Energy shift of H-atom electrons due to the blackbody photons

Miroslav Pardy

Department of Physical Electronics
and
Laboratory of Plasma Physics

Masaryk University
Kotlářská 2, 611 37 Brno, Czech Republic
e-mail:pamir@physics.muni.cz

May 10, 2016

Abstract

The electromagnetic shift of energy levels of H-atom electrons is determined by calculating the mean square amplitude of oscillation of an electron coupled to the relic photon fluctuations of the electromagnetic field. Energy shift of electrons in H-atom is determined in the framework of non-relativistic quantum mechanics.

The cosmical rays including relic photons were predicted by Gamow as a consequence of the Big Bang. The Mach cone is created when the high energy cosmical particles move with the speed greater than the velocity of sound in cosmical relic photon sea (Pardy, 2013a; 2013b).

The accidental discovery of the CMB in 1964 by American radio astronomers Arno Penzias and Robert Wilson was the culmination of work initiated in the 1940, and earned the discoverers the 1978 Nobel Prize. We consider here the influence of the heat bath of the relic photons on the energy shift of H-atom electrons.

Relic photons form so called blackbody, which has the distribution law of photons derived in 1900 by Planck (1900, 1901), (Schöpf, 1978). The derivation was based on the investigation of the statistics of the system of oscillators inside of the blackbody. Later Einstein (1917) derived the Planck formula from the Bohr model of atom where electrons have the discrete energies and the energy of the emitted photons are given by the Bohr formula $\hbar\omega = E_i - E_f$, E_i, E_f are the initial and final energies of electrons.

Now, let us calculate the modified Coulomb potential due to blackbody. The starting point of the determination of the energy shift in the H-atom is the potential $V_0(\mathbf{x})$, which is generated by nucleus of the H-atom. The potential at point $V_0(\mathbf{x} + \delta\mathbf{x})$, evidently is (Akhiezer, et al., 1953; Welton, 1948):

$$V_0(\mathbf{x} + \delta\mathbf{x}) = \left\{1 + \delta\mathbf{x}\nabla + \frac{1}{2}(\delta\mathbf{x}\nabla)^2 + ...\right\} V_0(\mathbf{x}). \tag{1}$$

If we average the last equation in space, we can eliminate so called the effective potential in the form

$$V(\mathbf{x}) = \left\{1 + \frac{1}{6}(\delta\mathbf{x})^2_T\Delta + ...\right\} V_0(\mathbf{x}), \tag{2}$$

where $\delta\mathbf{x})^2_T$ is the average value of te square coordinate shift caused by the thermal photon fluctuations. The potential shift follows from eq. (2):

$$\delta V(\mathbf{x}) = \frac{1}{6}(\delta\mathbf{x})^2_T\Delta V_0(\mathbf{x}). \tag{3}$$

The corresponding shift of the energy levels is given by the standard quantum mechanical formula (Akhiezer, et al., 1953)

$$\delta E_n = \frac{1}{6}(\delta\mathbf{x})^2_T(\psi_n\Delta V_0\psi_n). \tag{4}$$

In case of the Coulomb potential, which is the case of the H-atom, we have

$$V_0 = -\frac{e^2}{4\pi|\mathbf{x}|}. \tag{5}$$

Then for the H-atom we can write

$$\delta E_n = \frac{2\pi}{3}(\delta\mathbf{x})^2_T\frac{e^2}{4\pi}|\psi_n(0)|^2, \tag{6}$$

where we used the following equation for the Coulomb potential

$$\Delta\frac{1}{|\mathbf{x}|} = -4\pi\delta(\mathbf{x}). \tag{7}$$

Motion of electron in electric field is evidently described by elementary equation

$$\delta\ddot{\mathbf{x}} = \frac{e}{m}\mathbf{E}_T, \tag{8}$$

which can be transformed by the Fourier transformation into the folowing equation

$$|\delta \mathbf{x}_{T\omega}|^2 = \frac{1}{2}\left(\frac{e^2}{m^2\omega^4}\right)\mathbf{E}^2_{T\omega}, \tag{9}$$

where the index ω concerns the Fourier component of above functions.

