Small nonassociative corrections to the SUSY generators and cosmological constant

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Small nonassociative corrections for the SUSY operators $Q_{a,\dot{a}}$ are considered. The smallness is controlled by the ratio of the Planck length and a characteristic length $\ell_0 = \Lambda^{-1/2}$. Corresponding corrections of the momentum operator arising from the anticommutator of the SUSY operators are considered. The momentum operator corrections are defined via the anticommutator of the unperturbed SUSY operators $Q_{a,\dot{a}}$ and nonassociative corrections $Q_{1,a,\dot{a}}$. Choosing different anticommutators, one can obtain either a modified or q – deformed commutator of position x^μ and momentum operators P_ν.

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I. INTRODUCTION

Observational data [1]-[7] indicate that we live in the Universe expanding with acceleration. As a source of the acceleration, one can introduce either "dark energy" or modify Einstein gravity (for example, by considering $F(R)$ modified gravities). The list of models explaining the acceleration includes quintessence [8], a phantom scalar field [9], a tachyon scalar field [10], a Chaplygin gas [11]-[13], holographic dark energy [14]-[16], modified gravitational theories (including $F(R)$-gravities) [17] and so on.

All these models are dynamic in the sense that the present value of the cosmological constant Λ is explained in a dynamic way by using either some kind of matter or modified gravity. Another way is to postulate that Λ is indeed a fundamental constant. Following this way, it would be very interesting to understand what kind of physics is behind such approach. In Refs. [18, 19] we proposed the idea that the cosmological constant Λ can be connected with the appearance of nonassociativity (NA) in physics. In this model, the constant Λ controls the smallness of NA effects in quantum physics: the dimensionless quantity $l_{Pl}^2 \Lambda \approx 10^{-120}$ shows where the NA effects may occur. It may happen either on the huge scale $\ell_0 = 1/\sqrt{\Lambda} \approx 10^{26}$m (that means that there exists a maximal length ℓ_0) or with the small momentum $\mathcal{P}_0 \approx \hbar\sqrt{\Lambda} \approx 10^{-80}$g · m/s. We see for the first case that in Nature there exists a minimal 4D scalar curvature (a unique Lorentz invariant quantity having the dimensions cm^{-2}): $R_{min} \approx \Lambda$. It immediately leads to a very simple explanation for the acceleration of the present Universe: the Universe reaches the minimally possible curvature and has to stay in this state.

In the standard supersymmetry, the operator $Q_{a,\dot{a}}$ can be presented as a derivative with respect to coordinates $\theta^a, \theta^{\dot{a}}, x^\mu$. Here we want to consider a NA generalization of these generators when adding small NA terms. We will consider the consequences of the presence of such NA operators.

II. SMALL NONASSOCIATIVE CORRECTIONS TO THE STANDARD $Q_{a,\dot{a}}$ OPERATORS

In standard SUSY the operators $Q_{a,\dot{a}}$ and the momentum operator $P_\mu = -i\hbar\partial_\mu$ are connected by the expression

$$\{Q_a, Q_{\dot{a}}\} = 2\sigma^\mu_{a\dot{a}} P_\mu. \tag{1}$$

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In Refs. [18, 19] the NA generalization of the simplest SUSY algebra is proposed. Here we would like to investigate the case when the standard operators $Q_{a,\dot{a}}$ are slightly changed in a NA manner:

$$\tilde{Q}_a = Q_a + \xi Q_{1,a} + \xi^2 Q_{2,a} + \dots, \tag{2}$$

$$\tilde{Q}_{\dot{a}} = Q_{\dot{a}} + \xi Q_{1,\dot{a}} + \xi^2 Q_{2,\dot{a}} + \dots, \tag{3}$$

$$Q_a = \frac{\partial}{\partial \theta^a} - i\sigma^\mu_{a\dot{a}} \theta^{\dot{a}} \partial_\mu, \tag{4}$$

$$Q_{\dot{a}} = -\frac{\partial}{\partial \theta^{\dot{a}}} + i\theta^a \sigma^\mu_{a\dot{a}} \partial_\mu, \tag{5}$$

where $\xi = (l_P/\ell_0)^{1/3}$ is a small NA parameter, l_P is the Planck length, $\ell_0 = \Lambda^{-1/2}$, and $Q_{1,2,a,\dot{a}}$ are small additional NA terms for $Q_{a,\dot{a}}$. Henceforth we will work to the accuracy ξ.

