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The Electromagnetic Wave in the Dielectric and Magnetic Circuit of Alternating Current

Abstract

A solution to the Maxwell equations for dielectric and magnetic circuit of alternating current is presented. The structure of currents and energy flow is examined.

Contents

Part 1. Dielectric Circuit

1.1. Introduction

An electromagnetic field in vacuum is considered in [1]. The evident solution obtained in [1] is extended to a nonconducting dielectric medium with certain dielectric and magnetic permeability ε and μ, respectively. Therefore, the electromagnetic field does also exist in a capacitor as well. However, a considerable difference of the capacitor is that its field has a non-zero electrical intensity along on of the coordinates induced by an external source. The electromagnetic field in vacuum was examined on the basis of an assumption that an external source was absent.

The same can be said about an alternating current dielectric circuit. The system of Maxwell equations is applied to such a circuit. It is shown that an electromagnetic wave is also formed in this circuit. An important difference between this wave and the wave in vacuum is that the former has a longitudinal electrical intensity induced by an external power source.

Below are considered the Maxwell equations of the following form written in the CGS system (as in [1], but with ε and μ which are not equal to 1):

$$\mathrm{rot}(E)+\frac{\mu}{c}\frac{\partial H}{\partial t}=0, \tag{1}$$

$$\mathrm{rot}(H)-\frac{\varepsilon}{c}\frac{\partial E}{\partial t}=0, \tag{2}$$

$$\mathrm{div}(E)=0, \tag{3}$$

$$\mathrm{div}(H)=0, \tag{4}$$

where H, E are the magnetic intensity and the electrical intensity, respectively.

1.2. Maxwell Equations Solution

Let us consider solution to the Maxwell equations (1.1-1.4). In the cylindrical coordinate system $r,\ \varphi,\ z$, these equations take the form:

$$\frac{E_r}{r}+\frac{\partial E_r}{\partial r}+\frac{1}{r}\cdot\frac{\partial E_\varphi}{\partial \varphi}+\frac{\partial E_z}{\partial z}=0, \tag{1}$$

$$\frac{1}{r}\cdot\frac{\partial E_z}{\partial \varphi}-\frac{\partial E_\varphi}{\partial z}=v\frac{dH_r}{dt}, \tag{2}$$

$$\frac{\partial E_r}{\partial z}-\frac{\partial E_z}{\partial r}=v\frac{dH_\varphi}{dt}, \tag{3}$$

$$\frac{E_\varphi}{r}+\frac{\partial E_\varphi}{\partial r}-\frac{1}{r}\cdot\frac{\partial E_r}{\partial \varphi}=v\frac{dH_z}{dt}, \tag{4}$$

$$\frac{H_r}{r}+\frac{\partial H_r}{\partial r}+\frac{1}{r}\cdot\frac{\partial H_\varphi}{\partial \varphi}+\frac{\partial H_z}{\partial z}=0, \tag{5}$$

$$\frac{1}{r}\cdot\frac{\partial H_z}{\partial \varphi}-\frac{\partial H_\varphi}{\partial z}=q\frac{dE_r}{dt} \tag{6}$$

$$\frac{\partial H_r}{\partial z}-\frac{\partial H_z}{\partial r}=q\frac{dE_\varphi}{dt}, \tag{7}$$

$$\frac{H_\varphi}{r}+\frac{\partial H_\varphi}{\partial r}-\frac{1}{r}\cdot\frac{\partial H_r}{\partial \varphi}=q\frac{dE_z}{dt}, \tag{8}$$

where

$$v = -\mu/c, \tag{9}$$

$$q = \varepsilon/c, \tag{10}$$

$E_r,\ E_\varphi,\ E_z$ are the electrical intensity components,

$H_r,\ H_\varphi,\ H_z$ are the magnetic intensity components.

A solution should be found **for non-zero intensity component** E_z.

