Solving Coupled Hirota System by Using Reduced Differential Transform Method

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Abstract

*In this paper, Reduced Differential Transform Method (**RDTM**) has been successively used to find the numerical solutions of the coupled Hirota system (**CHS**). The results obtained by RDTM are compared with exact solutions to reveal that the RDTM is very accurate and effective. In our work, Maple 13 has been used for computations.*

keywords: Reduced differential transform method, coupled Hirota system, soliton solution.

1. Introduction

The nonlinear phenomena played a very significant role in the field of applied Mathematics and mathematical Physics. It is well known that many phenomena in scientific fields can be described by nonlinear partial differential equations. Since in the presence of computer programming software's, the solution of a linear equation is not a problem. But to solve non-linear problems

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analytically, it is still difficult for the mathematicians. The analytical methods are fastly developing, but still have some deficiencies and shortcomings, which do not satisfy the mathematician. The nonlinear models of real-life problems are still difficult to solve either numerically or theoretically. There has recently been much attention devoted to the search for better and more efficient solution methods in determining a solution, approximate or exact, analytical or numerical, to nonlinear models [3, 4, 9].

To describe nonlinear CHS, we start with Hirota equation [2, 5]:

$$\frac{\partial w}{\partial t} + 3\alpha |w|^2 \frac{\partial w}{\partial x} + \gamma \frac{\partial^3 w}{\partial x^3} = 0, -\infty < x < \infty,\ t > 0, \tag{1}$$

where w is a complex valued function of the spatial coordinate x and the time t, α and γ are positive real constants. This equation is an integrable equation which has a number of physical applications, such as the propagation of optical pluses in nematic liquid crystal waveguides. The Hirota equation is closely related to both the nonlinear Schrödinger (NLS) and modified Korteweg-de Vries (mKdV) equations, as it is complex generalization of the mKdV equation and it is a part of the NLS hierarchy of the integrable equation. Also, its soliton solution has a very similar form to the NLS soliton. The Hirota equation (1) has a two-parameter soliton family, with amplitude and velocity. The exact solution of Hirota equation (1) is

$$\begin{aligned} w(x,t) &= \beta \operatorname{sech}[k(x - Vt)] \exp(i\varphi), \\ \beta &= \sqrt{\frac{2\gamma}{\alpha}}\, k \quad , \ \varphi = a(x - bt), \\ V &= \gamma(k^2 - 3a^2),\ b = \gamma(3k^2 - a^2). \end{aligned} \tag{2}$$

β is the amplitude of the wave, k is related to the width of the wave envelope and V is the velocity [5]. The parameter a is the wave number of the phase and b is related to the frequency of the phase. Also the solution is $x = x_0$ at $t = 0$.

The Hirota equation (1) has been solved analytically by sine-cosine and tanh methods by Wazwaz [11] and showed that this equation admits sech-shaped soliton solutions whose amplitudes and velocities are free parameters, and tanh solution (kink type). Also solved by [10] by tanh method. Hirota method also used by [4] for solving (1). To avoid complex computation which needs too many calculations in the solution of (1), we assume

$$w(x,t) = u(x,t) + iv(x,t), \ \ i^2 = -1.$$

where $u(x,t)$ and $v(x,t)$ are real functions. After calculations, this will reduce Hirota equation (1) to the coupled Hirota system (CHS)

$$\begin{aligned} &\frac{\partial u}{\partial t} + 3\alpha\, f(u,v)\frac{\partial u}{\partial x} + \gamma\frac{\partial^3 u}{\partial x^3} = 0, \\ &\frac{\partial v}{\partial t} + 3\alpha\, f(u,v)\frac{\partial v}{\partial x} + \gamma\frac{\partial^3 v}{\partial x^3} = 0, \end{aligned} \tag{3}$$

where $f(u,v) = u^2 + v^2$.

