Planck Quantization of Newton and Einstein Gravitation

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Abstract

In this paper we rewrite the gravitational constant based on its relationship with the Planck length and, based on this, we rewrite the Planck mass in a slightly different form (that gives exactly the same value). In this way we are able to quantize a series of end results in Newton and Einstein's gravity theories. The formulas will still give exactly the same values as before, but everything related to gravity will then come in quanta. Numerically this only has implications at a quantum scale; for macro objects the discrete steps are so tiny that they are close to impossible to notice. Hopefully this can give additional insight into how well (ad-hoc) quantized Newton and Einstein gravity are potentially linked with the quantum world.

Key words: Quantized gravitation, gravitational constant, escape velocity, gravitational time dilation, Schwarzschild radius, Planck length, bending of light, Planck mass, Planck lenght.

1 Foundation

We suggest that the gravitational constant should be written as a function of Planck's reduced constant

$$G_p = \frac{\aleph^2 c^3}{\hbar} \tag{1}$$

where $\hbar$ is the reduced Planck's constant and c is the well tested round-trip speed of light. We could call this Planck's form of the gravitational constant. The parameter $\aleph$ is an unknown constant that is calibrated so that G_p matches our best estimate (measurment) for the gravitational constant.

As shown by Haug (2016) the Planck form of the gravitational constant enables us to rewrite the Planck length as

$$l_p = \sqrt{\frac{\hbar G_p}{c^3}} = \sqrt{\frac{\hbar \frac{\aleph^2 c^3}{\hbar}}{c^3}} = \aleph \tag{2}$$

and the Planck mass as

$$m_p = \sqrt{\frac{\hbar c}{G_p}} = \sqrt{\frac{\hbar c}{\frac{\aleph^2 c^3}{\hbar}}} = \frac{\hbar}{\aleph}\frac{1}{c} \tag{3}$$

Using the gravitational constant in the Planck form, as well as the rewritten Planck units, we are easily able to modify a series of end results from Newton and Einstein's gravitational theories to contain quantization as well.

2 Newton Universal Gravitational Force

The Newton gravitational force is given by

$$F_G = G_p \frac{m_1 m_2}{r^2}. \tag{4}$$

Using the gravitational constant of the form $G_p = \frac{\aleph^2 c^3}{\hbar}$ and the Planck mass of $m_p = \frac{\hbar}{\aleph}\frac{1}{c}$ we can rewrite the Newton gravitational force for two Planck masses as

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$$\begin{aligned} F_{GP} &= G_p \frac{m_p m_p}{r^2} \\ F_{GP} &= \frac{\aleph^2 c^3}{\hbar} \frac{\frac{\hbar}{\aleph}\frac{1}{c}\frac{\hbar}{\aleph}\frac{1}{c}}{r^2} = \frac{\hbar c}{r^2}. \end{aligned} \tag{5}$$

In the special case where $r = \aleph$ we get

$$F_{GP} = \frac{\hbar}{\aleph}\frac{c}{\aleph}. \tag{6}$$

It seems from this that gravity could be interpreted as hits per second. For large masses the form will be

$$\begin{aligned} F_{GP} &= G_p \frac{N m_p Y m_p}{r^2} \\ F_{GP} &= G_p \frac{N Y m_p^2}{r^2} \\ F_{GP} &= N Y \frac{\hbar}{r^2} c \end{aligned} \tag{7}$$

where N is the number of Planck masses in object one and Y is the number of Planck masses in object two. In the case when the two masses are of equal size we have

$$F_{GP} = N^2 \frac{\hbar}{r^2} c. \tag{8}$$

3 Escape Velocity at the Quantum Scale

The traditional escape velocity is given by

$$v_e = \sqrt{\frac{2GM}{r}} \tag{9}$$

where G is the traditional gravitational constant and M is the mass of the object we are "trying" to escape from, and r is the radius of that object. In other words, we stand at the surface of the object, for example a hydrogen atom or a planet. Based on the gravitational constant written in the Planck form we can find the escape velocity at Planck scale; see also the Appendix for a derivation from "scratch". It must be

$$\begin{aligned} v_{e,p} &= \sqrt{\frac{2G_p m_p}{r}} \\ v_{e,p} &= \sqrt{\frac{2N \frac{\aleph^2 c^3}{\hbar} \frac{\hbar}{\aleph}\frac{1}{c}}{r}} \\ v_{e,p} &= \sqrt{\frac{2N\aleph c^2}{r}} \\ v_{e,p} &= c\sqrt{\frac{2N\aleph c^2}{r}} \end{aligned} \tag{10}$$

where N is the number of Planck masses in the planet or mass in question.

