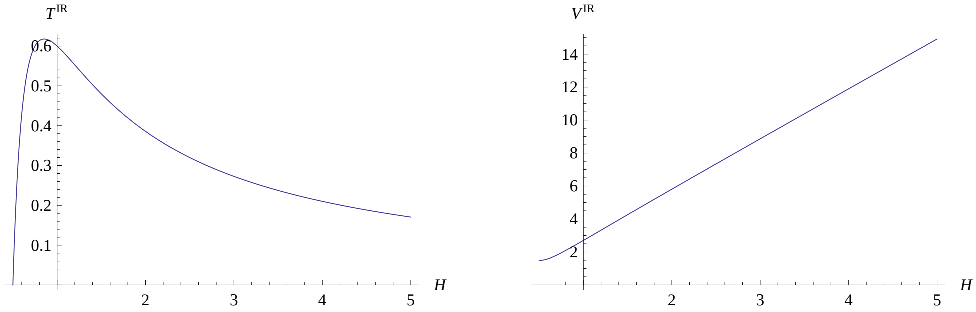

Fig. 1. Evolution of the warped brane tention and the self-interacting potential of fractal ghost DBI-essence in the infra-red regime for $\gamma = M_p = 1$ and $\lambda = 1.5$.

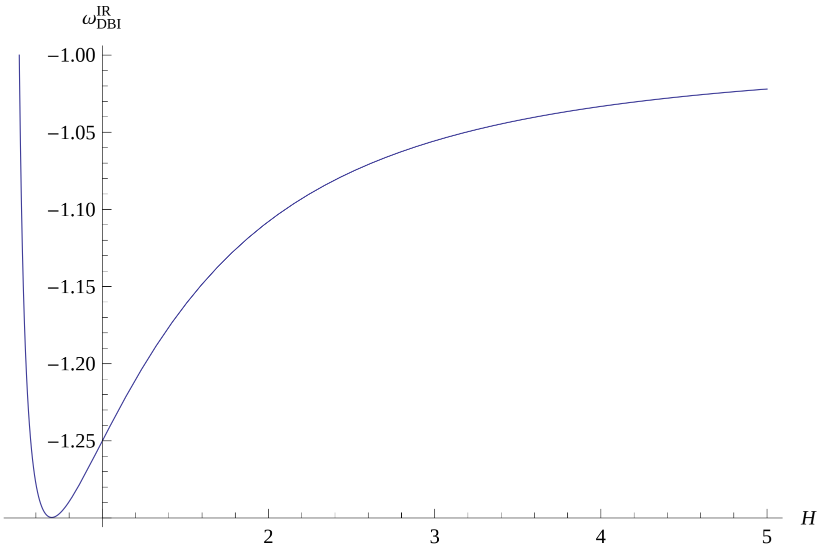

Fig. 2. Evolution of the equation-of-state parameter of the ghost DBI-essence in the fractal infra-red regime for $\gamma = M_p = 1$ and $\lambda = 1.5$.

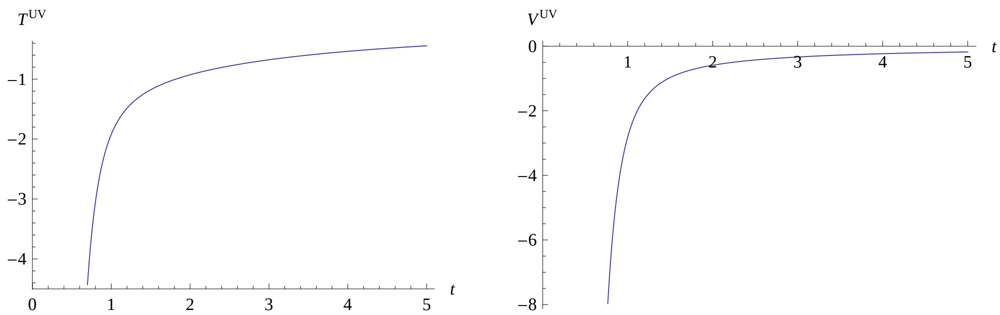

Fig. 3. Time evolution of the warped brane tension and the self-interacting potential of fractal ghost DBI-essence in the ultra-violet regime for $\upsilon = \gamma = M_p = 1$ and $\lambda = 1.5$.

