DECELERATION PARAMETER Q (Z) AND EXAMINING IF A JOINT DM-DE MODEL IS FEASIZBLE, WITH A REVISIT TO THE QUESTION OF COSMIC SINGULARITIES.

Andrew Beckwith
Chongqing University department of physics,
Emails: abeckwith@uh.edu, rwill9955b@gmail.com
Chongqing, PRC, 400044

Abstract

This paper is a revisit to a 2011 document , with the addition of results pertinent to singularities in the case of a single repeating universe, as well as when the multiverse voids the necessity of a classical GR singularity. When a classical singularity does not exist, it impacts the formation of a massive graviton for reasons brought up, and allows for re acceleration of the universe due to massive gravitons. The existence of massive gravitons would also affect initial entropy, and also lead *to the* datum, that a calculated inflaton $\phi(t)$ may re-emerge after fading out in the aftermath of inflation. The inflaton may be a contributing factor to, with non-zero graviton mass, in re acceleration of the universe a billion years ago. The inflaton is a source of re acceleration of the universe, especially if the effects of a re emergent inflaton are in tandem with the appearance of macro effects of a small graviton mass, leading to a speed up of the rate of expansion of the universe one billion years ago, at red shift value of Z ~ .423. We find that the graviton being massless or massive directly affects graviton contributions to re acceleration of the universe, with other phenomenological consequences. Finally we give our own counterpart as to how much space-time should be transferred to the present cosmological inflationary cycle which may permit preservation of Planks constant value, compliments Corda's brilliant "gravity's breath" document

Keywords: inflaton , non zero graviton mass, emergent structure, BBN, Singularities

1 Introduction

We begin with a brief model as to singular universe, versus a multiverse in terms of input into singularity construction. The singularity behavior envisioned in this document is given by the following argument, as given by Kauffman [1] , and the author in [2] . with the case of when one has Reexamining the question of a 'near singularity' in a multiverse. The case of when one has a massless graviton corresponds to the null condition of the universe, as outlined in the beginning of the 2nd section of this document, whereas we will examine the in the conclusions in the case of when a massive graviton exists one has reacceleration of the universe as due to massive gravitons. To summarize, what we find is. in order to determine the case of a massive graviton, if it exists or not, we first of all examine its relations to the existence, or lack of, of classical GR singularities. Perfect GR singularities will be shown to be leading to massless gravitons, which occurs in a repeating universe, whereas deviations from the GR singularity results are due to what is really Ergodic mixing of space time, [3], which we will find full justification for, which is, in turn sharply differentiated from [4], which does not take heed of the necessity of a multiverse.

2 Review of the formalism of congruence or lack of with singularities if a massive graviton exists, in early universe geometry

We follow the recent work of Kauffmann [1], which sets an upper bound to concentrations of energy, in terms of how he formulated the following equation put in below as Eq. (1). Eq. (1) specifies an inter-relationship between an initial radius R for an expanding universe, and a "gravitationally based energy" expression we will call $T_G(r)$ which lead to a lower bound to the radius of the universe at the start of the Universe's initial expansion, with manipulations. The term $T_G(r)$ is defined via (2) afterwards. We start off with Kauffmann's expression [1]

$$R \cdot \left(\frac{c^4}{G}\right) \geq \int_{|r''| \leq R} T_G\left(r + r''\right) d^3 r'' \tag{1}$$

Kauffmann [1] calls $\left(\frac{c^4}{G}\right)$ a "Planck force" which is relevant due to the fact we will employ (1) at the initial instant of the universe, in the Planckian regime of space-time. Also, we make full use of setting for small r, the following:

$$T_G\left(r + r''\right) \approx T_{G=0}\left(r\right) \cdot const \sim V(r) \sim m_{Graviton} \cdot n_{Initial-entropy} \cdot c^2 \tag{2}$$

I.e. what we are doing is to make the expression in the integrand proportional to information leaked by a past universe into our present universe, with Ng [5] [18] style quantum infinite statistics use of

$$n_{Initial-entropy} \sim S_{Graviton-count-entropy} \tag{3}$$

Then Eq.(3) will lead to

$$\begin{aligned} & R \cdot \left(\frac{c^4}{G}\right) \geq \int_{|r''| \leq R} T_G\left(r + r''\right) d^3 r'' \approx const \cdot m_{Graviton} \cdot \left[n_{Initial-entropy} \sim S_{Graviton-count-entropy} \right] \\ & \Rightarrow R \cdot \left(\frac{c^4}{G}\right) \geq const \cdot m_{Graviton} \cdot \left[n_{Initial-entropy} \sim S_{Graviton-count-entropy} \right] \\ & \Rightarrow R \geq \left(\frac{c^4}{G}\right)^{-1} \cdot \left[const \cdot m_{Graviton} \cdot \left[n_{Initial-entropy} \sim S_{Graviton-count-entropy} \right] \right] \end{aligned} \tag{4}$$

Here, $\left[n_{Initial-entropy} \sim S_{Graviton-count-entropy} \right] \sim 10^{5}$, $m_{Graviton} \sim 10^{-62}\ grams$, and we set Planck length as:

Planck length = l_{Planck} = 1.616199 × 10^{-35} meters **(5)**

where we set $l_{Planck} = \sqrt{\frac{\hbar G}{c^{3}}}$ with $R \sim l_{Planck} \cdot 10^{\alpha}$, and $\alpha > 0$. Typically $R \sim l_{Planck} \cdot 10^{\alpha}$ is about $10^{3} \cdot l_{Planck}$ at the outset, when the universe is the most compact. The value of *const* is chosen based on common assumptions about contributions from all sources of early universe entropy, and will be more rigorously defined in a later paper. We argue that the above methodology, giving a non zero initial starting point is made especially tendi ble if one is using a low temperature start, allowing for the existence of prior recycling universes gravitons to play a role, i.e. that in the single universe repeated again and again, there would be real issues as to the survival of the graviton allowing for the conclusion as to Eq. (4) What Eq.(4) is doing is to help us determine if conditions exist for a massive graviton versus a massless graviton. Should Eq.(4) be really congruent with the conditions for a massive graviton, then the inflaton picture, as given below, can contribute to DE being generated by massive gravitons.

3 Looking at measuring Gravity waves, and Gravitons, with mass.

We will start with a first-principle introduction to detection of gravitational wave density using the definition given by Maggiore [6]

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_{c}} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \Rightarrow h_{0}^{2}\Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_{f}}{10^{37}} \right] \cdot \left(\frac{f}{1kHz} \right)^{4} \qquad (6)$$

Where n_f is the frequency-based numerical count of gravitons per unit phase space. The author suggests that n_f may also depend upon the interaction of gravitons with neutrinos in plasma during early-universe nucleation, as modeled by M. Marklund *et al* [7]. Having said that, the question is, what sort of mechanism is appropriate for considering macro effects of gravitons, and the author thinks that he has one, i.e. reacceleration of the universe, as far as a function of graviton mass, i.e. what Beckwith is to modify is what what was in reference [8] Assume Snyder geometry and look at use of the following inequality for a change in the HUP [8],

$$\Delta x \geq \left[(1/\Delta p) + l_s^2 \cdot \Delta p\right] \equiv (1/\Delta p) - \alpha \cdot \Delta p \qquad (7)$$

and that the mass of the graviton is partly due to the stretching alluded to by Fuller and Kishimoto [9] a supposition the author is investigating for a modification of a joint KK tower of gravitons, as given by Maartens [10, 11] for DM. Assume the stretching of early relic neutrinos that would lead to the KK tower of gravitons--for when $\alpha < 0$, is,

$$m_n(Graviton) = \frac{n}{L} + 10^{-65} \text{ Grams} \qquad (8)$$

Note that Rubakov [12, 13, 14] writes KK graviton representation as, after using the following normalization $\int \frac{dz}{a(z)} \cdot \left[h_m(z) \cdot h_{\tilde{m}}(z)\right] \equiv \delta(m - \tilde{m})$ where J_1, J_2, N_1, N_2 are different forms of Bessel functions, to obtain the KK graviton/ DM candidate representation along RS dS brane world [12]

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \qquad (9)$$

This Eq. (8) and Eq. (9) is for KK gravitons having a TeV magnitude mass $M_Z \sim k$ (i.e. for mass values at .5 TeV to above a TeV in value) on a negative tension RS brane. What would be useful would be managing to relate this KK graviton, which is moving with a speed proportional to H^{-1} with regards to the negative tension brane with $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ as an initial starting value for the KK graviton mass, before the KK graviton, as a ‘massive’ graviton moves with velocity H^{-1} along the RS dS brane. If so, and if $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ represents an initial state, then one may relate the mass of the KK graviton, moving at high speed, with the initial rest mass of the graviton, which in four space in a rest mass configuration would have a mass lower in value, i.e. of $m_{graviton}(4-Dim \quad GR) \sim 10^{-48} eV$, as opposed to $M_X \sim \quad M_{KK-Gravito} \sim .5 \times 10^{9} eV$. Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky et.al. [15] whom argue for graviton mass using CMBR measurements, of $M_{KK-Graviton} \sim 10^{-20} eV$ Dubosky et. al.[15] results can be conflated with Alves et. al. [16] arguing that non zero graviton mass may lead to an acceleration of our present universe, in a manner usually conflated with DE, i.e. their graviton mass would be about $m_{graviton}(4-Dim \quad GR) \sim 10^{-48} \times 10^{-5} eV \sim 10^{65}$ grams. Also assume that to calculate the deceleration, the following modification of the HUP is used: [2] $\Delta x \geq \left[(1/\Delta p) + l_s^2 \cdot \Delta p \right] \equiv (1/\Delta p) - \alpha \cdot \Delta p$, where the LQG condition is $\alpha > 0$, and brane worlds have, instead, $\alpha < 0$. Also (10) will be the starting point used for a KK tower version of (10) below. So from Maarten's [10,11] paper,

$$\dot{a}^2 = \left[\left(\frac{\tilde{\kappa}^2}{3} \left[\rho + \frac{\rho^2}{2\lambda} \right] \right) a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K \right] \tag{10}$$

