It is Time to Replace Pi with Tau

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Pi is the constant ratio between the diameter and the circumference of a circle. This is a valuable ratio because if you know either one of the two lengths (the diameter or the circumference) you can compute the other.

$$\begin{aligned} \text{circumference} &= \text{diameter} \times \pi && (1) \\ \text{diameter} &= \text{circumference} \,/\, \pi && (2) \end{aligned}$$

Pi has been used for thousands of years to work with circles. However, today our mathematical system has evolved to use the radius (r) in our formulas and not the diameter. For example:

$$\begin{aligned} \text{diameter of a circle} &= 2\mathrm{r} && (3) \\ \text{area of a circle} &= \pi \mathrm{r}^2 && (4) \\ \text{volume of a sphere} &= \frac{4}{3}\pi \mathrm{r}^3 && (5) \end{aligned}$$

As we switched to using the radius in our formulas, the ratio between the diameter and circumference of a circle was no longer needed. What was needed was the ratio between the radius and circumference of a circle - which is called tau (τ). Tau has the value of approximately 6.28 (which is twice the value of pi).

$$\tau = 2\pi \qquad (6)$$

But instead of switching to tau, we used 2π in our formulas. Unfortunately this substitution (2π for τ) doesn't always work, and the three formulas above (that use the radius) illustrate the problem.

Those formulas are part of an important series in mathematics, yet this series has remained mysterious and not fully understood - because we are using π. When we use equivalent τ formulas, the patterns become apparent. These patterns are also found in Euler's expansion of e^x:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots \tag{7}$$

Remarkably, each term in Euler's series evolves using integration/derivatives. For example the derivative of the third term ($x^3/3!$) is the second term ($x^2/2!$) This type of integration has an elegant "balance" between the exponential and factorial terms - they are always equal.

The table below shows that our series of formulas evolves using a similar form of integration, and the "factorial" values are always equal to the "exponential" values of the radius. However it evolves in two "paths" which work differently with odd and even numbers. Each term in the series is written in three equivalent formulas to illustrate these patterns.

	Even Path			Odd Path		
Term number	"Factorial" formula	Tau formula	Pi formula	"Factorial" formula	Tau formula	Pi formula
0	$\frac{1}{0!}\tau^0\mathrm{r}^0$	1	1			
1				$\frac{2}{1!}\tau^0\mathrm{r}^1$	2r	2r
2	$\frac{1}{2!}\tau^1\mathrm{r}^2$	$\frac{1}{2}\tau\mathrm{r}^2$	$\pi\mathrm{r}^2$			
3				$\frac{2}{3!}\tau^1\mathrm{r}^3$	$\frac{2}{3}\tau\mathrm{r}^3$	$\frac{4}{3}\pi\mathrm{r}^3$
4	$\frac{1}{4\times2}\tau^2\mathrm{r}^4$	$\frac{1}{8}\tau^2r^4$	$\frac{1}{2}\pi^2\mathrm{r}^4$			
5				$\frac{2}{5\times3}\tau^2\mathrm{r}^5$	$\frac{2}{15}\tau^2\mathrm{r}^5$	$\frac{8}{15}\pi^2\mathrm{r}^5$
6	$\frac{1}{6\times4\times2}\tau^3\mathrm{r}^6$	$\frac{1}{48}\tau^3\mathrm{r}^6$	$\frac{1}{6}\pi^3\mathrm{r}^6$			
7				$\frac{2}{7\times5\times3}\tau^3\mathrm{r}^7$	$\frac{2}{105}\tau^3\mathrm{r}^7$	$\frac{16}{105}\pi^3\mathrm{r}^7$
8	$\frac{1}{8\times6\times4\times2}\tau^4\mathrm{r}^8$	$\frac{1}{384}\tau^4\mathrm{r}^8$	$\frac{1}{24}\pi^4\mathrm{r}^8$			
9				$\frac{2}{9\times7\times5\times3}\tau^4\mathrm{r}^9$	$\frac{2}{945}\tau^4\mathrm{r}^9$	$\frac{32}{945}\pi^4\mathrm{r}^9$

This pattern of integration/derivatives is apparent with the tau formula, but not the pi formula. Tau grows exponentially in this series. When it is replaced with 2π, the 2 also grows exponentially and that is the problem - the balance between the exponential and factorial terms is thrown off by some factor of 2.

Mother nature writes her formulas using tau! Using pi obscures those formulas to some extent, which is why it is time to replace 2π with τ.