E8 Root Vectors and Geometry of E8 Physics

Frank Dodd (Tony) Smith, Jr. - 2016 - viXra 1602.0319

Abstract - Introduction

This paper is an exposition of my view that the 240 E8 Root Vectors encode the basic structure of a Unified Theory of Fundamental Physics by forming a local classical Lagrangian for the Standard Model plus Gravity and Dark Energy.

The Root Vectors know where they belong in the Lagrangian because of their place in the geometric structure of E8 and its related symmetric spaces such as:
E8 / D8 = 128-dim (OxO)P2
E8 / E7 x SU(2) = 112-dim set of (QxO)P2 in (OxO)P2
D8 / D4 x D4 = 64-dim Gr(8,16)

By embedding each E8 local classical Lagrangian into a Cl(16) Real Clifford Algebra and taking the completion of the union of all tensor products of all the Cl(16)s you get a generalized hyperfinite II1 von Neumann factor Algebraic Quantum Field Theory that is a realistic global quantum theory for our universe.

The main body of the paper describes physical interpretations of the 240 Root Vectors with a rough qualitative description of how they are used in setting up calculations of force strengths, particle masses, Dark Energy : Dark Matter : Ordinary Matter ratios, Kobayashi-Maskawa parameters, etc. It is not intended to give full details of all calculations etc but is only intended to provide an expository overview of how the 240 E8 Root Vectors produce a realistic Unified Theory of Fundamental Physics.

The main body of the paper (46 pages) concludes with a summary of the results of those calculations. Details of the calculations, some related experimental results, etc, are given in a more lengthy set of appendices (216 pages) that can be read and evaluated by anyone who might be interested.

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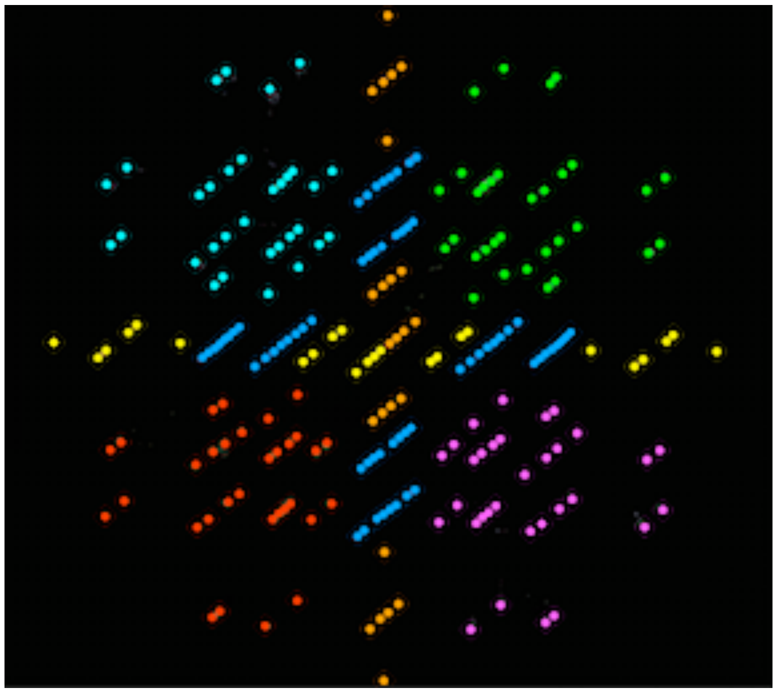
Appendices:

E8 Root Vectors and Maximal SubGroups

248-dim Lie Group E8 has 240 Root Vectors arranged on a 7-sphere S7 in 8-dim space.

Since it is hard to visualize points on S7 in 8-dim space,
I prefer to represent the 240 E8 Root Vectors in 2-dim / 3-dim space as



To understand the Geometry related to the 240 E8 Root Vectors, consider that

248-dim E8 = 120-dim Spin(16) D8 + 128-dim half-spinor of Spin(16) D8

and

240 E8 Root Vectors = 112 D8 Root Vectors + 128 D8 half-spinors

and
there are two ways to see a maximal symmetric subspace of E8 and E8 Root Vectors:

