

# AN ALTERNATIVE FORMULATION OF SPECIAL RELATIVITY

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This article presents an alternative formulation of special relativity which can be applied in any inertial reference frame. In addition, a new universal force is proposed.

## Introduction

The invariant mass ( $m$ ) and the frequency factor ( $f$ ) of a massive particle are given by:

$$m \doteq m_o$$

$$f \doteq \left(1 - \frac{\mathbf{v} \cdot \mathbf{v}}{c^2}\right)^{-1/2}$$

where ( $m_o$ ) is the rest mass of the massive particle, ( $\mathbf{v}$ ) is the velocity of the massive particle and ( $c$ ) is the speed of light in vacuum.

The invariant mass ( $m$ ) and the frequency factor ( $f$ ) of a non-massive particle are given by:

$$m \doteq \frac{h \kappa}{c^2}$$

$$f \doteq \frac{\nu}{\kappa}$$

where ( $h$ ) is the Planck constant, ( $\nu$ ) is the frequency of the non-massive particle, ( $\kappa$ ) is a positive universal constant with dimension of frequency and ( $c$ ) is the speed of light in vacuum.

In this article, a massive particle is a particle with non-zero rest mass and a non-massive particle is a particle with zero rest mass.

## The Alternative Kinematics

The special position ( $\bar{\mathbf{r}}$ ), the special velocity ( $\bar{\mathbf{v}}$ ) and the special acceleration ( $\bar{\mathbf{a}}$ ) of a ( massive or non-massive ) particle are given by:

$$\bar{\mathbf{r}} \doteq \int f \mathbf{v} dt$$

$$\bar{\mathbf{v}} \doteq \frac{d\bar{\mathbf{r}}}{dt} = f \mathbf{v}$$

$$\bar{\mathbf{a}} \doteq \frac{d\bar{\mathbf{v}}}{dt} = f \frac{d\mathbf{v}}{dt} + \frac{df}{dt} \mathbf{v}$$

where ( $f$ ) and ( $\mathbf{v}$ ) are the frequency factor and the velocity of the particle.

## The Alternative Dynamics

If we consider a ( massive or non-massive ) particle with invariant mass  $m$  then the linear momentum  $\mathbf{P}$  of the particle, the angular momentum  $\mathbf{L}$  of the particle, the net force  $\mathbf{F}$  acting on the particle, the work  $W$  done by the net force acting on the particle, and the kinetic energy  $K$  of the particle, for an inertial reference frame, are given by:

$$\mathbf{P} \doteq m \bar{\mathbf{v}} = m f \mathbf{v}$$

$$\mathbf{L} \doteq \mathbf{P} \times \mathbf{r} = m \bar{\mathbf{v}} \times \mathbf{r} = m f \mathbf{v} \times \mathbf{r}$$

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} = m \bar{\mathbf{a}} = m \left[ f \frac{d\mathbf{v}}{dt} + \frac{df}{dt} \mathbf{v} \right]$$

$$W \doteq \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 \frac{d\mathbf{P}}{dt} \cdot d\mathbf{r} = \Delta K$$

$$K \doteq m f c^2$$

where ( $f$ ,  $\mathbf{r}$ ,  $\mathbf{v}$ ,  $\bar{\mathbf{v}}$ ,  $\bar{\mathbf{a}}$ ) are the frequency factor, the position, the velocity, the special velocity and the special acceleration of the particle relative to the inertial reference frame and ( $c$ ) is the speed of light in vacuum. The kinetic energy ( $K_o$ ) of a massive particle at rest is ( $m_o c^2$ )

## The Kinetic Force

In an isolated system of ( massive or non-massive ) particles, the kinetic force  $\mathbf{K}_{ij}$  exerted on a particle  $i$  with invariant mass  $m_i$  by another particle  $j$  with invariant mass  $m_j$  is given by:

$$\mathbf{K}_{ij} = - \left[ \frac{m_i m_j}{M} (\bar{\mathbf{a}}_i - \bar{\mathbf{a}}_j) \right]$$

where  $\bar{\mathbf{a}}_i$  is the special acceleration of the particle  $i$ ,  $\bar{\mathbf{a}}_j$  is the special acceleration of the particle  $j$  and  $M (= \sum_z m_z)$  is the invariant mass of the isolated system of particles.

From the above equation it follows that the net kinetic force  $\mathbf{K}_i (= \sum_z \mathbf{K}_{iz})$  acting on the particle  $i$  is given by:

$$\mathbf{K}_i = - m_i \bar{\mathbf{a}}_i$$

where  $m_i$  is the invariant mass of the particle  $i$  and  $\bar{\mathbf{a}}_i$  is the special acceleration of the particle  $i$ .

