Time dependent analysis of the 3S2 Normal Mode

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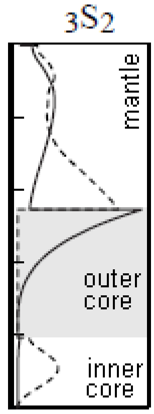
Abstract: The time dependent analysis of the normal mode $_3S_2$ near 1102.8 µHz allows a precise determination of the frequency and reveals a surprising phase modulation. This phase flip was first observed with the $_{10}S_2$ normal mode and, by careless use of FFT, it may be misinterpreted as a frequency splitting. The correct demodulation requires the very robust method of homodyn conversion.

Introduction



After earthquakes, the Earth vibrates like a bell and a set of different natural frequencies is recorded by various instruments. According to the theory, the mode $_3S_2$ may be well suited as a probe to study certain properties of the Earth's inner core. In the drawing, the dashed curve shows the assumed value of shear energy depending on the distance from the center of the earth. In the liquid outer core (shown in gray), this value is zero. The measured inversions of the phase can not be explained by an isotropic model of the Earth and may even be caused by properties of the inner Earth's core. Perhaps it is consistent with simple axisymmetric models of anisotropy.

The Preparation of the data records

During the first hours after a strong earthquake, the amplitude of the spectral line near 1103 µHz exceeds clearly the noise and can be easily observed. Later, one has to filter a very narrow frequency band in order to discover these weak oscillations at all. With careless use of standard filtering software, the huge dynamic range of more than 100 dB generates intermodulation and numeric noise in standard math co-processors, whereby the SNR deteriorates noticeably.

In a first step, the raw data (the most common sampling time is one second) of all available SG stations between 2004-12-26 and 2005-01-31 were made machine readable. The influence of atmosphere pressure variation on the gravity data was omitted because the air mass above the instruments changes much slower than the oscillation time of $_3S_2$ (about 15 minutes). Extensive experiments have shown that any admixture of air pressure data deteriorates the SNR, regardless of the sign and value of the prefactor.

The extremely strong tides are removed very easily with a feedforward comb filter. It has all the positive characteristics of an FIR filter, is extremly fast and does not generate nonlinear distortions of the data. The application is very simple: all samples are shifted by 454 seconds and subtracted from the original record. The ratio of this time difference to the period(s) of the oscillations produces the desired effect: The very intense amplitudes of the slow tides ($T \approx$ severals hours) are largely compensated, while the amplitude of the much faster $_3S_2$ oscillations ($T \approx$ 15 minutes) is doubled. A low-pass filter ($f <$ 4 ms), followed by a second "*shift and subtract*" procedure generates the final data string. A multiple application does not bring further improvement. After the comb filter has reduced the dynamic range of the data below 40 dB, the mean frequency of $_3S_2$ is shifted to the intermediate frequency 200 µHz and the bandwidth is limited to 14 µHz or less, using the mixing method[1]. The absolute frequency difference Δf between all spectral lines remains constant while the ratio $\Delta f / f$ increases, facilitating the separation of adjacent spectral lines. This simplifies the data analysis and allows to prolong the sampling rate of the record to 540 seconds without altering the information content. After the very positive experience with this type of processing during the analysis of the $_{10}S_2$ normal mode[2], the comparison with the CORMIN data was omitted in this paper. IIR filters were not used in this study.

