Short Note n°1: Number Pi

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abstract

In this note we show some formulas related with: Number Pi

Fórmula Para Pi

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Resumen-Abstract

En esta nota mostramos una fórmula para la constante Pi:

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right) = 3.141592\ldots$$

Introducción

Tres números reales: s, c, r , definidos como sigue:

(1) $$s = \frac{1}{3}\left(-1 + \frac{4\cdot 2^{2/3}}{\left(5+3i\sqrt{111}\right)^{1/3}} + \frac{\left(5+3i\sqrt{111}\right)^{1/3}}{2^{2/3}}\right)$$

(2) $$c = \sqrt{-\frac{\left(1+i\sqrt{3}\right)\left(-9+i\sqrt{111}\right)^{1/3}}{4\cdot 3^{2/3}} - \frac{1-i\sqrt{3}}{\left(3\left(-9+i\sqrt{111}\right)\right)^{1/3}}}$$

(3) $$r = \sqrt{\frac{5}{3} - \frac{7\cdot 2^{2/3}\left(1-i\sqrt{3}\right)}{3\left(67+3i\sqrt{111}\right)^{1/3}} - \frac{\left(1+i\sqrt{3}\right)\left(67+3i\sqrt{111}\right)^{1/3}}{3\cdot 2^{2/3}}}$$

Los valores aproximados de s, c, r , son:

(4) $$\begin{cases} s = 0.854637679718461\ldots \\ c = 0.519224841860869\ldots \\ r = 0.607537970981942\ldots \end{cases}$$

Los números s, c, r , satisfacen las siguientes ecuaciones polinomiales:

(5) $$2s^3 + 2s^2 - 2s + 1 = 0$$

(6) $$4c^6 - 4c^2 + 1 = 0$$

(7) $$r^6 - 5r^4 - r^2 + 1 = 0$$

Algunas fórmulas interesantes son:

(8) $$s = -\frac{1}{3} + \sqrt[3]{\frac{5}{54} + \frac{4}{3}\sqrt[3]{\frac{5}{54} + \frac{4}{3}\sqrt[3]{\frac{5}{54} + \cdots}}}$$

(9) $$c = \sqrt{\frac{1}{4} + \left(\frac{1}{4} + \left(\frac{1}{4} + \cdots\right)^3\right)^3}$$

(10) $$r = \sqrt{\frac{5-2\sqrt{7}}{3} + \sqrt{\frac{4(28\sqrt{7}-67)}{54\sqrt{7}-27\sqrt{\frac{4(28\sqrt{7}-67)}{54\sqrt{7}-27\sqrt{\frac{4(28\sqrt{7}-67)}{54\sqrt{7}-\cdots}}}}}}}$$

(11) $$s^2 + c^2 = 1$$

(12) $$r\,s - c = 0$$

Fórmula Para Pi

(14)

$$\pi = 8\sum_{n=0}^{\infty}\frac{1}{2n+1}r^{2n+1}\cos\left((2n+1)\cot^{-1}r\right) =$$

$$8\sum_{n=0}^{\infty}\frac{1}{2n+1}\left(\frac{c}{s}\right)^{2n+1}\sum_{k=0}^{n}(-1)^k\binom{2n+1}{2k}c^{2n-2k+1}s^{2k} =$$

$$8\sum_{n=0}^{\infty}\frac{1}{2n+1}\frac{(1-s^2)^{n+1}}{s^{2n+1}}\sum_{k=0}^{n}(-1)^k\binom{2n+1}{2k}(1-s^2)^{n-k}s^{2k} =$$

$$8\sum_{n=0}^{\infty}\frac{1}{2n+1}\left(\frac{c}{\sqrt{1-c^2}}\right)^{2n+1}\sum_{k=0}^{n}(-1)^k\binom{2n+1}{2k}c^{2n-2k+1}(1-c^2)^k$$

Referencias

[1] Valdebenito, E.:, Pi Handbook , manuscript , unpublished , 1989 , (20000 formulas).