Maartens [10,11] also gives a 2nd Friedman equation, as

$$\dot{H}^2 = \left[-\left(\frac{\tilde{\kappa}^2}{2} \cdot [p + \rho] \cdot \left[1 + \frac{\rho^2}{\lambda} \right] \right) + \frac{\Lambda \cdot a^2}{3} - 2\frac{m}{a^4} + \frac{K}{a^2} \right] \tag{11}$$

Also, if we are in the regime for which $\rho \cong -P$, for red shift values z between zero to 1.0-1.5 with exact equality, $\rho = -P$, for z between zero to **.5.** The net effect will be to obtain, due to (6)**,** and use $a \equiv [a_0 = 1]/(1+z)$. As given by Beckwith [17, 18]

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \equiv -1 - \frac{\dot{H}}{H^2} = -1 + \frac{2}{1 + \tilde{\kappa}^2 [\rho / m] \cdot (1+z)^4 \cdot (1 + \rho \;\; / 2\lambda)} \approx -1 + \frac{2}{2 + \delta(z)} \quad (12)$$

Eq. (12) assumes $\Lambda = 0 = K$, and the net effect is to obtain, a substitute for DE, by presenting how gravitons with a small mass done with $\Lambda \neq 0$, even if curvature **K** =0

4 Consequences of small graviton mass for reacceleration of the universe

In a revision of Alves *et. al* [16] *,* Beckwith [17,18] used a higher-dimensional model of the brane world and Marsden [10, 11] KK graviton towers. The density ρ of the brane world in the Friedman equation as used by Alves *et. al* [16] is use by Beckwith for a non-zero graviton [17,18]

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g \cdot (c=1)^6}{8\pi G (\hbar = 1)^2} \right] \cdot \left(\frac{1}{14 \cdot (1+z)^3} + \frac{2}{5 \cdot (1+z)^2} - \frac{1}{2} \right) \qquad (13)$$

I.e. Eq. (12), and eq. (13) above is making a joint DM and DE model, with all of Eq. (13) being for KK gravitons and DM, and 10^{-65} grams being a 4 dimensional DE. (11) is part of a KK graviton presentation of DM/ DE dynamics. Beckwith [17,18] found at z ~ **. 4**, a billion years ago, that acceleration of the universe increased, as shown in Fig. 1 [17,18] .

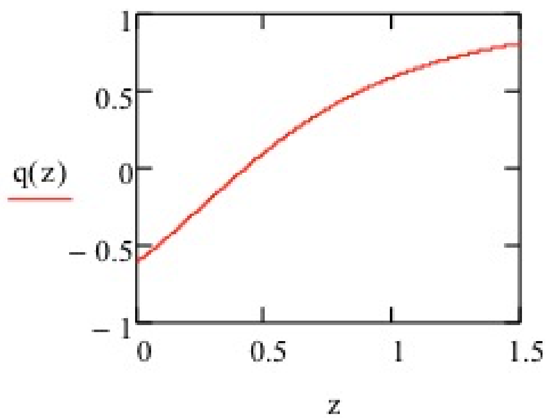

Fig. 1: Reacceleration of the universe based on Beckwith [17,18]**(note that q < 0 if z <.423)**

5. What if an inflaton partly re-emerges in space-time dynamics? At z ~ . 423?

Padmanabhan [19, 20] has written up how the 2nd Friedman equation as of (11), which for **z ~ . 423** may be simplified to read as [10,11]

$$\dot{H}^2 \cong \left[-2\frac{m}{a^4} \right] \tag{14}$$

Eq. (14) would lead to an inflaton value of, when put in, for scale factor behavior as given by $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$, of, and for the inflaton and inflation of [19, 20]

$$\phi(t) = \int dt \cdot \sqrt{-\frac{\dot{H}}{4\pi G}} \qquad (15)$$

Assuming a decline of $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$, Eq.(15) yields [19]

$$\phi(t) \sim \sqrt{\frac{2m}{4\pi G}} \cdot \left[2\varepsilon^{+}\right] \cdot t^{2 \cdot \varepsilon^{+}} \qquad (16)$$

As the scale factor of $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$ had time of the value of roughly $a(t) \propto t^{\lambda}, \lambda = (1/2) - \varepsilon^{+}, 0 \le \varepsilon^{+} << 1$ have a power law relationship drop below $a(t) \propto t^{1/2}$, the inflaton took Eq. (16)'s value which may have been a factor as to the increase in the rate of acceleration, as noted by the q factor, given in Fig. 1. Note that there have been analytical work projects relating the inflaton, and its behavior to entropy via noting that inflation stopped when the inflaton field settled down into a lower lower energy state. The way to relate an energy state to the inflaton is, if $a(t) = a_0 t^{\lambda}$, then in the early universe, one has a potential energy term of [19, 20]

$$V(\phi) = V_0 \cdot \exp\left[-\sqrt{\frac{16\pi G}{\lambda}} \cdot \phi(t)\right] \qquad (17)$$

A situation where both $\lambda = (1/2) - \varepsilon^{+}$ grows smaller, and, temporarily, $\phi(t)$ takes on Eq.(16)'s value, even if the time value gets large, and also, if acceleration of the cosmic expansion is taken into account, then there is infusion of energy by an amount dV. The entropy dS ≃dV/T, will lead, if there is an increase in V, as given by Eq.(17) a situation where there is an effective increase in entropy. If there is, as will be related to later, circumstances, where [5] $S \approx N =$ number of graviton states [17, 18] as will be derived in Eq. (17), then at least in higher dimensions, we have an argument that the re emergence of an inflaton, with a corresponding reduction of Eq. (17) in magnitude may be part of gravitons playing a role in the re acceleration of the universe.

5. Other than five dimensions for cosmology? Problems which need resolutions

If a way to obtain a graviton mass in four dimensions is done which fits in with the as given higher 5 dimensions specified by a slight modification of brane theory, or Maarten's cosmological evolution [10,11] equations, what benefits could this approach accrue for other outstanding problems in cosmology ? The author, Beckwith ,claims that a re do of the Friedmann equations would result in deceleration parameter q(z) similar to Fig. 1 above. Snyder geometry for the four dimensional case with would specify Friedmann equations along the lines of $\alpha > 0$ in Eq. (2) above. If one follows $\alpha < 0$, then the Friedmann equations appear as giving details to the following equation [21]

$$\Im = -\frac{1}{2}\int d^4 x h^0_{uv} \cdot T^{uv} \sim L^2 \approx \delta^+ \geq 0 \qquad (18)$$

The constructions done from sections 1 to 3 are for $\alpha < 0$. When $\alpha > 0$**,** the claim is that almost all the complexity is removed $\alpha > 0$, and what is left is a [21] **treatment** of the Friedmann equations, where he obtains, to first order, if ρ is a scalar field density,

$$\left(\frac{\dot{a}}{a}\right)^2 = [\kappa/3] \cdot \rho \qquad (19)$$

and

$$\left(\frac{\ddot{a}}{a}\right) = -[2 \cdot k/3] \cdot \rho \qquad (20)$$

The interpretation of ρ as a scalar field density [21], and if one does as Alves et al [16] uses Eq. (7) above. We need to interpret the role of ρ. In the LQG version by, Eq. (20) may be rewritten as follows: If conjugate momentum is in many cases, "almost" or actually a constant, using $\dot{\phi} = -[\hbar/i] \cdot [\partial/\partial p_\phi]$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv [\kappa/6] \cdot [p_\phi^2 / a^6] \tag{21}$$

Beckwith [17, 18] claims that the deceleration parameter q (z) incorporating Eq. (19), Eq. (20) and. Eq. (21) should give much the same behavior as Fig. 1 above. If so, then if one is differentiating between four and five dimensions by what is gained, in cosmology, one needs having it done via other criteria. The following is a real problem. As given by Maggiore [6], the massless equation of the graviton evolution equation takes the form

$$\partial_\mu \partial^\varpi h_{\mu\nu} = \sqrt{32\pi G} \cdot \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T^\mu_\mu \right) \tag{22}$$

When $m_{graviton} \neq 0$, the above becomes [6]

$$\left(\partial_\mu \partial^\varpi - m_{graviton}\right) \cdot h_{\mu\nu} = \left[\sqrt{32\pi G} + \delta^+\right] \cdot \left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu} T^\mu_\mu + \frac{\partial_\mu \partial_\nu T^\mu_\mu}{3 m_{graviton}} \right) \tag{23}$$

The mismatch between these two equations, when $m_{graviton} \to 0$, is due to $m_{graviton} h^\mu_\mu \neq 0$ as $m_{graviton} \to 0$, which is due to setting a value of $m_{graviton} \cdot h^\mu_\mu =$

$-\left[\sqrt{32\pi G} + \delta^+\right] \cdot T^\mu_\mu$ The semi classical method by t'Hooft [22, 23], using Eq. (22) and Eq. (23) is the solution. We generalize to higher dimensions the following diagram as given by Beckwith [24, 25]. Use an instanton- anti instanton structure, and t'Hooft [22, 23] equivalence classes along the lines of (24) below with equivalence class structure in the below wave functional to be set by a family of admissible values [24, 25] $\phi_0(x)$

$$\Psi_{i,f}\;[\phi(\mathbf{x})]\Big|_{\phi=\phi_{ci,cf}} = c_{i,f}\cdot\exp\left\{-\int d\mathbf{x}\;\alpha\left[\phi_{_{Ci,f}}(\mathbf{x})-\phi_0(\mathbf{x})\right]^2\right\} \qquad (24)$$