To write the equations in a concise form, the following designations are used below:

$$co = \cos(\alpha\varphi + \chi z + \omega t), \tag{11}$$

$$si = \sin(\alpha\varphi + \chi z + \omega t), \tag{12}$$

where $\alpha,\ \chi,\ \omega$ are constants. Let us write the unknown functions in the following form:

$$H_r. = h_r(r)co, \tag{13}$$

$$H_\varphi. = h_\varphi(r)si, \tag{14}$$

$$H_z. = h_z(r)si, \tag{15}$$

$$E_r. = e_r(r)si, \tag{16}$$

$$E_\varphi. = e_\varphi(r)co, \tag{17}$$

$$E_z. = e_z(r)co, \tag{18}$$

where $h(r),\ e(r)$ are function of the coordinate r.

Direct substitution enables us to ascertain that functions (13-18) convert the system of equations (1-8) with four arguments $r,\ \varphi,\ z,\ t$ in a system of equations with one argument r and unknown functions $h(r),\ e(r)$.

Appendix 1 proves that such a solution **does exist**. It takes the following form:

$$e_\varphi(r) = \mathrm{kh}(\alpha,\ \chi,\ r), \tag{20}$$

$$e_r(r) = \frac{1}{\alpha}\left(e_\varphi(r) + r \cdot e'_\varphi(r)\right), \tag{21}$$

$$e_z(r) = r \cdot e_\varphi(r) q/\alpha, \tag{22}$$

$$h_\varphi(r) = -\frac{\varepsilon\omega}{\chi c} e_r(r), \tag{23}$$

$$h_r(r) = \frac{\varepsilon\omega}{\chi c} e_\varphi(r), \tag{24}$$

$$h_z(r) \equiv 0. \tag{25}$$

where kh() – is the function determined in Appendix 2,

$$q = \left(\chi - \frac{\mu \varepsilon \omega^2}{c^2 \chi} \right). \tag{26}$$

Let us compare this solution with the solution for vacuum [1], see Table 1 in Section 2.2. A considerable difference between these solutions is evident.

1.3. Field Intensity and Energy Flows

As in [1], the energy Flow density along the coordinates is calculated by the formula

$$\overline{S} = \begin{bmatrix} \overline{S_r} \\ \overline{S_\varphi} \\ \overline{S_z} \end{bmatrix} = \eta \iint_{r,\varphi} \begin{bmatrix} s_r \cdot si^2 \\ s_\varphi \cdot si \cdot co \\ s_z \cdot si \cdot co \end{bmatrix} dr \cdot d\varphi . \tag{1}$$

where

$$\begin{aligned} s_r &= \left(e_\varphi h_z - e_z h_\varphi \right) \\ s_\varphi &= \left(e_z h_r - e_r h_z \right), \\ s_z &= \left(e_r h_\varphi - e_\varphi h_r \right) \end{aligned} \tag{2}$$

$$\eta = c/4\pi . \tag{3}$$

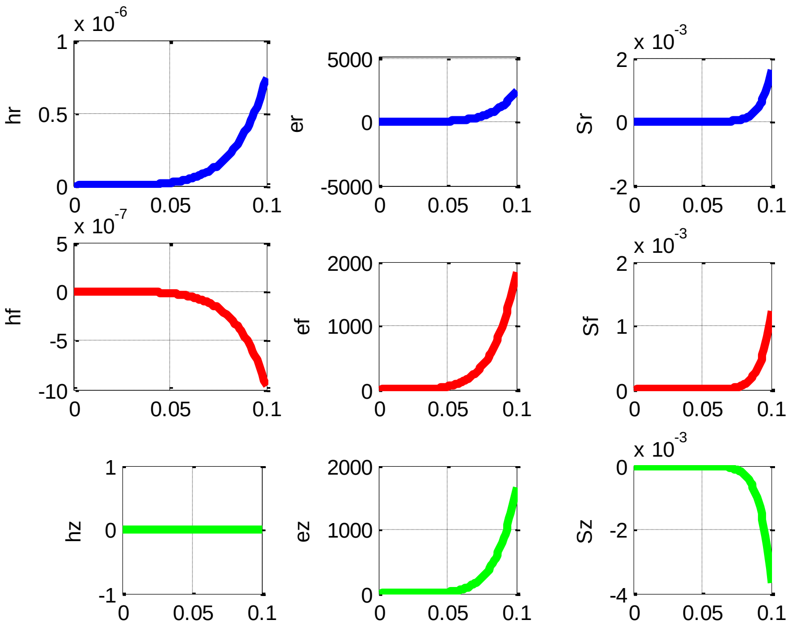

Fig.1. (SSB6(3).m)

Let us consider functions (2) and $e_r(r),\ e_\varphi(r),\ e_z(r)$, $h_r(r),\ h_\varphi(r),\ h_z(r)$. Fig. 1 shows, for example, these functions plotted for $A=1,\ \alpha=5.5,\ \mu=1,\ \varepsilon=2,\ \chi=50,\ \omega=300$.

