Primes obtained concatenating to the right with 1 the partial sums of repdigits

Abstract. In this paper I state the following conjecture: For any digit from 1 to 9 there exist a sequence with an infinity of prime terms obtained concatenating to the right with 1 the partial sums of the repdigits. Examples: for repunit numbers 1, 11, 111 (...), concatenating the sum S(3) = 1 + 11 + 111 = 123 to the right with 1 is obtained 1231, prime; for repdigit numbers 3, 33, 333, 3333 (...), concatenating the sum S(4) = 3 + 33 + 333 + 3333 = 3702 to the right with 1 is obtained 37021, prime.

Conjecture:

For any digit from 1 to 9 there exist a sequence with an infinity of prime terms p obtained concatenating to the right with 1 the partial sums of the repdigits. Examples: for repunit numbers 1, 11, 111 (...), concatenating the sum S(3) = 1 + 11 + 111 = 123 to the right with 1 is obtained 1231, prime; for repdigit numbers 3, 33, 333, 3333 (...), concatenating the sum S(4) = 3 + 33 + 333 + 3333 = 3702 to the right with 1 is obtained 37021, prime.

Primes p for the sums of the numbers (10^n − 1)/9:
(see sequence A014824 in OEIS for the partial sums of repunits)

:   11, 1231, 1234567891, 123456790123441, 12345679012345661, 1234567901234567901234567901201 (...)

Primes p for the sums of the numbers 2*(10^n − 1)/9:
(see sequence A099669 in OEIS for the partial sums of these repdigits)

:   241, 2469135802469101 (...)

Primes p for the sums of the numbers 3*(10^n − 1)/9:
(see sequence A099670 in OEIS for the partial sums of these repdigits)

:   31, 3691, 37021, 370370370370321, 3703703703703651, 370370370370370311, 3703703703703703641 (...)

Primes p for the sums of the numbers 4*(10^n − 1)/9:
(see sequence A099671 in OEIS for the partial sums of these repdigits)

:   41, 4938271561, 49382716001, 493827160441 (...)

Primes p for the sums of the numbers 5*(10^n − 1)/9:
(see sequence A099672 in OEIS for the partial sums of these repdigits)

:   601, 6151, 6172801 (...)

Primes p for the sums of the numbers 6*(10^n − 1)/9:
(see sequence A099673 in OEIS for the partial sums of these repdigits)

:   61 (...)

Primes p for the sums of the numbers 7*(10^n − 1)/9:
(see sequence A099674 in OEIS for the partial sums of these repdigits)

:   71, 86381, 864151, 86419691 (...)

Primes p for the sums of the numbers 8*(10^n − 1)/9:
(see sequence A099675 in OEIS for the partial sums of these repdigits)

:   6172801 (...)

Primes p for the sums of the numbers 9*(10^n − 1)/9:
(see sequence A099676 in OEIS for the partial sums of these repdigits)

:   11071, 111111111110971, 111111111111110941, 1111111111111111110881, 1111111111111111111110871 (...)