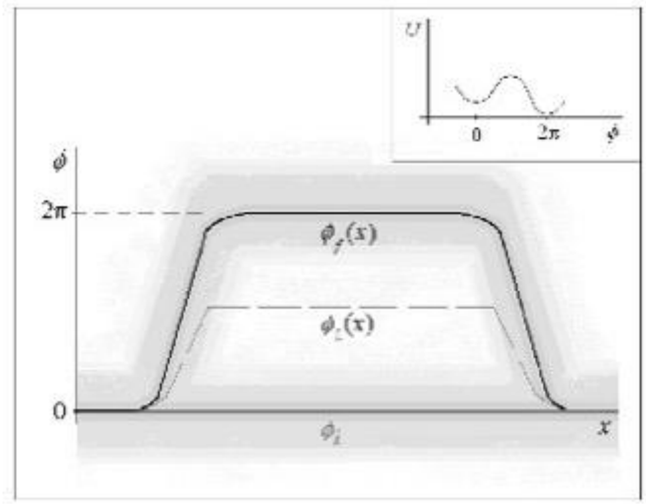

Fig. 2: The pop up effects of an intanton-anti-instanton in Euclidian space [24,25]

This discussion above, would be consistent upon having a graviton represented by not only Eq. (24). If one is adding the small mass of $m_n(Graviton) = \frac{n}{L} + 10^{-65}$ grams[3], with $m_0(Graviton) \approx 10^{-65}$ grams, then the problem being worked with is a source term problem of the form given by Peskins [26] as of the type

$$\psi_n(x) \equiv \int d^3p\cdot\frac{1}{(2\pi)^3}\cdot\frac{1}{\sqrt{2E_P}}\cdot\left\{\left(a_P + \frac{i}{\sqrt{2E_P}}\cdot FT(m_0(graviton))\right)\exp(-ipx) + H.C.\right\} \qquad (24a)$$

This is, using the language Rubakov [12] put up equivalent to obtain,

$$\psi_m(x) \approx h_m(x) + \int d^3 p \cdot \frac{1}{(2\pi)^3} \cdot \left(\frac{1}{\sqrt{2E_P}}\right)^2 \cdot \{(i \cdot FT(m_0(graviton)))\exp(-ipx) + H.C.\} \quad (24b)$$

If $m_0(graviton)$ is a constant, then the expression (24b) has delta functions. This is the field theoretic identification. Another way is to consider an instanton-anti instanton treatment of individual gravitons, and to first start with the supposed stretch out of gravitons to enormous lengths. Assuming $m_0(Graviton) \approx 10^{-65}$ grams for gravitons in 4 dimensions, the supposition by Bashinsky [27] and Beckwith[3] is that density fluctuations are influenced by a modification of cosmological density ρ in the Friedmann equations by the proportionality factor given by Bashinsky [27] , $\left[1-5\cdot(\rho_{neutrino}/\rho)+\vartheta\left([\rho_{neutrino}/\rho]^2\right)\right]$ This proportionality factor for ρ as showing up in the Friedmann equations should be taken as an extension of results from Marklund *et. al* [7], due to graviton-neutrino interactions as proposed by Marklund *et al* [7] , where neutrinos interact with plasmons and plasmons interact with gravitons. Thereby implying neutrino- graviton interactions Also, graviton wavelengths have the same order of magnitude of neutrinos. Note, from Valev [28] ,

$$\begin{aligned} & m_{graviton}\Big|_{RELATIVISTIC} < 4.4\times10^{-22} h^{-1} eV/c^2 \\ & \Leftrightarrow \lambda_{graviton} \equiv \frac{\hbar}{m_{graviton}\cdot c} < 2.8\times10^{-8}\, meters \end{aligned} \quad (25)$$

Extending M. Marklund *et al.* [7] and Valev [28], some gravitons may become larger [14], i.e.

$$\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton}\cdot c} < 10^4\, meters \text{ Or larger} \quad (26)$$

A way to accommodate this wave length as to an instanton-anti instanton packaging of gravitons, was to start with an analogy between Giovannini, [29] from a least action version of the Einstein – Hilbert action for 'quadratic' theories of gravity involving Euler- Gauss-Bonnet. Then Giovannini's [28] equation 6 corresponds to

$$\phi = \tilde{v} + \arctan\left((bw)^{v}\right) \tag{26a}$$

Givannini [28] represents of Eq. (26a) as a kink, and makes references to an anti-kink solution, in Fig. 1 in Givannini [28] **.** Furthermore the similarity between Eq. (26a) and $\phi_{+}(z,\tau) = 4 \cdot \arctan\left(\exp\left\{\frac{z+\beta\cdot\tau}{\sqrt{1-\beta^{2}}}\right\}\right)$ in Beckwith's [24, 25] treatment with regards to density wave physics instantons is obvious. If $\arctan\left((bw)^{v}\right)$ is part of representing a graviton as a kink-anti-kink combination, arising from a 5 dimensional line element**,** [28]

$$dS^{2} = a(w) \cdot \left[\eta_{uv} dx^{u} dx^{v} - dw^{2}\right] \tag{26b}$$

The end result of this would be to have an instaton-anti instanton structure as to emergence of a massive graviton if noting, that there is the possibility of using t'Hoofts[17] [22,23]classical embedding of "deterministic quantum mechanics" as a way to embed a nearly four dimensional graviton as having almost zero mass, in a larger non linear theory.

7. How DM would be influenced by gravitons

The interrelationship of structure of the profile of a DM cluster , with any perturbations DM density profile [29]

$$\delta \equiv -\left[\frac{3}{2} \cdot \Omega_{m} \cdot H^{2}\right]^{-1} \cdot \nabla^{2}\Phi \tag{27}$$

As told to the author by Matarre [29], in July, 2009, in Como Italy, the gravitational potential has, perturbative speaking an additional term f_{NL} added to variations in the gravitational potential term which Matarre [29] gave as

$$\Phi \equiv \Phi_L + f_{NL} \cdot \left[\Phi_L^2 - \left\langle \Phi_L^2 \right\rangle \right] + g_{NL} \cdot \Phi_L^3 \tag{28}$$

It is suggested that the function f_{NL} is largely due to entropy variations, some of which occurred during relic GW/graviton production. Here the expression $f_{NL} =$ variations from gaussianity. Furthermore, Φ_L is a linear Gaussian potential, and the overall gravitational potential is altered by inputs from f_{NL}. Note that neutrinos flavor physics oscillations are not very important in terms of f_{NL}, as specified in conversations. Beckwith had in September 23, 2009 in Erice with George Raffert [30] . Which leads to emphasizing the role of entropy processes due to graviton-neutrino physics, as $\bar{L} \to 0$ as written up by Beckwith [31]

8 1st part of massive graviton consequences

The real start to this investigation is to explain how, and why the star HE0107-5240 could form with so little lithium in the first place[31]. As stated by Fuller et al [9] neutrinos could interact with DM potential wells in ways Beckwith thinks could influence deviations from standard galaxy hierarchy formation models which will also have a counter part in deviations in the BBN nucleosynthesis of light elements,by examining the role of temperature fluctuations modeled on Eq.(29) below ,leading to fluctuations affecting BBN element rarity [31].

$$(\delta T / T) \cong (1/3) \cdot \left[\Phi_L + \tilde{f}_{NL} \cdot \left(\Phi_L^2 - \langle \Phi_L \rangle^2 \right) \right] \tag{29}$$

While Eq.(29) above would have its maximum impact for regions as of about red shift $Z \sim 1.5 - 2.0$, the impact of Eq. (29) would be as of red shifts $Z \sim 1000 - 1100$, with the corresponding $\tilde{f}_{NL}$ influenced by Bashinsky's [27] neutrino – gravition damping as stated by the coefficient of density fluctuation modified by

$\left[1-5\cdot(\rho_{neutrino}/\rho)+\vartheta\left([\rho_{neutrino}/\rho]^2\right)\right]$[27]. Note that $\tilde{f}_{NL}$ would be larger than f_{NL} of Eq. (28) and would be dominated by neutrino-graviton interactions, whereas f_{NL} would be dominated by graviton generated entropy, with neutrinos at $Z \sim 2.0$ hitting DM directly. We submit that a graviton with a small rest mass may be more amendable to such interaction with neutrinos, and that in addition Eq.(27), Eq.(28) and Eq.(29) may influence and affect structure formation as seen by the following diagram in figure 1. Note that this is assuming that early universe interactions which we are talking about eventually play out and reach, with the re acceleration of the universe, as outlined in the 1st half of our document to also be indirectly responsible for the famous "halo merging tree diagram we call Fig 3 below. At or about when $k \geq k_{equilibrium} \equiv \tau^{-1}_{equilibrium} \sim 10^{-2}\,Mpc^{-1}$ begins to delineate the neutrino-GW interaction becoming a significant damping impact upon each other, one would be seeing variations from the usual structure formation, as given by the following diagram.[32]

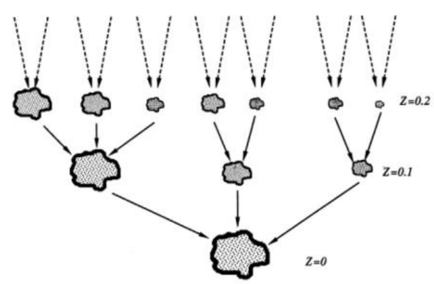

Figure 3 how we obtain 'bottom up' development of galactic super structure which ducplicates a diagram given in reference[32]

