The Gravitational Constant and the Planck's Units
A Simplification of the Quantum Realm[*]

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Abstract: In this paper, I suggest a new way to write the gravitational constant that makes all of the Planck units: Planck length, Planck time, Planck mass, and Planck energy more intuitive and simpler to understand. By writing the gravitational constant in a Planck functional form, we can rewrite all of the Planck units (without changing their values). Hopefully this can be a small step on the way to a better understanding of the quantum realm.

Résumé: Dans cet article, je propose une nouvelle faon d'écrire la constante gravitationnelle qui rend toutes les unités de Planck, la longueur de Planck, le temps de Planck, la masse de Planck, et l'énergie de Planck, plus intuitives et plus simples comprendre.

En crivant la constante gravitationnelle sous une forme fonctionnelle de Planck, nous pouvons réécrire toutes les unités de Planck (sans changer leur valeur). Nous espérons que cela puisse tre un petit pas vers une comprhension encore meilleure du domaine quantique.

Key words: Gravitational constant, Max Planck, Planck units: length, time, mass, energy, quantum physics.

I. A NEW PERSPECTIVE ON THE PLANCK UNITS

We suggest that Newton's gravitational constant[1] can be written as a function of Planck's reduced constant

$$G_p = \frac{l_p^2 c^3}{\hbar} \tag{1}$$

where $\hbar$ is the reduced Planck's constant [2], c is the well tested round-trip speed of light, and l_p is the Planck length [3]. We could call this Planck's form of the gravitational constant. The Planck length l_p is calibrated so that G_p matches our best estimate for the gravitational constant. We can use the gravitational constant to find the Planck length, or the Planck length to set the gravitational constant. In our view, the Planck form of the gravitational constant enables us to rewrite Planck's constants in a way that simplifies and gives deeper insight, potentially opening up the path for new interpretations in physics.

Based on this, the Planck length is given by

$$l_p = \sqrt{\frac{\hbar G_p}{c^3}} = \sqrt{\frac{\hbar \frac{l_p^2 c^3}{\hbar}}{c^3}} = l_p \tag{2}$$

Here the Planck length is simply our constant l_p. Further, the Planck time in this context is

$$t_p = \sqrt{\frac{\hbar G_p}{c^5}} = \sqrt{\frac{\hbar \frac{l_p^2 c^3}{\hbar}}{c^5}} = \frac{l_p}{c} \tag{3}$$

One could argue that rewriting the gravitational con-

Next the Planck mass in this context results in

$$m_p = \sqrt{\frac{\hbar c}{G_p}} = \sqrt{\frac{\hbar c}{\frac{l_p^2 c^3}{\hbar}}} = \frac{\hbar}{l_p}\frac{1}{c} \tag{4}$$

Based on the gravitational constant, the Planck energy can be simplified to

$$E_p = m_p c^2 = \sqrt{\frac{\hbar c}{G_p}} c^2 = \frac{\hbar}{l_p}\frac{1}{c} c^2 = \frac{\hbar}{l_p} c \tag{5}$$

And finally we can also rewrite the reduced Compton wavelength:

$$\frac{\hbar}{m_p c} = \frac{\hbar}{\frac{\hbar}{l_p}\frac{1}{c} c} = \frac{1}{\frac{1}{l_p}} = l_p \tag{6}$$

I summarize a series of rewritten Planck units in Table 1.

One interesting thing to note from the table is that in the Planck form of the Planck units, one has $c^{1.5}$, $c^{2.5}$, $c^{3.5}$ and $c^{4.5}$ as well as c^4, c^5, c^7, c^8 and it is very hard to find any intuition in c powered to such numbers. In the rewritten forms introduced in this paper, we only have c in most of the units, and c^2 for only the Planck power and Planck intensity. We have gotten rid of the square root as well as the high-powered, non-intuitive notation in the Planck units.

stant in this way creates a circular argument, since the

TABLE I. The table shows the standard Planck units and the units rewritten in the simpler and more intuitive form.