Let us recall the definition of an associator

$$[A, B, C] = (AB)\, C - A\, (BC)\,, \tag{6}$$

where A, B, C are nonassociative quantities. Evaluation of associators for $Q_{a,\dot{a}}$ with an accuracy of ξ^3 gives us

$$\left[\tilde{Q}_x, \tilde{Q}_y, \tilde{Q}_z\right] = \xi^3 \left[Q_{1,x}, Q_{1,y}, Q_{1,z}\right], \tag{7}$$

where the indices x, y, z are any combinations of dotted and undotted indices, and we took into account that the operators $Q_{a,\dot{a}}$ are associative ones but $Q_{1,a,\dot{a}}$ are nonassociative ones. Henceforth, for brevity, we omit the index 1: $Q_{1,a,\dot{a}} \rightarrow Q_{a,\dot{a}}$.

It is shown in Ref. [19] that the simplest nonassociative generalization of SUSY operators gives us the following 3-point associators

$$\xi^3 \left[Q_a, Q_b, Q_c\right] = \frac{\hbar}{\ell_0} \zeta_1 \left(Q_a \epsilon_{bc} - Q_c \epsilon_{ab}\right), \tag{8}$$

$$\xi^3 \left[Q_{\dot{a}}, Q_b, Q_c\right] = \frac{\hbar}{\ell_0} \zeta_2 Q_{\dot{a}} \epsilon_{bc}, \tag{9}$$

$$\xi^3 \left[Q_a, Q_{\dot{b}}, Q_c\right] = 0, \tag{10}$$

$$\xi^3 \left[Q_a, Q_b, Q_{\dot{c}}\right] = -\frac{\hbar}{\ell_0} \zeta_2 Q_{\dot{c}} \epsilon_{ab}, \tag{11}$$

$$\xi^3 \left[Q_a, Q_{\dot{b}}, Q_{\dot{c}}\right] = \frac{\hbar}{\ell_0} \zeta_3 Q_a \epsilon_{\dot{b}\dot{c}}, \tag{12}$$

$$\xi^3 \left[Q_{\dot{a}}, Q_b, Q_{\dot{c}}\right] = 0, \tag{13}$$

$$\xi^3 \left[Q_{\dot{a}}, Q_{\dot{b}}, Q_c\right] = -\frac{\hbar}{\ell_0} \zeta_3 Q_c \epsilon_{\dot{a}\dot{b}}, \tag{14}$$

$$\xi^3 \left[Q_{\dot{a}}, Q_{\dot{b}}, Q_{\dot{c}}\right] = \frac{\hbar}{\ell_0} \zeta_4 \left(Q_{\dot{a}} \epsilon_{\dot{b}\dot{c}} - Q_{\dot{c}} \epsilon_{\dot{a}\dot{b}}\right), \tag{15}$$

where $|\zeta_{1,2,3,4}| = 1$. Here we have inserted the coefficient ξ^3 on the left hand sides of the equations to have the same order of smallness of the left-hand and right-hand sides of equations (8)-(15), and

$$\epsilon_{\dot{a}\dot{b}} = \epsilon_{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{16}$$

$$\epsilon^{ab} = \epsilon^{\dot{a}\dot{b}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{17}$$

The dotted indices are lowered and raised by using $\epsilon_{\dot{a}\dot{b}}, \epsilon^{\dot{a}\dot{b}}$, and the undotted – by using $\epsilon_{ab}, \epsilon^{ab}$. Introducing dimensionless operators $\bar{Q}_x = (\hbar/l_P)^{1/2} Q_x$, one can show that the left-hand and right-hand sides of equations (8)-(15) have the same order.

III. NONASSOCIATIVE CORRECTIONS IN THE MOMENTUM OPERATOR

Now we can calculate the anticommutator (1) using the expansion (2):

$$\left\{\tilde{Q}_a, \tilde{Q}_{\dot{a}}\right\} = 2\sigma^\mu_{a\dot{a}} \tilde{P}_\mu = 2\sigma^\mu_{a\dot{a}} P_\mu + \xi \left\{\tilde{Q}_a, Q_{1,\dot{a}}\right\} + \dots = 2\sigma^\mu_{a\dot{a}} \left(P_\mu + \xi P_{1,\mu} + \dots\right). \tag{18}$$

This expression defines the generalized momentum operator $\tilde{P}_\mu$ with the NA corrections (the terms with ξ) which are negligibly small since $\xi \approx 10^{-20}$.

Let us consider the Heisenberg uncertainty principle with the generalized momentum operator $\tilde{P}_\mu$

$$\left[x^\mu, \tilde{P}_\nu\right] = i\hbar\delta^\mu_\nu + \xi\left[x^\mu, P_{1,\nu}\right] + \cdots . \tag{19}$$

The properties of the operator $Q_{1,a,\dot{a}}$ are determined by the associators (8)-(15) and the anticommutator

$$\left\{\tilde{Q}_a, Q_{1,\dot{a}}\right\} = 2\sigma^\mu_{a\dot{a}} P_{1,\mu}. \tag{20}$$

Let us consider different choices of the operator $P_{1,\mu}$.