We should keep in mind that the following holds, i.e. for flat space. That one will have Figure 3 in both flat and in curved space. Also note that, M. Marklund, G. Brodin, and P.K. Shukla [7] posted their own version of not only neutrino mass, as given by $m_\nu^2 = -g_{\alpha\ \beta}\, p^\alpha p^\beta$, where the overall mass is set by Note, here, that the potential for where the frequency comes from is, here , is $U = \hbar \cdot \omega_F$, and , according to Eberle and Ringwald et al. [33], may have lightest relic neutrino masses of the order of

$$m_{relic-neutrino} \propto .1eV/c^2 \tag{30}$$

as opposed to, as given by D, Valev [34]

$$m_{graviton} \leq 2\times 10^{-29}\breve{h}^{-1}eV/c^2 \tag{31}$$

Where $\breve{h} \approx .65$, is a dimensionless Hubble constant, Very roughly put, for relic early universe conditions, one may be seeing that the neutrino has $10^{28}-10^{29}$ the effective mass than a graviton. Furthermore, for a neutrino we have

$$\lambda_k \approx \frac{hc}{E_k} + \frac{hm_{vk}^2 c^5}{2E_k^3} \tag{32}$$

This will tie in directly with a neutrino mass limit we state as [7]

$$m_\nu^2 = -g_{\alpha\ \beta} p^\alpha p^\beta \equiv \left[\hbar \cdot \sqrt{|g_{00}| \cdot \omega_F^2 - g_{\alpha\beta} k^\alpha k^\beta - 2\omega_F g_{0\alpha} k^\alpha}\right]^2 .. \tag{33}$$

If, as if often expected in inflation, space becomes abruptly flat at the onset of inflation, then for a neutrino mass, as the $\bar{L} \xrightarrow[approach-to-s\tan dard-\mathrm{mod}el-physics]{} 0$ will then lead to the following inequality [7, 31]

$$\begin{aligned}
&m_\nu^2 \equiv \left[\hbar \cdot \sqrt{|g_{00}| \cdot \omega_F^2 - g_{\alpha\beta} k^\alpha k^\beta - 2\omega_F g_{0\alpha} k^\alpha}\right]^2 \\
&\xrightarrow[Flat-Space]{} \left[\hbar \cdot \sqrt{|g_{00}| \cdot \omega_F^2 - g_{\alpha\alpha}\left[k^\alpha\right]^2 - 2\omega_F g_{00} k^0}\right]^2 > 0 \\
&\Leftrightarrow |g_{00}| \cdot \omega_F^2 > g_{\alpha\alpha}\left[k^\alpha\right]^2 + 2\omega_F g_{00} k^0 \Rightarrow |g_{00}| \cdot \omega_F^2 > g_{\alpha\alpha}\left[k^\alpha\right]^2 + 2\omega_F g_{00} k^0
\end{aligned} \tag{34}$$

Now, how would variation from the above " halo Merging history tree', partly due to the modulation , via entropy, of DM structure formation, due to GW/gravitons affecting DM profile affect the concentration for lithium in stars, and perhaps lead to the famous 'lithium problem" being resolved ? We are investigating it. But we do think that having a graviton with mass is affecting the particulars of the 'halo mixing tree' diagram[32].

9. 2nd part of massive graviton consequences

Beckwith [35] has concluded that the only way to give an advantage to higher dimensions as far as cosmology would be to look at if a fifth dimension may present a way of actual information exchange to give the following parameter input from a prior to a present universe, i.e. the fine structure constant, as given by [35]

$$\tilde{\alpha} \equiv e^2 / \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\lambda}{hc} \qquad (35)$$

Eq. (35) above is in tandem, with examining if the following holds,i.e. for the consistency of physical law, namely from cycle to cycle is there a preservation of Planck's constant ? Namely

$$\hbar(prior - universe) = \hbar(present - universe) \qquad (35a)$$

The wave length as may be chosen to do such an information exchange would be part of a graviton as being part of an information counting algorithm as can be put below, namely: Argue that when taking the log, that the 1/N term drops out. As used by Ng [5]

$$Z_N \sim (1/N!) \cdot \left(V/\lambda^3\right)^N \qquad (36)$$

This, according to Ng [5], leads to entropy of the limiting value of, if $S = (\log[Z_N])$ will be modified by having the following done, namely after his use of quantum infinite statistics,

$$S \approx N \cdot \left(\log\left[V/\lambda^3\right] + 5/2\right) \approx N \tag{37}$$

Eventually, the author hopes to put on a sound foundation what 'tHooft [22,23] is doing with respect to t'Hooft [22,23] deterministic quantum mechanics and equivalence classes embedding quantum particle structures.. Doing so will answer the questions Kay [36][29] raised about particle creation, and the limitations of the particle concept in curved and flat space, i.e. the global hyperbolic space time which is flat everywhere expect in a localized "bump" of curvature. Furthermore, making a count of gravitons with $S \approx N \sim 10^{20}$ gravitons, [2] with $I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} \sim 10^{20}$ as implying at least one operation per unit graviton, with gravitons being one unit of information, per produced graviton[3]. This datum needs experimental confirmation and is important to astro physics linkage of DE with DM, in the future. Eq. (14) to. Eq.(17) if confirmed for Z ~ . 423 may prove that higher dimensions are necessary for cosmology.

10. 3rd massive graviton consequences , the need to find out the border of the introduction of where Quantum gravity emerges from a prior 'analog' structure may, if tied into questions of graviton mass determine if multiple universes are possible/ feasible.

Beckwith [37], in his FQXi document outlined a procedure where a graviton with mass may be indicative of the existence of multiple universes co existing. The details of the mapping of that multiple universe picture involve a transition from an analog physics (discrete, i.e. classical world picture) to one where octonian gravity is formed ,i.e. a quantum picture as a pre cursor to quantum gravity. The existence of a small mass may mean the extension of quantum physics to a larger embedding/ extension of quantum physics. Furthermore, keep in mind that tandem to that step of semi classical embedding of a graviton, that eventually we want to make explicit an idea by, T. Padmanabhan in DICE 2010[38] , as to finding "atoms of space time" permitting a thermodynamic treatment of emergent structure similar to Gibbs treatment of statistical physics. I.e. for finding out if the following is possible, ie. can an ensemble of gravitons, be used to construct an 'atom' of space time congruent with relic GW. That is our ultimate end, as to our research. That would make our inquiry of the nature of gravitons most worthwhile. This idea was presented at DICE 2010,[39] and we would like to refine it in our future research work. This would be in tandem of adapting the Kiefer, Polarski, and Starobinsky[40] presentation of the evolution of relic entropy via the evolution of phase spaces, with Γ/Γ_0 being the ratio of 'final (future)' / 'initial' phase space volume, for k modes of secondary GW background. From "atoms of space time" treatment of early universe space time geometry according to [40]

$$S(k) = \ln \frac{\Gamma}{\Gamma_0} \qquad (38)$$

This lead to the author, Beckwith to derive the following a important for structure formation, note the following about what happens if $g_* \geq 120$

$$\Omega \sim \frac{\left[\frac{\pi^2}{30} \cdot g_* \cdot \left[1.416 \times 10^{19} GeV\right]^4\right]}{3 \cdot \left[1.2 \times 10^{19} GeV\right]^2 H_{initial}^{\ 2}} \sim \frac{\frac{\pi^2}{30} \cdot g_* \cdot c^4}{3} \cdot \frac{\left[10^{19} GeV\right]^2}{H_{initial}^2} \geq 1 \qquad (39)$$

I.e. especially if the degrees of freedom rises above $g_* \geq 120$.

Note that $g_* \approx 120$ at T ~ 100 KeV .Unless the term for $H_{initial}$ were absolutely enormous, and if $g_* \approx 1000$, then $\Omega \geq 1$ could happen, which would be physically meaningless. The other situation is that there could be situations for which g_* would be undefined, especially if $T \leq 1.416 \times 0^{32} K \sim 10^{19} GeV$ were close to an equality. We state here unequivocally that Eq.(38) and Eq. (39) above are important, and that this has serious experimental import. Having said that, we will next go to what would be a way to determine if Gravitons can have mass(massive Gravitons). I.e. in the conclusions section, we radically extend the consequences if $g_* \geq 120$, with a speculation as to what could happen as to dark matter and dark energy contributions, which we think is important to the matter of singularities and their purported connection to a multiverse. But before we get to that matter, we will examine the role of partition functions, in terms of background which will lead to several pages later, to $g_* \geq 120$ contributions, especially for the regime of values, say of 1100 to 1200, which we think has to be seriously looked at.

