Bell Inequalities ?

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Dedicated to Marie-Louise Nykamp

Abstract

Recently in [3] it was shown that the so called Bell Inequalities are *irrelevant* in physics, to the extent that they are in fact *not* violated either by classical, or by quantum systems. This, as well known, is contrary to the claim of John Bell that the mentioned inequalities *would be* violated in certain quantum contexts. The relevant point to note in [3] in this regard is that Bell's mentioned claim, quite of a wider acceptance among quantum physicists, is due to a most simple, elementary and trivial *mistake* in handling some of the involved statistical data. A brief presentation, simplified perhaps to the maximum that still presents the essence of that mistake, can be found in [10], see also [9]. The present paper tries to help in finding a way to the understanding of the above by quantum physicists, an understanding which, typically, is obstructed by an instant and immense amount and variety of "physical intuitions" with their mix of "physics + philosophy" considerations which - as an unstoppable avalanche - ends up making a hopeless situation from one which, on occasion, may in fact be quite simple and clear, as shown in [3] to actually happen also with

the Bell Inequalities story. The timeliness of such an attempt here, needless to say not the first regarding the Bell Inequalities story, is again brought to the fore due to the no less than *three* most freshly claimed to be fundamental contributions to the Bell Inequalities story, [4,5,13], described and commented upon in some detail in [6].

1. The Bell Inequalities an unintended plagiarism of already longer known Pure Mathematics results ?

In [7], back in 1989, it was clearly and with abundant detail shown that the so called, and much celebrated Bell Inequalities happen to belong to a wider class of probabilistic type inequalities first established by George Boole and later considered and studied by a number of mathematicians. All that happened prior to and/or independently of quanta. In fact, the respective studies mentioned in [7] are of a *purely* mathematical nature, thus *without* absolutely any physical type considerations, be they classical or quantum. Indeed, Boole initiated all such inequalities in the Appendix of his much celebrated 1854 book "The Laws of Thought". Following it and considerably extending it, a purely mathematical study of a wide class of inequalities was presented in the work of a number of mathematicians and probabilists, see relevant details in [7].
Seemingly, John Bell, more than a whole century after Boole, and back in 1964, was not aware of any of the above when he worked out the inequalities associated nowadays with his name. But then, nowadays, such an oversight is simply, and not seldom, the rather innocent effect of the widely and wildly raging ultra-specialization in sciences ...

2. The 2011 Hans de Raedt, et. al., paper [3]

The paper [3] shows, among others, *two* facts which hardly anyone seems to know among quantum physicists, namely :

A) The Bell Inequalities are of a *purely* mathematical nature, involving nothing more than usual logic and a rather elementary algebra. In this regard, the relevant results of George Boole are also mentioned.

B) The conclusion of John Bell that certain quantum contexts violate the so called Bell Inequalities is an effect of a simple and trivial *error* in the statistical manipulation of data. Once that error is avoided, that claimed violation is no longer supported.

It follows therefore that, so far, one has *not* obtained any support for the Bell Inequalities being violated either by classical, or by quantum systems.

Therefore, in this regard, the Bell Inequalities are so far *irrelevant* in physics.

3. Fighting a conveniently set up "straw man" ...

Amusingly, in recent foundational debates regarding quanta one of the conflicting views got labeled as "Bell was wrong", see for instance subsection 2.8 in [6].

However, as follows from the above, this labeling is but an erroneous setup in which a mere "straw man" is created in order to be quite easily attacked, and hopefully also demolished ...

Indeed, as far as one can know in present mathematics and quanta, John Bell was *not* wrong either with the inequalities associated with him nowadays, or with the quantum systems he considered.
Instead, what *was* wrong was his particular statistical manipulation in which he made a simple and trivial error, an error which ever since is *endlessly* repeated by just about everybody involved in the issue ...

In this way, as far as known nowadays, the error of John Bell simply *cannot* be found in the mathematics of the Bell Inequalities, or in the behaviour of entangled quanta !

