# Time dependent analysis of the 2S3 Normal Mode

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**Abstract**: The time dependent analysis of the normal mode $_2S_3$ near 1244 µHz allows a precise determination of phase and frequency and reveals a phase modulation with a strong geographical dependence. All measurements are performed by a phase-locked loop.

# Introduction

After earthquakes, the Earth vibrates like a bell and a set of different natural frequencies is recorded by various instruments. One of them is $_2S_3$ and according to the theory, this mode has considerable shear–energy in the inner core of the earth, necessary to study certain properties of the inner core of the Earth like shape and rotation.

# The Preparation of the data records

In a first step, the raw data (the most common sampling time is one second) of all available SG stations between 2004-12-26 and 2005-01-31 were made machine readable. The influence of atmosphere pressure variation on the gravity data was omitted because the air mass above the instruments changes much slower than the oscillation time of $_2S_3$ (about 13 minutes). Extensive experiments have shown that any admixture of air pressure data deteriorates the SNR, regardless of the sign and value of the prefactor.

In order not to worsen the already low SNR of the SG records, the very intense tides must be effectively reduced by a comb filter in the first processing step. The huge dynamic range of the data (more than 100 dB) plus a safety margin of at least 40 dB inside the filter programs must not override the math co-processor of the computer in order to prevent the generation of intermodulation products which can not be distinguished from actual spectral lines. The application is very simple: all samples are shifted by 402 seconds and subtracted from the original record. The ratio of this time difference to the period(s) of the oscillations produces the desired effect: The very intense amplitudes of the slow tides (T $\approx$ several hours) are largely compensated, while the amplitude of the much faster $_2S_3$ oscillations is doubled. A low-pass filter ($f$ < 4 ms), followed by a second "*shift and subtract*" procedure generates the final data string. A multiple application does not bring further improvement. After the comb filter has reduced the dynamic range of the data below 40 dB, the mean frequency of $_3S_2$ is shifted to the intermediate frequency 200 µHz and the bandwidth is limited to 15 µHz, using the mixing method[1]. The absolute frequency difference $\Delta f$ between all spectral lines remains constant while the ratio $\Delta f/f$ increases, facilitating the separation of adjacent spectral lines. This simplifies the data analysis and allows to prolong the sampling rate of the record to 60 seconds without altering the information content.

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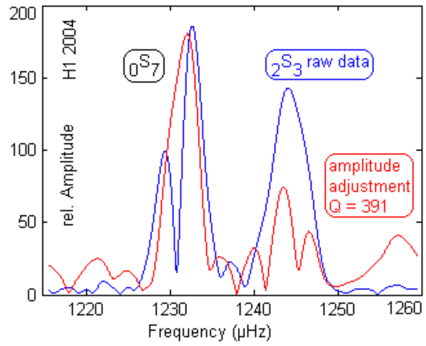
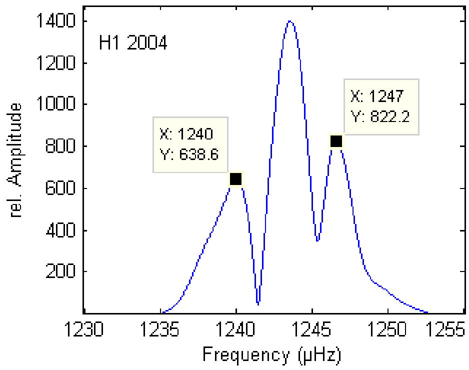
# The Spectrum




All normal modes are amplitude modulated with an envelope, which is usually composed of more components than the exponential decay alone. Any modulation increases inevitably the spectral half width of the spectral line. In the spectrum of the unchanged data, ${}_2S_3$ appears as a strikingly broad, unstructured bell curve (blue color). After compensating for the exponential decay of the amplitude by a suitable time-dependent factor (red color for Q = 391), the FWHM is reduced and due to the improved frequency resolution, ${}_2S_3$ disintegrates into three distinct peaks. A close look shows a slight reduction of the frequency, which is subsequently confirmed by other methods. The reason is that the amplitude correction hardly changes the data immediately after the earthquake. But later incoming data are weighted more heavily and may change the average frequency.