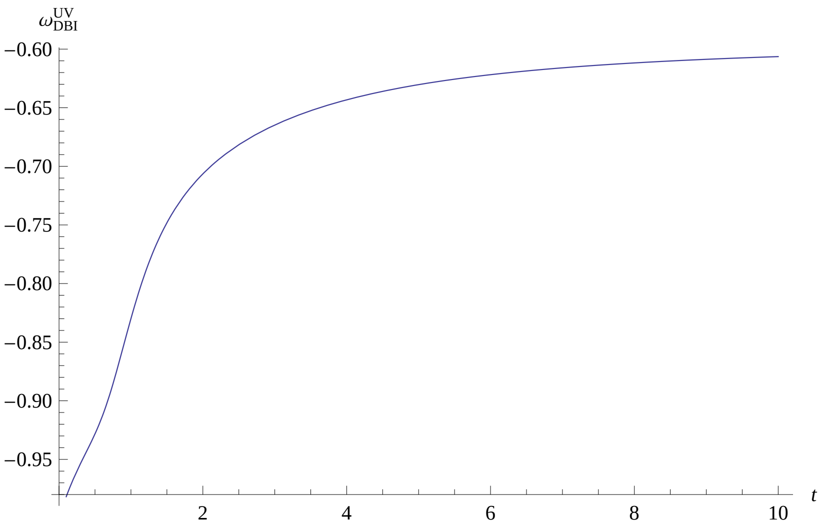

Fig. 4. Time evolution of the equation-of-state parameter of the ghost DBI-essence in the fractal ultra-violet regime for $\upsilon = \gamma = M_p = 1$ and $\lambda = 1.5$.

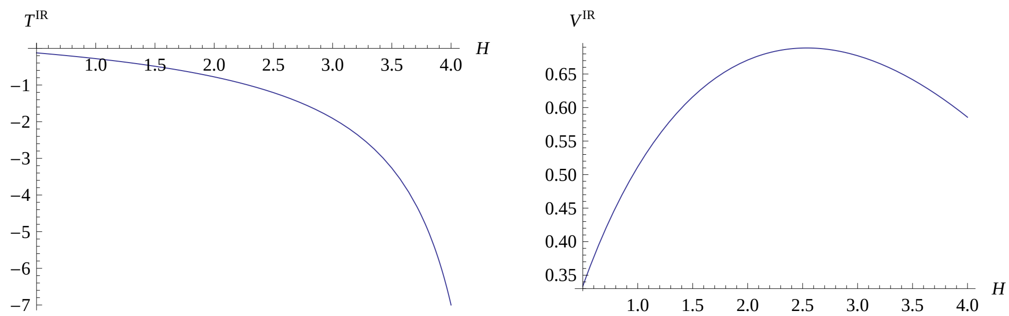

Fig. 5. Evolution of the warped brane tention and self-interacting potential of the ghost DBI-essence field in the fractal infra-red regime for the case $\gamma = 1$, $\Omega_G = 0.72$ and $\lambda = \dot{\phi} \neq constant$.