Units:	**"Normal"-form:**	**Simplified-form:**
Gravitational constant	$G \approx 6.67408 \times 10^{-11}$	$G_p = \frac{l_p^2 c^3}{\hbar}$
Planck length	$l_p = \sqrt{\frac{\hbar G}{c^3}}$	$l_p = l_p$
Planck time	$t_p = \sqrt{\frac{\hbar G}{c^5}}$	$t_p = \frac{l_p}{c}$
Planck mass	$m_p = \sqrt{\frac{\hbar c}{G}}$	$m_p = \frac{\hbar}{l_p}\frac{1}{c}$
Planck energy	$E_p = \sqrt{\frac{\hbar c^5}{G}}$	$E_p = \frac{\hbar}{l_p} c$
Relationship mass and energy	$E_p = m_p c^2$	$\frac{\hbar}{l_p} c = \frac{\hbar}{l_p}\frac{1}{c} c^2$
Reduced Compton wavelength	$\frac{\hbar}{m_p c}$	l_p
Planck area	$l_p^2 = \frac{\hbar G}{c^3}$	$l_p^2 = l_p^2$
Planck volume	$l_p^3 = \sqrt{\frac{\hbar^3 G^3}{c^9}}$	$l_p^3 = l_p^3$
Planck force	$F_p = \frac{E_p}{l_p} = \frac{\hbar}{l_p t_p} = \frac{c^4}{G}$	$F_p = \frac{\hbar}{l_p}\frac{c}{l_p}$
Planck power	$P_p = \frac{E_p}{t_p} = \frac{c^5}{G}$	$P_p = \frac{\hbar}{l_p}\frac{c^2}{l_p}$
Planck mass density	$\rho_p = \frac{m_p}{l_p^3} = \frac{c^5}{\hbar G^2}$	$\rho_p = \frac{\frac{\hbar}{l_p}\frac{1}{c}}{l_p^3} = \frac{\hbar}{l_p^4}\frac{1}{c} = \frac{\hbar}{l_p}\frac{1}{c l_p^3}$
Planck energy density	$\rho_p^E = \frac{E_p}{l_p^3} = \frac{c^7}{\hbar G^2}$	$\rho_p^E = \frac{\frac{\hbar}{l_p} c}{l_p^3} = \frac{\hbar}{l_p^4} c = \frac{\hbar}{l_p}\frac{c}{l_p^3}$
Planck intensity	$I_p = \rho_p c = \frac{c^8}{\hbar G^2}$	$I_p = \frac{\hbar}{l_p}\frac{c^2}{l_p^3}$
Planck frequency	$\omega_p = \frac{1}{t_p} = \sqrt{\frac{c^5}{\hbar G}}$	$\omega_p = \frac{1}{\frac{l_p}{c}} = \frac{c}{l_p}$
Planck pressure	$p_p = \frac{F_p}{l_p^2} = \frac{\hbar}{l_p^3 t_p} = \frac{c^7}{\hbar G^2}$	$p_p = \frac{\hbar}{l_p}\frac{c}{l_p^3}$

Planck length is derived from Newton's gravitational constant. We should consider this in a historical perspective. Newton's gravitational constant was discovered long before the Planck length was even considered (in 1906). The Planck length was derived from the gravitational constant, the speed of light, and the Planck constant. However, it could have taken place the other way around if the Planck length had been introduced as a "hypothetical" fundamental entity first. The fact that Newton's gravitational constant was discovered before the Planck length was established does not necessarily make it more fundamental than the Planck length. Newton's gravitational constant was likely discovered first because it was easier to measure; this is true even if Big G is hard to measure accurately; see [4–8]. Even so, Big G is easier to measure (indirectly) than the Planck length.

Recently Haug [9, 10] has given a new theoretical insight strongly suggesting that fundamental particles in Einstein's relativistic mass equation actually have a maximum velocity just below that of the speed of light. This can be seen as an additional "boundary condition" that affects the interpretation of Einstein's relativistic mass energy formula without actually changing the formula itself. The maximum velocity can be estimated accurately and is far above the velocity currently attained at the LHC. Still, what is most interesting in this context is that the Planck length can be found experimentally to be only a function of the speed of light c and the reduced Compton wavelength of the mass in question; the Haug formula for the Planck length is: $l_p = \bar{\lambda}\sqrt{1 - \frac{v_{max}^2}{c^2}}$. This adds support to the view that the Planck length could be just as fundamental as big G, if not more so. Although our technology is not advanced enough yet for precision in such analysis, it may be in the not too distant future.

One could argue that the equation is biting its own tail as the maximum velocity is a function of the Planck length, e.g. we have simply invented a circular solution to the Planck length with no real solution. However, this is a misconception. The important point is that v_{max} can be measured experimentally (At a minimum within a thought experiment, that is until our technology for accelerating particles get more advanced.) and we know that v_{max} is the composite structure, thus we can use this to extract l_p. We typically know the reduced Compton wavelength (of an electron, for example) and we know c per definition; based on this, we can extract l_p. Remarkably, we need no knowledge of G or $\hbar$ to find the find the Planck length and even the Planck mass.

The maximum velocity derived by Haug can also be written[1] as a function of G, c, $\hbar$ and m_e

$$v_{max} = c\sqrt{1 - \frac{Gm_e^2}{\hbar c}} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}_e^2}} \approx c\sqrt{1 - 1.7517 \times 10^{-45}} \tag{7}$$

Since v_{max} here is a function of the universal constants G, $\hbar$, and c one could try to argue that this is evidence

[1] Thanks to an anonymous referee for pointing this out.

that l_p must be a function of G and $\hbar$ and c and not that G is a function of l_p. In other words, that G must be a universal constant and l_p is just a derived constant. However, the beauty of equation 7 is that G and $\hbar$ cancel out and that we are left with v_{max} as a function of c, l_p only and the reduced Compton wavelength of the particle in question, $\bar{\lambda}$, and not of G and $\hbar$. It is worth pointing out that, for example, the reduced Compton wavelength of an electron can experimentally be found completely independent of any knowledge of G, see [11]. To find l_p one need the reduced Compton wavelength that can be found totally independent on G as well at the maximum velocity for an electron, v_{max}. This maximum velocity has to be found experimentally. This maximum velocity for an electron are very close to c, but still higher than velocities one operate with at LHC. However, the fact that something is predicted and not found yet is not a sufficient argument for rejecting a theory, this should simply encorrage further investigation.