No, not at all, since that error is in the simple and trivial manipulation of statistical data, as brought to light in [3].

Consequently, even those who claim - including John Bell himself - that the Bell Inequalities were violated, are simply talking nonsense, since all that happened was that the respective claimed violation was obtained *based* alone on a simple and trivial *error* in the statistical handling of data. Thus the thing which is clearly *violated* and keeps being violated, are a few of the simple and trivial rules of statistics regarding finite sets of data.

That is, therefore, the *only* meaning so far of that label of "Bell was wrong" !

4. Some further clarifications for the possible benefit of physicists ...

Let us stop for a short while, step back even if one single step only, and try and have a brief deeper look into logic, mathematics and physics, all of which are of course essentially involved in issues such as the alleged violation of the Bell Inequalities ...
A more general such an approach regarding logic, mathematics and physics can be found in [11,12] which, however, is not sine qua non in this paper.

a) As of today, it is not known whether usual mathematics, such as for instance that which alone is involved in establishing the mentioned Boole type inequalities, thus the Bell Inequalities as well, is contradiction free. On the other hand, the chances that it is so are not at all negligible in view of the fact that, after many millennia, no contradiction was ever found in mathematics.

b) If in physics, chemistry, or some other realm outside of mathematics, one finds a valid statement, say, S which contradicts a certain valid statement, say, M in mathematics, then the very same statement S will contradict *all* possible other valid statements M* in mathematics. Indeed, this fact is an immediate logical consequence of the assumption - accepted so far - that mathematics is contradiction free, see Appendix 1. Consequently, if certain quantum considerations do violate the Bell Inequalities, then the very same quantum consideration *must* also vio-

late *all* other valid mathematical statements, in particular, they must violate all such equations like $0 = 0$, or $1 = 1$, and so on ...
Therefore, there is *no* point in endlessly insisting that the Bell Inequalities may have some most privileged and rather unique status, being so far the *only* purely mathematical statements which - allegedly - are violated by quanta.

c) In case in some realm, say, R outside of mathematics one may eventually find a valid statement which happens to contradict a valid mathematical statement, then one or more of the following situations could be possible :

c1) usual logic may be inadequate for mathematics and/or for that realm R,

c2) mathematics and/or that realm R may be contradictory,

c3) the given mathematical modelling of that realm R may be unsatisfactory.

d) In case mathematics and/or quantum theory would happen to be contradictory in themselves, a way out is mentioned in Appendix 2.

5. A ... naive ... question to quantum physicists ...

In view of b) in section 4 above, the following holds :

The Bell Inequalities are violated by quanta $\iff$

$\iff$ The equation $0 = 0$ is violated by quanta

So then, may I kindly ask :

> Why do quantum physicists not keep making noise about
> the equation $0 = 0$ being violated by the quanta ?

After all, it would make quite a few things involved sound so much more simple ...

And who knows, it may possibly make them also so much more clear ...

Appendix 1

We start by as simple a sketch of a proof as possible of the statement in b) in section 4. This proof is of a purely logical nature, and thus of a far more general scope than the issues related to the Bell Inequalities. The general background is the so called *Predicate Calculus* in Mathematical Logic. As is well known, usual *axiomatic mathematical theories* are formulated within that background.

We are interested in the following facts in Predicate Calculus.

Given two well formed formulas, or in short, wff-s, p and q, then $p \Longrightarrow q$ is also a wff. Furthermore, $p \Longrightarrow q$ is *true*, if and only if the wff

(*1) $\qquad ((non\, p) \text{ or } q)$

is *true*.

Let us suppose that

(*2) $\qquad s \Longrightarrow (non\, b)$

is *true*, for certain given wff-s s and b. Then in view of (*1), it follows that $((non\, s) \text{ or } (non\, b))$ is also *true*, since in general, $non\,(non\, p)$ and p are simultaneously *true*, for every wff p.