11 Working with a partition function argument in the case of a multiverse

This section is to determine if gravitons have mass and backs the assertion made earlier that multiverse construction has massive gravitons. Note that this section is directly linked to the first part of this document, as to what was done by the author to extend Kauffman's work [1]

We assume that there are no fewer than N universes undergoing Penrose 'infinite expansion' (Penrose, 2006) [41, 42, 43] contained in a mega universe structure. Furthermore, each of the N universes has black hole evaporation, with the Hawking radiation from decaying black holes. If each of the N universes is defined by a partition function, called $\left\{\Xi_i\right\}_{i\equiv N}^{i\equiv 1}$, then there exist an information ensemble of mixed minimum information correlated as about $10^7 - 10^8$ bits of information per partition function in the set $\left\{\Xi_i\right\}_{i\equiv N}^{i\equiv 1}\Big|_{before}$, so minimum information is conserved between a set of partition functions per universe [44]

$$\left\{\Xi_i\right\}_{i\equiv N}^{i\equiv 1}\Big|_{before} \equiv \left\{\Xi_i\right\}_{i\equiv N}^{i\equiv 1}\Big|_{after} \tag{40}$$

However, there is non-uniqueness of information put into each partition function $\{\Xi_i\}_{i\equiv N}^{i\equiv 1}$

. Furthermore Hawking radiation from the black holes is collated via a strange attractor collection in the mega universe structure to form a new big bang for each of the N universes represented by $\{\Xi_i\}_{i\equiv N}^{i\equiv 1}$. Verification of this mega structure compression and expansion of information with a non-uniqueness of information placed in each of the N universes favors ergodic mixing treatments of initial values for each of N universes expanding from a singularity beginning. The n_f value, will be using $S_{entropy} \sim n_f$. . How to tie in this energy expression, as in Eq. (40) will be to look at the formation of a nontrivial gravitational measure as a new big bang for each of the N universes as by $n(E_i)\cdot$ the density of states at a given energy E_i for a partition function. **(**Poplawski**, 2011) [45]**

$$\{\Xi_i\}_{i\equiv 1}^{i\equiv N} \propto \left\{ \int_0^\infty dE_i \cdot n(E_i) \cdot e^{-E_i} \right\}_{i\equiv 1}^{i\equiv N} . \tag{41}$$

Each of E_i identified with Eq. (41) above, are with the iteration for N universes (Penrose, 2006) [41, 42, 43, 44] Then the following holds, namely [44]

Claim 1, [44]

$$\frac{1}{N}\cdot\sum_{j=1}^{N}\Xi_j\Big|_{j-before-nucleation-regime} \xrightarrow{vacuum-nucleation-tranfer} \Xi_i\Big|_{i-fixed-after-nucleation-regime} \tag{42}$$

For N number of universes, with each $\Xi_j\Big|_{j-before-nucleation-regime}$ for j = 1 to N being the partition function of each universe just before the blend into the RHS of Eq. (42) above for our present universe. Also, each of the independent universes given by $\Xi_j\Big|_{j-before-nucleation-regime}$ are constructed by the absorption of one to ten million black holes taking in energy**. I.e. (Penrose, 2006)** [41,42,43,44]. Furthermore, the main point is similar to what was done in [18] in terms of general ergodic mixing

Claim 2 [44]

$$\Xi_j \Big|_{j-before-nucleation-regime} \approx \sum_{k=1}^{Max} \tilde{\Xi}_k \Big|_{black-holes-jth-universe} \tag{43}$$

Claim 3 The idea here is to use what is known as CCC cosmology [41,42,43,44]

First. Have a big bang (initial expansion) for the universe. After redshift z = 10, a billion years ago, SMBH formation starts. Matter- energy is vacuumed up by the SMBHs, which at a much later date than today (present era) gather up all the matter-energy of the universe and recycles it in a cyclic conformal translation, as follows, namely

$$\begin{aligned} &E = 8\pi \cdot T + \Lambda \cdot g \\ &E = source \quad for \quad gravitational \quad field \\ &T = mass \quad energy \quad density \\ &g = gravitational \quad metric \end{aligned} \tag{44}$$

$$\Lambda = vacuum \quad energy, rescaled \quad as \quad follows$$

$$\Lambda = c_1 \cdot [Temp]^{\beta} \tag{45}$$

c_1 is a constant. Then the main methodology in the Penrose proposal has been in Eq. (45) evaluating a change in the metric g_{ab} by a conformal mapping $\hat{\Omega}$ [43, 44] to

$$\hat{g}_{ab} = \hat{\Omega}^2 g_{ab} \tag{46}$$

Penrose's suggestion has been to utilize the following[43]

$$\hat{\Omega} \xrightarrow[ccc]{} \hat{\Omega}^{-1} \tag{47}$$

Infall into cosmic black hopes has been the main mechanism which the author asserts would be useful for the recycling apparent in Eq.(47) above with the caveat that $\hbar$ is kept constant from cycle to cycle as represented by a restatement of Eq.(35a) as in the multiverse as

$$\hbar_{old-\text{cosmology}-cycle} = \hbar_{present-\text{cosmology}-cycle} \tag{48}$$

Eq. (47) is to be generalized, as given by a weighing averaging as given by Eq.(42) where the averaging is collated over perhaps thousands of universes, call that number N, with an ergodic mixing of all these universes, with the ergodic mixing represented by Eq.(42) to generalize Eq.(47) from cycle to cycle.

12 Why this just outlined multiverse averaging procedure implies a graviton with mass. Also why a single repeating universe has no massive gravitons

In this chapter, we are looking at a generalization of Kolb and Turner's[46] gravitational radiation result which is given as

$$\frac{k}{\rho_{Horizon}} \cdot \frac{d\rho_{graviton}}{dk} \sim \frac{4}{3\pi} \cdot \left(\frac{H}{m_{Planck}} \right)^2 \tag{49}$$

In the immediate aftermath of inflation, and just before inflation, we generalize $\frac{4}{3\pi} \cdot \left(\frac{H}{m_{Planck}} \right)^2$ as a constant, as well as approximate $\rho_{Horizon}$ as a constant, with also putting in [34]

$$\lambda_{graviton} \equiv \frac{2\pi}{k} = \frac{\hbar}{c \cdot m_{graviton}} \tag{50}$$

Then we have that if 'Before' is just before the formation of the present universe, and 'Final' is just after the formation of the present universe

$$\log \frac{\rho_{graviton}\big|_{Before}}{\rho_{graviton}\big|_{Final}} = \frac{4\rho_{Horizon}}{3\pi} \cdot \left(\frac{H}{m_{Planck}} \right)^2 \cdot \left[\frac{1}{m_{graviton}\big|_{Before}} - \frac{1}{m_{graviton}\big|_{Final}} \right] \tag{51}$$

Claim 4, in the case of a single repeating Universe, the RHS of (51) is zero, leading to $m_{graviton}\big|_{Before} = m_{graviton}\big|_{Final}$ implying that the mass of a graviton in a single repeating universe is zero.

Proof: We will use the following value of the net energy, i.e. if $\beta = \dfrac{1}{k_{Boltzman} \cdot T(temp)}$

$$\bar{E}_{N(\,fixed\,)} = -\frac{\partial}{\partial\beta}\left\{\Xi_{i=N(\,fixed\,)}\right\} \propto -\frac{\partial}{\partial\beta}\left\{\int_{0}^{\infty} dE_i \cdot n(E_i) \cdot e^{-E_i}\right\}_{i=N(\,fixed\,)} \qquad (52)$$

Now define an average gravitational energy as given by having a single universe, denoted by N (fixed), i.e. one universe out of N of them [maybe infinite] given as

$$\bar{\rho}_{graviton}\big|_{N(\,fixed\,)} = \frac{\bar{E}_{graviton}\big|_{N(\,fixed\,)}}{V(volume)\big|_{N(\mathrm{fixed})} \cdot c^2} \qquad (53)$$

This is the single universe , repeated, i.e. in this case, we assume that the Volume, per single repeating universe, is the same for a regime of the BB immediately before and after the cosmic explosion. Hence, we have that

In terms of equipartition function definitions, and to rewrite Eq.(52)as in the case of a multiverse, i.e. one out of N ‘universes’

$$\bar{E}_{N(\,fixed\,)} \sim \frac{1}{N(\,full-range)} \cdot \frac{\sum_{r=1}^{\infty}\left[\exp(-\beta \cdot \sum_{j=1}^{N} E_j)\right]_r \cdot \left[-\beta \cdot \sum_{j=1}^{N} E_j\right]_r}{\sum_{r=1}^{\infty}\left[\exp(-\beta \cdot \sum_{j=1}^{N} E_j)\right]_r} \qquad (54)$$

It so happens, then that there are r ‘states’ per universe, and an infinite number of them.Then the average graviton radiation density would be, for r =1 to infinite number of energy states per Nth universe, with the label N(full-range) being the number of universe domains in a multiverse.

$$\bar{\rho}_{graviton}\Big|_{N(\,fixed\,)} = \frac{1}{N(\,full-range)} \cdot \frac{\sum_{r=1}^{\infty}\left[\exp(-\beta\cdot\sum_{j=1}^{N}E_j)\right]_r \cdot \left[-\beta\cdot\sum_{j=1}^{N}E_j\right]_r}{V(volume)\big|_{N(\text{fixed})}\cdot c^2\sum_{r=1}^{\infty}\left[\exp(-\beta\cdot\sum_{j=1}^{N}E_j)\right]_r} \quad (55)$$

In terms of the averaging procedure of Eq. (42), we then have the initial and final states for the multiverse as

$$\bar{\rho}_{graviton}\Big|_{N(\,fixed\,)}\Big|_{before-BB} \neq \bar{\rho}_{graviton}\Big|_{N(\,fixed\,)}\Big|_{after-BB} \quad (56)$$

$$\left.\frac{1}{N(\,full-range)} \cdot \frac{\sum_{r=1}^{\infty}\left[\exp(-\beta\cdot\sum_{j=1}^{N}E_j)\right]_r \cdot \left[-\beta\cdot\sum_{j=1}^{N}E_j\right]_r}{V(volume)\big|_{N(\text{fixed})}\cdot c^2\sum_{r=1}^{\infty}\left[\exp(-\beta\cdot\sum_{j=1}^{N}E_j)\right]_r}\right|_{Just-Before-present-universe} \quad (57)$$

$$\xrightarrow[Transfer-to-present-universe]{} \left.\frac{\sum_{r=1}^{\infty}\left[\exp(-\beta\cdot E_{N(\,fixed\,)}\right]_r \cdot \left[-\beta\cdot E_{N(\,fixed\,)}\right]_r}{V(volume)\big|_{N(\text{fixed})}\cdot c^2\sum_{r=1}^{\infty}\left[\exp(-\beta\cdot E_{N(\,fixed\,)}\right]_r}\right|_{N(\,fixed\,)-start-\inf}$$