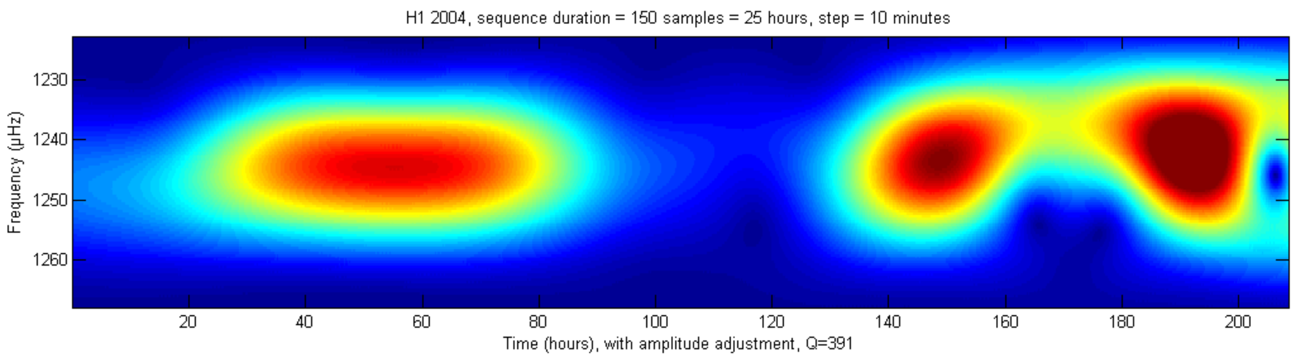

Since the adjacent normal mode ${}_0S_7$ requires a different time-dependent factor, the half-width of this spectral line is broadened. That does not bother the following investigations, because at the relatively low intermediate frequency of 200 µHz, high quality filters can be constructed. A very selective sinc filter with a rectangular passband prevents any influence on the measurement of ${}_2S_3$ inside the computer. However, a mutual influence of the two closely spaced resonances by nonlinearities within the earth can not be excluded.

The maximum useful sequence length of 233 hours (subsequently, the SNR is prohibitive) leads to a frequency resolution of 1.2 µHz and allows to determine the difference between the side bands and the center frequency. The modulation frequency 3.3 µHz corresponds to a modulation period of 84 hours. Surprisingly, the measurement of the ${}_3S_2$ mode yielded almost exactly the same value[2]. Since the Earth's inner core affects both normal modes, this result may help to determine its structure, shape and movement (rotation or vibration).

Spectra are always calculated from the magnitude of the complex Fourier coefficients. Since this operation deletes the phase information, it can not be judged whether the sidebands are created by an amplitude modulation or a phase modulation. Indeed, it is a combination of both.

# The Temporal Evolution of ${}_2S_3$

After compensating the exponential decay of the amplitude by a suitable time-dependent factor (*Q*-factor = 391), the spectrograms of all SG recordings in Central Europe reveal an aperiodic amplitude modulation.



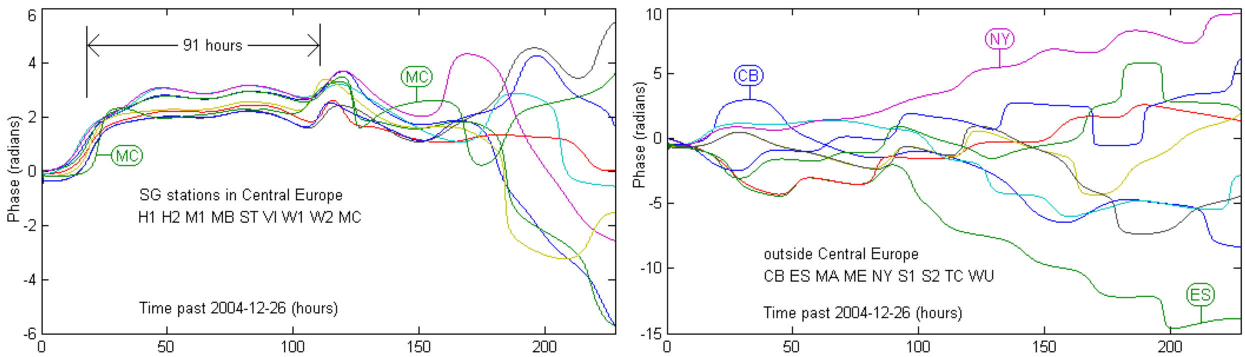

The spectrogram is composed of 1251 individual spectra, each one generated from a sequence of 150 samples (zero-padded to a total length 4096) and whose start times advance in steps of 600 s. The slow frequency drift is barely visible in this figure, it is computed more precisely below.

Amazingly and with few exceptions (the noisy recordings of the stations ES, MA and ME and NY), most spectrograms show the same peculiar temporal structure, which seems not to depend on the geographical location of the station. About 120 hours after the earthquake, there is always a minimum. Before, there is an elongate amplitude maximum and later, there is a twin maximum. All spectra with a starting point 220 hours past the EQ are omitted due to the poor SNR.

# Phase-locked Loop Detection

In the first days after a strong earthquake, the vibrations of ${}_2S_3$ can be identified clearly and with good signal-to-noise ratio. But despite all efforts, no meaningful results could be achieved with Fourier analysis or homodyne detection. The strong influence of program parameters (such as bandwidth) could not be satisfactorily reduced and the consistency of the results of adjacent stations was insufficient. These problems are most likely caused by the fact that both methods have been designed to analyze a *constant* frequency and are disturbed by the rapid phase change by about 90 degrees that is recorded by many SG stations 17 hours after the earthquake. Therefore, a PLL program was developed to track the oscillation phase precisely. The PLL consists of three standard blocks of digital data processing: A VCO, a phase frequency discriminator and a PID controller.