A particular interesting case is when we only have one Planck mass $N = 1$ and $r = 2\aleph$ (this is actually the Schwarzschild radius of a Planck mass object). This gives us

$$\begin{aligned} v_{e,p} &= c\sqrt{\frac{2 \times 1 \times \aleph}{2\aleph}} \\ v_{e,p} &= c \end{aligned} \tag{11}$$

The escape velocity for a particle with Planck mass with radius $2\aleph$ is c. Next we will see if the formula above can also be used to calculate the escape velocity of Earth. The Earth's mass is $5,972 \times 10^{24}$ kg. We must convert this to the number of Planck masses. The Planck mass is

$$m_p = \frac{\hbar}{\aleph^2}\frac{1}{c} \approx 2,17651 \times 10^{-8}$$

The Earth's mass in terms of the numbers of Planck masses must be $\frac{5.972\times10^{24}}{2,17651\times10^{-8}} \approx 2.74388 \times 10^{32}$. Further the radius of the Earth is $r \approx 6371000$ meters. We can now just plug this into the Planck scale escape velocity:

$$v_{e,p,N} = c\sqrt{\frac{2N\aleph}{r}} \tag{12}$$

where the N simply is the numbers of Planck masses. This gives us

$$\begin{aligned}
v_{e,p} &= c\sqrt{\frac{2N\aleph}{r}} \\
v_{e,p} &= 299792458 \times \sqrt{\frac{2 \times 2.74388 \times 10^{32} \times 1.61622837 \times 10^{-35}}{6371000}} \approx 11185.7\text{meters/second}
\end{aligned}$$

Which is equal to 40,269 km/s, which is the well known escape velocity from the Earths gravitational field. We think our new way of looking at gravity could have consequences for the understanding of gravity. Gravitation must come in discrete steps and the escape velocity must also come in discrete steps for a given radius; this is because the amount of matter likely comes in discrete steps.

4 Gravitational Time Dilation at Planck Scale

Einstein's gravitational time dilation is given by

$$t_0 = t_f\sqrt{1 - \frac{2GM}{rc^2}} = \sqrt{1 - \frac{v_e^2}{c^2}} \tag{13}$$

where v_e is the traditional escape velocity. We can rewrite this in the form of quantized escape velocity (derived above).

$$\begin{aligned}
t_o &= t_f\sqrt{1 - \frac{v_{e,p,N}^2}{c^2}} \\
t_o &= t_f\sqrt{1 - \frac{\left(c\sqrt{2N\frac{\aleph}{r}}\right)^2}{c^2}} \\
t_o &= t_f\sqrt{1 - \frac{2N\aleph}{r}}
\end{aligned}$$

Let's see if we can calculate the time dilation at, for example, the surface of the Earth from Planck scale gravitational time dilation. The Earth's mass is $5,972 \times 10^{24}$ kg. And again, the Earth's mass in terms of the Planck mass must be $\frac{5.972\times10^{24}}{2,17651\times10^{-8}} \approx 2.74388 \times 10^{32}$. Further, the radius of the Earth is $r \approx 6371000$ meters. We can now just plug this into the quantized gravitational time dilation

$$\begin{aligned}
t_o &= t_f\sqrt{1 - \frac{2N\aleph}{r}} \\
t_o &= t_f\sqrt{1 - \frac{2 \times 2.74388 \times 10^{32} \times 1.61622837 \times 10^{-35}}{6371000}} \approx t_f \times 0.999999999303915
\end{aligned}$$

That is for every second that goes by in outer space (a clock far away from the massive object), 0.99999999930391500 seconds goes by on the surface of the Earth. That is for every year in in outer space (very far from the Earth), there is about 22 milliseconds left to reach a Earth year. This is naturally the same as we would get with Einstein's formula. Still, the new way of writing the formula gives additional insight.