Now, substituting (  $\mathbf{F}_i = m_i \bar{\mathbf{a}}_i$  ) and rearranging, we obtain:

$$\mathbf{T}_i \doteq \mathbf{K}_i + \mathbf{F}_i = 0$$

Therefore, in an isolated system of ( massive or non-massive ) particles, the total force  $\mathbf{T}_i$  acting on a particle  $i$  is always zero.

In this article, the linear momentum of an isolated system of ( massive or non-massive ) particles is conserved (  $\sum_z m_z \bar{\mathbf{v}}_z = \text{constant}$  )

## Bibliography

**A. Einstein**, Relativity: The Special and General Theory.

**E. Mach**, The Science of Mechanics.

**W. Pauli**, Theory of Relativity.

**A. French**, Special Relativity.

## Appendix I

### System of Equations I

$$\begin{array}{ccccc}
 & & & & \boxed{[1]} \\
 & & & & \downarrow dt \downarrow \\
 \boxed{[4]} & \leftarrow \times \mathbf{r} \leftarrow & & & \boxed{[2]} \\
 \downarrow dt \downarrow & & & & \downarrow dt \downarrow \\
 \boxed{[5]} & \leftarrow \times \mathbf{r} \leftarrow & \boxed{[3]} & \rightarrow \int d\mathbf{r} \rightarrow & \boxed{[6]}
 \end{array}$$

$$[1] \quad \frac{1}{\mu} \left[ \int \mathbf{P} dt - \iint \mathbf{F} dt dt \right] = 0$$

$$[2] \quad \frac{1}{\mu} \left[ \mathbf{P} - \int \mathbf{F} dt \right] = 0$$

$$[3] \quad \frac{1}{\mu} \left[ \frac{d\mathbf{P}}{dt} - \mathbf{F} \right] = 0$$

$$[4] \quad \frac{1}{\mu} \left[ \mathbf{P} - \int \mathbf{F} dt \right] \times \mathbf{r} = 0$$

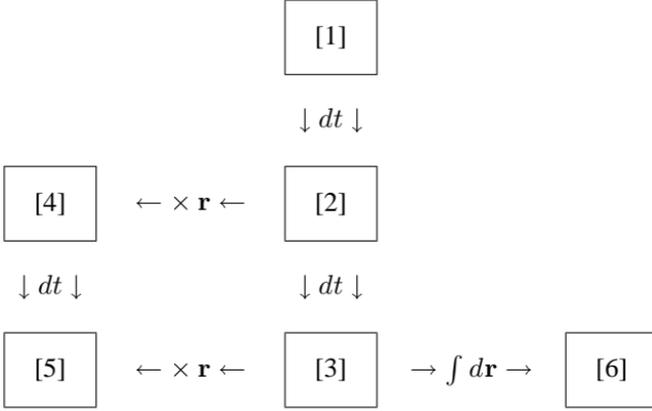
$$[5] \quad \frac{1}{\mu} \left[ \frac{d\mathbf{P}}{dt} - \mathbf{F} \right] \times \mathbf{r} = 0$$

$$[6] \quad \frac{1}{\mu} \left[ \int \frac{d\mathbf{P}}{dt} \cdot d\mathbf{r} - \int \mathbf{F} \cdot d\mathbf{r} \right] = 0$$

$[\mu]$  is an arbitrary constant with dimension of mass (M)

## Appendix II

### System of Equations II



$$[1] \quad \frac{1}{\mu} \left[ m \bar{\mathbf{r}} - \iint \mathbf{F} dt dt \right] = 0$$

$$[2] \quad \frac{1}{\mu} \left[ m \bar{\mathbf{v}} - \int \mathbf{F} dt \right] = 0$$

$$[3] \quad \frac{1}{\mu} \left[ m \bar{\mathbf{a}} - \mathbf{F} \right] = 0$$

$$[4] \quad \frac{1}{\mu} \left[ m \bar{\mathbf{v}} - \int \mathbf{F} dt \right] \times \mathbf{r} = 0$$

$$[5] \quad \frac{1}{\mu} \left[ m \bar{\mathbf{a}} - \mathbf{F} \right] \times \mathbf{r} = 0$$

$$[6] \quad \frac{1}{\mu} \left[ m f c^2 - \int \mathbf{F} \cdot d\mathbf{r} \right] = 0$$

$[\mu]$  is an arbitrary constant with dimension of mass (M)