N.S.B. Letters NSBL-QF-041

Divergence-Free Quantum Gravity in a Scalar Model

N.S. Baaklini
nsbqft@aol.com
http://nsbcosmic.neocities.org

Abstract

We present results of applying our divergence-free effective action quantum field theory techniques to a scalar model with gravity-like, non-polynomial interactions characterized by a dimensional coupling constant. This treatment would give a clear perspective regarding the viability of applying the divergence-free approach to quantum gravity. Issues regarding the masslessness of the effective graviton, while the virtual counterpart is massive, as well as, regarding the invariance of the basic Lagrangian, are discussed.

1 Introduction

In our divergence-free effective action approach to quantum field theory,[1, 2, 3, 4, 5] we have a consistent manner of evading the loop divegences, not only in theories with dimensionless couplings, like a scalar model with quartic coupling, quantum electrodynamics, and non-Abelian gauge theories, but also in theories with dimensional couplings, like quantum gravity. Our purpose in this article is to demonstrate results of applying our methods to a scalar model with a non-polynomial Lagrangian characterized by a gravity-like dimensional coupling. We employ this model for computational simplicity, and would return to present corresponding results in quantum gravity itself, in a number of other articles.

In the following sections, we shall begin by presenting the Lagrangian and the associated Feynman rules. This will be followed by giving the results of applying our divergence-free methods to several loop computations. We shall conclude with a brief discussion of the results. While we do suppress much of the detailed derivations, our results are obtained by direct application of our methods, and must be clear from earlier articles. However, all the details may be found in comprehensive reports that are available elsewhere.[6] Our following presentation, however, would illustrate the one-to-one correspondence between our results and those of the ordinary Feynman graphic counterparts.

2 The Model Lagrangian and Graphical Rules

The Lagrangian density of our scalar model is given by the following expression:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 \, e^{\kappa\phi} \tag{1}$$

This describes the non-polynomial interactions of a massless scalar field ϕ, with a gravity-like coupling constant κ. Notice that the masslessness of our Goldstone-like field is associated with the invariance of the above Lagrangian density (up to an overall constant that does not affect the equations of motion) under the shift $\phi \to \phi + c$, where c is an x-independent constant. In order to develop perturbative loop computations, the above (non-polynomial) exponential coupling may be expanded in an infinite series of vertices:

$$\frac{1}{2}(\partial\phi)^2 \left\{ 1 + \kappa\phi + \frac{1}{2}\kappa^2\phi^2 + \frac{1}{3!}\kappa^3\phi^3 + \dots \right\} \tag{2}$$

The usual (Minkowskian) momentum-space Feynman graphic rules that can be associated with the above system are as follows:

- For every internal line or bare propagator with momentum p, we write

  $$\frac{1}{-p^2 + m^2}$$

  Notice that we have introduced the mass m as an infrared regulating parameter. This mass would only concern the virtual field, and in the framework of a massless gauge field would not break the effective gauge invariance.

- For every symmetrized n-leg vertex, with respective ingoing momenta $p, q, r, \cdots$, we write

  $$-\frac{1}{2}\kappa^{(n-2)} \left(p^2 + q^2 + r^2 + \cdots \right)$$

- We must associate a factor of i for each propagator, a factor of i for each vertex, and an overall factor of $-i$ for each graph.

- We must supply the appropriate combinatoric factors for each graph.

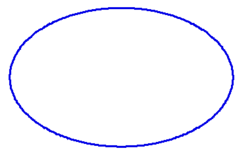
- Most importantly, we must supply the appropriate *regularizing parameters* and the corresponding *pole-removing operators*, together with the gamma functions factors, and Feynman parameter combinations, all according to our divergence-free methods.[1]

In the following section, we shall display associated graphics and computational results suppressing all details.

3 Vacuum Contributions

3.1 One-Loop Contribution

The 1-loop vacuum contribution corresponds to the following no-vertex graph:

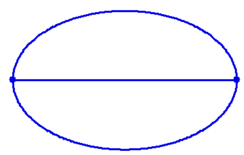


With the associated result

$$\frac{m^4(3-2\ln(m^2))}{128\pi^2} \tag{3}$$

3.2 Two-Loop Contributions

For 2-loop contributions we have two graphs. The first one,

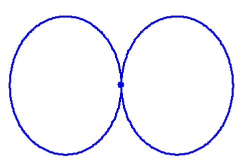


gives the result:

$$\frac{m^6\kappa^2}{\pi^4}\left(\frac{7139}{44236800}-\frac{13589}{5160960}\ln(m^2)+\frac{23}{12288}\ln^2(m^2)\right) \tag{4}$$

We should note here that the above contribution is not exact; it does involve an approximation "to 6th order in overlapping momenta".