A. The case of $P_{1,\mu} = P_\mu f(P^2)$

In this case, using the relation $[A, BC] = B[A, C] + [A, B]C$, we obtain

$$\left[x^\mu, P_{1,\nu}\right] = \left[x^\mu, P_\nu\right] f\left(P^2\right) + P_\mu\left[x^\mu, f\left(P^2\right)\right]. \tag{21}$$

To be specific, let us consider the case $f(P^2) = P_\mu P^\mu/P_0^2$, where $P_0 = \hbar/l_P$. Substituting it into (21), we obtain

$$\left[x^\mu, P_\nu\frac{P_\alpha P^\alpha}{P_0^2}\right] = \frac{i\hbar}{P_0^2}\left(\delta^\mu_\nu P^2 + 2P^\mu P_\nu\right). \tag{22}$$

Finally, we have

$$\left[x^\mu, \tilde{P}_\nu\right] = i\hbar\left[\delta^\mu_\nu + \frac{\xi}{P_0^2}\left(\delta^\mu_\nu P^2 + 2P^\mu P_\nu\right)\right]. \tag{23}$$

We see that it is a generalized uncertainty principle [20]. The difference from the standard approach is that the coefficient ξ in the NA approach (23) is negligibly small and is connected with the constant Λ.

B. The case of $P_{1,\mu} = \alpha x_\mu f(P^2) + \beta f(P^2) x_\mu$

Analogously to (21), we have

$$\left[x^\mu, P_{1,\nu}\right] = \alpha x_\nu\left[x^\mu, f(P^2)\right] + \beta\left[x^\mu, f(P^2)\right] x_\nu. \tag{24}$$

For the simplest case $f(P^2) = P_\alpha P^\alpha/(2i\hbar)$ we obtain

$$\left[x^\mu, P_{1,\nu}\right] = \alpha x^\mu P_\nu + \beta P_\nu x^\mu. \tag{25}$$

Substituting it into (19), we have

$$\left[x^\mu, \tilde{P}_\nu\right] = i\hbar\delta^\mu_\nu + \xi\left(\alpha x^\mu P_\nu + \beta P_\nu x^\mu\right), \tag{26}$$

which can be rewritten as

$$\left[x^\mu, \tilde{P}_\nu\right]_q = i\hbar\delta^\mu_\nu, \tag{27}$$

where a $q-$ deformed commutator is

$$\left[x^\mu, \tilde{P}_\nu\right]_q = \left(1 - \alpha\xi\right) x^\mu P_\nu - \left(1 + \beta\xi\right) P_\nu x^\mu. \tag{28}$$

For the case $\alpha = \beta$

$$q = 1 - \alpha\xi, \quad \frac{1}{q} \approx 1 + \alpha\xi. \tag{29}$$

Then (27) and (28) take the form

$$\left[x^\mu, \tilde{P}_\nu\right]_q = q x^\mu \tilde{P}_\nu - \frac{1}{q}\tilde{P}_\nu x^\mu = i\hbar\delta^\mu_\nu. \tag{30}$$

We can make a mild conjecture that at least in this direction (non-associativity $\rightarrow q-$ deformation) this appears to be the case in this specific case and might be true in general too (for details of $q-$ deformation, see the textbooks [21, 22]).

IV. DISCUSSION AND CONCLUSIONS

We have investigated small nonassociative corrections for the SUSY generators $Q_{a,\dot{a}}$. The corrections are controlled by the nonassociative parameter ξ. We have considered the corrections with an accuracy of ξ. It is shown that such corrections give rise to the appearance of additional terms in the momentum operator. These terms modify the commutator $[x^\mu, P_\nu]$, which depends on the properties of the commutator between the unperturbed SUSY generator $Q_{a,\dot{a}}$ and the NA corrections $Q_{1,a,\dot{a}}$.

In our approach the cosmological constant $\Lambda = \ell_0^{-2}$ is a NA parameter controlling the smallness of NA effects in quantum physics. The corresponding dimensionless parameter is $\xi = (l_P/\ell_0)^{1/3}$. But now one surprising effect can be observed on huge scales when a huge value of ℓ_0 gives rise to an extremely small inverse quantity – the scalar curvature $R_{min} = \ell_0^{-2} = \Lambda$. Physically, it means that the 4-dimensional Ricci scalar curvature of the Universe should satisfy the inequality $R_{4D} \gtrsim R_{min}$. Thus, in our model Λ is a constant which is associated with the NA effects in cosmology.

Another interesting effect of the model under consideration is that the small NA corrections in the SUSY operators $Q_{a,\dot{a}}$ give rise to modifications of a quantum commutator of position and momentum operators. These modifications depend on the properties of an anticommutator of unperturbed and perturbed SUSY operators.

It must be noted that all the NA effects are extremely small because of the smallness of the NA parameter $\xi \approx 10^{-20}$.

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