Divergence-Free Non-Linear Scalar Model

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Abstract

We present results of applying our divergence-free effective action quantum field theory techniques to a scalar model with nonlinear interactions governed by a dimensional coupling constant. This gives an example of the applicability of our divergence-free methods, and the viability of theories that are often disregarded due to the outstanding problem of nonrenormalizable divergences. Our results demonstrate that the (Goldstone) scalar would remain massless in the effective quantum action, while the original vertices, governed by nonlinear invariance, would preserve their form.

1 Introduction

The divergence-free effective action approach to quantum field theory,[1, 2, 3, 4, 5] manages to evade the loop divegences, not only in traditionally acceptable (renormalizable) theories like quantum electrodynamics, but also in theories with dimensional couplings, like quantum gravity. In this article we give another example of the applicability of our methods to a scalar model with nonlinear interactions having a dimensional coupling constant, much in the same spirit like the gravity-like scalar model.[6]

After presenting the Lagrangian and the associated Feynman rules, we give the results of applying our divergence-free methods to several loop computations, concluding with a brief discussion of the results. Again, we shall suppress much of the detailed derivations, leaving them to comprehensive reports that are available elsewhere.[7]

2 The Model Lagrangian and Graphical Rules

The Lagrangian of our non-linear scalar model derives from the O_2 symmetric expression:

$$\mathcal{L} = \frac{1}{2}(\partial A)^2 + \frac{1}{2}(\partial B)^2 \tag{1}$$

with two scalar fields A and B. This is subjected to the constraint $A^2 + B^2 = 1/\kappa^2$, where κ has the dimension of length (or inverse mass). Using the constaint to eliminate

B, we obtain

$$\mathcal{L} = \frac{1}{2}(\partial A)^2 \left(\frac{1}{1 - \kappa^2 A^2} \right) = \frac{1}{2}(\partial A)^2 \left(1 + \kappa^2 A^2 + \kappa^4 A^4 + \cdots \right) \tag{2}$$

This is characterized by an infinite series of vertices of even order (quartic, hexic, etc.). The (Minkowskian) momentum-space Feynman graphic rules that can be associated with the above system are as follows:

- For every internal line or bare propagator with momentum p, we write

  $$\frac{1}{-p^2 + m^2}$$

  Notice that we have introduced the mass m as an infrared regulating parameter. This mass would only concern the virtual field, and we shall see whether our (Goldstone) scalar would remain massless in the effective action.

- For every symmetrized n-leg vertex (with $n = 4, 6, \cdots$), with respective ingoing momenta $p, q, r, \cdots$, we write

  $$-\frac{(n-2)!}{2} \kappa^{(n-2)} \left(p^2 + q^2 + r^2 + \cdots \right)$$

- We must associate a factor of i for each propagator, a factor of i for each vertex, and an overall factor of $-i$ for each graph.

- We must supply the appropriate combinatoric factors for each graph.

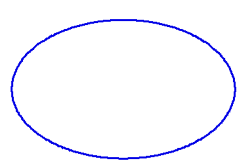
- Most importantly, we must supply the appropriate *regularizing parameters* and the corresponding *pole-removing operators*, together with the gamma functions factors, and Feynman parameter combinations, all according to our divergence-free methods.[1]

In the following section, we shall display associated graphics and computational results suppressing all details.

3 Vacuum Contributions

3.1 One-Loop Contribution

The 1-loop vacuum contribution corresponds to the following no-vertex graph:

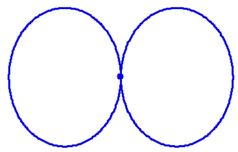


With the associated result

$$\frac{m^4}{128\pi^2}\left(3-2\ln(m^2)\right) \tag{3}$$

3.2 Two-Loop Contributions

For 2-loop contributions we have the one graph

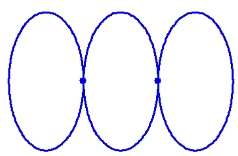


giving the result:

$$-\frac{m^6\kappa^2}{1024\pi^4}\left(-1+\ln(m^2)\right)\left(-3+2\ln(m^2)\right) \tag{4}$$

3.3 Three-Loop Contributions

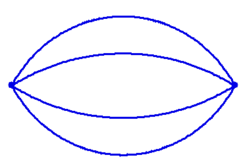
There are three graphs for the 3-loop vacuum contribution. The 1st graph



gives exactly

$$-\frac{m^8\kappa^4}{65536\pi^6}\left(3+45\ln(m^2)-78\ln^2(m^2)+32\ln^3(m^2)\right) \tag{5}$$