The second 2-loop graph



gives the exact result:

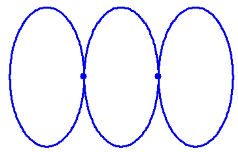
$$-\frac{m^6\kappa^2}{2048\pi^4}(-1+\ln(m^2))(-3+2\log(m^2)) \tag{5}$$

Adding the two, 2-loop vacuum contributions, we have:

$$\frac{m^6\kappa^2}{\pi^4}\left(-\frac{57661}{44236800}-\frac{989}{5160960}\ln(m^2)+\frac{11}{12288}\ln^2(m^2)\right) \tag{6}$$

3.3 Three-Loop Contributions

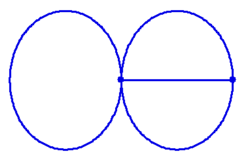
There are eight graphs for 3-loop vacuum contribution. The 1st graph



gives exactly

$$-\frac{m^8\kappa^4}{262144\pi^6}\left(3+45\ln(m^2)-78\ln^2(m^2)+32\ln^3(m^2)\right) \tag{7}$$

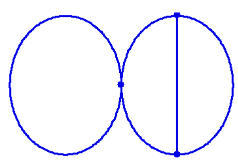
The 2nd graph



gives (approx.)

$$-\frac{m^8\kappa^4}{4954521600\pi^6}\left(50410+1143091\ln(m^2)-2002740\ln^2(m^2)+831600\ln^3(m^2)\right) \tag{8}$$

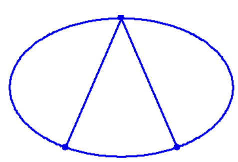
The 3rd graph



gives (approx.)

$$\frac{m^8\kappa^4}{3303014400\pi^6}\left(1085622+313837\ln(m^2)-2751540\ln^2(m^2)+1436400\ln^3(m^2)\right) \tag{9}$$

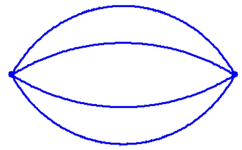
The 4th graph



gives (approx.)

$$\frac{m^8\kappa^4}{6606028800\pi^6}\left(2478555+352514\ln(m^2)-5692680\ln^2(m^2)+3024000\ln^3(m^2)\right) \tag{10}$$

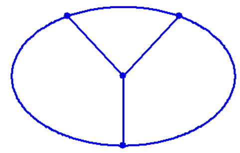
The 5th graph



gives (approx.)

$$-\frac{m^8\kappa^4}{39636172800\pi^6}\left(-610259+3134320\ln(m^2)-4512480\ln^2(m^2)+1814400\ln^3(m^2)\right) \tag{11}$$

The 6th graph

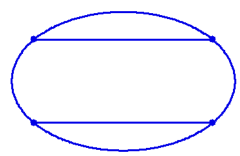


gives (approx.)

$$-\frac{m^8\kappa^4}{37748736\pi^6}\left(-13089-15242\ln(m^2)+49956\ln^2(m^2)+7272\ln^3(m^2)\right) \tag{12}$$

The above contributions is much more approximated than the others (to 2nd order 'in overlap' rather than 6th).

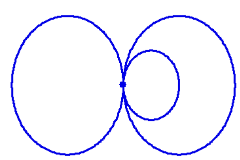
The 7th graph



gives (approx.)

$$-\frac{m^8\kappa^4}{554906419200\pi^6}\left(28896167-148362600\ln(m^2)-118972980\ln^2(m^2)+572065200\ln^3(m^2)\right) \tag{13}$$