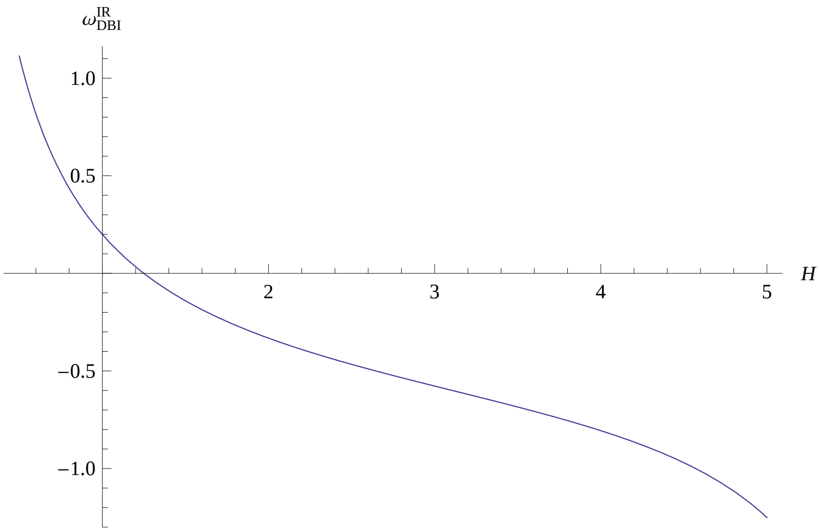

Fig. 6. Evolution of equation-of-state parameter of the ghost DBI-essence dark energy for the infra-red fractal profile for the case $\gamma = 1$, $\Omega_G = 0.72$ and $\lambda = \dot{\phi} \neq constant$.

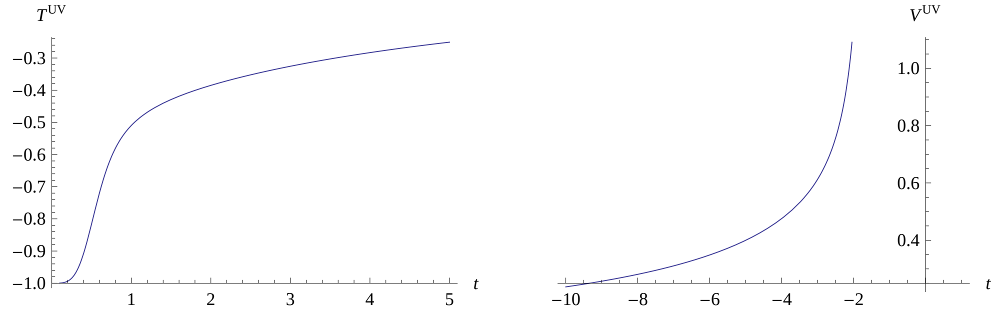

Fig. 7. Time evolution of the warped brane tention and self-interacting potential of the ghost DBI-essence in the ultra-violet fractal sector for the case $\gamma = 1$, $\Omega_G = 0.72$ and $\lambda = \dot{\phi} \neq constant$.