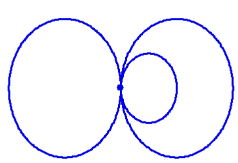
The 2nd graph



gives (approx.)

$$-\frac{m^8\kappa^4}{14863564800\pi^6}\left(-144203+3957528\ln(m^2)-6588360\ln^2(m^2)+2721600\ln^3(m^2)\right) \tag{6}$$

The 3rd graph



gives

$$\frac{m^8\kappa^4}{16384\pi^6}\left(-1+\ln(m^2)\right)^2\left(-3+2\ln(m^2)\right) \tag{7}$$

Adding the three, 3-loop vacuum contributions, we get

$$-\frac{m^8\kappa^4}{14863564800\pi^6}\left(8700997-7609272\ln(m^2)-5227560\ln^2(m^2)+4536000\ln^3(m^2)\right) \tag{8}$$

3.4 Fixing the Vacuum

Adding the 1-loop, the 2-loop, and the 3-loop vacuum contributions, we obtain

$$\begin{cases} \frac{m^4}{128\pi^2}\left\{3-2\ln(m^2)\right\} \\ -\frac{m^6\kappa^2}{1024\pi^4}\left\{-1+\ln(m^2)\right\}\left\{-3+2\ln(m^2)\right\} \\ -\frac{m^8\kappa^4}{14863564800\pi^6}\left\{8700997-7609272\ln(m^2)-5227560\ln^2(m^2)+4536000\ln^3(m^2)\right\} \end{cases} \tag{9}$$

The above series (to order κ^4) should be equated to zero and inverted to give an expression for $\ln(m^2)$. This procedure would eliminate $\ln(m^2)$ from the theory by fixing the arbitrary energy scale than governs the argument of the logarithm. We obtain, to order κ^4, the following result:

$$\ln(m^2)=\frac{3}{2}-\frac{834079m^4\kappa^4}{232243200\pi^4}+\cdots \tag{10}$$

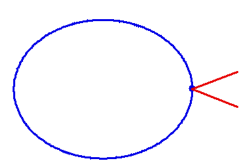
We note the absence of an order κ^2 term.

4 Bilinear Contributions

In this section, we present contributions to the bilinear kernel; corrections to the Lagrangian of second order in the scalar field. We compute 1-loop and 2-loop contributions.

4.1 One-Loop Contributions

We have the one graph



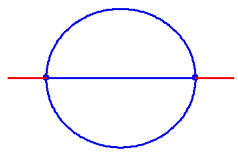
giving:

$$\frac{m^2\kappa^2}{32\pi^2}\left\{-1+\ln(m^2)\right\}r^2+\frac{m^4\kappa^2}{64\pi^2}\left\{-3+2\ln(m^2)\right\} \tag{11}$$

Here r is the external momentum carried by the effective field. Notice that in our graphics, internal lines are depicted in blue, external insertions in red.

4.2 Two-Loop Contributions

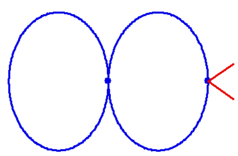
There are three graphs that give the 2-loop bilinear contributions. We shall give the results that correspond to these graphs, computing only to 2nd order in the external momentum. For the 1st graph,



we obtain (approx.)

$$\begin{aligned}&\tfrac{m^4\kappa^4}{135475200\pi^4}\{-148916-437535\ln(m^2)+529200\ln^2(m^2)\}r^2\\&+\tfrac{m^6\kappa^4}{77414400\pi^4}\{49973-815340\ln(m^2)+579600\ln^2(m^2)\}\end{aligned}\tag{12}$$

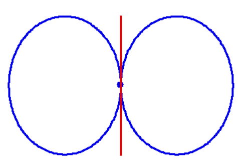
For the 2nd graph,



we obtain

$$\begin{aligned}&\tfrac{5m^6\kappa^4}{1024\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\}\\&+\tfrac{m^4\kappa^4}{1024\pi^4}\{-1+2\ln(m^2)\}\{-4+3\ln(m^2)\}r^2\end{aligned}\tag{13}$$