This would be due to the behavior of $\left[-\beta\cdot\sum_{j=1}^{N}E_j\right]_r$ before the big bang, which will lead to

$$\bar{\rho}_{graviton}\Big|_{N(\,fixed\,)}\Big|_{before-BB} = \left.\frac{1}{N(\,full-range)} \cdot \frac{\sum_{r=1}^{\infty}\left[\exp(-\beta\cdot\sum_{j=1}^{N}E_j)\right]_r \cdot \left[-\beta\cdot\sum_{j=1}^{N}E_j\right]_r}{V(volume)\big|_{N(\text{fixed})}\cdot c^2\sum_{r=1}^{\infty}\left[\exp(-\beta\cdot\sum_{j=1}^{N}E_j)\right]_r}\right|_{Just-Before-present-universe} \quad (58)$$

Which should be compared to

$$\bar{\rho}_{graviton}\Big|_{N(fixed)}\Big|_{after-BB} = \left. \frac{\sum_{r=1}^{\infty}\left[\exp(-\beta\cdot E_{N(fixed)}\right]_r \cdot \left[-\beta\cdot E_{N(fixed)}\right]_r}{V(volume)\big|_{N(\text{fixed})}\cdot c^2 \sum_{r=1}^{\infty}\left[\exp(-\beta\cdot E_{N(fixed)}\right]_r} \right|_{N(fixed)-start-\inf} \tag{59}$$

Eq.(58) and Eq.(59) above are not the same value, hence the results given inEq (56). Hence the masses of the gravitons would not be the same. By Eq.(51)

Note that Feynman and Hibbs [47] have a different way of writing a net energy as can be written using E_i as the total energy of the i universe, and $E_r =$ energy of the r th sub domain of the ith universe .. I.e. two different energy expressions.

$$\Xi_i\big|_{i-fixed,\text{one-universe}} \propto \left.\left\{\int_0^{\infty} dE_i \cdot n(E_i)\cdot e^{-E_i}\right\}\right|_{i-fixed,\text{one-universe}} \sim \sum_r \exp\left[-\beta E_r\right] \tag{60}$$

Then, using Feynman and Hibbs [47], the net energy can be written as

$$\begin{aligned} \bar{E} &\equiv \frac{\sum_{r=1}^{\infty} E_r \cdot \exp\left(-\beta\cdot E_r\right)}{\Xi_i\big|_{i-fixed,\text{one-universe}}} \\ &\propto \frac{\left.\sum_{r=1}^{\infty} E_r \cdot \exp\left(-\beta\cdot E_r\right)\right|_{r-subdomains,\text{i-fixed,one-univerese}}}{\left.\left\{\int_0^{\infty} dE_i \cdot n(E_i)\cdot e^{-E_i}\right\}\right|_{i-fixed,\text{one-universe}}} \end{aligned} \tag{61}$$

The results as outlined above are, again then, more obvious.

13. Conclusions- and further tests as far as upper bounds to a Graviton mass. With consequences

First of all, the contributions of Gravitons to re acceleration of the universe is outlined as a consequence of massive gravitons. In addition the graviton mass of a non zero value is central to the process of entropy generation which leads to our next comment which is a further research project in its own right. For what it is worth, we will address an extension of an entropy versus graviton production linkage implied in the first linkage. This entropy versus gravition linkage, as seen below will imply a non zero initial radius for the universe. Before that is brought up, we should consider entropy generation with an initial cosmological "constant" (vacuum energy) at the start of inflation.

13a. Difficulty in visualizing what g_* is in the initial phases of inflation.

Secondly, we look for a way to link initial energy states, which may be pertinent to entropy, in a way which permits an increase in entropy from about 10^{10} at the start of the big bang to about 10^{100} today.

One such way to conflate entropy with an initial cosmological constant may be of some help, i.e. if $V_4\big|_{Threshold-volume-for-quantum-effects} \sim \left(10^{-4}\,cm\right)^3$ or smaller, i.e. in between the threshold value, and the cube of Planck length, one may be able to look at coming up with an initial value for a cosmological constant as given by Λ_{Max} as given by Padmanabhan [17]

$$\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \sim T^{00} V_4 \equiv \rho \cdot V_4 = E_{total} \tag{62}$$

Then making the following identification of total energy with entropy via looking at Λ_{Max} models, i.e. consider Park's model of a cosmological "constant" parameter scaled via background temperature [48]

$$\Lambda_{Max} \sim c_2 \cdot T^{\tilde{\beta}} \tag{63}$$

A linkage between energy and entropy, may be seen in the following construction, namely looking at what Kolb put in [46] , i.e.

$$\rho = \rho_{radiation} = (3/4) \cdot \left[\frac{45}{2\pi^2 g_*} \right]^{1/3} \cdot S^{4/3} \cdot r^{-4} \tag{64}$$

Here, the idea would be, possibly to make the following equivalence, namely look at, as was recently derived by the author

$$\left[\left[\frac{\Lambda_{Max} r^4}{8\pi G} \right] \cdot (4/3) \cdot \left[\frac{2\pi^2 g_*}{45} \right]^{1/3} \right]^{3/4} \sim S_{initial} \tag{65}$$

Note that in the case that quantum effects become highly significant, that the contribution as given by $V_4\big|_{Threshold-volume-for-quantum-effects} \sim \left(10^{-4} cm\right)^3$ and potentially much smaller, as in the threshold of Plancks length, going down to possibly as low as 4.22419 × 10^{-105} m ^{3} = 4.22419 × 10^{-96} cm ^{3} leads us to conclude that even with very high temperatures, as an input into the initial entropy, that $S_{initial} \approx 10^{10}$ is very reasonable. Note though that Kolb and Turner [46] , however, have that g_* is at most about 120, whereas the author, in conversation with H. De La Vega, in 2009 [49] indicated that even the exotic theories of g_* have an upper limit of about 1200, and that it is difficult to visualize what g_* is in the initial phases of inflation. De La Vega stated in Como Italy, that he, as a conservative cosmologist, viewed defining g_* in the initial phases of inflation as impossible[49] .If the DM and DE contributions to g_* are allowed, then this supposition as given by [49] is then drawn into question.

One could argue that the presence of a non zero initial radius, that one is using circular reasoning to confirm the existence of non zero initial entropy, whereas we claim that non zero entropy is necessary in information exchange. How we break out of the alleged circular reasoning is to go back again to the datum of (48), namely we assert non zero initial entropy, to exchange information, in order to seed having the following hold from cycle to cycle.

$$\hbar_{old-\text{cosmology}-cycle} = \hbar_{present-\text{cosmology}-cycle} \tag{48}$$

The following will be what is to be worked upon, namely for now assuming that we can break down the degrees of freedom question as follows,

$$g_*\big|_{initial} \cong \left(g_*\big|_{Baryons-initial} \sim 110-120 \right) + \left(g_*\big|_{(DM-DE)-initial} \sim 1000 \right) \equiv 1100-1200 \tag{66}$$

The figure for the first entry is from Kolb and Turner, and what we assume we have to investigate is the bona fides of looking at what happens due to

$$g_*\big|_{(DM-DE)-initial} = -g_*\big|_{Baryons-initial} + \left(\frac{45}{2\pi^2}\right)\cdot\left(\frac{3}{4}\right)^3 \cdot \frac{\left(S_{initial}\right)^4}{\left[\dfrac{\Lambda_{\max}\cdot r^4}{8\pi G}\right]^3} \tag{67}$$

The details of this derivation would assume, that there would be a multiverse, that secondly there would be an initial entropy, and most likely, there would be a non zero initial radii for the start of our present universe. Finally, this is a phenomenological prediction which should be tested, namely, experimental tests which may permit upper bound tests as to the mass of a graviton. The following section makes references to interstellar tests which give upper bound values, which may indicate how the approximation by [1] may be utilized. Note that this following discussion does not take into account theories as to

13b. How the CMBR permits, via maximum frequency, and maximum wave amplitude values, an upper bound value for massive graviton mass m_g

Camp and Cornish (2004) [50] use the typical transverse gravitational gauge h_{ij} with a typically traceless value summed as $0 + h_{+} - h_{+} + 0$ and off diagonal elements of h_{x} on each side of the diagnonal to mix with a value of

$$h_{ij} \equiv \frac{G_N}{c^4} \cdot \frac{2}{r} \cdot \left[\frac{d^2}{dt^2} Q_{ij} \right]_{retarded}^{TT} \tag{68}$$

This assumes r is the distance to the source of gravitational radiation, with the *retarded* designation on Eqn. (68) denoting $\frac{d}{dt}$ replaced by a retarded time derivative $\frac{d}{d[t-(r/c)]}$, while TT means take the transverse projections and substract the trace. Here, we call the quadrupole moment, with $\rho(t,x)$ a density measurement. Now, the following value of the Q_{ij} as given gives a luminosity function L, where R is the 'characteristic size' of a gravitational wave source. Note that if M is the mass of the gravitating system [50].

$$Q_{ij} = \int d^3x \left[x_i x_j - \frac{1}{3} \cdot \delta_{ij} \cdot x^2 \right] \cdot \rho(t,x) \tag{69}$$

$$L \approx \frac{1}{5} \cdot \frac{G_N}{c^5} \cdot \frac{d^3 Q_{ij}}{dt^3} \cdot \frac{d^3 Q^{ij}}{dt^3} \cong \frac{\pi \cdot c^5}{G_N} \cdot \left(\frac{G_N M}{R \cdot c^2} \right)^2 \tag{70}$$

After certain considerations reported by Camp and Cornish (2004) , one can recover a net GW amplitude

$$h \sim 2 \cdot \left[\frac{G_N \cdot M}{R \cdot c^2} \right] \cdot \left[\frac{G_N \cdot M}{r \cdot c^2} \right] \tag{71}$$

This last equation requires that $R > R_G = \frac{G_N M}{c^2} \equiv$ gravitational radius of a system, with a black hole resulting if one sets $R < R_G = \frac{G_N M}{c^2}$. Note that when $R \sim R_G = \frac{G_N M}{c^2}$ we are at an indeterminate boundary where one may pick our system as having black hole properties.