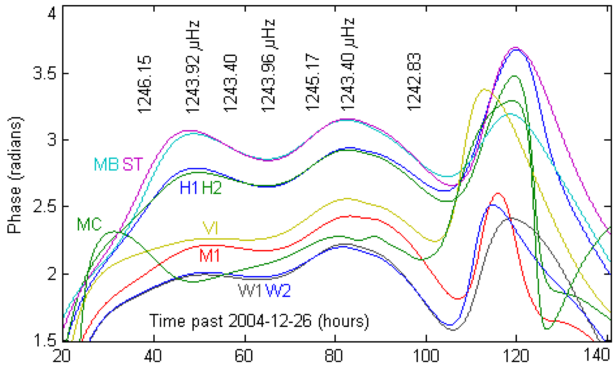
Normally, the VCO starts with the initial frequency 1244 µHz and zero phase. Then, with every zero crossing of the filtered signal, the phase of the VCO is adjusted to minimize the deviation from the measured phase. The careful choice of controller parameters ensures that the phase error is always less than 0.05 radians and all phase errors fall below the noise level within 30 time steps. Frequency deviations are inherently excluded. The calculated phase is very reliable because it does neither depend on the amplitude of the signal nor on the values of the three controller coefficients.

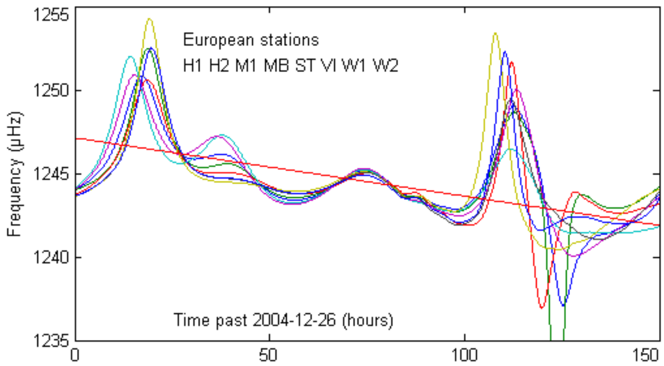



The comparison of the results of 18 stations shows several interesting relations:

- During the first 150 hours after the earthquake, the phases of seven Central European stations are virtually identical (Details are discussed below). MC seems to lie at the edge of the area, ME and NY must be excluded. Towards the end of the recordings, the signal-to-noise ratio deteriorates, whereby strong phase changes occur.
- The measured phases of all Central European stations jump twice and synchronously by about 90 degrees. Between these two events at an interval of 91 hours, the phases are nearly constant. The almost constant phase in this area shows the low frequency deviation from the initial value 1244 µHz.
- The timing of the second phase jump about 120 hours after the earthquake correlates well with the striking amplitude minimum of all European stations.
- At some points of the earth's surface (ES and NY), the phase changes approximately proportional to time. The resulting frequency offset from the reference value 1244 µHz is calculated below.

# Central European recordings

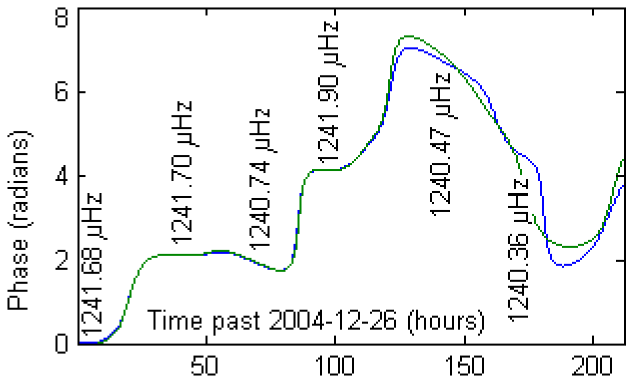



Differentiating the phase information yields the instantaneous frequency deviation $f = \frac{1}{2\pi}\frac{d\varphi}{dt}$ with respect to the initial VCO reference 1244 µHz. The frequency of ${}_2S_3$ is not constant and the slow frequency drift is described by the formula $f_{2S3} = 1247.1\,\mu Hz - 0.035\,\mu Hz \frac{t}{hours}$ . The superimposed short-term and very strong frequency modulation is the cause that the usual methods FFT and direct conversion fail and provide no reasonable results in spite of good SNR.

It is striking that the relatively widely spaced stations MB and ST recorded almost identical phases. This feature may be a key to reconstruct the nodal lines of ${}_2S_3$ .