The 8th graph



gives exactly

$$\frac{m^8\kappa^4}{131072\pi^6}(-1+\ln(m^2))^2(-3+2\ln(m^2)) \tag{14}$$

Adding the eight, 3-loop vacuum contributions, we get

$$-\frac{m^8\kappa^4}{110981283840\pi^6}\left(\begin{array}{l}-107588351-44292384\ln(m^2)\\ \\ +226573452\ln^2(m^2)+72288720\ln^3(m^2)\end{array}\right) \tag{15}$$

3.4 Fixing the Vacuum

Adding the 1-loop, the 2-loop, and the 3-loop vacuum contributions, we obtain

$$\begin{cases} \frac{m^4}{128\pi^2}\left\{3 - 2\ln(m^2)\right\} + \\ \frac{m^6\kappa^2}{309657600\pi^4}\left\{-403627 - 59340\ln(m^2) + 277200\ln^2(m^2)\right\} + \\ \frac{m^8\kappa^4}{110981283840\pi^6}\left\{107588351 + 44292384\ln(m^2) - 226573452\ln^2(m^2) - 72288720\ln^3(m^2)\right\} \end{cases} \tag{16}$$

The above series (to order κ^4) should be equated to zero and inverted to give an expression for $\ln(m^2)$. This procedure would eliminate $\ln(m^2)$ from the theory by fixing the arbitrary energy scale than governs the argument of the logarithm. We obtain, to order κ^4, the following result:

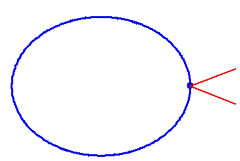
$$\ln(m^2) = \frac{3}{2} + \frac{131063m^2\kappa^2}{4838400\pi^2} - \frac{18393440911m^4\kappa^4}{55738368000\pi^4} + \cdots \tag{17}$$

4 Bilinear Contributions

In this section, we present contributions to the bilinear kernel; those loop corrections to the Lagrangian of second order in the scalar field. We compute 1-loop and 2-loop contributions.

4.1 One-Loop Contributions

We have two graphs for the bilinears in 1-loop. The 1st graph

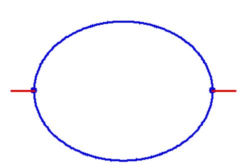


gives to 2nd order in the the external momentum (or derivative in the effective action):

$$\frac{m^2\kappa^2}{64\pi^2}\left\{-1 + \ln(m^2)\right\} r^2 + \frac{m^4\kappa^2}{128\pi^2}\left\{-3 + 2\ln(m^2)\right\} \tag{18}$$

Here r is the external momentum carried by the effective field. The above graph doesn't give a higher order in r. Notice that in our graphics, while internal lines are depicted in blue, external insertions are depicted in red.

The 2nd graph



gives to order r^4,

$$\frac{3m^4\kappa^2}{128\pi^2}\left\{3-2\ln(m^2)\right\}-\frac{5m^2\kappa^2}{128\pi^2}\left\{-1+\ln(m^2)\right\}r^2-\frac{\kappa^2}{256\pi^2}\ln(m^2)r^4 \tag{19}$$

Notice that the term involving r^4, would correspond to the curvature squared corrections in a tensorial gravitational theory.

Adding the two 1-loop contributions, we obtain

$$\frac{m^4\kappa^2}{64\pi^2}\left\{3-2\ln(m^2)\right\}-\frac{3m^2\kappa^2}{128\pi^2}\left\{-1+\ln(m^2)\right\}r^2-\frac{\kappa^2}{256\pi^2}\ln(m^2)r^4 \tag{20}$$

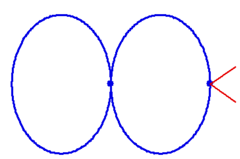
Notice, at this point, that putting the pertinent result $\ln(m^2)=3/2$, obtained in a preceding section, would eliminate the effective mass term, giving the 1-loop bilinear contribution:

$$-\frac{3m^2\kappa^2}{256\pi^2}r^2-\frac{3\kappa^2}{512\pi^2}\ln(m^2)r^4 \tag{21}$$

In a tensorial gravitational theory, this corresponds to the result that fixing the vacuum means eliminating the cosmological constant, giving a massless graviton.