Let now $\mathcal{T}$ be the set of all *true* wff-s. Then for every wff-s x and y, we have

(*3) $\qquad (x \in \mathcal{T},\ x \Longrightarrow y) \quad \Longrightarrow \quad y \in \mathcal{T}$

is *true*. Furthermore, let us suppose that in (*2) we have

(*4) $\qquad s, b \in \mathcal{T}$

Then we show that the wff

(*5) $\qquad \forall\ a \in \mathcal{T}\ :\ s \Longrightarrow (non\, a)$

is *true*.

Indeed, assume that the wff in (*5) is not true, then

$$\exists\ a_0 \in \mathcal{T}\ :\ non\,(s \Longrightarrow (non\, a_0))$$

hence (*1) implies the *true* wff

$$\exists\ a_0 \in \mathcal{T}\ :\ s \text{ and } a_0$$

It follows that b, a_0, $s \in \mathcal{T}$, and then (*2), (*3) give the contradiction that b, $non\, b \in \mathcal{T}$.

And now, a more detailed argument.

Let us start by noticing the often missed fact that Axiomatic Mathematical Theories as mere Models !
And this should, among others, be made clear to physicists as well, since the axiomatizations of various branches of physics has had - and does have also today - a strong tendency, at least since Newton's "Principia Mathematica".

Indeed, ever since Euclid axiomatized Geometry more than two millennia ago, there has been a widespread and strong tacit tendency, and not only among mathematicians, to identify the respective specific, particular axiomatic theory with Geometry as such. No lesser a philosopher than Immanuel Kant (1724-1804) considered the resulting Euclidean Geometry to be the *only* possible one in the whole Creation. Therefore the shock in the early 1800s when non-Euclidean geometries have been discovered.

A similar phenomenon happened more than a century back with the

Peano Axioms of the natural numbers which were supposed to express absolutely all the relevant properties of such numbers. And yet, we had to face the related shock of the Gödel Incompleteness Theorem which, in the early 1930s, came as a total surprise.

The moral, of course, is that, on one hand, we may have a concept like "geometry" or "number", for instance, while on the other hand, we can have one or another specific, particular axiomatic mathematical theory which aims to describe such a most abstract concept. Thus here we deal with a rather inevitable divide

Abstract Concept .... *Axiomatic Theory*

and the gap between these two sides may be hard to bridge, let alone eliminate. In conclusion, we have to note that

- Each *Axiomatic Theory* can only be a particular, specific *MODEL* of a given *Abstract Concept*

Let us now have a brief look at what is in fact an *axiomatic mathematical theory*, or more generally, an *axiomatic theory*.

One starts such an axiomatic theory with setting up a *formal deductive system*. Namely, let A be an *alphabet* which can be given by any nonvoid finite or infinite set. Then a procedure is given according to which one can in a nite number of steps effectively construct - by using the symbols in A - a set $\mathcal{F}$ of *well formed formulas*, or in short, wff-s.

Next, one chooses a nonvoid set $\mathcal{R}$ of *logical deduction rules* which operate as follows

(A1.1) $$\mathcal{F} \supseteq P \xrightarrow{\mathcal{R}} Q \subseteq \mathcal{F}$$

that is, from any set P of wff-s which are the *premises*, it leads to a corresponding set Q of wff-s which are all the *logical consequences* of P.

Further, it will be convenient to assume that, for every set of well formed formulas $P \subseteq \mathcal{F}$, we have

(A1.2) $\qquad P \subseteq \mathcal{R}(P) = \mathcal{R}(\mathcal{R}(P))$

in other words, the premises P themselves are supposed to be among the logical consequences $\mathcal{R}(P)$, and in addition, these logical consequences $\mathcal{R}(P)$ of P contain all the possible logical consequences of P, thus the iteration of the logical deduction rules $\mathcal{R}$ does not produce further logical consequences of P. Clearly, condition (A1.2) does not lead to a loss of generality regarding $\mathcal{R}$ in (A1.1). Indeed, if the relation