# Special Case S1 S2

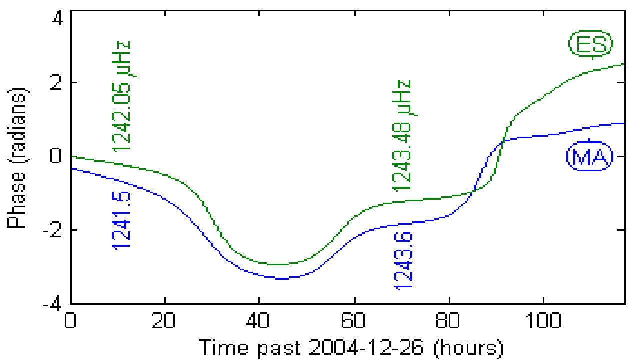



During the first 130 hours after the earthquake, the phases of the two South African stations S1 and S2 are almost indistinguishable because the two spheres are located in one single instrument.

Starting with the VCO reference 1244 µHz, the slope of the phase curve (not shown) is always negative. That means that the average frequency is significantly lower than that of the Central European stations. After reducing the reference frequency to an estimated value of 1241.8 µHz and restarting the PLL, the phase is constant in several segments. The smaller distance to the strong ${}_0S_7$ lines requires a reduction of bandwidth (13 µHz).

Apparently there are three separate time periods in which both receivers spheres of this station register the frequency 1241.75 µHz. These areas are separated by very rapid phase changes of 2.05 radians each. The SNR deteriorates 160 hours after the earthquake so that the following data become inaccurate.

# Special Case MA - ES

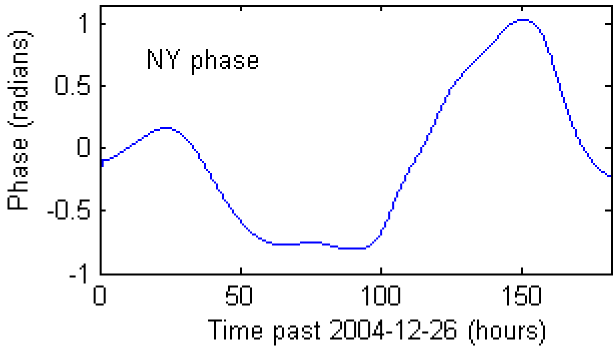



Similar to the European stations, the distance between the adjacent Japanese stations MA and ES causes an approximately constant phase difference. The frequency drift is positive

$$\Delta f \approx +0.030\,\mu Hz \frac{t}{hours}$$

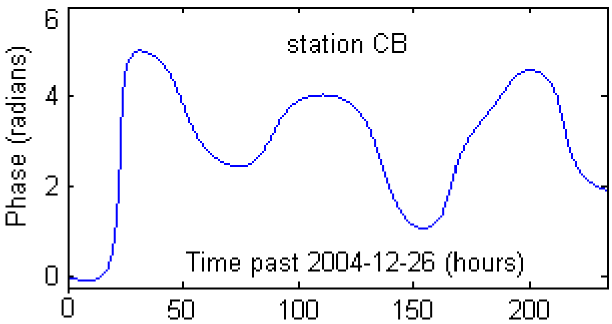
## Special Case NY

The data from NY differ in two respects from those of all other stations. The amplitude is much lower and the phase curve shows no rapid changes. The profile shown in the image is caused primarily by the low signal-to-noise ratio. NY is the only station that allows a precise frequency determination over the whole range with acceptable SNR. The jackknife method yields $f_{2S3}$(NY) = 1245.772 ± 0.013 μHz. This is the highest frequency in the whole set of recordings measured by 18 SG stations.




## Special Case CB

The signal received by the station CB signal shows a nearly regular phase modulation with the period T ≈ 85 hours around the mean frequency 1242 μHz. A time-dependent value can not be determined because the rapid phase changes lead to unrealistically large frequency excursions. This reason also prevents a reliable frequency determination with the other two methods FFT and direct conversion.




Since the frequency spacing to ${}_0S_7$ is very small, the filter bandwidth was reduced to 12.4 μHz in order to avoid interference. Any further reduction erases the phase modulation. Remarkably, the normal mode ${}_3S_2$ is modulated with the same rhythm[2].

Seventeen hours after the earthquake, there is a very abrupt phase change by Δφ = 5 radians, nearly anti-phase. Because simultaneously a lesser phase increase is observed in Central Europe and South Africa, a common cause must be suspected.

## Summary

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[3]. Almost all the calculations were performed using Matlab. The programs used therein may be requested from the author.

[1] H. Weidner, A New method for High-resolution Frequency Measurements,
http://viXra.org/abs/1506.0005
[2] H. Weidner, Time dependent analysis of the 3S2 Normal Mode, http://viXra.org/abs/1603.0324
[3] The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html