We know that the numerical methods do not require discretization of space-time variables or linearization of the nonlinear equations. The RDTM was first proposed by the Turkish mathematician Keskin [6, 7, 8] in 2009. It has received much attention since it has applied to solve a wide variety of linear and nonlinear problems by many authors (for more details we refer the reader to see [1] and the references there are). In this paper, we shall extend the technique of RDTM to solve (3) numerically. The paper has been organized as follows. In **Section 2**, we begin by introducing the definition and the basic mathematical operations of RDTM. In **Section 3**, RDMT applied to Solve the CHS (3). In **Section 4**, In this section, numerical example solved to show the effectiveness of this method. Finally, some conclusions are provided in **Section 5**.

2. Basic Idea of the Reduced Differential Transform Method

To describe RDTM in a similar manner of [6], consider a function of two variables $u(x,t)$ and suppose that it can be represented as a product of two single-variable functions, i.e. $u(x,t) = f(x)g(t)$. Based on the properties of one-dimensional differential transform, the function $u(x,t)$ can be represented as

$$u(x,t) = \sum_{i=0}^{\infty} F(i)x^i \sum_{j=0}^{\infty} G(j)t^j = \sum_{k=0}^{\infty} U_k t^k, \tag{4}$$

where $U_k(x)$ is called t-dimensional spectrum function of $u(x,t)$.

The basic definitions and operations of RDTM are introduced as follows [1, 6, 7, 8]:

Definition 2.1. If function $u(x,t)$ is analytic and differentiated continuously with respect to time t and space x in the domain of interest, then let

$$U_k(x) = \frac{1}{k!}\frac{\partial^k}{\partial x^k}u(x,t)\mid_{t=0}, \tag{5}$$

where the t-dimensional spectrum function $U_k(x)$ is the transformed function. In this work, the lowercase $u(x,t)$ represent the original function, while the uppercase $U_k(x)$ stand for the transformed function.

Definition 2.2. The differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k, \tag{6}$$

From the combination of Equations (5) and (6), it follows that

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\frac{\partial^k}{\partial t^k}u(x,t)\mid_{t=0} t^k. \tag{7}$$

From the above two equations, it can be found that the concept of the RDTM is derived from the Taylor series expansion.

To illustrate the basic concepts of the RDTM, consider the following nonlinear partial differential equation written in an operator form

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = g(x,t), \tag{8}$$

with initial condition

$$u_0(x,t) = u(x,0) = f(x), \tag{9}$$

where $L = \frac{\partial}{\partial t}$, R is a linear operator, $Nu(x,t)$ is a nonlinear term and $g(x,t)$ is an inhomogeneous term.

According to the RDTM and Table (1), we can construct the following iteration formula:

$$(k+1)U_{k+1}(x) = G_k(x) - RU_k(x) - NU_k(x), \tag{10}$$

where $U_k(x)$, $RU_k(x)$, $NU_k(x)$ and $G_k(x)$ are the transformations of the functions $Lu(x,t)$, $Ru(x,t)$, $Nu(x,t)$ and $g(x,t)$ respectively.
From initial condition (9), we write

$$U_0(x) = f(x). \tag{11}$$

Substituting Equation (11) into (10) and by straightforward iterative calculation, we get the following $U_k(x)$ values. Then the inverse transformation of the set of values $\{U_k(x)\}_{k=0}^{n}$ gives the n-terms approximate solution as:

$$\tilde{u}_n(x,t) = \sum_{k=0}^{n} U_k(x)t^k, \tag{12}$$

where n is order of approximate solution.
Therefore the exact solution of the problem is given by

$$u(x,t) = \lim_{n\to\infty} \tilde{u}_n(x,t). \tag{13}$$

The mathematical operations performed by RDTM are listed in Table (1).
Table (1) The fundamental operations of RDTM.