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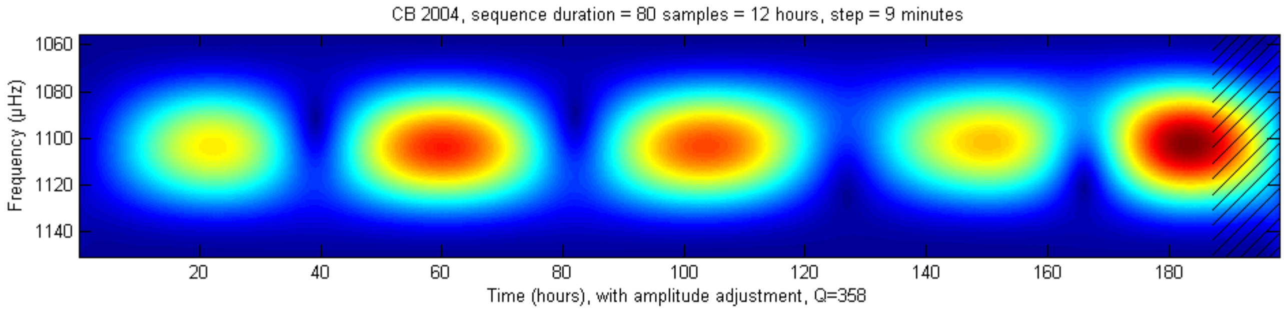
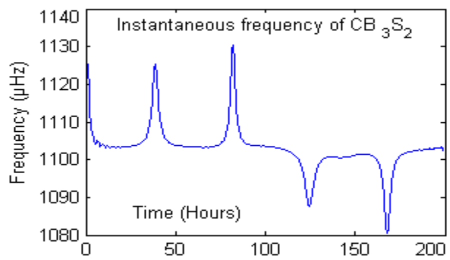
The Temporal Evolution of $_3S_2$

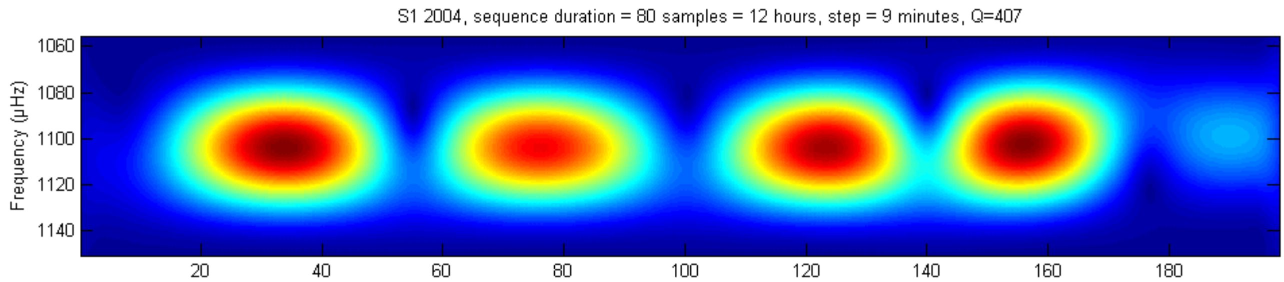
The amplitudes of all normal modes decrease exponentially, leading to a substantial increase in spectral half width (as with any modulation). Compensating this amplitude change by a suitable time-dependent factor decreases the FWHM and allows the separation of adjacent spectral lines. By the way, this ensures a good time independence of color in the spectrogram. In the figure below, the exponential decay of the $_3S_2$ amplitude was compensated in accordance with the *Q*-factor of 358. The optimal value is sometimes hard to find, because it depends on the SNR.

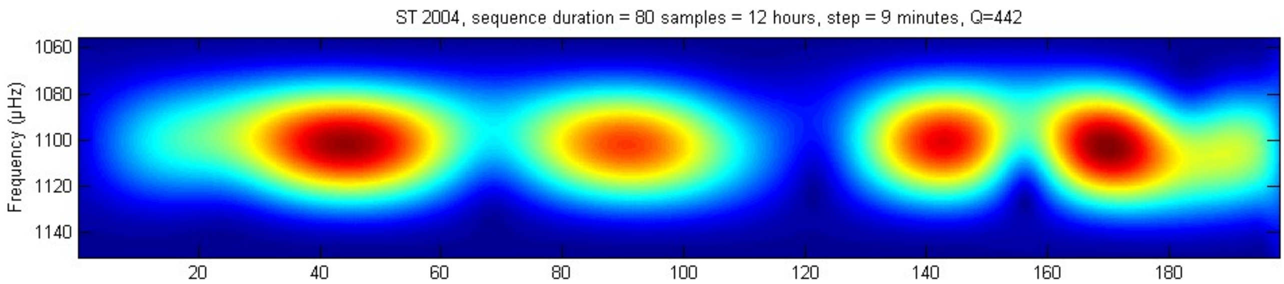


The spectrogram is composed of 1321 individual spectra, each one generated from a sequence of 80 samples (zero-padded to a total length 4096) and whose start times advance in steps of 540 seconds. The representation within the shaded area on the very right must be ignored, because about 200 hours after the earthquake, the $_3S_2$ oscillation disappears in the noise.