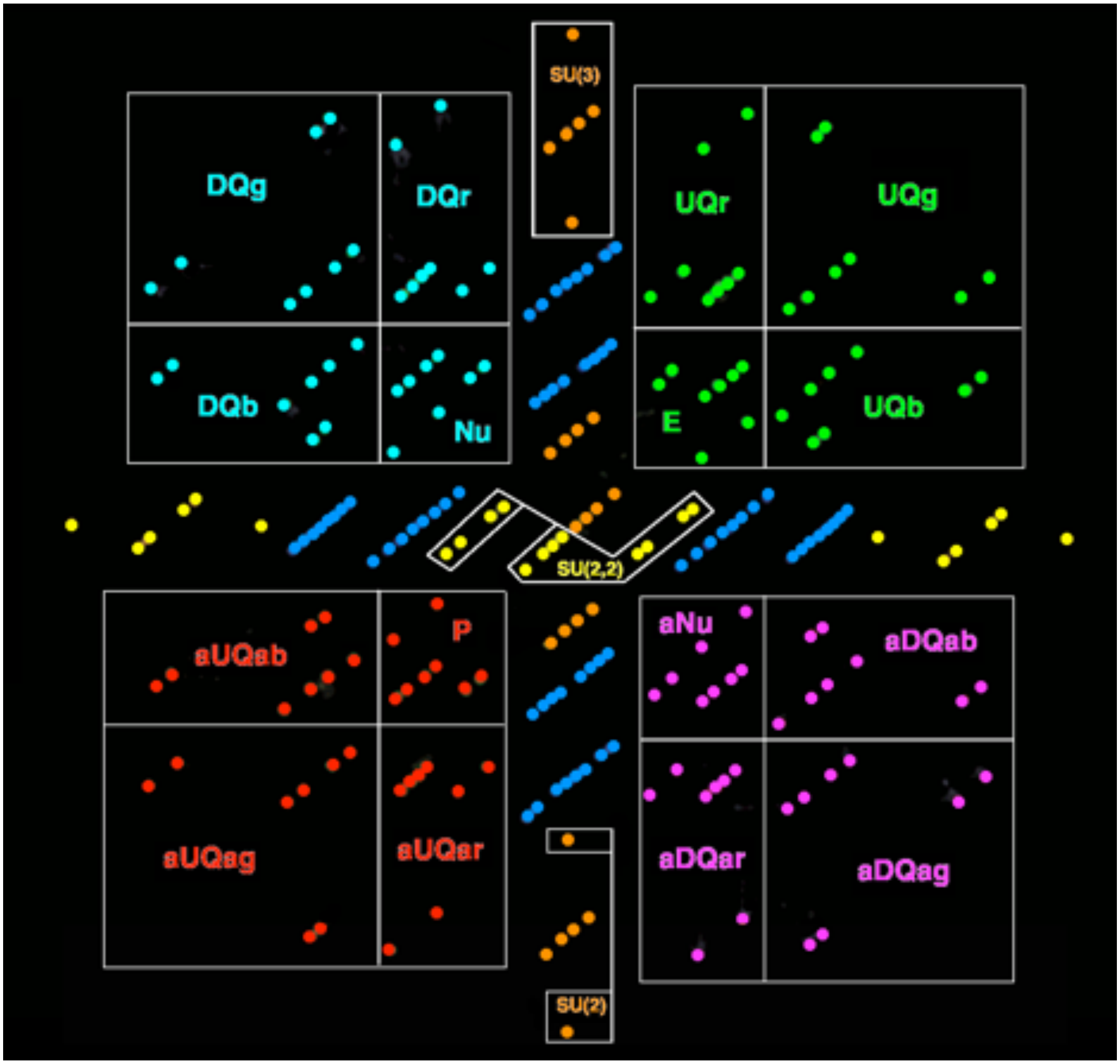
the symmetric space corresponding to the 128 D8 half-spinors

E8 / D8 = 128-dim Octonion-Octonionic Projective Plane (OxO)P2

and
the symmetric space corresponding to the 112 D8 Root Vectors

E8 / E7 x SU(2) = 112-dim set of (QxO)P2 in OxO)P2
where (QxO)P2 = Quaternion-Octonion Projective Planes

Geometric Structure leads to physical interpretation of the E8 Root Vectors as:



E = electron, UQr = red up quark, UQg = green up quark, UQb = blue up quark
Nu = neutrino, DQr = red down quark, DQg = green down quark, DQb = blue down quark
P = positron, aUQar = anti-red up antiquark,
aUQag = anti-green up antiquark, aUQab = anti-blue up antiquark
aNu = antineutrino, aDQar = anti-red down antiquark,
aDQag = anti-green down antiquark, aDQab = anti-blue down antiquark