5 The Schwarzschild Radius

The Schwarzschild radius of a mass M is given by

$$r_s = \frac{2GM}{c^2} \tag{14}$$

Rewritten into the quantum realm as described in this article it must be

$$\begin{aligned} r_s &= \frac{2G_pM}{c^2} \\ r_s &= \frac{2G_pNm_p}{c^2} \\ r_s &= \frac{2\frac{\aleph^2c^3}{\hbar}N\frac{\hbar}{\aleph}\frac{1}{c}}{c^2} \\ r_s &= 2N\aleph. \end{aligned} \tag{15}$$

For a clock at the Schwarzschild radius we get a time dilation of

$$t_o = t_f\sqrt{1-\frac{2N\aleph}{2N\aleph}} = 0. \tag{16}$$

At the Schwarzschild radius, time stands still. For a radius shorter than that the gravitational time dilation equation above breaks down[1]

6 Gravitational Acceleration/Field

The gravitational acceleration field in modern physics is given by

$$g = \frac{GM}{r^2}. \tag{17}$$

This can be rewritten in quantized form as

$$\begin{aligned} g &= \frac{G_pM}{r^2} \\ g &= \frac{\frac{\aleph^2c^3}{\hbar}N\frac{\hbar}{\aleph}\frac{1}{c}}{r^2} \\ g &= \frac{N\aleph}{r^2}. \end{aligned} \tag{18}$$

7 Gravitational Parameter

The standard gravitational parameter is given by

$$\mu = GM \tag{19}$$

This can be rewritten in quantized form as

$$\begin{aligned} \mu_p &= G_pM \\ \mu_p &= G_pNm_p \\ \mu_p &= \frac{\aleph^2c^3}{\hbar}N\frac{\hbar}{\aleph}\frac{1}{c} \\ \mu_p &= N\aleph c^2. \end{aligned} \tag{20}$$

8 Quantized Gravitational Bending of Light

The angle of deflection in Einstein's General relativity theory is given by

$$\delta_{GR} = \frac{4GM}{c^2r}$$

This can be rewritten as

[1]Except if we assume the $\aleph$ represents the radius of an indivisible particle. Thus if we move away from the point particle concept, this would simply mean that we could not go below the Planck scale Schwarzschild Radius.

$$\delta_{GRH} = \frac{4G_pM}{c^2r}$$
$$\delta_{GRH} = \frac{4G_pNm_p}{c^2r}$$
$$\delta_{GRH} = \frac{4\frac{\aleph^2c^3}{\hbar}N\frac{\hbar}{\aleph}\frac{1}{c}}{c^2r}$$
$$\delta_{GRH} = \frac{4N\aleph}{r} \tag{21}$$

where N is the number of Planck masses making up the mass we are interested in. From the formula above, this means that the deflection of angles comes in quanta. Lets also "control" that our Planck scale deflection rooted in Planck and GR is consistent for large bodies like for example the Sun. The solar mass is $M_s \approx 1.988 \times 10^{30}$ kg. The Sun's mass in terms of the number of Planck masses must be $\frac{1.988\times10^{30}}{2,17651\times10^{-8}} \approx 9.134 \times 10^{37}$. Further, the radius of the Sun is $r_s \approx 696342000$ meters. We can now just plug this into the Planck scale deflection:

$$\delta_{GRH} = \frac{4N\aleph}{r} = \frac{4 \times 9.134 \times 10^{37} \times 1.61622837 \times 10^{-35}}{696342000} \approx 8.48 \times 10^{-06} \tag{22}$$

If we multiply this by $\frac{648000}{\pi}$ we get a bending of light of about 1.75 arcseconds or $\frac{1.75}{3600}$ of a degree. This is the same as has been confirmed by experiments and helped make Einstein famous, as Newton gravitation supposedly only predicted half of the bending of light. Newton bending of light is given by

$$\delta_{Newton} = \frac{2GM}{c^2r} \tag{23}$$

See for example Soares (2009) and Momeni (2012) for derivations of bending of light under Newton gravitation.

8.1 Table Summary

The table below summarizes our rewriting of some gravitational formulas. The output is still the same, but based on this view of gravity, masses, gravitational time dilation, and even escape velocity all come in discrete steps:

Table 1: The table shows some of the standard gravitational relationships given by Newton and Einstein and their expression in quantized form.