1.4. Discussion

Further conclusions are similar to those of [1]. Thus, an electromagnetic wave propagates via a dielectric circuit and, in particular, through a capacitor connected to an AC circuit, and the mathematical description of this wave is the solution of the Maxwell equations. In this case, the field intensity, the displacement current, and the energy Flow propagate in the dielectric along a spiral path.

Part 2. Magnetic Circuit

2.1. Introduction

Part 1 deals with the electromagnetic field in an AC dielectric circuit. The electromagnetic filed in an AC magnetic circuit can be examined using the same approach. The simplest example of such a circuit is an AC solenoid. However, if the dielectric circuit has a longitudinal electrical field intensity component induced by an external power source, the magnetic circuit features a longitudinal magnetic field component induced by an external power source and transmitted to circuit with the solenoid coil.

In this case, the Maxwell equations outlined in Part 1, are also used - see (1.1.1-1.1.4).

2.2. Maxwell Equations Solution

Part 1 demonstrates that in the cylindrical coordinate system $r,\ \varphi,\ z$ these equations take the form (1.2.1-1.2.18). Their solution should be found for non-zero field intensity H_z. Here, the functions $h(r),\ e(r)$ are of different form. Similar to the above-mentioned, it can be demonstrated that the solution **does also exist** in this case. Its form is as follows:

$$e_z(r)\equiv 0, \tag{20}$$

$$h_\varphi(r)=\mathrm{kh}(\alpha,\ \chi,\ r), \tag{21}$$

$$h_r(r)=-\frac{1}{\alpha}\left(h_\varphi(r)+r\cdot h'_\varphi(r)\right), \tag{22}$$

$$h_z(r) = r \cdot h_\varphi(r) q / \alpha, \tag{23}$$

$$e_\varphi(r) = \frac{\mu\omega}{\chi c} h_r(r), \tag{24}$$

$$e_r(r) = -\frac{\mu\omega}{\chi c} h_\varphi(r). \tag{25}$$

Let us compare this solution with the solution obtained in Part 1, see Table 1. The similarity of these solutions is evident.

Table 1.

	Vacuum	Dielectric circuit	Magnetic circuit
e_r	$Ar^{\alpha-1}$	$\mathrm{A} \cdot \mathrm{kh}(\alpha,\ \chi,\ r)$	$-A\frac{\mu\omega}{\chi c} h_\varphi(r)$
e_φ	$Ar^{\alpha-1}$	$\frac{A}{\alpha}\left(e_\varphi(r) + r \cdot e'_\varphi(r)\right)$	$A\frac{\mu\omega}{\chi c} h_r(r)$
e_z	**0**	$A \cdot r \cdot e_\varphi(r)\frac{q}{\alpha}$	**0**
h_r	$-e_\varphi(r)$	$A\frac{\varepsilon\omega}{c\chi} e_\varphi(r)$	$-\frac{A}{\alpha}\left(h_\varphi(r) + r \cdot h'_\varphi(r)\right)$
h_φ	$-h_r(r)$	$-A\frac{\varepsilon\omega}{c\chi} e_r(r)$	$\mathrm{Akh}(\alpha,\ \chi,\ r)$
h_z	**0**	**0**	$Ar \cdot h_\varphi(r) q / \alpha$

2.3. Field Intensity and Energy Flows

As it is done in Part 1, the density of energy flows along the coordinate axes are calculated by the formulae (1.3.1 – 1.3.3). Let us consider functions (1.3.2) and $e_r(r),\ e_\varphi(r),\ e_z(r)$, $h_r(r),\ h_\varphi(r),\ h_z(r)$. Fig. 1 shows these functions for $A=1,\ \alpha=5.5,\ \mu=1,\ \varepsilon=2,\ \chi=50,\ \omega=300$. These variables are chosen identical to those used in Part 1 for comparison of the obtained results.