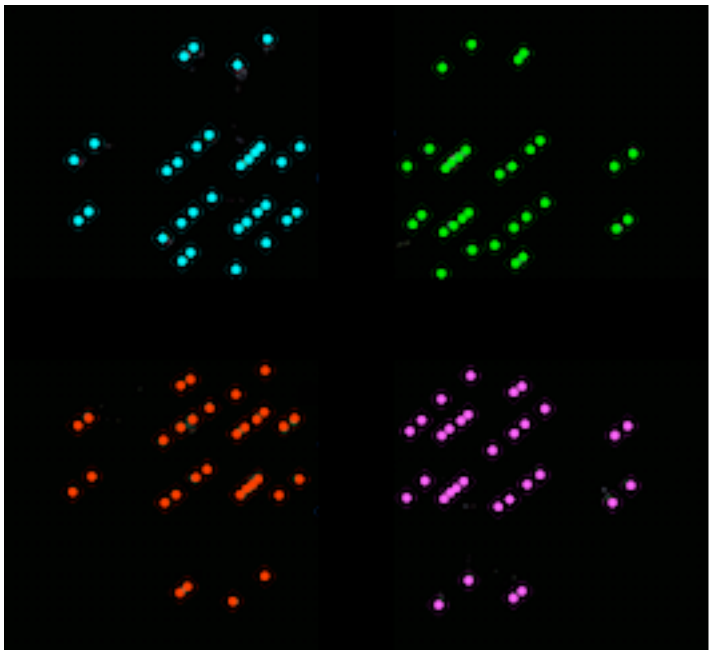
Each Lepton and Quark has 8 components with respect to 4+4 dim Kaluza-Klein
6 orange SU(3) and 2 orange SU(2) represent Standard Model root vectors
24-6-2 = 16 orange represent U(2,2) Conformal Gravity Ghosts
12 yellow SU(2,2) represent Conformal Gravity SU(2,2) root vectors
24-12 = 12 yellow represent Standard Model Ghosts
32+32 = 64 blue represent 4+4 dim Kaluza-Klein spacetime position and momentum

Octonionic D8 and Spinor Fermions and Coleman-Mandula

First consider the symmetric space corresponding to the 128 D8 half-spinors

E8 / D8 = 128-dim Octonion-Octonionic Projective Plane (OxO)P2

These are the 128 (OxO)P2 half-spinor E8 Root Vectors on a S7 in 8-dim space:



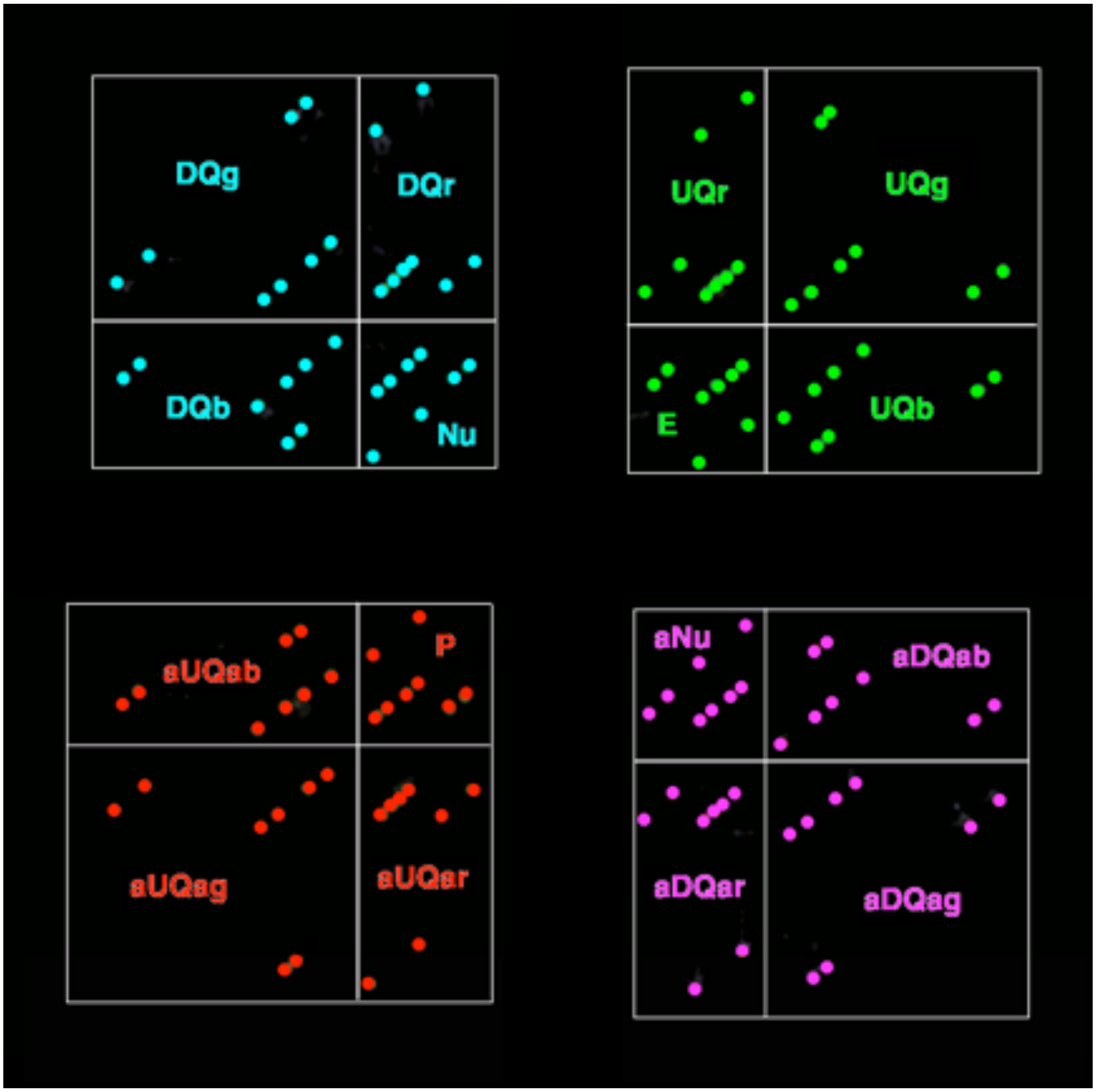
Since D8 Spin(16) is the local isotropy group of E8 / D8 = (OxO)P2
the 128 = 64+64 = 8x8 + 8x8 half-spinor Root Vectors have Octonionic Symmetry
and can represent