$$\forall \, P \subseteq \mathcal{F} \; : \; P \subseteq \mathcal{R}(P)$$

is not satisfied, then this relation will obviously be satisfied by the modification of $\mathcal{R}$ given by $\mathcal{R}^*(P) = P \bigcup \mathcal{R}(P)$. Also, if the relation

$$\forall \, P \subseteq \mathcal{F} \; : \; \mathcal{R}(\mathcal{R}(P)) = \mathcal{R}(P)$$

is not satisfied, then this relation will obviously be satisfied by the modification of $\mathcal{R}$ given by

$$\mathcal{R}^{**}(P) = \mathcal{R}(P) \bigcup \mathcal{R}(\mathcal{R}(P)) \bigcup \mathcal{R}(\mathcal{R}(\mathcal{R}(P))) \bigcup \ldots$$

And now at last come the *axioms* which can be any nonvoid subset $\mathcal{A} \subseteq \mathcal{F}$ of wff-s.

With the above established, the respective *axiomatic theory* follows easily as being the *smallest* subset $\mathcal{T} \subseteq \mathcal{F}$ with the properties

(A1.3) $\qquad \mathcal{A} \subseteq \mathcal{T}$

(A1.4) $\qquad \mathcal{T} \supseteq P \stackrel{\mathcal{R}}{\longrightarrow} Q \subseteq \mathcal{T}$

in which case the wff-s in $\mathcal{T}$ are called the *theorems* of the given *axiomatic theory* defined by the *axioms* $\mathcal{A}$.

In view of (A1.3), clearly, all axioms in $\mathcal{A}$ are also theorems.

Now an essential fact is that the set $\mathcal{T}$ of theorems depends not only on the axioms in $\mathcal{A}$, but also on the logical deduction rules $\mathcal{R}$, and prior to that, on the set $\mathcal{F}$ of well formed formulas. Consequently, it is appropriate to write

(A1.5) $\quad \mathcal{T}_{\mathcal{F},\mathcal{R}}(\mathcal{A})$ or more simply $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$

for the set $\mathcal{T}$ of theorems.

Here are some of the relevant questions which can arise regarding such axiomatic systems :

- are the axioms in $\mathcal{A}$ independent of one another ?
- are the axioms in $\mathcal{A}$ consistent, that is, do they not conflict with one another ?
- are the axioms in $\mathcal{A}$ complete ?

Independence means that for no axiom $\alpha \in \mathcal{A}$, do we have $\mathcal{T}_{\mathcal{R}}(\mathcal{A}) = \mathcal{T}_{\mathcal{R}}(\mathcal{B})$, where $\mathcal{B} = \mathcal{A} \setminus \{\alpha\}$. In other words, the axioms in $\mathcal{A}$ are minimal in order to obtain the theorems in $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$. This condition can be formulated equivalently, but more simply and sharply, by saying that for no axiom $\alpha \in \mathcal{A}$, do we have $\alpha \in \mathcal{T}_{\mathcal{R}}(\mathcal{B})$, where $\mathcal{B} = \mathcal{A} \setminus \{\alpha\}$.

As for consistency, it means that there is no theorem $\tau \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$, such that for its negation $non\,\tau$, we have $non\,\tau \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$.

Completeness, in one of its possible formulations, means that, given any additional axiom $\beta \in \mathcal{F} \setminus \mathcal{A}$ which is independent from $\mathcal{A}$, that is, for which $\beta \notin \mathcal{T}_{\mathcal{R}}(\mathcal{A})$, then this extended set of axioms $\mathcal{B} = \mathcal{A} \bigcup \{\beta\}$ is inconsistent.