4.2 Two-Loop Contributions

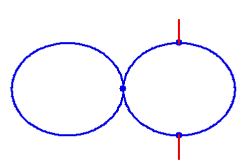
There are eleven graphs that give the 2-loop bilinear contributions. We shall give the results that correspond to these graphs, computing only to 2nd order in the external momentum. For the 1st graph,



we obtain exactly

$$\frac{5m^6\kappa^4}{4096\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\}+\frac{m^4\kappa^4}{4096\pi^4}\{-1+2\ln(m^2)\}\{-4+3\ln(m^2)\}r^2 \tag{22}$$

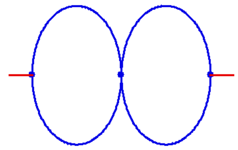
For the 2nd graph,



we obtain

$$-\frac{9m^6\kappa^4}{2048\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\}-\frac{5m^4\kappa^4}{8192\pi^4}\{6-15\ln(m^2)+8\ln^2(m^2)\}r^2 \tag{23}$$

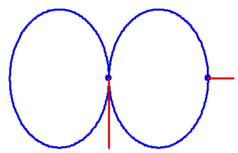
For the 3rd graph,



we obtain

$$-\frac{3m^6\kappa^4}{1024\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\}-\frac{3m^4\kappa^4}{4096\pi^4}\{-1+2\ln(m^2)\}\{-4+3\ln(m^2)\}r^2 \quad (24)$$

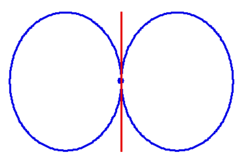
For the 4th graph,



we obtain

$$\frac{5m^6\kappa^4}{2048\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\}+\frac{m^4\kappa^4}{4096\pi^4}\{-2+3\ln(m^2)\}\{-5+4\ln(m^2)\}r^2 \quad (25)$$

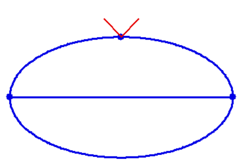
For the 5th graph,



we obtain

$$-\frac{m^6\kappa^4}{4096\pi^4}\{-1+\ln(m^2)\}\{-3+2\ln(m^2)\}-\frac{m^4\kappa^4}{4096\pi^4}\{-1+\ln(m^2)\}^2r^2 \quad (26)$$

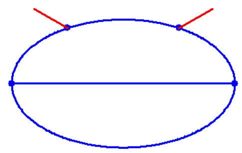
For the 6th graph,



we obtain (approx.)

$$\begin{aligned}&\tfrac{m^6\kappa^4}{72253440\pi^4}\{170237+316764\ln(m^2)-352800\ln^2(m^2)\}\\&+\tfrac{m^4\kappa^4}{103219200\pi^4}\{265193+46080\ln(m^2)-214200\ln^2(m^2)\}r^2\end{aligned} \quad (27)$$

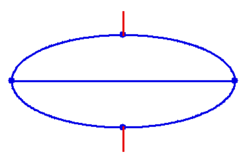
For the 7th graph,



we obtain (approx.)

$$\begin{aligned}&\tfrac{m^6\kappa^4}{72253440\pi^4}\{-8500531-6243020\ln(m^2)+10760400\ln^2(m^2)\}\\&+\tfrac{m^4\kappa^4}{95374540800\pi^4}\{-820116259+21614040\ln(m^2)+611226000\ln^2(m^2)\}r^2\end{aligned} \tag{28}$$

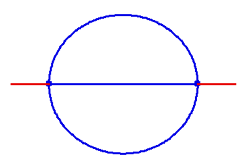
For the 8th graph,



we obtain (approx.)

$$\begin{aligned}&\tfrac{m^6\kappa^4}{90316800\pi^4}\{-1366067-1840545\ln(m^2)+2138850\ln^2(m^2)\}\\&+\tfrac{m^4\kappa^4}{216760320\pi^4}\{-3198332-412335\ln(m^2)+3188430\ln^2(m^2)\}r^2\end{aligned} \tag{29}$$

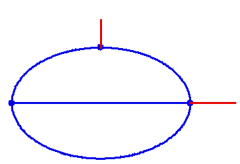
For the 9th graph,



we obtain (approx.)

$$\begin{aligned}&\tfrac{m^6\kappa^4}{309657600\pi^4}\{49973-815340\ln(m^2)+579600\ln^2(m^2)\}\\&+\tfrac{m^4\kappa^4}{541900800\pi^4}\{-148916-437535\ln(m^2)+529200\ln^2(m^2)\}r^2\end{aligned} \tag{30}$$