Now for stars, Camp and Cornish (2004) [50] give us that

$$h \approx 10^{-21} \cdot \left[\frac{15 \cdot Mpc}{r} \right] \cdot \left[\frac{M}{2.8 M_{solar-mass}} \right]^2 \cdot \left(\frac{90km}{R} \right) \tag{72}$$

$$f \equiv \text{Frequency} \approx \sqrt{\frac{M}{2.8 M_{solar-mass}}} \cdot \sqrt{\frac{90km}{R}} \cdot 100Hz \tag{73}$$

As well as a mean time τ_{GW} for half of gravitational wave potential energy to be radiated away as

$$\tau_{GW} \approx \frac{R}{2\pi \cdot c} \cdot \left[\frac{G_N M}{R \cdot c^2} \right]^{-3} \sim \left(\frac{R}{90km} \right)^4 \cdot \left[\frac{2.8 M_{solar-mass}}{M} \right]^3 \cdot \left(\frac{1}{2} \cdot \sec \right) \tag{74}$$

The assumption we make is that if we model $R \sim R_G = \dfrac{G_N M}{c^2}$, for a sufficiently well posed net mass M that the star formulas roughly hold for early universe conditions, provided that we can have a temperature T for which we can use the approximation $\approx \sqrt{\frac{M}{2.8 M_{solar-mass}}} \cdot \sqrt{\frac{90km}{R}} \cdot 100Hz$ that we also have $\left[\frac{T}{TeV} \right] \sim 10^{13}$ or higher, so, that at a minimum we recover Grishchuck's [51] value of

$$\begin{aligned} f_{Peak} &\approx \left(10^{-3} Hz\right) \cdot \left[\frac{T}{TeV} \right] \sim 10^{10} Hz \\ &\approx \sqrt{\frac{M}{M_{solar-mass}}} \cdot \sqrt{\frac{90km}{R}} \end{aligned} \tag{75}$$

Eq. (75) places , for a specified value of R, which can be done experimentally, an upper bound as far as far as what a mass M would be . Can this be exploited to answer the question of if or not there is a minimum value for the Graviton mass? The key to the following discussion will be that

$$\sqrt{\frac{M}{2.8 M_{solar-mass}}} \cdot \sqrt{\frac{90km}{R}} \approx 10^8 \text{, or larger} \tag{76}$$

13. C. Inter relationship between graviton mass m_g and the problem of a sufficient number of bits of $\hbar$ from a prior multiverse contribution to the present universe, to preserve continuity between fundamental constants namely Planck's constant.

P. Tinyakov (2006) [52] gives that there is, with regards to the halo of sub structures in the local Milky Way galaxy an amplitude factor for gravitational waves of

$$< h_{ij} > \sim 10^{-10} \cdot \left[\frac{2 \cdot 10^{-4} Hz}{m_{graviton}} \right] \tag{77}$$

If we use LISA values for the Pulsar Gravitational wave frequencies , this may mean that the massive graviton is ruled out. On the other hand $\sqrt{\frac{M}{2.8 M_{solar-mass}}} \cdot \sqrt{\frac{90km}{R}} \approx 10^8$ as proportional to the initial entropy leads to looking at, if

$$< h_{ij} > \sim h \sim 10^{-5} \cdot \left[\frac{15Mpc}{r} \right]^{1/2} \cdot \left[\frac{M}{2.8 \cdot M_{solar-mass}} \right]^{1/2} \approx 10^{-30} \tag{78}$$

If the radius is of the order of $r \geq$ 10 billion light-years ~ 4300 *Mpc* or much greater, so then we have, as an example

$$< h_{ij} > \sim 10^{-10} \cdot \left[\frac{2 \cdot 10^{-4} Hz}{m_{graviton}} \right] \approx 5.9 \cdot 10^{-7} \cdot \left[\frac{M}{2.8 \cdot M_{solar-mass}} \right]^{1/2} \tag{79}$$

$$\left[\frac{10^{-7} Hz}{m_{graviton}} \right] \approx \left[\frac{5.9}{\sqrt{5.6}} \right] \cdot \sqrt{\frac{M}{M_{solar-mass}}} \tag{80}$$

This Eq. (71) is in units where $\hbar = c = 1$.

If 10^{-60} **grams per graviton, and 1 electron volt is in rest mass, so** $1.6 \times 10^{-33}\, grams$ $\Rightarrow gram = 6.25 \times 10^{32} eV$**. Then**

$$\left[\frac{10^{-7} Hz}{m_{graviton}}\right]$$
$$\equiv\left[\frac{10^{-7} Hz\cdot\left[6.582\times 10^{-15} eV\cdot s\right]}{\left[10^{-60} grams \equiv 6.25\times 10^{-28} eV\right]\cdot\left[2.99\times 10^{9} meter/\sec\right]^{2}}\right] \quad (81)$$
$$\sim\frac{10^{-22}}{10^{-9}}\sim 10^{-13}$$

Then, there exist

$$M \sim 10^{-26} M_{solar-mass} \approx 1.99\times 10^{33-26} \equiv 1.99\times 10^{7} grams. \quad (82)$$

Conceivably this mass M would be transferred from a a prior multiverse to a present universe, and may have been enough to preserve the value of Planck's constant in the sense of what is represented in (48), as given above.This has much to do with the assumptions as given in [52],[53] and [54] and should be experimentally tested as soon as possible. In particular the value of Eq. (81) is a counter part to the values calculated in [54] and while different in absolute magnitude, the same procedure is in common between Eq.(82) and reference [54].

Of special note, is [55], namely that gravitational waves, have been discovered so that one can say with confidence, that LIGO

Quote:

Observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of $1.0\times 10^{\wedge -21}$. *It matches the waveform predicted by general relativity*

Hence, we have a pretty good idea that at least the outward forms of General relativity have been experimentally vetted. This needs to be contrasted with [29], in that if there are Gaussianity or non Gaussianity issues to contend with, as far as gravitational waves, that the data of [54] be vetted In addition, comparing the above predictions as given in this document should also be checked as to fidelity with [56], which significantly has

Quote:

We constrain the graviton Compton wavelength in a hypothetical theory of gravity in which the graviton is massive and place a 90%-confidence lower bound of 10^13 km. Within our statistical uncertainties, we find no evidence for violations of general relativity in the genuinely strong-field regime of gravity.

i.e. General relativity appears to hold up, well, but in terms of configuring admissible values of a massive graviton, as alluded to in this document, it would be appropriate to review data as to the presumed Compton wavelength of a 'massive graviton' and to insure that it is commensurate with section **13 c** above. I.e. we view that it is, but that in the future great refinements as to section **13 c** should be done, once the procedures of [56] are refined via additional experimentation.

Finally, and not least is, that the ultimate goal should be to determine the utility of not only [56] but of [57], i.e. to determine if scalar-tensor gravity, which would be commensurate with 3, instead of 2 polarization states for gravitation, or classical General relativity is favored by the data . Correct review of [55] and [56] plus refinements of section **13 c** will hopefully allow researchers to determine this, and it would be through utilization of

Quote:

accurate angular and frequency dependent response functions of interferometers for GWs arising from various Theories of Gravity, i.e. General Relativity and Extended Theories of Gravity, will be the definitive test for General Relativity.

The good news is that we are through [55] and [56] learning enough so as to make this determination, and it has to do with refinement of enough information to look at frequency response functions, which was a particular focal point of [55] as to their very careful LIGO work.

In doing all of this it is useful to keep in mind that [55] to [57] plus review of section **13 c** above will permit the following, namely as was stressed in an interaction the author had with the editors of this journal, that

quote

the realization of gravitational wave astronomy will be important for discriminating among general relativity and other gravity theories

The above section **13 c** and references [55] to [57], if considerably refined, will lead to such a goal being accomplished. The author looks forward to this happy occurrence once it commences with the birth of gravitational wave astronomy.

Acknowledgements

This work is supported in part by National Nature Science Foundation of China grant No. 11375279

References

[1] S. Kauffman, " A Self Gravitational Upper bound On Localized Energy Including that of Virtual Particles and Quantum Fields, which Yield a Passable Dark Energy Density Estimate", http://arxiv.org/abs/1212.0426
[2] A. Beckwith, "What violations of the Null Energy Condition tell us about information exchange between prior to present universes? How to obtain spectral index confirmation?" http://vixra.org/abs/1102.0039
[3] H.A. Dye, "On the Ergodic Mixing Theorem", Transactions of the American Mathematical society H. A. Dye Vol. 118, (Jun., 1965), pp. 123-130
[4] C. Gao, " A Model of Nonsingular Universe", Entropy 2012, 14, 1296-1305.
[5] Y, Jack, Ng,"Article: Spacetime Foam: From Entropy and Holography to Infinite Statistics and Nonlocality" Entropy 2008, 10(4), 441-461; DOI: 10.3390/e10040441
[6] Maggiore, M. "Gravitational Waves, Volume 1 : Theory and Experiment", Oxford, 2008
[7] M. Marklund, G. Brodin, and P. Shukla, Phys. Scr. T82 130-132 (1999)
[8] Marco Valerio Battisti , " Cosmological bounce from a deformed Heisenberg Algebra", PRD 79, 083506, (2009)
[9] G. Fuller, C. Kishimoto, Phys. Rev. Lett. 102 (2009) 201303.