Original Function	Reduced,Transformed Function
$u(x,t)$	$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial x^k}u(x,t)\right]_{t=0}$
$w(x,t) = u(x,t) \pm v(x,t)$	$W_k(x) = U_k(x) \pm V_k(x)$
$w(x,t) = \alpha u(x,t)$	$W_k(x) = \alpha U_k(x)$, (α is constant)
$w(x,t) = x^m t^n$	$W_k(x) = x^m \delta(k-n) = \begin{cases} x^m, k=n \\ 0, \text{ otherwise} \end{cases}$
$w(x,t) = x^m t^n u(x,t)$	$W_k(x) = x^m U_{k-n}(x)$
$w(x,t) = u(x,t)v(x,t)$	$W_k(x) = \sum_{r=0}^{k} U_r(x)V_{k-r}(x) = \sum_{r=0}^{k} V_r(x)U_{k-r}(x)$
$w(x,t) = \frac{\partial^k}{\partial t^k}u(x,t)$	$W_k(x) = \frac{(k+r)!}{k!}U_{k+r}(x)$
$w(x,t) = \frac{\partial}{\partial x}u(x,t)$	$W_k(x) = \frac{\partial}{\partial x}U_k(x)$

The proofs of Table (1) are available in Ph.D. thesis by Keskin [9].

3. Applying Reduced Transformed Method to Solve the Coupled Hirota System

In this section, the RDTM are used to solve CHS (3) as follows:
According to the basic properties of the RDTM, Table (1) and equation (10), we can find the transformed form of CHS (3) as:

$$(k+1)U_{k+1}(x) = -3\alpha\left[A_k(x) + B_k(x)\right] - \gamma\frac{\partial^3 U_k(x)}{\partial x^3}, \tag{14}$$

$$(k+1)V_{k+1}(x) = -3\alpha\left[C_k(x) + D_k(x)\right] - \gamma\frac{\partial^3 V_k(x)}{\partial x^3}, \tag{15}$$

where the t-dimensional spectrum function $U_k(x)$ and $V_k(x)$ are the transformed function, $A_k(x)$, $B_k(x)$, $C_k(x)$ and $D_k(x)$ are transformed form of the nonlinear terms.

Starting with the initial approximations $u_0(x,t) = U_0(x)$ and $v_0(x,t) = V_0(x)$ of (CHS), by straightforward iterative steps, we obtain the following $U_k(x)$ and $V_k(x)$; $k = 0,1,2,\cdots,n$ values:

$$A_k(x) = \sum_{r=0}^{k}\sum_{s=0}^{r}\left[U_{k-r}(x)U_{r-s}(x)\frac{\partial U_s(x)}{\partial x}\right], \tag{16}$$

$$B_k(x) = \sum_{r=0}^{k}\sum_{s=0}^{r}\left[V_{k-r}(x)V_{r-s}(x)\frac{\partial U_s(x)}{\partial x}\right], \tag{17}$$

$$C_k(x) = \sum_{r=0}^{k}\sum_{s=0}^{r}\left[U_{k-r}(x)U_{r-s}(x)\frac{\partial V_s(x)}{\partial x}\right], \tag{18}$$

$$D_k(x) = \sum_{r=0}^{k}\sum_{s=0}^{r}\left[V_{k-r}(x)V_{r-s}(x)\frac{\partial V_s(x)}{\partial x}\right]. \tag{19}$$

From the Equations (14) and (15), put $k = 0$, respectively, we obtain:

$$U_1(x) = -3\alpha\left[A_0(x) + B_0(x)\right] - \gamma\frac{\partial^3 U_0}{\partial x^3}, \tag{20}$$

and

$$V_1(x) = -3\alpha\left[C_0(x) + D_0(x)\right] - \gamma\frac{\partial^3 V_0}{\partial x^3}. \tag{21}$$

Now, from the Equations (16)-(19), put $k = 0$, respectively, we have:

$$A_0(x) = U_0^2\frac{\partial U_0}{\partial x}, \quad B_0(x) = V_0^2\frac{\partial U_0}{\partial x}, \tag{22}$$

$$C_0(x) = U_0^2\frac{\partial V_0}{\partial x}, \quad D_0(x) = V_0^2\frac{\partial V_0}{\partial x}. \tag{23}$$

Substituting the Equations (22) and (23) into the Equations (20) and (21), respectively, we get:

$$U_1(x) = -3\alpha\left[U_0^2 + V_0^2\right]\frac{\partial U_0}{\partial x} - \gamma\frac{\partial^3 U_0}{\partial x^3},$$

and

$$V_1(x) = -3\alpha\left[U_0^2 + V_0^2\right]\frac{\partial V_0}{\partial x} - \gamma\frac{\partial^3 V_0}{\partial x^3},$$

