Pi Formulas , Part 19

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abstract

In this note we show some formulas related with the constant Pi

Algunas Fórmulas Que Involucran La Constante Pi

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05-10-2014 7:41:34

Resumen. En esta nota mostramos algunas fórmulas que involucran la constante Pi:

$$\pi = 4\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = 3.14159265\ldots$$

Introducción. Sea $\{c_n : n \in \mathbb{N} \cup \{0\}\}$, la sucesión definida por la recurrencia:

$$c_n = 1 - \sum_{k=1}^{n}\frac{c_{n-k}}{k} \quad , \qquad c_0 = 1$$

$$\{c_n\} = \left\{1, 0, \frac{1}{2}, \frac{1}{6}, \frac{1}{3}, \frac{13}{60}, \frac{97}{360}, \frac{571}{2520}, \frac{1217}{5040}, \frac{3391}{15120}, \ldots\right\}$$

En esta nota mostramos una colección de fórmulas que involucran la clásica constante Pi,trascendental e irracional.

Notación: $z = x + i\,y$, $Re(z) = x, Im(z) = y$, $i = \sqrt{-1}$

Fórmulas

(1) $$\pi = (4 + 2\ln 2)\tan\left(\sum_{n=0}^{\infty}\frac{c_n}{n+1}Im\left(\left(\frac{1+i}{2}\right)^{n+1}\right)\right)$$

(2) $$\pi = (6 + 3\ln 3)\tan\left(\sum_{n=0}^{\infty}\frac{c_n}{n+1}Im\left(\left(\frac{1}{2}+\frac{i}{2\sqrt{3}}\right)^{n+1}\right)\right)$$

(3) $$\pi = \left(8 + 4\ln\left(2+\sqrt{2}\right)\right)\tan\left(\sum_{n=0}^{\infty}\frac{c_n}{n+1}Im\left(\left(\frac{1+i\left(\sqrt{2}-1\right)}{2}\right)^{n+1}\right)\right)$$

(4) $$\pi = \left(12 + 6\ln\left(2+\sqrt{3}\right)\right)\tan\left(\sum_{n=0}^{\infty}\frac{c_n}{n+1}Im\left(\left(\frac{1+i\left(2-\sqrt{3}\right)}{2}\right)^{n+1}\right)\right)$$

(5) $$\pi = \left(6 + 3\ln\frac{4}{3}\right)\tan\left(\sum_{n=0}^{\infty}\frac{c_n}{n+1}Im\left(\left(\frac{1+i\sqrt{3}}{4}\right)^{n+1}\right)\right)$$

(6) $$\pi = \left(8 + 4\ln\left(4-2\sqrt{2}\right)\right)\tan\left(\sum_{n=0}^{\infty}\frac{c_n}{n+1}Im\left(\left(\frac{2-\sqrt{2}+i\sqrt{2}}{4}\right)^{n+1}\right)\right)$$

(7) $$\pi = \left(12 + 6\ln\left(8-4\sqrt{3}\right)\right)\tan\left(\sum_{n=0}^{\infty}\frac{c_n}{n+1}Im\left(\left(\frac{2-\sqrt{3}+i}{4}\right)^{n+1}\right)\right)$$

(8) $$\pi = \left(6 - 3\ln\frac{4}{3}\right)\tan\left(\frac{1}{\sqrt{3}}\sum_{n=0}^{\infty}\frac{(-1)^n c_{2n}\,3^{-n}}{2n+1}\right)$$

(9) $$\pi = \left(8 - 4\ln\left(4-2\sqrt{2}\right)\right)\tan\left(\sum_{n=0}^{\infty}\frac{(-1)^n c_{2n}\left(\sqrt{2}-1\right)^{2n+1}}{2n+1}\right)$$

(10) $$\pi = \left(12 - 6\ln\left(8-4\sqrt{3}\right)\right)\tan\left(\sum_{n=0}^{\infty}\frac{(-1)^n c_{2n}\left(2-\sqrt{3}\right)^{2n+1}}{2n+1}\right)$$

(11) $$\left(1+\frac{1}{2}\ln 2\right)^2 + \frac{\pi^2}{16} = exp\left(2\sum_{n=0}^{\infty}\frac{c_n}{n+1}\,Re\left(\left(\frac{1+i}{2}\right)^{n+1}\right)\right)$$

(12) $$\left(1+\frac{1}{2}\ln 3\right)^2 + \frac{\pi^2}{36} = exp\left(2\sum_{n=0}^{\infty}\frac{c_n}{n+1}\,Re\left(\left(\frac{1}{2}+\frac{i}{2\sqrt{3}}\right)^{n+1}\right)\right)$$

(13) $$\left(1+\frac{1}{2}\ln\left(2+\sqrt{2}\right)\right)^2 + \frac{\pi^2}{64} = exp\left(2\sum_{n=0}^{\infty}\frac{c_n}{n+1}\,Re\left(\left(\frac{1+i\left(\sqrt{2}-1\right)}{2}\right)^{n+1}\right)\right)$$

(14) $$\left(1+\frac{1}{2}\ln\left(2+\sqrt{3}\right)\right)^2 + \frac{\pi^2}{144} = exp\left(2\sum_{n=0}^{\infty}\frac{c_n}{n+1}\,Re\left(\left(\frac{1+i\left(2-\sqrt{3}\right)}{2}\right)^{n+1}\right)\right)$$

$$(15)\qquad \left(1+\frac{1}{2}\ln\frac{4}{3}\right)^2+\frac{\pi^2}{36}=exp\left(2\sum_{n=0}^{\infty}\frac{c_n}{n+1}\ Re\left(\left(\frac{1+i\sqrt{3}}{4}\right)^{n+1}\right)\right)$$

$$(16)\qquad \left(1+\frac{1}{2}\ln\left(4-2\sqrt{2}\right)\right)^2+\frac{\pi^2}{64}=exp\left(2\sum_{n=0}^{\infty}\frac{c_n}{n+1}\ Re\left(\left(\frac{2-\sqrt{2}+i\sqrt{2}}{4}\right)^{n+1}\right)\right)$$

$$(17)\qquad \left(1+\frac{1}{2}\ln\left(8-4\sqrt{3}\right)\right)^2+\frac{\pi^2}{144}=exp\left(2\sum_{n=0}^{\infty}\frac{c_n}{n+1}\ Re\left(\left(\frac{2-\sqrt{3}+i}{4}\right)^{n+1}\right)\right)$$

$$(18)\qquad \left(1-\frac{1}{2}\ln\frac{4}{3}\right)^2+\frac{\pi^2}{36}=exp\left(\sum_{n=0}^{\infty}\frac{(-1)^{n+1}c_{2n+1}\ 3^{-n-1}}{n+1}\right)$$

$$(19)\qquad \left(1-\frac{1}{2}\ln\left(4-2\sqrt{2}\right)\right)^2+\frac{\pi^2}{64}=exp\left(\sum_{n=0}^{\infty}\frac{(-1)^{n+1}c_{2n+1}\left(\sqrt{2}-1\right)^{2n+2}}{n+1}\right)$$

$$(20)\qquad \left(1-\frac{1}{2}\ln\left(8-4\sqrt{3}\right)\right)^2+\frac{\pi^2}{144}=exp\left(\sum_{n=0}^{\infty}\frac{(-1)^{n+1}c_{2n+1}\left(2-\sqrt{3}\right)^{2n+2}}{n+1}\right)$$

Referencias

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