For the 3rd graph,



we obtain

$$\begin{aligned}&-\tfrac{3m^6\kappa^4}{512\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\}\\&-\tfrac{3m^4\kappa^4}{512\pi^4}\{-1+\ln(m^2)\}^2r^2\end{aligned}\tag{14}$$

Adding the above 3 contributions, we obtain for the total 2-loop bilinear correction:

$$\begin{aligned}&\tfrac{m^6\kappa^4}{77414400\pi^4}\{-176827-437340\ln(m^2)+428400\ln^2(m^2)\}\\&+\tfrac{m^4\kappa^4}{135475200\pi^4}\{-413516-305235\ln(m^2)+529200\ln^2(m^2)\}r^2\end{aligned} \tag{15}$$

4.3 Fixing the Bilinear

Adding the 1-loop and the 2-loop bilinear corrections, and replacing $\ln(m^2)$, to the needed order in κ, as obtained before (through fixing the vacuum to zero), $\ln(m^2)\to\frac{3}{2}$, we obtain for the bilinear correction, to order κ^4:

$$\frac{131063m^6\kappa^4}{77414400\pi^4}+\left(\frac{m^2\kappa^2}{64\pi^2}+\frac{638663m^4\kappa^4}{270950400\pi^4}\right)r^2 \tag{16}$$

We shall return to discuss some features of this result later on.

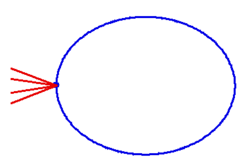
5 Quartilinear Contributions

In this section we give the results of 1-loop and 2-loop corrections to the effective quartilinear vertex.

5.1 One-Loop Contributions

In this case, we have two graphic contributions. We give the results to 2nd order in the external momenta r, s, t, u. Symmetrization in the latter will be done after adding the two contributions.

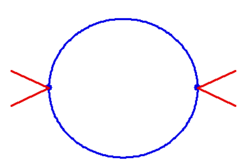
For the 1st graph,



we obtain

$$\begin{aligned}&\tfrac{m^4\kappa^4}{64\pi^2}\{-3+2\ln(m^2)\}\\&+\tfrac{m^2\kappa^4}{32\pi^2}\{-1+\ln(m^2)\}(r^2+s^2+t^2+r\cdot s+r\cdot t+s\cdot t)\end{aligned} \tag{17}$$

For the 2nd graph,



we obtain

$$
\begin{aligned}
&-\tfrac{3m^4\kappa^4}{128\pi^2}\{-3+2\ln(m^2)\}\\
&-\tfrac{m^2\kappa^4}{128\pi^2}\{-1+\ln(m^2)\}(5r^2+5s^2+4t^2+6r\cdot s+4r\cdot t+4s\cdot t)
\end{aligned}
\tag{18}
$$

Adding the above 2 contributions, and letting $\ln(m^2) \to 3/2$, which value is pertinent to the present order in κ, we obtain

$$
-\frac{m^2\kappa^4}{256\pi^2}(r^2+s^2+2r\cdot s) \tag{19}
$$

Now symmetrizing with respect to r, s, t, u and dividing by 4!, we obtain

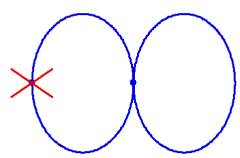
$$
-\frac{m^2\kappa^4}{768\pi^2}(r^2+s^2+t^2+u^2) \tag{20}
$$

Comparing this with the symmetrized quartilinear vertex of the original Lagrangian, we obtain the value $(m^2\kappa^2/32\pi^2)$ as the correcting 1-loop contribution. We shall discuss this result later on.

5.2 Two-Loop Contributions

In this case, we have seven graphic contributions. We give the results to 2nd order in the external momenta r, s, t, u. Symmetrization in the latter will be done after adding the two contributions.

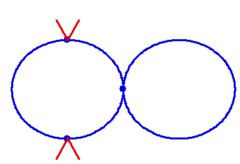
For the 1st graph,



we obtain

$$
\begin{aligned}
&\tfrac{5m^6\kappa^6}{1024\pi^2}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\}\\
&+\tfrac{m^4\kappa^6}{1024\pi^2}\{-1+2\ln(m^2)\}\{-4+3\ln(m^2)\}(r^2+s^2+t^2+r\cdot s+r\cdot t+s\cdot t)
\end{aligned}
\tag{21}
$$