From the Equations (14) and (15), put $k = 1$, respectively, we obtain:

$$U_2(x) = \left(\frac{1}{2}\right)\left(-3\alpha\left[A_1(x) + B_1(x)\right] - \gamma\frac{\partial^3 U_1}{\partial x^3}\right), \tag{24}$$

and

$$V_2(x) = \left(\frac{1}{2}\right)\left(-3\alpha\left[C_1(x) + D_1(x)\right] - \gamma\frac{\partial^3 V_1}{\partial x^3}\right). \tag{25}$$

Now, from the Equations (16)-(19), put $k = 1$, respectively, we have:

$$A_1(x) = 2U_0U_1\frac{\partial U_0}{\partial x} + U_0^2\frac{\partial U_1}{\partial x}, \quad B_1(x) = 2V_0V_1\frac{\partial U_0}{\partial x} + V_0^2\frac{\partial U_1}{\partial x}, \tag{26}$$

$$C_1(x) = 2U_0U_1\frac{\partial V_0}{\partial x} + U_0^2\frac{\partial V_1}{\partial x}, \quad D_1(x) = 2V_0V_1\frac{\partial V_0}{\partial x} + V_0^2\frac{\partial V_1}{\partial x}, \tag{27}$$

Substituting the Equations (26) and (27) into the Equations (24) and (25), respectively, we get:

$$U_2(x) = \left(\frac{1}{2}\right)\left(-3\alpha\left[2\left(U_0U_1 + V_0V_1\right)\frac{\partial U_0(x)}{\partial x} + \left(U_0^2 + V_0^2\right)\frac{\partial U_1(x)}{\partial x}\right] - \gamma\frac{\partial^3 U_1(x)}{\partial x^3}\right),$$

and

$$V_2(x) = \left(\frac{1}{2}\right)\left(-3\alpha\left[2\left(U_0U_1 + V_0V_1\right)\frac{\partial V_0(x)}{\partial x} + \left(U_0^2 + V_0^2\right)\frac{\partial V_1(x)}{\partial x}\right] - \gamma\frac{\partial^3 V_1(x)}{\partial x^3}\right).$$

$\vdots$

and so on, in the same manner, the rest of components can be obtained.
Taking the inverse transformation of the set of values $\{U_k(x)\}_{k=0}^{n}$ and $\{V_k(x)\}_{k=0}^{n}$ gives n-terms approximate solutions as follows:

$$\tilde{u}_n(x,t) = \sum_{k=0}^{n} U_k(x)t^k = U_0(x) + U_1(x)t + U_2(x)t^2 + \cdots + U_n(x)t^n,$$

$$\tilde{v}_n(x,t) = \sum_{k=0}^{n} V_k(x)t^k = V_0(x) + V_1(x)t + V_2(x)t^2 + \cdots + V_n(x)t^n,$$

Therefore, the exact solution of the problem is readily obtained as follows [6]:

$$u(x,t) = \lim_{n\to\infty} \tilde{u}_n(x,t) = \lim_{n\to\infty} \left(\sum_{k=0}^{n} U_k(x)t^k \right),$$
$$v(x,t) = \lim_{n\to\infty} \tilde{v}_n(x,t) = \lim_{n\to\infty} \left(\sum_{k=0}^{n} V_k(x)t^k \right).$$

4. Numerical Applications

In this section, we will apply RDTM to solve the nonlinear CHS, and present numerical results to verify the effectiveness of this method, we take the following example:

Example 1:

Consider the following nonlinear CHS:

$$\frac{\partial u}{\partial t} + 3\alpha \left(u^2 + v^2\right) \frac{\partial u}{\partial x} + \gamma \frac{\partial^3 u}{\partial x^3} = 0,$$
$$\frac{\partial v}{\partial t} + 3\alpha \left(u^2 + v^2\right) \frac{\partial v}{\partial x} + \gamma \frac{\partial^3 v}{\partial x^3} = 0,$$

with the initial condition

$$u_0(x,t) = u(x,0) = \sqrt{\frac{2\gamma}{\alpha}}\, k \,\mathrm{sech}(kx) \cos(ax),$$

and

$$v_0(x,t) = v(x,0) = \sqrt{\frac{2\gamma}{\alpha}}\, k \operatorname{sech}(kx) \sin(ax),$$

where k and a are arbitrary constants.