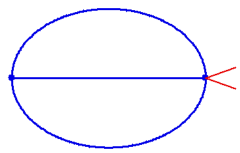
For the 10th graph,



we obtain (approx.)

$$\frac{m^6\kappa^4}{18063360\pi^4}\{170237+316764\ln(m^2)-352800\ln^2(m^2)\} + \frac{m^4\kappa^4}{361267200\pi^4}\{3831509+756420\ln(m^2)-3351600\ln^2(m^2)\}r^2 \tag{31}$$

For the 11th graph,



we obtain (approx.)

$$\frac{m^6\kappa^4}{309657600\pi^4}\{49973-815340\ln(m^2)+579600\ln^2(m^2)\} + \frac{m^4\kappa^4}{154828800\pi^4}\{-33461-114900\ln(m^2)+126000\ln^2(m^2)\}r^2 \tag{32}$$

Adding the above 11 contributions, we obtain for the total 2-loop bilinear correction:

$$\frac{m^6\kappa^4}{2167603200\pi^4}\{-57453629+15533700\ln(m^2)+21873600\ln^2(m^2)\} + \frac{m^4\kappa^4}{95374540800\pi^4}\{-1343634023+831139680\ln(m^2)+613166400\ln^2(m^2)\}r^2 \tag{33}$$

4.3 Fixing the Bilinear

Adding the 1-loop and the 2-loop bilinear corrections, and replacing $\ln(m^2)$, to the needed order in κ, as obtained before (through fixing the vacuum to zero):

$$\ln(m^2) \to \frac{3}{2} + \frac{131063m^2\kappa^2}{4838400\pi^2}$$

we obtain for the bilinear correction, to order κ^4:

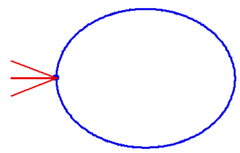
$$\frac{4409213m^6\kappa^4}{722534400\pi^4} + \left(-\frac{3m^2\kappa^2}{256\pi^2} + \frac{1222148791m^4\kappa^4}{95374540800\pi^4}\right)r^2 + \left(-\frac{3\kappa^2}{512\pi^2} - \frac{131063m^2\kappa^4}{1238630400\pi^4}\right)r^4 \tag{34}$$

We shall return to discuss some features of this result later on.

5 Trilinear Contributions

In this section we give the results of 1-loop corrections to the effective trilinear vertex. In this case, we have three graphic contributions. We give the results to 2nd order in the external momenta r, s, t. Symmetrization in the latter will be done after adding the three contributions.

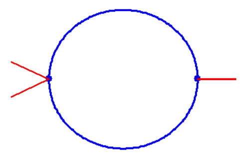
For the 1st graph,



we obtain

$$\frac{m^4\kappa^3}{384\pi^2}\{-3+2\ln(m^2)\}+\frac{m^2\kappa^3}{192\pi^2}\{-1+\ln(m^2)\}(r^2+s^2+r\cdot s) \qquad (35)$$

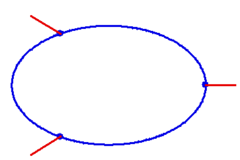
For the 2nd graph,



we obtain

$$-\frac{3m^4\kappa^3}{128\pi^2}\{-3+2\ln(m^2)\}-\frac{m^2\kappa^3}{128\pi^2}\{-1+\ln(m^2)\}(5r^2+4s^2+4r\cdot s) \qquad (36)$$

For the 3rd graph,



we obtain

$$\frac{m^4\kappa^3}{32\pi^2}\{-3+2\ln(m^2)\}+\frac{5m^2\kappa^3}{128\pi^2}\{-1+\ln(m^2)\}(r^2+s^2+r\cdot s) \qquad (37)$$

Adding the above 3 contributions, and letting $\ln(m^2)\to 3/2$, which value is pertinent to the present order in κ, we obtain

$$\frac{m^2\kappa^3}{768\pi^2}(2r^2+5s^2+5r\cdot s) \qquad (38)$$

Now symmetrizing with respect to r, s, t and dividing by $3!$, we obtain

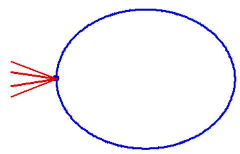
$$\frac{m^2\kappa^3}{512\pi^2}(r^2+s^2+t^2) \qquad (39)$$

Comparing this with the symmetrized trilinear vertex of the original Lagrangian, we obtain the value $(-3m^2\kappa^2/128\pi^2)$ as the correcting 1-loop contribution. We shall discuss this result later on.