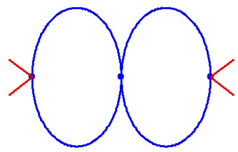
For the 2nd graph,



we obtain

$$
\begin{aligned}
&-\tfrac{9m^6\kappa^6}{1024\pi^2}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\}\\
&-\tfrac{m^4\kappa^6}{4096\pi^2}\{6-15\ln(m^2)+8\ln^2(m^2)\}(5r^2+5s^2+4t^2+6r\cdot s+4r\cdot t+4s\cdot t)
\end{aligned}
\tag{22}
$$

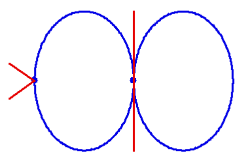
For the 3rd graph,



we obtain

$$-\tfrac{m^4\kappa^6}{512\pi^4}(9m^2+3r^2+3s^2+2t^2+4r\cdot s+2r\cdot t+2s\cdot t)$$
$$+\tfrac{m^4\kappa^6}{2048\pi^4}(60m^2+33r^2+33s^2+25t^2+41r\cdot s+25r\cdot t+25s\cdot t)\ln(m^2) \quad (23)$$
$$-\tfrac{m^4\kappa^6}{1024\pi^4}(12m^2+9r^2+9s^2+7t^2+11r\cdot s+7r\cdot t+7s\cdot t)\ln^2(m^2)$$

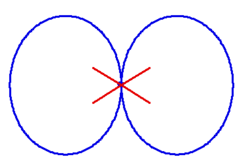
For the 4th graph,



we obtain

$$\tfrac{3m^4\kappa^6}{512\pi^4}(15m^2+5r^2+5s^2+4t^2+6r\cdot s+4r\cdot t+4s\cdot t)$$
$$-\tfrac{3m^4\kappa^6}{1024\pi^4}(50m^2+23r^2+23s^2+16t^2+24r\cdot s+16r\cdot t+16s\cdot t)\ln(m^2) \quad (24)$$
$$+\tfrac{3m^4\kappa^6}{256\pi^4}(5m^2+3r^2+3s^2+2t^2+3r\cdot s+2r\cdot t+2s\cdot t)\ln^2(m^2)$$

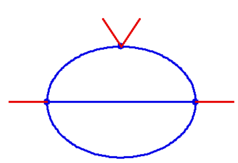
For the 5th graph,



we obtain

$$-\tfrac{15m^6\kappa^6}{1024\pi^2}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\}$$
$$-\tfrac{15m^4\kappa^6}{1024\pi^2}\{-1+\ln(m^2)\}^2(r^2+s^2+t^2+r\cdot s+r\cdot t+s\cdot t) \quad (25)$$

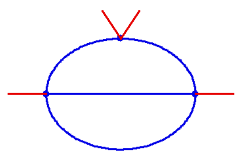
For the 6th graph,



we obtain

$$\frac{m^4\kappa^6}{180633600\pi^4}\begin{pmatrix} 3566024m^2+2582071r^2+3424074s^2+3424074t^2 \\ +2582071r\cdot s+2582071r\cdot t+3108321s\cdot t \end{pmatrix}$$
$$+\frac{m^4\kappa^6}{860160\pi^4}(29820m^2+4458r^2+4645s^2+4645t^2+4458r\cdot s+4458r\cdot t+6280s\cdot t)\ln(m^2) \tag{26}$$
$$-\frac{m^4\kappa^6}{1024\pi^4}(40m^2+17r^2+19s^2+19t^2+17r\cdot s+17r\cdot t+21s\cdot t)\ln^2(m^2)$$

For the 7th graph,



we obtain

$$\frac{m^4\kappa^6}{135475200\pi^4}\begin{pmatrix} 349811m^2-595664r\cdot r-468454s\cdot s-468454t\cdot t \\ -468454r\cdot s-468454r\cdot t-468454s\cdot t \end{pmatrix}$$
$$-\frac{m^4\kappa^6}{322560\pi^4}(13589m^2+4167r\cdot r+3830s\cdot s+3830t\cdot t+3830r\cdot s+3830r\cdot t+3830s\cdot t)\ln(m^2)$$
$$+\frac{m^4\kappa^6}{768\pi^4}(23m^2+12r\cdot r+10s\cdot s+10t\cdot t+10r\cdot s+10r\cdot t+10s\cdot t)\ln^2(m^2) \tag{27}$$

