Pi Formulas , Part 12 : Special Function

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abstract

In this note we give some formulas related to the constant Pi

NÚMERO π , LA FUNCIÓN $v(x)=exp\left(tan^{-1}(x)\right)$,$x\in\mathbb{R}$

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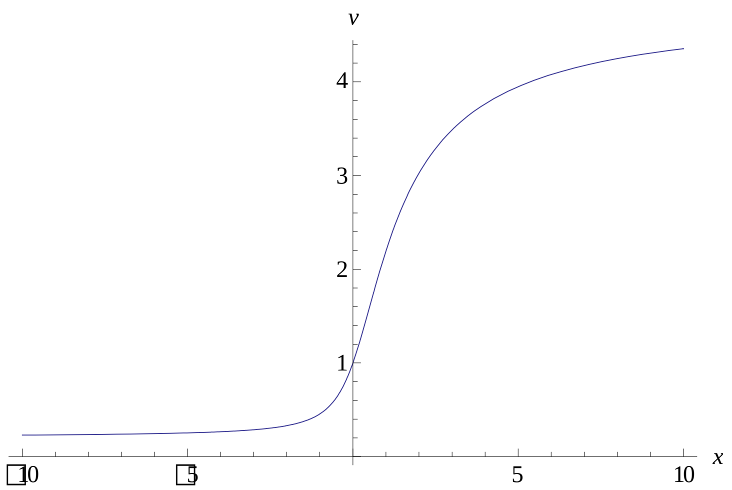
Resumen. Se muestran algunas fórmulas relacionadas con la función:

$$v(x)=exp\left(tan^{-1}(x)\right)\ ,x\in\mathbb{R}$$

1. INTRODUCCIÓN.

En esta nota se muestran fórmulas relacionadas con la función $v(x)$.



2. FÓRMULAS.

2.1. $tan^{-1}(x)=ln\left(v(x)\right)$

2.2. $\dfrac{1}{v(x)}\dfrac{dv}{dx}=\dfrac{1}{1+x^2}$

2.3. $v(\infty)=exp\left(\dfrac{\pi}{2}\right)$, $v(-\infty)=exp\left(-\dfrac{\pi}{2}\right)$

2.4. $v(0)=1$, $v(1)=exp\left(\dfrac{\pi}{4}\right)$

2.5. $x=tan\left(ln\left(v(x)\right)\right)$

2.6. $\dfrac{dv}{dx}=v(x)\left(cos\left(ln\left(v(x)\right)\right)\right)^{2}$

2.7. $v(-x)=\left(v(x)\right)^{-1} \quad ,x>0$

2.8. $exp\left(-\dfrac{\pi}{2}\right)<v(x)<exp\left(\dfrac{\pi}{2}\right) \quad ,x\in\mathbb{R}$

2.9. $exp(\pi)=\left(v(x)\right)^{2}\left(v\left(\dfrac{1}{x}\right)\right)^{2} \quad ,x>0$

2.10. $v(a)=a\Rightarrow a=3.6925856...$

2.11. $$v(x)v(y)=\begin{cases} v\left(\dfrac{x+y}{1-xy}\right) & ,xy<1 \\ e^{\pi}v\left(\dfrac{x+y}{1-xy}\right) & ,x>0,xy>1 \\ e^{-\pi}v\left(\dfrac{x+y}{1-xy}\right) & ,x<0,xy>1 \end{cases}$$

2.12. $$v(x)\left(v(y)\right)^{-1}=\begin{cases} v\left(\dfrac{x-y}{1+xy}\right) & ,xy>-1 \\ e^{\pi}v\left(\dfrac{x-y}{1+xy}\right) & ,x>0,xy<-1 \\ e^{-\pi}v\left(\dfrac{x-y}{1+xy}\right) & ,x<0,xy<-1 \end{cases}$$

2.13.