6 Quartilinear Contributions

In this section we give the results of 1-loop corrections to the effective quartilinear vertex. In this case, we have five graphic contributions. We give the results to 2nd order in the external momenta r, s, t, u. Symmetrization in the latter will be done after adding the five contributions.

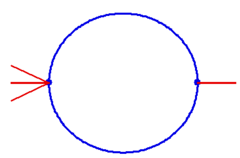
For the 1st graph,



we obtain

$$\frac{m^4\kappa^4}{1536\pi^2}\{-3+2\ln(m^2)\}+\frac{m^2\kappa^4}{768\pi^2}\{-1+\ln(m^2)\}(r^2+s^2+t^2+r\cdot s+r\cdot t+s\cdot t) \tag{40}$$

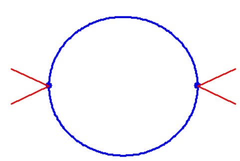
For the 2nd graph,



we obtain

$$-\frac{m^4\kappa^4}{128\pi^2}\{-3+2\ln(m^2)\}-\frac{m^2\kappa^4}{384\pi^2}\{-1+\ln(m^2)\}(5r^2+4s^2+4t^2+4r\cdot s+4r\cdot t+4s\cdot t) \tag{41}$$

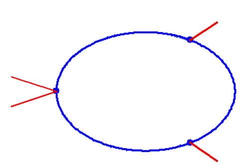
For the 3rd graph,



we obtain

$$-\frac{3m^4\kappa^4}{512\pi^2}\{-3+2\ln(m^2)\}-\frac{m^2\kappa^4}{512\pi^2}\{-1+\ln(m^2)\}(5r^2+5s^2+4t^2+6r\cdot s+4r\cdot t+4s\cdot t) \tag{42}$$

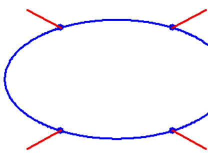
For the 4th graph,



we obtain

$$\frac{3m^4\kappa^4}{64\pi^2}\{-3+2\ln(m^2)\}+\frac{3m^2\kappa^4}{256\pi^2}\{-1+\ln(m^2)\}(5r^2+5s^2+5t^2+6r\cdot s+5r\cdot t+5s\cdot t) \tag{43}$$

For the 5th graph,



we obtain

$$-\frac{5m^4\kappa^4}{128\pi^2}\{-3+2\ln(m^2)\}-\frac{5m^2\kappa^4}{128\pi^2}\{-1+\ln(m^2)\}(r^2+s^2+t^2+r\cdot s+r\cdot t+s\cdot t) \tag{44}$$

Adding the above 5 contributions, and letting $\ln(m^2)\to 3/2$, which value is pertinent to the present order in κ, we obtain

$$-\frac{m^2\kappa^4}{3072\pi^2}(3r^2-s^2-4t^2-16r\cdot s-4r\cdot t-4s\cdot t) \tag{45}$$

Now symmetrizing with respect to r,s,t,u and dividing by 4!, we obtain

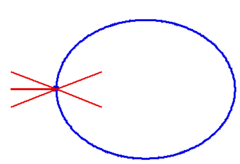
$$-\frac{m^2\kappa^4}{2048\pi^2}(r^2+s^2+t^2+u^2) \tag{46}$$

Comparing this with the symmetrized quartilinear vertex of the original Lagrangian, we obtain the value $(3m^2\kappa^2/128\pi^2)$ as the correcting 1-loop contribution. We shall discuss this result later on.

7 Quintilinear Contributions

In this section we give the results of 1-loop corrections to the effective quintilinear vertex. In this case, we have seven graphic contributions. We give the results to 2nd order in the external momenta r,s,t,u,v. Symmetrization in the latter will be done after adding the seven contributions.