Adding the above 7 contributions, we obtain for the total 2-loop quartilinear contribution:

$$\frac{m^4\kappa^6}{541900800\pi^4}\begin{pmatrix} 20035316m^2+8274157r\cdot r+11309006s\cdot s+9986006t\cdot t \\ +10105997r\cdot s+7459997r\cdot t+9038747s\cdot t \end{pmatrix}$$
$$-\frac{m^4\kappa^6}{2580480\pi^4}\begin{pmatrix} 82252m^2+57132r\cdot r+53875s\cdot s+20485t\cdot t \\ +42466r\cdot s+21046r\cdot t+15580s\cdot t \end{pmatrix}\ln(m^2) \tag{28}$$
$$+\frac{m^4\kappa^6}{3072\pi^4}(2m^2+21r\cdot r+7s\cdot s-17t\cdot t+r\cdot s-11r\cdot t-23s\cdot t)\ln^2(m^2)$$

5.3 Fixing the Quartilinear

Adding the 1-loop and the 2-loop quartilinear contributions, and letting

$$\ln(m^2)\to\frac{3}{2}-\frac{834079m^4\kappa^4}{232243200\pi^4}$$

which value is pertinent to the present order in κ, then symmetrizing the momentum-dependent part with respect to r,s,t,u and dividing by 4!, we obtain

$$-\left(\frac{m^2\kappa^4}{768\pi^2}+\frac{1841299m^4\kappa^6}{1083801600\pi^4}\right)(r^2+s^2+t^2+u^2) \tag{29}$$

Comparing this with the symmetrized quartilinear vertex of the original Lagrangian, we obtain the value

$$\left(\frac{m^2\kappa^2}{32\pi^2}+\frac{638663m^4\kappa^4}{135475200\pi^4}\right)$$

as the correcting 1-loop contribution. We shall discuss this result later on.

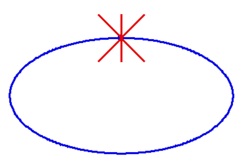
6 Hexilinear Contributions

In this section we give the results of 1-loop and two-loop corrections to the effective hexilinear vertex.

6.1 One-Loop Contributions

In this case, we have three graphic contributions. We give the results to 2nd order in the external momenta r, s, t, u, v, w. Symmetrization in the latter will be done after adding the three contributions.

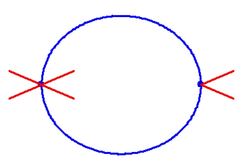
For the 1st graph,



we obtain

$$\begin{aligned}&\tfrac{m^4\kappa^6}{64\pi^2}\{-3+2\ln(m^2)\}\\&+\tfrac{m^2\kappa^6}{32\pi^2}\{-1+\ln(m^2)\}\begin{pmatrix}r^2+s^2+t^2+u^2+v^2+r\cdot s+r\cdot t+r\cdot u+r\cdot v\\ +s\cdot t+s\cdot u+s\cdot v+t\cdot u+t\cdot v+u\cdot v\end{pmatrix}\end{aligned} \tag{30}$$

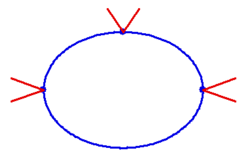
For the 2nd graph,



we obtain

$$\begin{aligned}&\tfrac{3m^4\kappa^6}{64\pi^2}\{-3+2\ln(m^2)\}\\&-\tfrac{m^2\kappa^6}{64\pi^2}\{-1+\ln(m^2)\}\begin{pmatrix}5r^2+5s^2+4t^2+4u^2+4v^2+6r\cdot s+4r\cdot t+4r\cdot u+4r\cdot v\\ +4s\cdot t+4s\cdot u+4s\cdot v+4t\cdot u+4t\cdot v+4u\cdot v\end{pmatrix}\end{aligned} \tag{31}$$

For the 3rd graph,



we obtain

$$\frac{m^4\kappa^6}{32\pi^2}\{-3+2\ln(m^2)\}$$
$$+\frac{m^2\kappa^6}{128\pi^2}\{-1+\ln(m^2)\}\left(\begin{array}{l}5r^2+5s^2+5t^2+5u^2+4v^2+6r\cdot s+5r\cdot t+5r\cdot u+4r\cdot v\\ +5s\cdot t+5s\cdot u+4s\cdot v+6t\cdot u+4t\cdot v+4u\cdot v\end{array}\right) \tag{32}$$

Adding the above three 1-loop contributions, using $\ln(m^2)\to\frac{3}{2}$, symmetrizing with respect to the six external momenta r,s,t,u,v,w, and dividing by 6!, we obtain

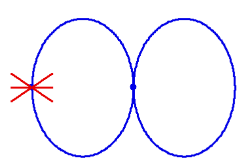
$$-\frac{m^2\kappa^6}{1920\pi^2}(r^2+s^2+t^2+u^2+v^2+w^2) \tag{33}$$

Hence, the one-loop correction to the hexilinear vertex is $(m^2\kappa^2/32\pi^2)$. We shall return later for a discussion of this result.