4. Discussion

Further conclusions are similar to the conclusions of Part 1. Thus, an electromagnetic wave propagates in an AC magnetic circuit, and the mathematical description of this wave is a solution to the Maxwell equations. In this case, the field intensity and the energy Flow follow a spiral trajectory in the considered circuit.

Appendices

Appendix 1.

A solution to equations (2.1-2.8) is considered to be in the form of functions (2.13-2.18). Derivatives with respect to *r* will be denoted with primes. Let us re-write equations (2.1-2.8) considering (2.11, 2.12) in the form

$$\frac{e_r(r)}{r}+e_r'(r)-\frac{e_\varphi(r)}{r}\alpha-\chi\cdot e_z(r)=0\,, \tag{1}$$

$$-\frac{1}{r}\cdot e_z(r)\alpha+e_\varphi(r)\chi-\frac{\mu\omega}{c}h_r=0, \tag{2}$$

$$e_r(r)\chi-e_z'(r)+\frac{\mu\omega}{c}h_\varphi=0, \tag{3}$$

$$\frac{e_\varphi(r)}{r}+e_\varphi'(r)-\frac{e_r(r)}{r}\cdot\alpha+\frac{\mu\omega}{c}h_z=0, \tag{4}$$

$$\frac{h_r(r)}{r}+h_r'(r)+\frac{h_\varphi(r)}{r}\alpha+\chi\cdot h_z(r)=0\,, \tag{5}$$

$$\frac{1}{r}\cdot h_z(r)\alpha-h_\varphi(r)\chi-\frac{\varepsilon\omega}{c}e_r=0, \tag{6}$$

$$-h_r(r)\chi-h_z'(r)+\frac{\varepsilon\omega}{c}e_\varphi=0, \tag{7}$$

$$\frac{h_\varphi(r)}{r}+h_\varphi'(r)+\frac{h_r(r)}{r}\cdot\alpha+\frac{\varepsilon\omega}{c}e_z(r)=0\,. \tag{8}$$

The correspondence between the formula numbers in Part 2 and in this Appendix is as follows:

Part 2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8
App. 1	1	5	6	7	8	6	7	8

Formulae (1 – 8) will be transformed below. In doing so, the formula numbering will be retained after transformation (to make easier to follow the sequence of transformations), and only new formulae will take the next number.

Assume that

$$h_z(r)=0\,. \tag{9}$$

From (6, 7) it follows that:

$$h_\varphi(r)=-\frac{\varepsilon\omega}{c}e_r(r)\frac{1}{\chi} \tag{6}$$

$$h_r(r) = \frac{\varepsilon\omega}{c} e_\varphi(r) \frac{1}{\chi} \tag{7}$$

Let us compare (1, 8):

$$\frac{e_r(r)}{r} + e_r'(r) - \frac{e_\varphi(r)}{r}\alpha - \chi \cdot e_z(r) = 0\,, \tag{1}$$

$$\frac{h_\varphi(r)}{r} + h_\varphi'(r) + \frac{h_r(r)}{r} \cdot \alpha + \frac{\varepsilon\omega}{c} e_z(r) = 0, \tag{8}$$

From (6, 7) it follows that (1, 8) are identical. Then (8) can be deleted. Then compare (4) with (5):

$$\frac{e_\varphi(r)}{r} + e_\varphi'(r) - \frac{e_r(r)}{r} \cdot \alpha = 0, \tag{4}$$

$$\frac{h_r(r)}{r} + h_r'(r) + \frac{h_\varphi(r)}{r}\alpha = 0\,. \tag{5}$$

From (6, 7) it follows (4) and (5) are identical. Hence, equation (5) can be deleted. The remaining equations are as follows:

$$\frac{e_r(r)}{r} + e_r'(r) - \frac{e_\varphi(r)}{r}\alpha - \chi \cdot e_z(r) = 0\,, \tag{1}$$