It is obvious, therefore, that in setting up axiomatic systems, there is a lot of freedom in choosing the alphabet A, the well formed formulas $\mathcal{F}$, the logical deduction rules $\mathcal{R}$ and the axioms $\mathcal{A}$, all of which influence the resulting theorems $\mathcal{T}$. However, such a freedom is not necessarily a complete blessing when it comes to express all the possible relevant properties of such abstract concepts as “geometry”, “numbers”, and so on. Indeed, each particular such choice may not only miss on certain relevant properties, but may actually introduce some strange and

unintended ones as well.
In this regard, in modern times, it was the philosophy of neo-positivism, or the so called third positivism, which in the early 20th century brought to attention the fact that the very structure of language can significantly influence thinking and the results of thought, and in particular, can lead to pseudo-problems. Not much later, in linguistics, a similar idea arose with the Sapir-Whorf Hypothesis about the relativity of language.

A corresponding recognition in mathematics, as mentioned, started to emerge in the early 1800s, even if tentatively, with the non-Euclidean geometries, and was later confirmed by further modern developments of various axiomatic mathematical theories, a most important moment in this regard being Gödel's Incompleteness Theorem, in the early 1930s.

In conclusion, it is appropriate to realize even if in the day to day activity of the so called "working mathematicians" and "working physicists" it may still be disregarded that an axiomatic theory most likely fails to express all of the properties of the domain of mathematics or physics which it is supposed to model, and in fact, may even introduce inappropriate properties.

And now, based on the above, we can give a simple proof of the fact presented in b) in section 4 above.

Let us consider the usual mathematical theory - actually but not necessarily reducible to elementary algebra - which is involved in the considerations related to the Bell Inequalities. Then we have a corresponding alphabet A, set $\mathcal{F}$ of well formed formulas, logical deduction rules $\mathcal{R}$, and lastly, set of axioms $\mathcal{A}$.
All of these will result in the corresponding set $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$ of mathematical theorems.
And obviously, one of these theorems is precisely the statement of the Bell Inequalities, a statement which, for convenience we shall denote by

(A1.6) $\qquad \beta\iota \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

Let us now consider a formulation of usual quantum theory which allows the mathematical modelling of the quantum contexts that - allegedly - lead to the violation of the Bell Inequalities. Then the mentioned mathematical modelling corresponds to a set $\mathcal{S}$ of well formed formulas which is a subset of $\mathcal{F}$.

In the above framework, the alleged violation of the Bell Inequalities means the following :

There exists a well formed formula $\sigma \in \mathcal{S}$, such that

(A1.7) $\quad \sigma \implies non\, \beta\iota$

Then we show that we have

(A1.8) $\quad \forall\ \alpha \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})\ :\ \sigma \implies non\, \alpha$

Indeed, we recall that for every $p, q \in \mathcal{F}$, we have

(A1.9) $\quad (p \implies q) \in \mathcal{T}_{\mathcal{R}}(\mathcal{A}) \iff ((non\, p) \text{ or } q) \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

hence

(A1.10) $\quad non\,(p \implies q) \in \mathcal{T}_{\mathcal{R}}(\mathcal{A}) \iff (p \text{ and } (non\, q)) \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

Let us now assume that (A1.8) does not hold, thus we have

(A1.11) $\quad non\,(\sigma \implies non\, \alpha_0) \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

for a certain $\alpha_0 \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$. But then the relation (A1.10) gives

(A1.12) $\quad \sigma,\ \alpha_0 \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

which means that, in particular, we have

(A1.13) $\quad \sigma \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$

and we obtained a contradiction, since we have now that $\beta\iota$, α_0, $\sigma \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$, and then, in view of (A1.7), $\mathcal{T}_{\mathcal{R}}(\mathcal{A})$ is contradictory, in view of the following property shared by all axiomatic sets of theorems

$$\text{(A1.14)} \qquad (\xi \implies \chi) \implies \chi \in \mathcal{T}_{\mathcal{R}}(\mathcal{A})$$

for all $\xi \in \mathcal{T}_{\mathcal{R}}(\mathcal{A}), \ \chi \in \mathcal{F}$.

Appendix 2

It is remarkable that due to practical requirements in theoretical computer science, there have for more than two decades by now been developments in dealing with and making use of *inconsistent* axiomatic theories. Details in this regard can be found in [11,12].

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