Pi Formulas , Part 24

Edgar Valdebenito

abstract

In this note we show some formulas related with the constant Pi

Número π, Integral de Función Racional Cuártica

EDGAR VALDEBENITO
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Resumen. Se muestra una fórmula que involucra el número π .

1 Introducción

Recordamos un clásico resultado:

$$\int_{-\infty}^{\infty} \frac{dx}{\left(x-(A+Bi)\right)\left(x-(A-Bi)\right)\left(x-(C+Di)\right)\left(x-(C-Di)\right)} =$$

$$= \frac{\pi\left(B+D\right)}{BD\left(\left(A-C\right)^2+\left(B+D\right)^2\right)}$$

$$A,B,C,D \in \mathbb{R} \ , B>0, D>0$$

2 Fórmula

Sean $A,B,C,D \in \mathbb{R}, B>0, D>0$; Sean a_0, a_1, a_2, a_3, a_4 , definidos como sigue:

$$a_0 = \left(A^2+B^2\right)\left(C^2+D^2\right)$$
$$a_1 = -2A\left(C^2+D^2\right)-2C\left(A^2+B^2\right)$$
$$a_2 = A^2+B^2+C^2+D^2+4AC$$
$$a_3 = -2\left(A+C\right)$$
$$a_4 = 1$$

Sean $p>0, q>0$, tales que:

$$\left|\frac{a_1 x + a_2 x^2 + a_3 x^3 + x^4}{a_0}\right| < 1 \quad , \quad 0 < x < p$$

$$\left|\frac{-a_1 x + a_2 x^2 - a_3 x^3 + x^4}{a_0}\right| < 1 \quad , \quad 0 < x < p$$

$$\left|a_3 x + a_2 x^2 + a_1 x^3 + a_0 x^4\right| < 1 \quad , \quad 0 < x < \frac{1}{q}$$

$$\left|-a_3 x + a_2 x^2 - a_1 x^3 + a_0 x^4\right| < 1 \quad , \quad 0 < x < \frac{1}{q}$$

Sea R definido por:

$$R = \frac{B + D}{BD\left((A - C)^2 + (B + D)^2\right)}$$

Sea $I(a_0, a_1, a_2, a_3, a_4, s, m)$ la función definida por:

$$I(a_0, a_1, a_2, a_3, a_4, s, m) = \sum_{n=0}^{\infty} \frac{(-1)^n}{a_0^{n+1}} \sum_{k1=0}^{n} \sum_{k2=0}^{k1} \sum_{k3=0}^{k2} T\begin{pmatrix} n, k1, k2, k3 \\ a_1, a_2, a_3, a_4, s, m \end{pmatrix}$$

donde

$$T = \binom{n}{k1}\binom{k1}{k2}\binom{k2}{k3} a_1^{n-k1} a_2^{k1-k2} a_3^{k2-k3} a_4^{k3} \frac{s^{n+k1+k2+k3+m+1}}{n + k1 + k2 + k3 + m + 1}$$

con $s > 0, m > 0$.

Se tiene:

$$\pi R = I\left(a_0, a_1, a_2, a_3, 1, p, 0\right) + I\left(1, a_3, a_2, a_1, a_0, \frac{1}{q}, 2\right) +$$

$$+ I\left(a_0, -a_1, a_2, -a_3, 1, p, 0\right) + I\left(1, -a_3, a_2, -a_1, a_0, \frac{1}{q}, 2\right) +$$

$$+ \int_p^q \frac{dx}{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + x^4} + \int_p^q \frac{dx}{a_0 - a_1 x + a_2 x^2 - a_3 x^3 + x^4}$$

3 Referencias

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