$$e^{\pi}=\left(v\left(\frac{1}{2}\right)\right)^{4}\left(v\left(\frac{1}{3}\right)\right)^{4}$$

$$=\left(v\left(\frac{1}{3}\right)\right)^{8}\left(v\left(\frac{1}{7}\right)\right)^{4}$$

$$=\left(v\left(\frac{1}{5}\right)\right)^{16}\left(v\left(\frac{1}{239}\right)\right)^{-4}$$

$$=\left(v\left(\frac{1}{\sqrt{2}}\right)\right)^{4}\left(v\left(\frac{1}{2\sqrt{2}}\right)\right)^{2}$$

$$=\left(v\left(\frac{1}{2}\right)\right)^{4}\left(v\left(\frac{1}{5}\right)\right)^{4}\left(v\left(\frac{1}{8}\right)\right)^{4}$$

$$=\left(v\left(\frac{1}{5}\right)\right)^{16}\left(v\left(\frac{1}{70}\right)\right)^{-4}\left(v\left(\frac{1}{99}\right)\right)^{4}$$

$$=\left(v\left(\frac{1}{18}\right)\right)^{48}\left(v\left(\frac{1}{57}\right)\right)^{32}\left(v\left(\frac{1}{239}\right)\right)^{-20}$$

$$=\left(v\left(\frac{1}{38}\right)\right)^{88}\left(v\left(\frac{7}{601}\right)\right)^{68}\left(v\left(\frac{7}{8149}\right)\right)^{40}$$

2.14. Para $|x|<1$, se tiene:

$$v(x)=exp\left(tan^{-1}(x)\right)=\sum_{n=0}^{\infty}\frac{b_n}{n!}x^n$$

$$b_0=b_1=b_2=1$$

$$b_{n+1}=b_n-n(n+1)b_{n-1}\quad ,n\geq 2$$

2.15.

$$v(x)=\sum_{n=0}^{\infty}\frac{c_n}{n!}\frac{(x-a)^n}{\left(1+a^2\right)^n}$$

$$c_0=c_1=v(a)$$

$$c_{n+1}=(1-2an)c_n-n(n-1)\left(1+a^2\right)c_{n-1}\quad ,n\geq 1$$

2.16.

$$v(x)=\prod_{n=1}^{\infty}v\left(\frac{x}{n(n+1)+x^2}\right)\quad ,x\in\mathbb{R}$$

2.17.

$$v(x)+v\left(\frac{1}{x}\right)=2+\frac{\pi}{2}+\sum_{n=2}^{\infty}\frac{\left(tan^{-1}(x)\right)^{n}+\left(tan^{-1}\left(\frac{1}{x}\right)\right)^{n}}{n!} \quad , x\in\mathbb{R}$$

2.18.

$$v(x)=exp\left(tan^{-1}(x)\right)=\prod_{n=0}^{\infty}\left(\prod_{k=0}^{n}(k+2)^{(-1)^{k}/2k+1}\right)^{x^{2n+1}\left(\frac{1}{ln(n+2)}-\frac{x^{2}}{ln(n+3)}\right)}$$

$$|x|<1$$

2.19. Para $|x|<1$, se tiene:

$$v(x)=exp\left(tan^{-1}(x)\right)=\prod_{n=0}^{\infty}\left(\frac{1+x^{2n+1}}{1-x^{2n+1}}\right)^{\frac{a_n}{2}}$$

$$a_0=1$$

$$a_n=\frac{(-1)^{n}-1}{2n+1}-\sum_{(m,k)\in A_n}\frac{a_m}{2k+1} \quad ,n\geq 1$$

$$A_n=\left\{(m,k):m\in\mathbb{N},k\in\mathbb{N}_0,(2m+1)(2k+1)=2n+1\right\}$$

$$a_1=-\frac{2}{3},a_2=0,a_3=-\frac{2}{7},a_4=\frac{2}{9},a_5=-\frac{2}{11},a_6=a_7=a_8=0$$

$$a_9=-\frac{2}{19},a_{10}=\frac{2}{21},....$$

$$e^{tan^{-1}(x)}=\left(\frac{1+x}{1-x}\right)^{1/2}\left(\frac{1+x^{3}}{1-x^{3}}\right)^{-1/3}\left(\frac{1+x^{7}}{1-x^{7}}\right)^{-1/7}\left(\frac{1+x^{9}}{1-x^{9}}\right)^{1/9}....$$

2.20. Para $|x|<1$, se tiene:

$$ln\left(\frac{dv}{dx}\right)=ln\frac{exp\left(tan^{-1}(x)\right)}{1+x^{2}}=\sum_{n=0}^{\infty}(-1)^{n}\left(\frac{x^{2n+1}}{2n+1}-\frac{x^{2n+2}}{n+1}\right)$$

3. REFERENCIAS.

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