Units:	**Newton and Einstein form:**	**Quantized-form:**
Gravitational constant	$G \approx 6.67408 \times 10^{-11}$	$G_p = \frac{\aleph^2c^3}{\hbar}$
Newton's gravitational force	$F_G = G\frac{M_1M2}{r^2}$	$F_G = NY\frac{\hbar c}{r^2}$
Newton's gravitational force	$F_G = G\frac{m_1m2}{\aleph^2}$	$F_G = G_p\frac{m_pm_p}{r^2} = \frac{\frac{\aleph^2c^3}{\hbar}\frac{\hbar}{\aleph}\frac{1}{c}\frac{\hbar}{\aleph}\frac{1}{c}}{\aleph^2} = \frac{\hbar}{\aleph}\frac{c}{\aleph}$
Escape velocity from a single Planck mass		$v_{e,p} = c\sqrt{\frac{2\aleph}{r}}$
Escape velocity from any mass	$v_e = \sqrt{\frac{2GM}{r}}$	$v_{e,p,N} = c\sqrt{\frac{2N\aleph}{r}}$
Escape velocity at $r = 2\aleph$		$v_{e,p} = c\sqrt{\frac{2\aleph}{2\aleph}} = c$
Planck mass with velocity at $r < 2\aleph$	Cannot escape=min Black Hole	Depends on quantum interpretation
Gravitational time dilation	$t_0 = t_f\sqrt{1 - \frac{2GM}{c^2}} = \sqrt{1 - \frac{v_e^2}{c^2}}$	$t_o = t_f\sqrt{1 - \frac{2N\aleph}{r}}$
Schwarzschild radius	$r_s = \frac{2GM}{c^2}$	$r_s = 2N\aleph$
Gravitational acceleration field	$g = \frac{GM}{r^2}$	$g_p = \frac{N\aleph}{r^2}$
Gravitational parameter	$\mu = GM$	$\mu_p = N\aleph c^2$.
Bending of light	$\delta_{GR} = \frac{4GM}{c^2r}$	$\delta_{GRH} = \frac{4N\aleph}{r}$
Black holes	Possible	Depends on quantum interpretation

9 Conclusion

By making the gravitational constant a function form of the reduced Planck constant one can easily rewrite many of the end results from Newton and Einstein's gravitation in quantized form. Even if this is seen as an ad-hoc method, it could still give new insight into what degree quantized Newton's gravitation and General relativity are consistent with the quantum realm.

10 Appendix: Escape velocity

Derivation of the escape velocity from Planck scale

$$\begin{aligned} E &= \frac{1}{2}mv^2 - \frac{GmM_s}{r} \\ E &= \frac{1}{2}\frac{\hbar}{\aleph}\frac{1}{c}v^2 - \frac{\frac{\aleph^2 c^3}{\hbar}\frac{\hbar}{\aleph}\frac{1}{c}N\frac{\hbar}{\aleph}\frac{1}{c}}{r} \\ E &= \frac{1}{2}\frac{\hbar}{\aleph}\frac{1}{c}v^2 - N\frac{\hbar}{r}c \end{aligned} \tag{24}$$

This we have to set to 0 and solve with respect to v to find the escape velocity:

$$\begin{aligned} \frac{1}{2}\frac{\hbar}{\aleph}\frac{1}{c}v^2 - N\frac{\hbar}{r}c &= 0 \\ v^2 &= 2\frac{N\frac{\hbar}{r}c}{\frac{\hbar}{\aleph}\frac{1}{c}} \\ v^2 &= 2\frac{N\aleph c^2}{r} \\ v^2 &= c\sqrt{\frac{2N\aleph}{r}}. \end{aligned} \tag{25}$$

References

EINSTEIN, A. (1916): "Näherungsweise Integration der Feldgleichungen der Gravitation," *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin.*

HAUG, E. G. (2016): "The Gravitational Constant and the Planck's Constants. A Deeper Understanding of the Quantum Realm," *www.ViXra.org March 13 2016.*

MOMENI, D. (2012): "Bending of Light a Classical Analysis," *Working paper*, pp. 1–4.

NEWTON, I. (1686): *Philosophiae Naturalis Principia Mathematics.* London.

PLANCK, M. (1901): "Ueber das Gesetz der Energieverteilung im Normalspectrum," *Annalen der Physik*, 4.

SCHWARZSCHILD, K. (1916a): "Über das Gravitationsfeld einer Kugel aus Inkompressibler Flussigkeit nach der Einsteinschen Theorie," *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik*, p. 424.

——— (1916b): "Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie," *Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik*, p. 189.

SOARES, D. S. L. (2009): "Newtonian Gravitational Deflection of Light Revisited," *Working Paper version 4.*