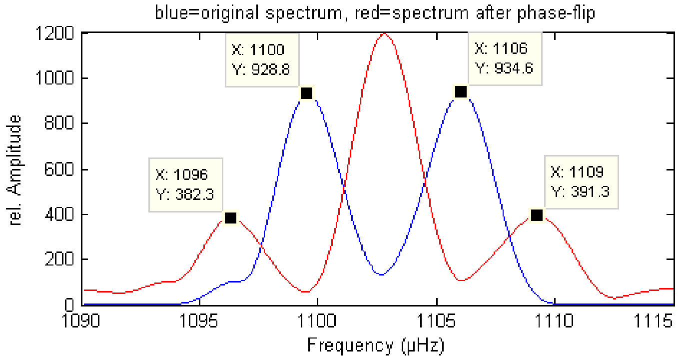
Using the complex results of the FFT along a horizontal line at f = 1100 µHz, 1321 instantaneous phases can be calculated. The time derivative of this phase sequence provides the instantaneous frequency for each time point. The figure shows that near an amplitude minimum, the frequency is calculated incorrectly because of the bad SNR. Below we will see that the actual reason is a phase inversion. Ignoring these defects and confining to regions of high amplitude, the weighted average provides f_{mean} ≈ 1103 µHz. From the Küpfmüller's uncertainty principle follows that the short sequence length of only 12 hours limits the frequency resolution to approximately 23 µHz.





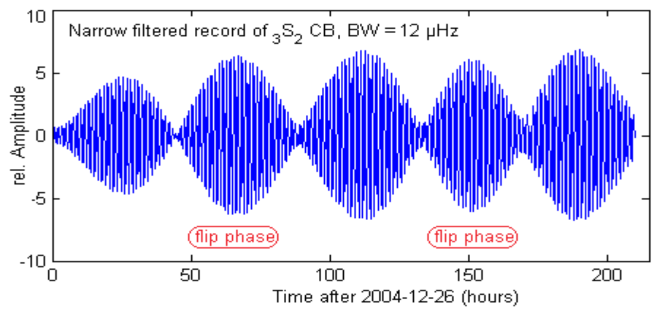
The comparison of three very widely separated stations CB, S1 and ST shows that the distinct amplitude maxima repeat at intervals of approximately 43 hours. Obviously, the absolute timing is determined by the geographical location of the recording SG station.

The Spectrum of ${}_3S_2$



To achieve a high time resolution, the above spectrograms were produced with short data sequences of only twelve hours. On closer inspection it turns out that each of the 1321 spectra shows a *single* peak near 1103 Hz, there is no splitting into two frequencies. Because of the very short sequence length, this peak has a large FWHM of more than 20 µHz, as seen in the spectrograms above.

That changes little, as long as the length of the sequences is less than about 50 hours. But if one calculates the total spectrum over the entire period of 210 hours, the Fourier analysis provides a result with a double peak (blue curve). Why? It is impossible to change the frequency of a normal mode of the earth by extending the measurement period. Incrementing the sequence length in small steps indicates the cause. As soon as the sequence length exceeds 60 hours, the first signs of a double peak appear. With increasing length, the split gets more pronounced. But that concerns only the indicated amplitudes, not the frequency difference between the (blue) maxima. The careless use of non-adapted or overlong sequence lengths in the spectral analysis is probably the reason that ${}_3S_2$ is classified as “anomalously” split mode[3].



Looking at the time series of the recorded data, one recognizes an amplitude profile which is well known in radio technology, called [Double-sideband with suppressed carrier]() (DSB-SC). That is a is a special case of amplitude modulation with two characteristic features: First, the (positive) envelope is not sinusoidal but the minima are much sharper formed than the maxima. Strictly speaking, the envelope with the correct phase relation traverses the zero line, changing its sign. Second, a each zero crossing of the envelope, there is a phase jump of the underlying oscillation by π (180 degrees). Without exception, both criteria are confirmed by the measurements of all eighten SG stations.