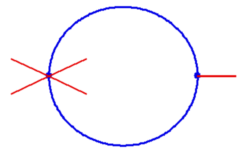
For the 1st graph,



we obtain

$$\begin{aligned}&\tfrac{m^4\kappa^5}{7680\pi^2}\{-3+2\ln(m^2)\}\\&+\tfrac{m^2\kappa^5}{3840\pi^2}\{-1+\ln(m^2)\}(r^2+s^2+t^2+u^2+r\cdot s+r\cdot t+r\cdot u+s\cdot t+s\cdot u+t\cdot u)\end{aligned} \tag{47}$$

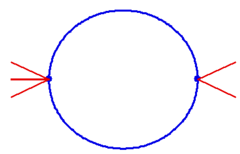
For the 2nd graph,



we obtain

$$-\frac{m^4\kappa^5}{512\pi^2}\{-3+2\ln(m^2)\}$$
$$-\frac{m^2\kappa^5}{1536\pi^2}\{-1+\ln(m^2)\}(5r^2+5s^2+5t^2+5u^2+6r\cdot s+6r\cdot t+6r\cdot u+6s\cdot t+6s\cdot u+6t\cdot u) \tag{48}$$

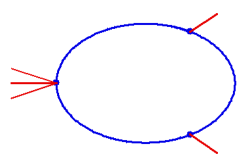
For the 3rd graph,



we obtain

$$-\frac{m^4\kappa^5}{256\pi^2}\{-3+2\ln(m^2)\}$$
$$-\frac{m^2\kappa^5}{768\pi^2}\{-1+\ln(m^2)\}(5r^2+5s^2+5t^2+4u^2+6r\cdot s+6r\cdot t+4r\cdot u+6s\cdot t+4s\cdot u+4t\cdot u) \tag{49}$$

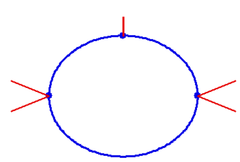
For the 4th graph,



we obtain

$$\frac{m^4\kappa^5}{64\pi^2}\{-3+2\ln(m^2)\}$$
$$+\frac{m^2\kappa^5}{256\pi^2}\{-1+\ln(m^2)\}(5r^2+5s^2+5t^2+5u^2+6r\cdot s+6r\cdot t+5r\cdot u+6s\cdot t+5s\cdot u+5t\cdot u) \tag{50}$$

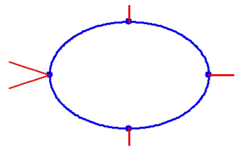
For the 5th graph,



we obtain

$$\frac{3m^4\kappa^5}{128\pi^2}\{-3+2\ln(m^2)\}$$
$$+\frac{3m^2\kappa^5}{512\pi^2}\{-1+\ln(m^2)\}(5r^2+5s^2+5t^2+4u^2+6r\cdot s+5r\cdot t+4r\cdot u+5s\cdot t+4s\cdot u+4t\cdot u) \tag{51}$$

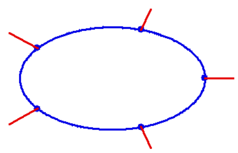
For the 6th graph,



we obtain

$$\begin{aligned}&-\tfrac{5m^4\kappa^5}{64\pi^2}\{-3+2\ln(m^2)\}\\&-\tfrac{m^2\kappa^5}{64\pi^2}\{-1+\ln(m^2)\}(5r^2+5s^2+5t^2+5u^2+6r\cdot s+5r\cdot t+5r\cdot u+5s\cdot t+5s\cdot u+5t\cdot u)\end{aligned} \tag{52}$$

For the 7th graph,



we obtain

$$\begin{aligned}&\tfrac{3m^4\kappa^5}{64\pi^2}\{-3+2\ln(m^2)\}\\&+\tfrac{5m^2\kappa^5}{128\pi^2}\{-1+\ln(m^2)\}(r^2+s^2+t^2+u^2+r\cdot s+r\cdot t+r\cdot u+s\cdot t+s\cdot u+t\cdot u)\end{aligned} \tag{53}$$

Adding the above 7 contributions, and letting $\ln(m^2)\to 3/2$, which value is pertinent to the present order in κ, we obtain

$$\frac{m^2\kappa^5}{15360\pi^2}(2r^2+2s^2+2t^2-33u^2-58r\cdot s+17r\cdot t-38r\cdot u+17s\cdot t-38s\cdot u-38t\cdot u) \tag{54}$$

Now symmetrizing with respect to r, s, t, u, v and dividing by 5!, we obtain

$$\frac{m^2\kappa^5}{10240\pi^2}(r^2+s^2+t^2+u^2+v^2) \tag{55}$$

Comparing this with the symmetrized quintilinear vertex of the original Lagrangian, we obtain the value $(-3m^2\kappa^2/128\pi^2)$ as the correcting 1-loop contribution. We shall discuss this result later on.