6.2 Two-Loop Contributions

In this case, we have 16 graphic contributions. For space economy, we give the results without external momenta. However, we shall return later to discuss the momentum-dependent results as well.

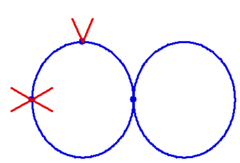
For the 1st graph,



we obtain

$$\frac{5m^6\kappa^8}{1024\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\} \tag{34}$$

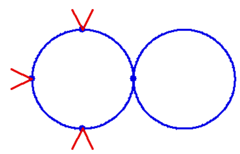
For the 2nd graph,



we obtain

$$\frac{9m^6\kappa^8}{512\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\} \tag{35}$$

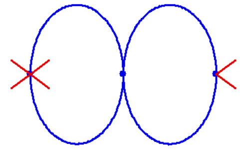
For the 3rd graph,



we obtain

$$\frac{7m^6\kappa^8}{512\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\} \tag{36}$$

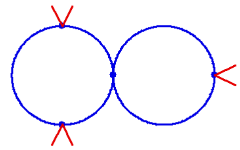
For the 4th graph,



we obtain

$$-\frac{3m^6\kappa^8}{256\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\} \tag{37}$$

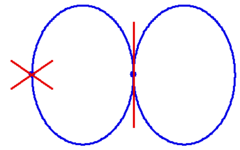
For the 5th graph,



we obtain

$$\frac{21m^6\kappa^8}{1024\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\} \tag{38}$$

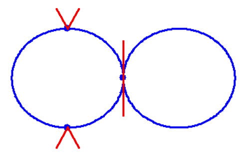
For the 6th graph,



we obtain

$$\frac{15m^6\kappa^8}{512\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\} \tag{39}$$

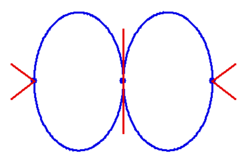
For the 7th graph,



we obtain

$$-\frac{27m^6\kappa^8}{512\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\} \tag{40}$$

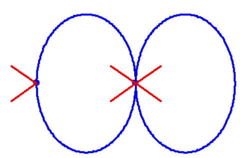
For the 8th graph,



we obtain

$$-\frac{9m^6\kappa^8}{256\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\} \tag{41}$$

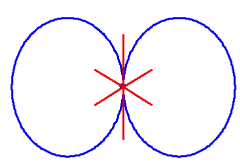
For the 9th graph,



we obtain

$$\frac{75m^6\kappa^8}{1024\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\} \tag{42}$$

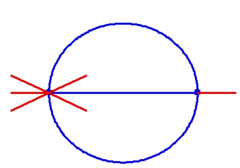
For the 10th graph,



we obtain

$$-\frac{7m^6\kappa^8}{256\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\} \tag{43}$$

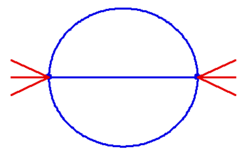
For the 11th graph,



we obtain

$$\frac{m^6\kappa^8}{12902400\pi^4}\{49973 - 815340\ln(m^2) + 579600\ln^2(m2)\} \tag{44}$$

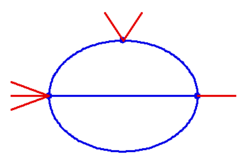
For the 12th graph,



we obtain

$$\frac{m^6\kappa^8}{19353600\pi^4}\{49973 - 815340\ln(m^2) + 579600\ln^2(m2)\} \tag{45}$$

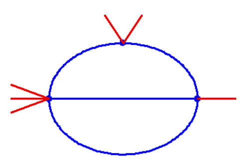
For the 13th graph,



we obtain

$$\frac{m^6\kappa^8}{2257920\pi^4}\{170237 + 316764\ln(m^2) - 352800\ln^2(m2)\} \tag{46}$$