$$-\frac{1}{r} \cdot e_z(r)\alpha + e_\varphi(r)\chi - \frac{\mu\omega}{c} h_r = 0, \tag{2}$$

$$e_r(r)\chi - e_z'(r) + \frac{\mu\omega}{c} h_\varphi = 0, \tag{3}$$

$$\frac{e_\varphi(r)}{r} + e_\varphi'(r) - \frac{e_r(r)}{r} \cdot \alpha = 0, \tag{4}$$

$$h_\varphi(r) = -\frac{\varepsilon\omega}{c} e_r(r) \frac{1}{\chi}, \tag{6}$$

$$h_r(r) = \frac{\varepsilon\omega}{c} e_\varphi(r) \frac{1}{\chi}\,. \tag{7}$$

Substitute (6, 7) in (2, 3):

$$-\frac{1}{r} \cdot e_z(r)\alpha + e_\varphi(r)\chi - \frac{\mu\omega}{c}\frac{\varepsilon\omega}{c} e_\varphi(r)\frac{1}{\chi} = 0, \tag{2}$$

$$e_r(r)\chi - e_z'(r) - \frac{\mu\omega}{c}\frac{\varepsilon\omega}{c} e_r(r)\frac{1}{\chi} = 0, \tag{3}$$

or

$$\frac{\alpha}{r} \cdot e_z(r) = e_\varphi(r)\left(\chi - \frac{\mu\omega}{c}\frac{\varepsilon\omega}{c}\frac{1}{\chi}\right) \tag{2}$$

$$e'_z(r) = e_r(r)\left(\chi - \frac{\mu\omega}{c}\frac{\varepsilon\omega}{c}\frac{1}{\chi}\right) \tag{3}$$

The remaining equations are as follows:

$$\frac{e_r(r)}{r} + e'_r(r) - \frac{e_\varphi(r)}{r}\alpha - \chi \cdot e_z(r) = 0\,, \tag{1}$$

$$\frac{\alpha}{r} \cdot e_z(r) = e_\varphi(r)\left(\chi - \frac{\mu\omega}{c}\frac{\varepsilon\omega}{c}\frac{1}{\chi}\right) \tag{2}$$

$$e'_z(r) = e_r(r)\left(\chi - \frac{\mu\omega}{c}\frac{\varepsilon\omega}{c}\frac{1}{\chi}\right) \tag{3}$$

$$\frac{e_\varphi(r)}{r} + e'_\varphi(r) - \frac{e_r(r)}{r} \cdot \alpha = 0, \tag{4}$$

$$h_\varphi(r) = -\frac{\varepsilon\omega}{c} e_r(r)\frac{1}{\chi}, \tag{6}$$

$$h_r(r) = \frac{\varepsilon\omega}{c} e_\varphi(r)\frac{1}{\chi}. \tag{7}$$

Let us denote:

$$q = \left(\chi - \frac{\mu\omega}{c}\frac{\varepsilon\omega}{c}\frac{1}{\chi}\right) \tag{11}$$

From (1, 2, 11) it can be found that:

$$\frac{e_r(r)}{r} + e'_r(r) - \frac{e_\varphi(r)}{r}\alpha - \chi r \cdot e_\varphi(r) q / \alpha = 0\,, \tag{12}$$

From (4) it can be found that:

$$e_r(r) = \frac{1}{\alpha}\left(e_\varphi(r) + r \cdot e'_\varphi(r)\right) \tag{13}$$

$$e'_r(r) = \frac{1}{\alpha}\left(2e'_\varphi(r) + r \cdot e''_\varphi(r)\right) \tag{14}$$

From (12-14) it can be found that:

$$\frac{1}{\alpha}\left(\frac{e_\varphi(r)}{r} + e'_\varphi(r)\right) + \frac{1}{\alpha}\left(2e'_\varphi(r) + r \cdot e''_\varphi(r)\right) - \frac{e_\varphi(r)}{r}\alpha - \frac{q\chi}{\alpha} r \cdot e_\varphi(r) = 0 \tag{15}$$