Note: The exact solutions of the Hirota equation (1) is given by (2), where α and $\gamma > 0$, are arbitrary constants.

Solution:

Applying RDTM (see Section 3), by using Equations (14)-(15) with the initial conditions $u_0(x,t) = U_0(x)$ and $v_0(x,t) = V_0(x)$, we have:

$$U_1(x) = \frac{\gamma^{\frac{3}{2}} k \sqrt{\frac{2}{\alpha}}}{\cosh(kx)^2} \left[(k^3 - 3a^2k) \cos(ax) \sinh(kx) + (3ak^2 - a^3) \sin(ax) \cosh(kx) \right],$$

$$V_1(x) = \frac{\gamma^{\frac{3}{2}} k \sqrt{\frac{2}{\alpha}}}{\cosh(kx)^2} \left[(k^3 - 3a^2k) \sin(ax) \sinh(kx) + (a^3 - 3ak^2) \cos(ax) \cosh(kx) \right],$$

$$\begin{aligned} U_2(x) = \frac{-\gamma^{\frac{5}{2}} k \sqrt{\frac{2}{\alpha}}}{2\cosh(kx)^3} \Big[&(a^6 - 15a^4k^2 + 15a^2k^4 - k^6) \sin(ax) \cosh(kx)^2 \\ &+ (6ak^5 - 20a^3k^3 + 6a^5k) \cos(ax) \cosh(kx) \sinh(kx) \\ &+ (18a^4k^2 - 12a^2k^4 + 2k^6) \sin(ax) \Big], \end{aligned}$$

$$\begin{aligned} V_2(x) = \frac{-\gamma^{\frac{5}{2}} k \sqrt{\frac{2}{\alpha}}}{2\cosh(kx)^3} \Big[&(a^6 - 15a^4k^2 + 15a^2k^4 - k^6) \sin(ax) \cosh(kx)^2 \\ &+ (6ak^5 - 20a^3k^3 + 6a^5k) \cos(ax) \cosh(kx) \sinh(kx) \\ &+ (18a^4k^2 - 12a^2k^4 + 2k^6) \sin(ax) \Big]. \end{aligned}$$

$\vdots$

Hence, the approximate solutions by RDTM of order two are:

$$u(x,t) = U_0(x) + U_1(x)t + U_2(x)t^2,$$

$$v(x,t) = V_0(x) + V_1(x)t + V_2(x)t^2.$$

Note: The results obtained by RDTM for Example 1 for $\alpha = 2$,$\gamma = 1$,$k = 0.2$ and $a = 0.5$, are listed in Tables (2)-(3) and plotted in Figures 1 and 2.

Table (2) Comparison of the exact solution of $u(x,t)$ with the approximate solution, absolute errors and least square error obtained by RDTM for different values of $-4 \leq x \leq 4$ and $0 \leq t \leq 1$.