Analysis of the *modified* data

In order to prove the periodic phase reversal, the sign in every second oscillation packet (the segments marked in red) is reversed. That leads to the *red* spectrum above with *one* central peak and two sideband frequencies. The exact frequencies of those three maxima (*with* phase flip!) are:

Lower Sideband (red)	Carrier (red)	Upper Sideband (red)
1096.35 µHz	1102.81 µHz	1109.29 µHz

From the difference between the sidebands and the carrier frequency, the (visual) oscillation period of the modulation can be calculated, yielding 42.9 hours. But that is only half the story, because these results were obtained with *modified* data. So far, the results are valid only if one ignores the regular phase reversal.

The split into two peaks is produced by the measurement procedure and is apparently no intrinsic feature of the natural resonance of the earth. It is the result of inappropriate sequence length ignoring the phase jumps. The analysis of the normal mode ${}_{10}S_2$ (special case CB)[2] shows the same dependence of the FFT result as a function of the sequence length.

Analysis of the unchanged data

The analysis of the actual measured data (the only changes: compensation of exponential decreasing amplitude) gives the blue spectrum above with the two peaks 1099.57 µHz and 1106.04 µHz. Following the DSB-SC approach, those two frequencies may be regarded as the result of a mixing process of two oscillations with the individual frequencies f_c and f_m, described by the formula

$$A = ½ \cdot A_c A_m (\cos(2\pi t(f_c - f_m)) + \cos(2\pi(f_c + f_m)))$$

The solution of $f_c - f_m$ = 1099.57 µHz and $f_c + f_m$ = 1106.04 µHz yields the (suppressed) carrier frequency f_c = 1102.81 µHz and the modulation frequency f_m = 3.24 µHz. Applying a well known trigonometric identity, we get the mathematical interpretation as a nonlinear process

$$A = A_c \cos(2\pi f_c t) \cdot A_m \cos(2\pi f_m t)$$

The physical interpretation is not as simple, because there is no known mechanism inside or outside the earth with the very low frequency f_m, corresponding to an oscillation period T_m = 85.86 hours. Perhaps this periodicity can be explained by the relative rotation of the inner Earth's core relative to the mantle.

It must be emphasized that all values above are largely independent of the program parameters, as long as the bandwidth remains between 10 µHz and 40 µHz. A too low bandwidth cuts off the side-bands and distorts and smears the signal. With excessive bandwidth, the signal soon gets unreadable because of too much noise.

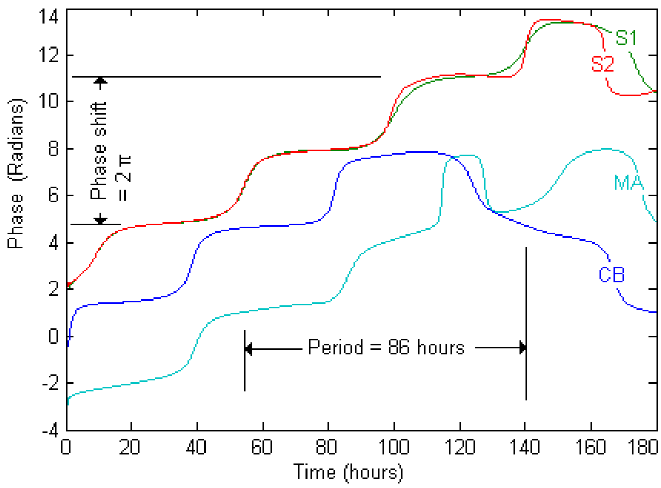
Coherent Detection

The decline of the $_3S_2$ amplitude is compensated by multiplication with the factor $\exp\left(\frac{\omega t}{2Q}\right)$. The elimination of this modulation reduces the bandwidth of the signal and allows to narrow the band-width of the filters in order to remove unwanted noise. A look at the spectrograms shows that the normal mode $_3S_2$ seems to be amplitude modulated and the phase changes periodically. The poor resolution of the FFT does not allow to exclude a frequency modulation. Any type of modulation produces sideband frequencies and it is important to identify these influences.

In the first attempts to decipher the signal (before regular phase inversion has been detected), the strongest frequency component was tracked by a Phase-locked loop, because the "message content" can be derived from the phase control signal. During the lengthy investigation both PI and also PID controllers were used and optimized. But unfortunately, the results depend heavily on the particular choice of the control parameters. Also, the Costas loop has been studied thoroughly, since it was designed to cope with phase jumps. But this method also fails to meet the expectations. Here too, any change of the parameters influences very strongly the results. Common cause is that both prin-ciples are based on feedback. Every phase inversion generates a step response, producing unde-sirable control fluctuations that last for a long time. And during the short data gaps in the vicinity of phase reversals, the control loop tries to break away.