On the basis of the Bethe idea of the influence of vacuum fluctuations on the energy shift of electron (Bethe, 1947), the following elementary relations was used by Welton (1948), Akhiezer et al. (1953) and Berestetzkii et al. (1999):

$$\frac{1}{2}\mathbf{E}^2_{\omega} = \frac{\hbar\omega}{2} \tag{10}$$

and in case of the thermal bath of the blackbody, the last equation is of the following form (Isihara, 1971):

$$\mathbf{E}^2_{T\omega} = \varrho(\omega) = \left(\frac{\hbar\omega^3}{\pi^2c^3}\right)\frac{1}{e^{\frac{\hbar\omega}{kT}}-1}, \tag{11}$$

because the Planck law in (11) was written as

$$\varrho(\omega) = G(\omega) < E_\omega >= \left(\frac{\omega^2}{\pi^2c^3}\right)\frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}}-1}, \tag{12}$$

where the term

$$< E_\omega >= \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}}-1} \tag{13}$$

is the average energy of photons in the blackbody and

$$G(\omega) = \frac{\omega^2}{\pi^2c^3} \tag{14}$$

is the number of electromagnetic modes in the interval $\omega, \omega + d\omega$.

Then,

$$(\delta \mathbf{x}_{T\omega})^2 = \frac{1}{2}\left(\frac{e^2}{m^2\omega^4}\right)\left(\frac{\hbar\omega^3}{\pi^2c^3}\right)\frac{1}{e^{\frac{\hbar\omega}{kT}}-1}, \tag{15}$$

where $(\delta \mathbf{x}_{T\omega})^2$ involves the number of frequences in the interval $(\omega, \omega + d\omega)$.

So, after some integration, we get

$$(\delta \mathbf{x})^2_T = \int_{\omega_1}^{\omega_2}\frac{1}{2}\left(\frac{e^2}{m^2\omega^4}\right)\left(\frac{\hbar\omega^3}{\pi^2c^3}\right)\frac{1}{e^{\frac{\hbar\omega}{kT}}-1} = \frac{1}{2}\left(\frac{e^2}{m^2}\right)\left(\frac{\hbar}{\pi^2c^3}\right)F(\omega_2-\omega_1), \tag{16}$$

where $F(\omega)$ is the primitive function of the omega-integral

$$J = \frac{1}{\omega}\frac{1}{e^{\frac{\hbar\omega}{kT}}-1}, \tag{17}$$

which cannot be calculated by the elementary integral methods and it is not involved in the tables of integrals.

Frequencies ω_1 and ω_2 will be determined with regard to the existence of the fluctuation field of thermal photons. It was determined in case of the Lamb shift (Bethe, 1947 ; Welton, 1947) by means of the physical analysis of the interaction of the Coulombic atom with the surrounding fluctuation field. We suppose here that the Bethe and Welton arguments are valid and so we take the frequencies in the Bethe-Welton form. In other words, electron cannot respond to the fluctuating field if the frequency which is much less than the atom binding energy given by the Rydberg constant (Rohlf, 1994) $E_{Rydberg} = \alpha^2 mc^2/2$. So, the lower frequency limit is

$$\omega_1 = E_{Rydberg}/\hbar = \frac{\alpha^2 mc^2}{2\hbar}, \tag{18}$$

where $\alpha \approx 1/137$ is so called the fine structure constant.

The specific form of the second frequency follows from the elementary argument, that we expect the effective cutoff, since we must neglect the relativistic effect in our non-relativistic theory. So, we write

$$\omega_2 = \frac{mc^2}{\hbar}. \tag{19}$$

If we take the thermal function of the form of the geometric series

$$\frac{1}{e^{\frac{\hbar\omega}{kT}} - 1} = q(1 + q^2 + q^3 + ...); \quad q = e^{-\frac{\hbar\omega}{kT}}, \tag{20}$$

$$\int_{\omega_1}^{\omega_2} q(1 + q^2 + q^3 + ...)\frac{1}{\omega}d\omega = \ln|\omega| + \sum_{k=1}^{\infty} \frac{(-\frac{\hbar\omega}{kT})^k}{k!k} + ...; \quad q = e^{-\frac{\hbar\omega}{kT}} \tag{21}$$

and the first thermal contribution is

$$Thermal\ contribution = \ln\frac{\omega_2}{\omega_1} - \frac{\hbar}{kT}(\omega_2 - \omega_1), \tag{22}$$

Then, with eq. (6)

$$\delta E_n \approx \frac{2\pi}{3}\left(\frac{e^2}{m^2}\right)\left(\frac{\hbar}{\pi^2 c^3}\right)\left(\ln\frac{\omega_2}{\omega_1} - \frac{\hbar}{kT}(\omega_2 - \omega_1)\right)|\psi_n(0)|^2, \tag{23}$$

where (Sokolov et al., 1962)

$$|\psi_n(0)|^2 = \frac{1}{\pi n^2 a_0^2} \tag{24}$$

with

$$a_0 = \frac{\hbar^2}{me^2}. \tag{25}$$

Let us only remark that the numerical form of eq. (23) has deep experimental astrophysical meaning.