For the solution and analysis of this equation, see Appendix 2. This solution cannot be presented as an analytical expression. Let us call this solution as a function

$$e_\varphi(r) = \mathrm{kh}(\alpha,\ \chi,\ r), \tag{16}$$

and its derivative as a function

$$e'_\varphi(r) = \mathrm{kh1}(\alpha,\ \chi,\ r). \tag{17}$$

With the known functions (16, 17), the remaining functions can also be found. Thus, all the functions can be determined from the following equations:

$$h_z(r) \equiv 0\,, \tag{9}$$

$$e_\varphi(r) = \mathrm{kh}(\alpha,\ \chi,\ r), \tag{16}$$

$$e'_\varphi(r) = \mathrm{kh1}(\alpha,\ \chi,\ r), \tag{17}$$

$$e_r(r) = \frac{1}{\alpha}\left(e_\varphi(r) + r \cdot e'_\varphi(r)\right), \tag{13}$$

$$e'_r(r) = \frac{1}{\alpha}\left(2e'_\varphi(r) + r \cdot e''_\varphi(r)\right), \tag{14}$$

$$e_z(r) = r \cdot e_\varphi(r)\frac{q}{\alpha}, \tag{2}$$

$$e'_z(r) = e_r(r)q\,, \tag{3}$$

$$h_\varphi(r) = -\frac{\varepsilon\omega}{c}e_r(r)\frac{1}{\chi}, \tag{6}$$

$$h_r(r) = \frac{\varepsilon\omega}{c}e_\varphi(r)\frac{1}{\chi}. \tag{7}$$

For the accuracy of the obtained solution, see Appendix 3.

Appendix 2.

Let us consider equation (15) from Appendix 1:

$$\frac{1}{\alpha}\left(\frac{e_\varphi(r)}{r} + e'_\varphi(r)\right) + \frac{1}{\alpha}\left(2e'_\varphi(r) + r \cdot e''_\varphi(r)\right) - \frac{e_\varphi(r)}{r}\alpha - \frac{q\chi}{\alpha} r \cdot e_\varphi(r) = 0. \tag{1}$$

Its simplification gives:

$$\left(\frac{e_\varphi(r)}{r} + e'_\varphi(r)\right) + \left(2e'_\varphi(r) + r \cdot e''_\varphi(r)\right) - \frac{e_\varphi(r)}{r}\alpha^2 - q\chi r \cdot e_\varphi(r) = 0$$

$$e_\varphi(r)\left(\frac{-\alpha^2 + 1}{r} - q\chi r\right) + 3e'_\varphi(r) + r \cdot e''_\varphi(r) = 0\,,$$

$$e''_\varphi(r) = e_\varphi(r)\left(\frac{\alpha^2 - 1}{r^2} + q\chi\right) - \frac{3}{r}e'_\varphi(r). \tag{2}$$

Equation (2) has not an analytical solution. But the following functions can be calculated numerically

$$e_\varphi(r) = \mathrm{kh}(\alpha,\ \chi,\ r) \tag{3}$$

$$e'_\varphi(r) = \mathrm{kh1}(\alpha,\ \chi,\ r) \tag{4}$$

$$e_{\varphi}''(r) = \mathrm{kh2}(\alpha,\ \chi,\ r) \tag{5}$$

For an example, Fig. 2 shows these functions for $(\alpha = 5.5,\ \chi = 50)$ at a radius of $R = 0.1$.