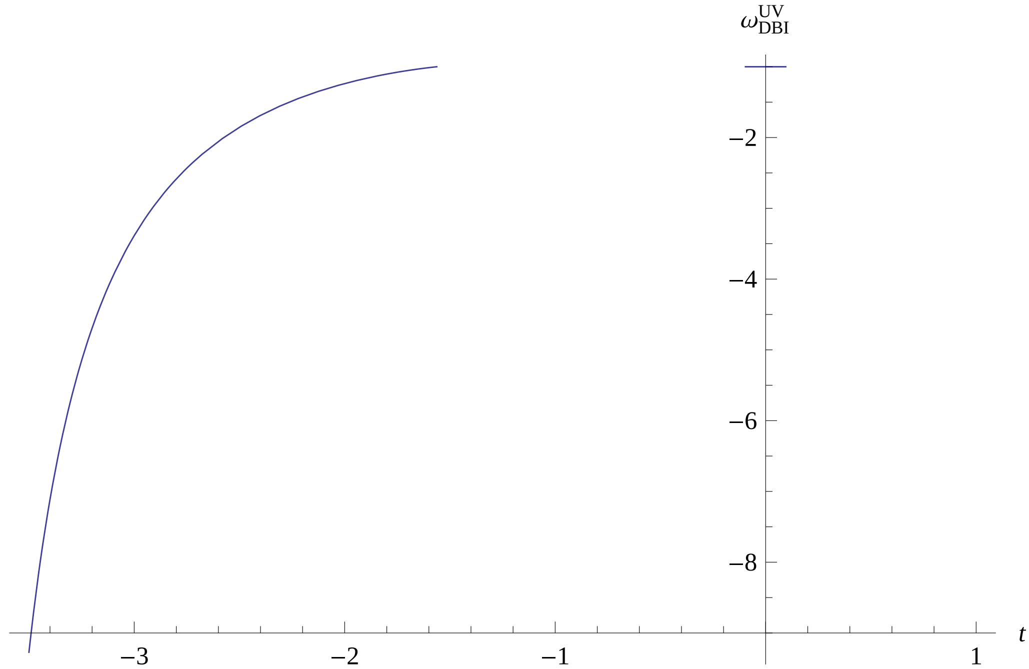

Fig. 8. Time evolution of equation-of-state parameter of the ghost DBI-essence in the ultra-violet fractal territory for the case $\gamma = 1$, $\Omega_G = 0.72$ and $\lambda = \dot{\phi} \neq constant$.

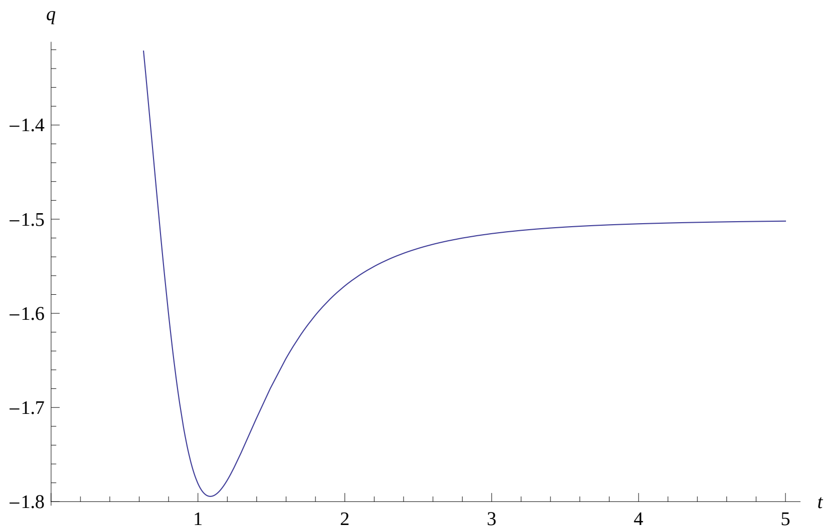

Fig. 9. Time evolution of the fractal deceleration parameter.

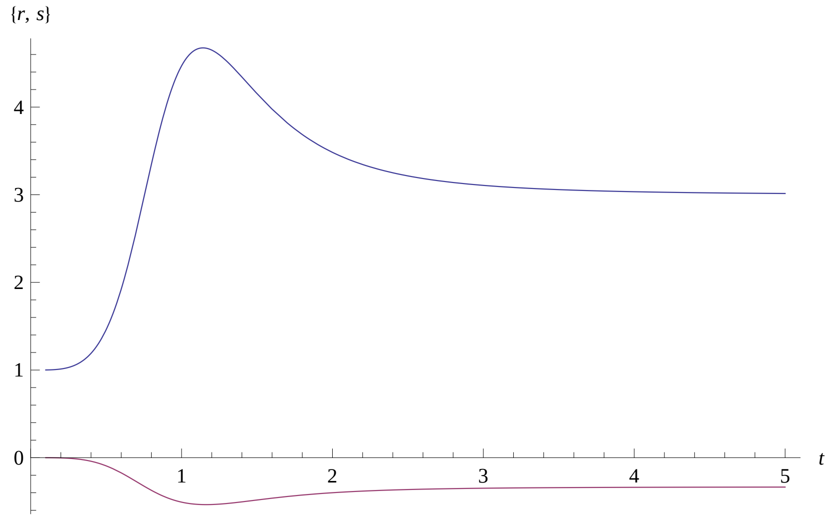

Fig. 10. Time evolution of the fractal statefinder parameters: the upper and bottom graphics are plotted for r and s, respectively.