In article by author (Pardy, 1994), which is the continuation of author articles on the finite-temperature Čerenkov radiation and gravitational Čerenkov radiation (Pardy, 1989a; ibid., 1989b), the temperature Green function in the framework of the Schwinger source theory was derived in order to determine the Coulomb and Yukawa potentials at finite-temperature using the Green functions of a photon with and without radiative corrections, and then by considering the processes expressed by the Feynman diagrams.

The determination of potential at finite temperature is one of the problems which form the basic ingredients of the quantum field theory (QFT) at finite temperature. This theory was formulated some years ago by Dolan and Jackiw (1974), Weinberg (1974) and Bernard (1974) and some of the first applications of this theory were the calculations of the temperature behavior of the effective potential in the Higgs sector of the standard model.

Information on the systematic examination of the finite temperature effects in quantum electrodynamics (QED) at one-loop order was given by Donoghue, Holstein and Robinett (1985). Partovi (1994) discussed the QED corrections to Planck's radiation law and photon thermodynamics,

A similar discussion of QED was published by Johansson, Peressutti and Skagerstam (1986) and Cox et al. (1984).

Serge Haroche (2012) and his research group in the Paris microwave laboratory used a small cavity for the long life-time of photon quantum experiments performed with the Rydberg atoms. We consider here the gas of relic photons (at temperature T) as the preamble for new experiments for the determination of the energy shift of H-atom electrons interacting with the relic photon gas. It is not excluded, that the experiments performed by the well educated experts will be the Nobelian ones.

References

Berestetzkii, V. B., Lifshitz, E. M. and Pitaevskii, L. P. Quantum electrodynamics, (Butterworth-Heinemann, Oxford, 1999).

Bethe, H. A. (1947). The electromagnetic shift of energy levels, Phys. Rev. **72**, 339.

Bernard. C. W. (1974). Feynman rules for gauge theories at finite temperature, Phys. Rev. D **9**, 3312.

Cox, P. H., Hellman, W. S. and Yildiz, A. (1984). Finite temperature corrections to field theory: electron mass, magnetic moment, and vacuum energy, Ann. Phys. (N.Y.) **154**, 211.

Dolan, L. and Jackiw, R. (1974). Symmetry behavior at finite temperature, Phys. Rev. D **9**, 3320.

Donoghue, J. F., Holstein, B. R. and Robinett, R. W. (1985).Quantum electrodynamics at finite temperature, Ann. Phys. (NY) **164** No. 2, 233.

Einstein, A. (1917). Zur Quantentheorie der Strahlung, Physikalische Zeitschrift, **18**, 121.

Haroche S. (2012). The secrets of my prizewinning research, Nature, **490**, 311.

Isihara, A. Statistical mechanics, (Academic Press, New York,London, 1971).

Johansson, A. E., Peressutti, G. and Skagerstam, B. S. (1986). Quantum field theory at finite temperature: renormalization and radiative corrections, Nucl. Phys. B **278**, 324.

Pardy, M. (1989). Finite-temperature Čerenkov radiation, Phys. Lett. A **134** No. 6 357.

Pardy, M. (1989). Finite-temperature gravitational Čerenkov radiation, International Journal of Theor. Physics, **34**, No. 6, 951.

Pardy, M. (1994). The two-body potential at finite temperature, CERN.TH.7397/94.

Pardy, M. (2013a). Velocity of sound in the relic photon sea, arXiv: General Physics (physics.gen-ph)/1303.3201.

Pardy, M. (2013b). Velocity of sound in the blackbody photon gas, Results in Physics **3**, 70.

Partovi, H. M. (1994). QED corrections to Plancks radiation law and photon thermodynamics, Phys. Rev. D **50**, 1118.

Planck, M. (1900). Zur Theorie des Gesetzes der Energieverteilung im Normalspektrum, Verhandlungen deutsch phys. Ges. **2**, 237.; ibid: (1901). Ann. Phys. **4**, 553.

Rohlf, J. W. Modern physics from α *to* Z^0, (John Wiley & Sons LTD., London - New York, 1994).

Schöpf, H-G. Theorie der Wärmestrahlung in historisch-kritischer Darstellung, (Alademie/Verlag, Berlin, 1978).

Sokolov, A. A., Loskutov, Yu. M. and Ternov, I. M. Quantum mechanics, (State Pedagogical Edition, Moscow, 1962). (in Russian).

Weinberg, S. (1974). Gauge and global symmetries at high temperature, Phys. Rev. D **9**, 3357.

Welton, Th. (1948). Some observable effects of the quantum-mechanical fluctuations of the electromagnetic field, Phys. Rev. **74**, 1157.