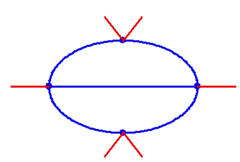
For the 14th graph,



we obtain

$$\frac{m^6\kappa^8}{9031680\pi^4}\{170237 + 316764\ln(m^2) - 352800\ln^2(m2)\} \tag{47}$$

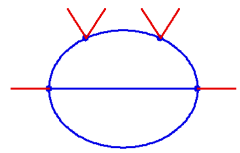
For the 15th graph,



we obtain

$$\frac{m^6\kappa^8}{25804800\pi^4}\{1805681 - 4717500\ln(m^2) + 2444400\ln^2(m2)\} \tag{48}$$

For the 16th graph,



we obtain

$$\frac{m^6\kappa^8}{180633600\pi^4}\{-8500531 - 6243020\ln(m^2) + 10760400\ln^2(m2)\} \tag{49}$$

Adding the above 16 two-loop contributions, and using $\ln(m^2) \to \frac{3}{2}$, we obtain:

$$-\frac{5748611m^6\kappa^8}{270950400\pi^4} \tag{50}$$

Notice, however, that the first 10 contributions, being evaluated exactly do vanish upon substituting the value of $\ln(m^2)$. The above remaining value is only an artifact of the approximations involved in evaluating the last 6 contributions (manifesting overlapping momenta). Hence we can say that the momentum-independent contributions to the hexilinear vertices must be vanishing, if could be evaluated exactly. We shall return later to discuss this issue much further. As a matter of fact, we have computed the much elaborate momentum-dependent contributions as well, but these are included in a much more detailed report.[7]

7 Discussion

The model treated in this article concerns a scalar field with nonlinear interactions governed by an O_2 symmetric constraint and a dimensional coupling. Our treatment should have given support to the idea that such models could be made viable, at the quantum field theoretical level, by utilizing the powerful approach[1] of divergence-free effective action. We conclude this article with a few observations regarding our results.

First of all, our computations of the corrections to the bilinears have demonstrated the masslessness of the effective (Goldstone) scalar field, at least at the one-loop level. This follows from demanding a vanishing vacuum contribution, fixing the scale via $\ln(m^2)$, where m is an infrared regulating parameter, being the mass of the virtual quantum propagating in the internal lines. However, the two-loop correction to the bilinears does seem to show that the effective field has acquired a mass term $\sim (m^4\kappa^4)m^2$. However, a closer inspection would reveal that this situation is only an artifact of the approximation needed in evaluating those 2-loop integrals with overlapping momenta.

We have also given the results of computing one-loop corrections to quartilinear, and hexilinear vertices. This work helped us to check whether the form of the Lagrangian (governed by the nonlinear invariance under O_2) is preserved. Our results seem to demonstrate that the corrections to all these vertices has the same value $(m^2\kappa^2/32\pi^2)$. Our results could be compared to those of the gravity-like scalar model.[6]

Again, whereas we could not supply, here, all pertinent computational details associated with each graphic contribution, interested readers may be able to acquire complete and updated reports on this and other related investigations from the author.[7]

References

[1] N.S. Baaklini, "Effective Action Framework for Divergence-Free Quantum Field Theory", *N.S.B. Letters*, **NSBL-QF-010**; http://www.vixra.org/abs/1312.0056

[2] N.S. Baaklini, "The Divergence-Free Effective Action for a Scalar Field Theory", *N.S.B. Letters*, **NSBL-QF-014**, http://www.vixra.org/abs/1312.0065

[3] N.S. Baaklini, "The Divergence-Free Effective Action for Quantum Electrodynamics", *N.S.B. Letters*, **NSBL-QF-015**, http://www.vixra.org/abs/1401.0013

[4] N.S. Baaklini, "Framework for the Effective Action of Quantum Gauge and Gravitational Fields", *N.S.B. Letters*, **NSBL-QF-011**, http://www.vixra.org/abs/1401.0121

[5] N.S. Baaklini, "Graphs and Expressions for Higher-Loop Effective Quantum Action", *N.S.B. Computing*, **NSBC-QF-005**, http://www.vixra.org/abs/1402.0085

[6] N.S. Baaklini, "Divergence-Free Quantum Gravity in a Scalar Model", *N.S.B. Computing*, **NSBC-QF-041**, http://www.vixra.org/abs/1604.0115

[7] *Divergence-Free Quantum Field Theory* by N.S. Baaklini (Complete Updated Development)

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