[10] R, Maartens *Brane world cosmology*, pp 213-247 from the conference The physics of the Early Universe, editor Papantronopoulos, (Lect. notes in phys., Vol 653, Springer Verlag)
[11] R. Maartens, Brane-World Gravity, http://www.livingreviews.org/lrr-2004-7 (2004)
[12] V. Rubakov, "Classical Theory of Gauge Fields," Princeton University Press, Princeton, 2002.
[13] V. Rubakov, Proc. Les Houches Summer School on Theoretical Physics. Session 84: Particle Physics Beyond the Standard Model, 2005.
[14] V. A. Rubakov , P. G. Tinyakov ," Infrared-modified gravities and massive gravitons" Phys.Usp.51:759-792,2008, http://arxiv.org/abs/0802.4379
[15] S. Dubovsky, R. Flauger, A. Starobinsky and I. Tkachev, report UTTG-06-09, TCC-23- 09. http://arxiv.org/abs/0907. 1658
[16] M. Alves, O. Miranda. and J. de Araujo, J. ," Can Massive Gravitons be an Alternative to Dark Energy?", arXiv:0907.5190 (July 2009)
[17] A.W. Beckwith, "DECELERATION PARAMETER Q(Z) IN FOUR AND FIVE DIMENSIONAL GEOMETRIES, AND IMPLICATIONS OF GRAVITON MASS IN MIMICKING DE IN BOTH GEOMETRIES", http://vixra.org/pdf/1002.0056v1.pdf
[18] A,W, Beckwith, Identifying a Kaluza Klein Treatment of a Graviton Journal of Cosmology, Vol 13**.; http://journalofcosmology.com/BeckwithGraviton.pdf**
[19] T. Padmanabhan, *An Invitation to Astrophysics*, World Scientific series in Astronomy and Astrophysics, Vol. 8
[20] T. Padmanabhan, *Theoretical Astrophysics,Volume 1, Astrophysical Processes*, Cambridge University press, Cambridge, UK, 2000
[21] V. Taveras, "Corrections to the Friedmann Equations from LQG for a Universe with a Free Scalar Field"; Phys. Rev. D78, 064072 (2008); http://arxiv.org/abs/0807.3325
[22] G. t'Hooft., "The mathematical basis for Deterministic Quantum Mechanics," http://arxiv.org/PS_cache/quant-ph/pdf/0604/0604008v2.pdf, (2006).
[23] G.t'Hooft, "Determinism beneath Quantum Mechanics", http://arxiv.org/PS_cache/quant-ph/pdf/0212/0212095v1.pdf
[24] A.W. Beckwith, "Classical and Quantum Models of Density Wave Transport, a comparative study", PhD dissertation, December 2001
[25] A.W. Beckwith, 'A New S-S' Pair Creation Rate Expression Improving Upon Zener Curves for I-E Plots", Mod.Phys.Lett. B20 (2006) 849-861, http://arxiv.org/abs/math-ph/0411045 ;
[26] M. Peskins, *Introduction to Quantum field theory,* Frontiers in Physics(1995)
[27] S. Bashinsky, http://arxiv.org/abs/astro-ph/0505502 (2005).
[28] M. Giovannini, "Non Topological Gravitating defects in 5 dimensional space", Class. Quantum. Grav. 23, 2006 , L73-80

[29] Sabino Matarrese, Lecture, International school of astro particle physics, Como, Italy, 10 July 2009.
[30] G. Raffert, Erice nuclear physics school, Private conversation, 23 September 2009.
[31] A. Beckwith, "Energy, Neutrino physics and the Lithium Problem: Why there are stars with essentially no lithium due to serious Lithium deficiency in certain spatial regions in the Early Universe?", progress in particle and nuclear physics(2010), 64,(2010),p 426-428
[32] Risa h. Wechsler, PhD Thesis, June 2001: Dark Halo Merging and Galaxy Formation , http://risa.stanford.edu/thesis.php
[33] B. Eberle, A. Ringwald, L. Song, T. J. Weiler, "Relic Neutrino Absorption Spectroscopy" , Phys.Rev. D70 (2004) 023007,Report number: DESY 03-219, SLAC-PUB-10302; http://arxiv.org/abs/hep-ph/0401203
[34] D. Valev, Aerospace Res. Bulg. 22:68-82, 2008; ; http://arxiv.org/abs/hep-ph/0507255
[35] A. Beckwith, http://vixra.org/abs/0912.0012, v 6 (newest version).
[36] B. Kay, Quantum Field theory in Curved Spacetime, pp 180-190, in the Encyclopedia of Mathematical physics, Vol 5, General Relativity; Quantum Gravity; String theory and M. Theory, With J-P Franciose, G. Naber, and T. Tsun as editors
[37] A.W. Beckwith," Is Nature Fundamentally Continuous or Discrete, and How Can These Two Different But Very Useful Conceptions be Fully Reconciled? (Condensed Version)", submitted for FQXi contest as of February 2011, http://vixra.org/abs/1102.0019
[38] T. Padmanabhan , "Lessons from Classical Gravity about the Quantum Structure of Spacetime", http://arxiv.org/pdf/1012.4476.pdf
[39] A. W. Beckwith, "How to use the cosmological Schwinger principle for energy flux, entropy, and "atoms of space–time" to create a thermodynamic space–time and multiverse", http://iopscience.iop.org/1742-6596/306/1/012064/
[40] Kiefer, Polarski, and Starobinsky," Quantum-to-classical transition for fluctuations in the early Universe", Int.J.Mod.Phys. D7 (1998) 455-462, http://arxiv.org/abs/gr-qc/9802003
[41] R. Penrose, "Conformal Cyclic Cosmology, Dark Matter, and Black Hole Evaporation", August 11, 2007 ,IGC *Inaugural* Conference. *State* College, PA. *Penn State University ... August* 7 – *August 11, 2007*.
[42] R. Penrose," Before the Big Bang, An outrageous new perspective and its implications for Particle physics", Proceedings of the European Particle Accelerator Conference (EPAC, 2006), pp 2759-2763, 2006
[43] R. Penrose, *Cycles of Time, an Extrardinary New View of the Universe* Alfred A. Knoff press, 2011
[44] A. Beckwith, "Analyzing Black Hole Super radiance Emission of Particles/ Energy from a Black Hole as a Gedankernsperiment to get Bounds on the Mass of a Graviton" ; Hindawi Publishing Corporation, , Advances in High Energy Physics, Volume 2014,

Article ID 230713, 7 pages; http://www.researchgate.net/publication/261709517_httpwww.hindawi.comjournalsahep2014230713
[45] N. Poplawski, “Cosmological constant from QCD vacuum and torsion,”Annalen der Physik, vol. 523, pp. 291–295, 2011. ArXiv:1005.0893 [gr-qc]
[46] Kolb and Turner, *The Early Universe*, Frontiers in Physics, Addison And Welsley Publishing company, Advanced Book program, Menlo Park, California , 1990
[47] R. Feynman, A. Hibbs *Quantum Mechanics and Path Integrals* Dover Publications, Inc, Mineola, New York, 2005
[48] D.K. Park, H. Kim., and S. Tamarayan, “Nonvanishing Cosmological Constant of Flat Universe in Brane world Scenarios,” Phys.Lett. B535, 5-10 (2002)
[49], Hector De La Vega, lecture on cosmology, and cosmological evolution, as given as a lecture at the ISAPP
(International School of Astro Particle Physics (2009)), “ Cosmic Microwave Background and Fundamental Interaction Physics”, in Como, Italy 8-16 July 2009. Personal observations given to Author
[50] J. Camp and N. Cornish ., “Gravitational wave Astronomy”,pp 525-577, as part of “ Annual Review of Nuclear and Particle Science”, Volume 54, with Kayser, B., Holstein, B., and Jawahery, A. as editors. Menlo Park, California, 2004
[51] L. Grishkuk, “Discovering Relic Gravitational Waves in Cosmic Microwave Background Radiation”, http://arxiv.org/abs/0707.3319 (2008)
[52] P. Tinyakov, “Giving mass to the graviton,” in Particle Physics,and Cosmology, the Fabric of Space-Time, F. Bernadeau, C.Grojean, and J. Dalibard, Eds., pp. 471–499, Elsevier, Oxford,UK, 2007, part of Les Houches, Session 86.
[53] M Novello and R P Neves,”The mass of the graviton and the cosmological constant”, Class. Quantum Grav. 20 No 6 (21 March 2003) L67-L73
[54] C. Corda, “Primordial Gravity’s breath”, http://arxiv.org/pdf/1110.1772
[55] B. P. Abbott *et al.* (LIGO Scientific Collaboration and Virgo Collaboration)

Phys. Rev. Lett. **116**, 061102 – Published 11 February 2016

[56] The LIGO Scientific Collaboration, the Virgo Collaboration,"Tests of general relativity with GW150914", http://arxiv.org/abs/1602.03841

[57] Christian Corda, "Interferometric detection of gravitational waves: the definitive test for General Relativity", Int. J. Mod. Phys. D18:2275-2282,2009, http://arxiv.org/abs/0905.2502