In contrast, the direct conversion leads to very good results and is robust even with very noisy data. Main advantage over all other methods is that are only two parameters to play with: The bandwidth of the filter and the inital frequency of the iterative search. The method was described in detail here[2]. The *in-phase* and the *quadrature* signal are used to calculate the phase as a function of time. After lock-in (zero-beat) to an unmodulated frequency, the phase is constant, that is a horizontal line. But in case of $_3S_2$, each phase inversion produces a step of height π. Because at the same time the amplitude of $_3S_2$ falls below the noise level, the direction of the step is random and may be posi-tive or negative. Choosing a wide bandwidth, the steps look rectangular, but they are very noisy and often disturbed by additional jumps. Decreasing the bandwidth makes the corners rounded and smooth. The optimum bandwidth for $_3S_2$ seems to be about 20 µHz.

Preliminary estimates show that it takes about 86 hours until the signal has completed two phase inversions. The exact frequency of the normal mode ${}_3S_2$ is determined by iteration. Frequency and phase of a local oscillator are varied until the long-term mean value at the output of the mixer is zero. This is called *zero beat,* we have lock-in. Normally, this signal (called *in-phase* component) fluctuates - either by noise or by frequency modulation. But the repeated phase jumps in the ${}_3S_2$ records prevent zero beat. In order to work around this problem, the iteration criterion is changed. Now the frequency of the local oscillator is varied until after two phase inversions, the ampliude is high *and* the phase difference equals either zero or 2π. The main advantage in comparison with the Fourier analysis is that the result does *not* depend on the very marked fluctuations of the ${}_3S_2$ amplitude. Here are the results:



SG Station	Frequency (µHz)	Period (hours)	Rel. amplitude	First Maximum (hour)	Q factor
CB	1102.874	81.3	13	23.5	358
ES	1102.749	81.63	7	25.67	462
H1	1102.546	98.8	3	43	507
H2	1102.507	100.13	3.8	43.67	452
M1	1102.578	81.5	5.6	41.5	365
MA	1102.788	84.13	4	21.67	472
MB	1102.733	95	2	44.5	612
MC	1102.032	92	6	41.5	385
ME	1102.519	90	1.9	26.67	484
NY	1102.511	90.33	1.1	21.17	650
S1	1102.976	90.33	12	33.67	416
S2	1102.998	91.03	12.5	33.17	424
ST	1102.956	94.37	3.8	44.33	416
TC	1102.523	82.67	10	39.33	385
VI	1103.018	90.8	3.4	33	483
W1	1102.875	94.8	3.2	37.5	484
W2	1102.835	95.2	2.7	38.5	484
WU	1102.575	83.33	6	33.67	904
average	1102.755 ± 0.047	89.85			

The average frequency was computed with the jackknife method. During the regular phase reversal of the oscillation, the measurable amplitude is very small. This pretends a very strong periodic amplitude modulation, which does not exist in reality. No signs of a frequency modulation were discovered. Within the large range of (1103 ± 40) µHz, no reproducible spectral lines were identified which are somehow linked to ${}_3S_2$.

Summary

The measured values differ significantly from the predictions of the PREM model (f_{3S2} = 1106.21 μHz)[4][5]. In no study, the time-dependent flip of the phase is mentioned and probably it was never contemplated.

Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[6].

[1] H. Weidner, A New method for High-resolution Frequency Measurements, http://viXra.org/abs/1506.0005
[2] H. Weidner, Time dependent analysis of the 10S2 Quintet, http://viXra.org/abs/1602.0357
[3] G. Roult, Observation of split modes from the 26th December 2004 Sumatra-Andaman megaevent, https://www.researchgate.net/publication/223015769
[4] A. Mäkinen, A. Deuss, Normal mode splitting function measurements of anelasticity and attenuation in the Earth's inner core, Geophys. J. Int. (2013) 194, 401-416
[5] J. Durek, B. Romanowicz, Inner core anisotropy inferred by direct inversion of normal mode spectra, Geophys. J. Int. (1999) 139, 599-622
[6] The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html