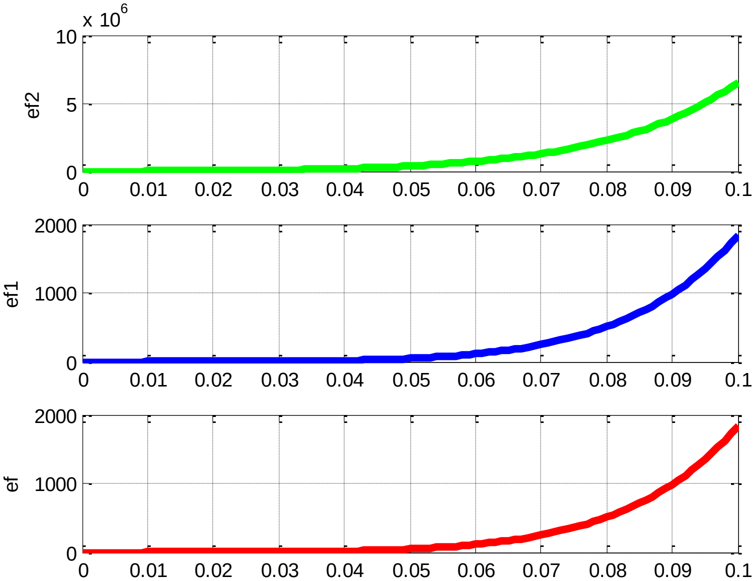

Fig.2. (SSMB6.13)

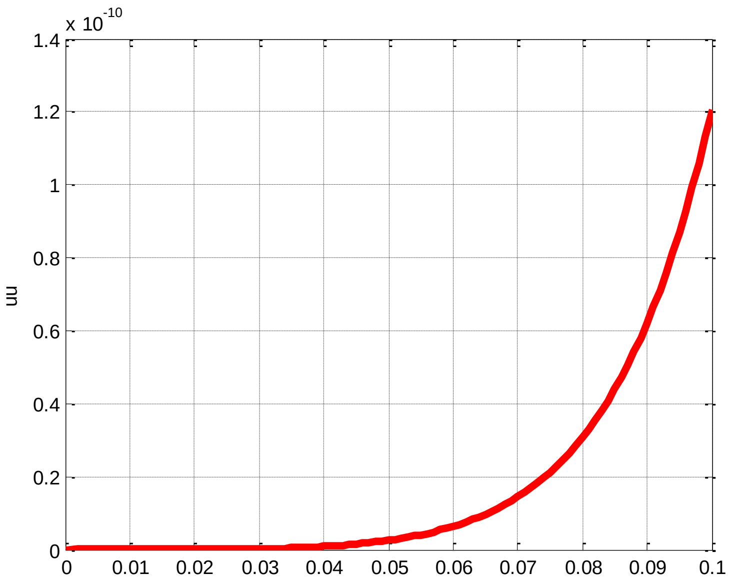

Fig.3. (SSMB6.14)

Appendix 3.

Substitution of the functions found in Appendix 1 in equations (1-8) enables us to determine a RMS residual error of these equations. Fig. 3 shows this residual error for $(\alpha = 5.5, \ \chi = 50)$ at a radius of $R = 0.1$.

A RMS residual error of these equations can be found as a function of one or other variable. Fig. 4 shows the residual error as a function of α for $\chi = 50$ at a radius of $R = 0.1$. Here, the upper window presents the residual error value, and lower window the residual error logarithm.

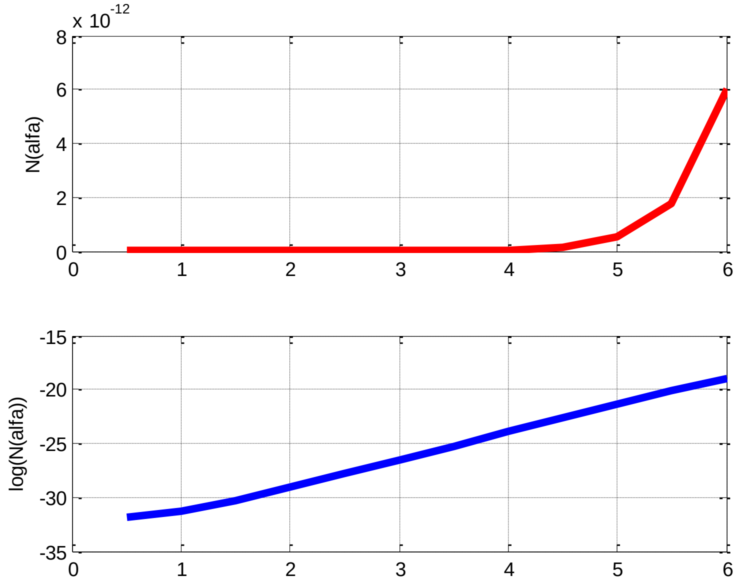

Fig.4. (SSMB6.333)

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