Time t	Exact Solution	Approximate Solution of the RDTM	Absolute Errors $\mid u_{Exact} - u_{RDTM} \mid$	Least Square Error
$x = -4$				
0	-0.06223059112	-0.06223059112	0.0000E+00	11.8202E-09
0.2	-0.06159459550	-0.06159492461	3.2911E-07	
0.4	-0.06087357030	-0.06087614029	2.5700E-06	
0.6	-0.06006578734	-0.06007423817	8.4508E-06	
0.8	-0.05916973998	-0.05918921825	1.9478E-05	
1	-0.05818416660	-0.05822108052	3.6914E-05	
$x = -2$				
0	0.09995673184	0.09995673184	0.0000E+00	3.7318E-08
0.2	0.10304245920	0.10304379310	1.3339E-06	
0.4	0.10608958040	0.10610029250	1.0712E-05	
0.6	0.10908995990	0.10912623020	3.6270E-05	
0.8	0.11203540760	0.11212160600	8.6198E-05	
1	0.11491773070	0.11508641990	1.6869E-04	
$x = 0$				
0	0.2	0.2	0.0000E+00	5.3195E-10
0.2	0.1999024782	0.1999024440	3.4200E-08	
0.4	0.1996103218	0.1996097760	5.4580E-07	
0.6	0.1991247547	0.1991219960	2.7587E-06	
0.8	0.1984478028	0.1984391040	8.6988E-06	
1	0.1975822747	0.1975611000	2.1175E-05	
$x = 2$				
0	0.099956731840	0.09995673184	0.0000E+00	3.4180E-08
0.2	0.096840430140	0.09683910874	1.3214E-06	
0.4	0.093701436340	0.09369092381	1.0513E-05	
0.6	0.090547440920	0.09051217705	3.5264E-05	
0.8	0.087385904220	0.08730286847	8.3036E-05	
1	0.084224022340	0.08406299805	1.6102E-04	
$x = 4$				
0	-0.06223059112	-0.06223059112	0.0000E+00	2.7610E-09
0.2	-0.06223059112	-0.06223059112	0.0000E+00	
0.4	-0.06325537422	-0.06325257073	2.8035E-06	
0.6	-0.06364851734	-0.06363888383	9.6335E-06	
0.8	-0.06396529804	-0.06394207913	2.3219E-05	
1	-0.06420821168	-0.06416215662	4.6055E-05	

Table (3) Comparison of the exact solution of $v(x,t)$ with the approximate solution, absolute errors and least square error obtained by RDTM for different values of $-4 \leq x \leq 4$ and $0 \leq t \leq 1$.

Time t	Exact Solution	Approximate Solution of the RDTM	Absolute Errors $\mid v_{Exact} - v_{RDTM} \mid$	Least Square Error
$x = -4$				
0	-0.1359763223	-0.1359763223	0.0000E+00	2.7380E-08
0.2	-0.1393458350	-0.1393469389	1.1039E-06	
0.4	-0.1427031908	-0.1427121353	8.9445E-06	
0.6	-0.1460413479	-0.1460719112	3.0563E-05	
0.8	-0.1493529467	-0.1494262668	7.3320E-05	
1	-0.1526303286	-0.1527752022	1.4487E-04	
$x = -2$				
0	-0.1556733863	-0.1556733863	0.0000E+00	6.7132E-09
0.2	-0.1559812729	-0.1559819203	6.4740E-07	
0.4	-0.1561417698	-0.1561467943	5.0245E-06	
0.6	-0.1561515918	-0.1561680082	1.6416E-05	
0.8	-0.1560079740	-0.1560455620	3.7588E-05	
1	-0.1557087083	-0.1557794557	7.0747E-05	
$x = 0$				
0	0	0	0.0000E+00	2.5422E-08
0.2	0.00259887862	0.0026000000	1.1214E-06	
0.4	0.00519103813	0.0052000000	8.9619E-06	
0.6	0.00776980512	0.0078000000	3.0195E-05	
0.8	0.01032859693	0.0104000000	7.1403E-05	
1	0.01286096549	0.0130000000	1.3903E-04	
$x = 2$				
0	0.1556733863	0.1556733863	0.0000E+00	1.1360E-08
0.2	0.1552218762	0.1552211923	6.8390E-07	
0.4	0.1546309478	0.1546253381	5.6097E-06	
0.6	0.1539052008	0.1538858240	1.9377E-05	
0.8	0.1530495848	0.1530026498	4.6935E-05	
1	0.1520693535	0.1519758155	9.3538E-05	
$x = 4$				
0	0.1359763223	0.1359763223	0.0000E+00	2.1350E-08
0.2	0.1326013605	0.1326002853	1.0752E-06	
0.4	0.1292273116	0.1292188281	8.4835E-06	
0.6	0.1258601838	0.1258319506	2.8233E-05	
0.8	0.1225056265	0.1224396526	6.5974E-05	
1	0.1191689280	0.1190419344	1.2699E-04	