There exists an alternative way to find l_p that is not dependent on G or $\hbar$. Further [9, 10] shows two other ways to derive v_{max} totally independent on G and $\hbar$. We are not questioning if G is a universal constant, we are asking if G could be a universal composite constant consisting of even more fundamental constants, and we have based on this report reason to think these are c, $\hbar$, and l_p.

We may never be able to measure the Planck length directly, but only indirectly through G or hopefully also through some other measurements such as recently suggested by Haug. That we today can measure G and not l_p independently of G yet does not necessarily mean that l_p is less fundamental than the gravitational constant.

It is interesting to note that the Planck length can also be obtained by the modification of Stoney's natural units [12] relating Newton's constant G to electromagnetic constants:

$$l_p = \sqrt{\frac{Gk_e e^2}{\alpha c^4}} \tag{8}$$

Since we have: $e = \sqrt{\frac{\hbar}{c}}\sqrt{\alpha}\sqrt{10^7}$, and Coulomb's constant, $k_e = c^2 10^{-7}$, we get

$$\begin{aligned} l_p &= \sqrt{\frac{Gk_e e^2}{\alpha c^4}} \\ l_p &= \sqrt{\frac{Gc^2 \times 10^{-7}\left(\sqrt{\frac{\hbar}{c}}\sqrt{\alpha}\sqrt{10^7}\right)^2}{\alpha c^4}} \\ l_p &= \sqrt{\frac{Gc^2 \times 10^{-7}\frac{\hbar}{c}\alpha \times 10^7}{\alpha c^4}} \\ l_p &= \sqrt{\frac{G\hbar}{c^3}} \end{aligned} \tag{9}$$

From the rewriting above, we see that that fine structure constants appear in several places and cancel each other out, basically illustrating the Planck relationship described in 1906. In other words, the Planck length (and thereby the gravitational constant) is not directly dependent on electromagnetic constants and we do not seem to lose any information by writing Newton's gravitational constant in the form $G = \frac{l_p^2 c^3}{\hbar}$. Naturally this does not exclude the possibility of other relationships existing between electromagnetism and gravity, but an in-depth discussion of such ideas is outside the scope of this paper.

In the Appendix we have derived the same relationship for big G based on dimensional analysis. Since dimensional analysis has certain limitations and weaknesses, it should not be used alone as a "proof" that this is an important relationship. However, it is an additional tool that can support the idea that big G written in this form could be highly relevant, particularly for simplifying many of the Planck units.

The approach to writing the gravitational constant as shown here could have important benefits for physics because it can be used to simplify Planck units and to quantize many gravitational formulas. This could lead to new intuition and interpretations about the depth of reality. Haug [13] has recently shown how the same rewriting of Newton's gravitational constant can be used to simplify and quantize Newton and Einstein's gravitation theories without changing their output values.

II. SUMMARY

By making the gravitational constant a functional form of the reduced Planck constant, we can rewrite the Planck units into simpler and more intuitive forms. As a minimum these should be somewhat easier to remember and work with. It should be easier to interpret c than, for example, $c^{4.5}$. Hopefully the rewritten and simplified Planck units can, over time, be a step in the right direction in helping us to better understand the quantum realm.

APPENDIX: DIMENSIONAL ANALYSIS

If we assume the Planck length could be an even more fundamental constant than G, then we can also find G through "traditional" dimensional analysis. Here we will assume that the speed of light c, the Planck length l_p, and the reduced Planck constant $\hbar$ are the three fundamental Universal constants. The dimensions of G and the three fundamental constants are

$$[G] = \frac{L^3}{MT^2} = L^3 M^{-1} T^{-2} \tag{10}$$

$$[\hbar] = M\frac{L^2}{T} = ML^2T^{-1} \tag{11}$$

$$[c] = \frac{L}{T} = LT^{-1} \tag{12}$$

$$[l_p] = L \tag{13}$$

Based on this, we have

$$\begin{aligned} G &= l_p^\alpha c^\beta \hbar^\gamma \\ \frac{L^3}{MT^2} &= L^\alpha \left(\frac{L}{T}\right)^\beta \left(M\frac{L^2}{T}\right)^\gamma \end{aligned} \tag{14}$$

Based on this, we obtain the following three equations

$$\text{Lenght}: \; 3 = \alpha + \beta + 2\gamma \tag{15}$$
$$\text{Mass}: \; -1 = \gamma \tag{16}$$
$$\text{Time}: \; -2 = -\beta - \gamma \tag{17}$$

This gives us

$$\alpha = 2$$

$$\beta = 3$$

$$\gamma = -1$$

which means

$$G = l_p^\alpha c^\beta \hbar^\gamma = \frac{l_p^2 c^3}{\hbar} \tag{18}$$

As stated previously, dimensional analysis should be used with great care. Still, we think as a tool it adds support to our theory that it could be useful to write Newton's gravitational constant in this form.

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