Why Now is the Time to Replace Pi with Tau

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Tau (τ) is a constant approximately equal to 6.28 - it is twice the value of pi (π). There are many good reasons why we should stop using pi and replace it with tau, tau is: simpler, more intuitive, easier to teach and learn, etc. However some people are content with pi and contend there is no real need to switch to tau because, “there is nothing that tau can do in a formula that 2π can’t also do”. That was true in the past - but not now.

It is easy to see how we got off on the wrong foot using pi. Thousands of years ago, people realized that there was a constant ratio between the diameter and the circumference of a circle (pi). But today, our mathematical system has evolved to use the radius (r) in our formulas and not the diameter. For example:

$$\begin{aligned} \text{diameter of a circle} &= 2\mathrm{r} && (1) \\ \text{area of a circle} &= \pi\mathrm{r}^2 && (2) \\ \text{volume of a sphere} &= \frac{4}{3}\pi\mathrm{r}^3 && (3) \end{aligned}$$

As we switched to using the radius in our formulas, we should have switched to using tau as well. Surprisingly, a convincing “proof” that we should be using tau, can be found in the above three formulas. They are part of a common but mysterious series. This series is actually very similar to Euler’s expansion of e^x :

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots \tag{4}$$

Remarkably, each term in Euler’s series is a derivative of the next. For example the derivative of $x^3/3!$ is $x^2/2!$ There is an eloquent “balance” between the exponential and factorial values - they are always equal. While it is not obvious, our series of math formulas evolves in a very similar manner.

The table below shows how the math series evolves from term to term alternating between an odd and an even path. The value of each term is simplified into a tau and a pi formula.

To get a better idea of how the terms are evolving in this series, start at the bottom of the "Even path" column and look upward through the terms. Notice that the "factorials" and the exponents of r work differently in this environment - odd numbers are ignored on the even path and even numbers are ignored on the odd path.

So this math series evolves with a version of derivatives and balanced exponents/factorials. For example, term number four (in the Even path column) is a derivative of term six (remember to skip odd numbers in the derivative).

Term number	Even path	Tau formula	Pi formula	Odd path	Tau formula	Pi formula
0	$\frac{1}{0!}\tau^0\mathrm{r}^0$	1	1			
1				$\frac{2}{1!}\tau^0\mathrm{r}^1$	2r	2r
2	$\frac{1}{2}\tau^1\mathrm{r}^2$	$\frac{1}{2}\tau\mathrm{r}^2$	$\pi\mathrm{r}^2$			
3				$\frac{2}{3}\tau^1\mathrm{r}^3$	$\frac{2}{3}\tau\mathrm{r}^3$	$\frac{4}{3}\pi\mathrm{r}^3$
4	$\frac{1}{4\times2}\tau^2\mathrm{r}^4$	$\frac{1}{8}\tau^2r^4$	$\frac{1}{2}\pi^2\mathrm{r}^4$			
5				$\frac{2}{5\times3}\tau^2\mathrm{r}^5$	$\frac{2}{15}\tau^2\mathrm{r}^5$	$\frac{8}{15}\pi^2\mathrm{r}^5$
6	$\frac{1}{6\times4\times2}\tau^3\mathrm{r}^6$	$\frac{1}{48}\tau^3\mathrm{r}^6$	$\frac{1}{6}\pi^3\mathrm{r}^6$			
7				$\frac{2}{7\times5\times3}\tau^3\mathrm{r}^7$	$\frac{2}{105}\tau^3\mathrm{r}^7$	$\frac{16}{105}\pi^3\mathrm{r}^7$
8	$\frac{1}{8\times6\times4\times2}\tau^4\mathrm{r}^8$	$\frac{1}{384}\tau^4\mathrm{r}^8$	$\frac{1}{24}\pi^4\mathrm{r}^8$			
9				$\frac{2}{9\times7\times5\times3}\tau^4\mathrm{r}^9$	$\frac{2}{945}\tau^4\mathrm{r}^9$	$\frac{32}{945}\pi^4\mathrm{r}^9$

However, this pattern of derivatives can only be faithfully represented with a tau formula. Tau grows exponentially in this series. When it is replaced with 2π, the 2 also grows exponentially and that is the problem. The factorial terms are thrown off by some factor of 2 when pi is used.

So this eloquent derivative pattern went unnoticed in one of our most common series of formulas - because we have been using pi, and mother nature writes her formulas using tau! Her formulas are only fully legible to us when we use tau. That is why now is the time to replace 2π with τ.