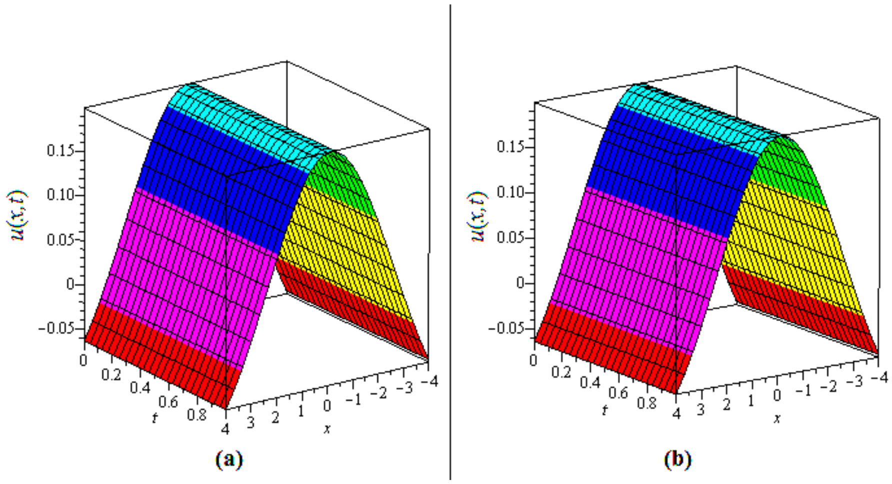

Figure 1. Plots of results of Example 1 for $-4 \leq x \leq 4$, $0 \leq t \leq 1$, $\alpha = 2$, $\gamma = 1$, $k = 0.2$ and $a = 0.5$.

(a) Exact solution of $u(x,t)$,

(b) Approximate solution of $u(x,t)$ by RDTM,

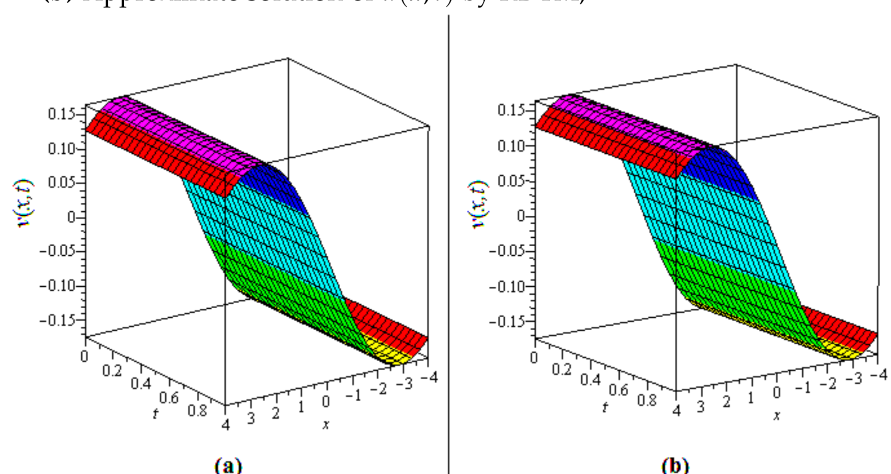

Figure 2. Plots of results for the previous Example 1 for $-4 \leq x \leq 4$, $0 \leq t \leq 1$, $\alpha = 2$, $\gamma = 1$, $k = 0.2$ and $a = 0.5$.

(a) Exact solution of $v(x,t)$,

(b) Approximate solution of $v(x,t)$ by RDTM,

5. Conclusion:

In this paper, we proposed the RDTM for solving the initial value problems associated with the CHS. The numerical results showed that the RDTM performed well for the considerable problem and the results are quite reliable. Also, we noted that the RDTM needs less work in comparison with the traditional methods. Therefore, this method can be applied to many complicated linear and nonlinear problems and does not require linearization, discretization or perturbation. The results show that RDTM is powerful mathematical tool for solving systems of nonlinear partial differential equations. Finally, from the Figures 1 and 2, it is clearly seen that the RDTM approximation and the exact solution are in good agreement.

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