8 Discussion

The model treated in this article concerns a scalar field with non-polynomial interactions governed by a dimensional coupling. This treatment should have given a good perspective regarding quantum gravity which has a similar, yet much more complicated, sort of interaction vertices. In fact, the presented scalar model derives its Lagrangian from that of pure Einstein gravity, replacing the metric $g_{\mu\nu}$ by $\eta_{\mu\nu}\exp(\kappa\phi)$, where $\eta_{\mu\nu}$ is the Minkowskian metric.

The application of our divergence-free effective action framework[1] to the scalar model, of this present article, should support the claim that the divergences of quantum gravity would be evaded likewise, providing an attractive resolution to the associated outstanding problem.

Now let us note a few observations regarding the results presented in this article.

First of all, our computations of the corrections to the bilinears have demonstrated the masslessness of the effective field (scalar graviton), at least at the one-loop level. This follows from demanding a vanishing vacuum contribution, fixing the scale via $\ln(m^2)$, where m is an infrared regulating parameter, being the mass of the virtual quantum propagating in the internal lines. Correspondingly, in a true (tensorial) quantum gravitational model, demanding the vanishing of the cosmological term would guarantee the masslessness of the effective graviton. However, the two-loop correction to the bilinears does seem to show that the effective field has acquired a mass term $\sim (m^4\kappa^4)m^2$. However, a closer inspection would reveal that this situation is only an artifact of the approximation needed in evaluating those 2-loop integrals with overlapping momenta.

We have also given the results of computing one-loop corrections to trilinear, quartilinear, and quintilinear vertices. Our purpose of doing such exercise was to check whether the form of the Lagrangian (governed by the invariance under $\phi \to \phi+\text{const}$) is preserved. Our results seem to demonstrate that the corrections to all these vertices has the same magnitude $\sim (3m^2\kappa^2/128\pi^2)$. However, we notice that whereas the correction is positive for even orders (bilinears, quartilinears, etc.), it turns out to be negative for odd counterparts (trilinears, quintilinears, etc.). We checked our computations carefully, however, it seems that this result is genuine, and should be the subject of further thought and consideration. In the case of the tensorial Einstein theory, we would expect that general covariance is so strong that the form of the original Lagrangian cannot be spoiled.

An important demonstration of this article was the presentation of the results of the divergence-free development, which is based on expanding regularized effective propagators, and effective vertices, in terms of ordinary Feynman graphs. Whereas we could not supply, here, all pertinent computational details to be associated with each graphic contribution, interested readers may be able to acquire complete and updated reports on this and other related investigations from the author.[6]

References

[1] N.S. Baaklini, "Effective Action Framework for Divergence-Free Quantum Field Theory", *N.S.B. Letters*, **NSBL-QF-010**; http://www.vixra.org/abs/1312.0056

[2] N.S. Baaklini, "The Divergence-Free Effective Action for a Scalar Field Theory", *N.S.B. Letters*, **NSBL-QF-014**, http://www.vixra.org/abs/1312.0065

[3] N.S. Baaklini, "The Divergence-Free Effective Action for Quantum Electrodynamics", *N.S.B. Letters*, **NSBL-QF-015**, http://www.vixra.org/abs/1401.0013

[4] N.S. Baaklini, "Framework for the Effective Action of Quantum Gauge and Gravitational Fields", *N.S.B. Letters*, **NSBL-QF-011**, http://www.vixra.org/abs/1401.0121

[5] N.S. Baaklini, "Graphs and Expressions for Higher-Loop Effective Quantum Action", *N.S.B. Computing*, **NSBC-QF-005**, http://www.vixra.org/abs/1402.0085

[6] *Divergence-Free Quantum Field Theory* by N.S. Baaklini (Complete Updated Development)

For Those Who Seek True Comprehension of Fundamental Theoretical Physics