8 components of 8 Generation-1 Fermion Particles (green/cyan dots)
plus
8 components of 8 Generation-1 Fermion AntiParticles (red/magenta dots)

Fermion Types are represented by Octonion Basis Elements:

1 - Neutrino
i - Red Down Quark
j - Green Down Quark
k - Blue Down Quark
E - Electron
I - Red Up Quark
J - Green Up Quark
K - Blue Up Quark

In this view, the physical interpretation of the 128 Fermion Root Vectors is



The Octonionic 8 Gen-1 Fermion Particles and 8 Gen-1 Fermion AntiParticles have Lorentz structure of spinor representations for 8-dim Spin(1,7) spacetime (since D8 = Cl(16) = Cl(8) x Cl(8) by 8-Periodicity and Cl(1,7) = Cl(0,8) = M(16,R) = 16x16 Real Matrix Algebra) and therefore satisfy the Coleman-Mandula Theorem

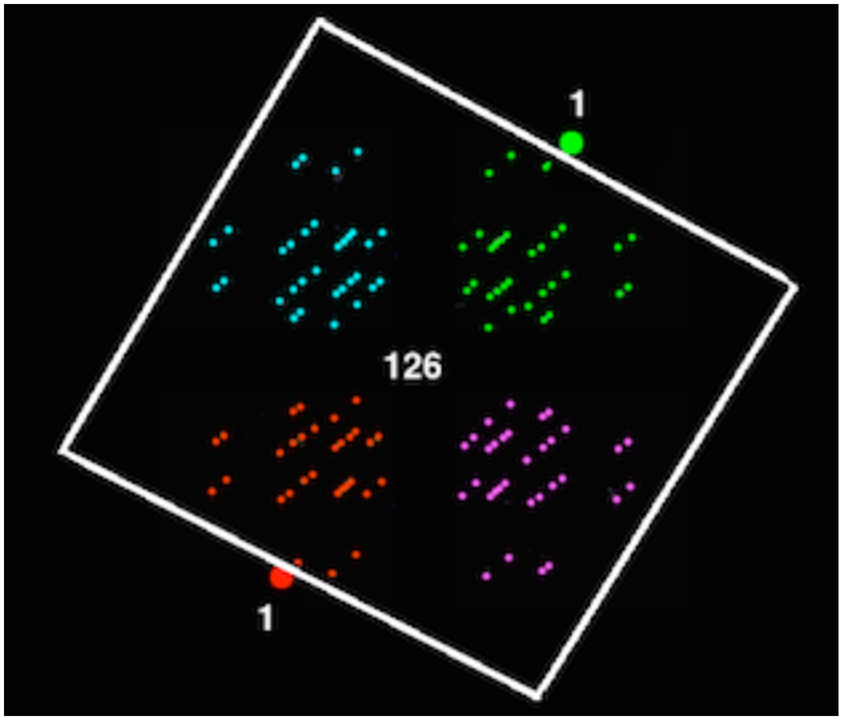
(see Appendix - Coleman-Mandula)

Creation-Annihilation Operators for the 8 components of the 8+8 Fermions are the odd-grade-+/-1 part of the E8 Maximal Contraction generalized Heisenberg Algebra h92 x A7 = 28 + **64** + ((SL(8,R)+1) + **64** + 28

(see Rutwig Campoamor-Stursberg in “Contractions of Exceptional Lie Algebras and SemiDirect Products” (Acta Physica Polonica B 41 (2010) 53-77)

Quaternionic E7 and SU(2)

If you pick two antipodal Root Vectors (the green dot at 1 and the red dot at another 1)



then you get 126 Root Vectors in a 7-dim hyperplane perpendicular to the antipodal axis so that

the green 1 dot and the red 1 dot are Root Vectors of 3-dim SU(2)
and
the 126 Root Vectors in the 7-dim hyperplane are Root Vectors of 133-dim E7

therefore

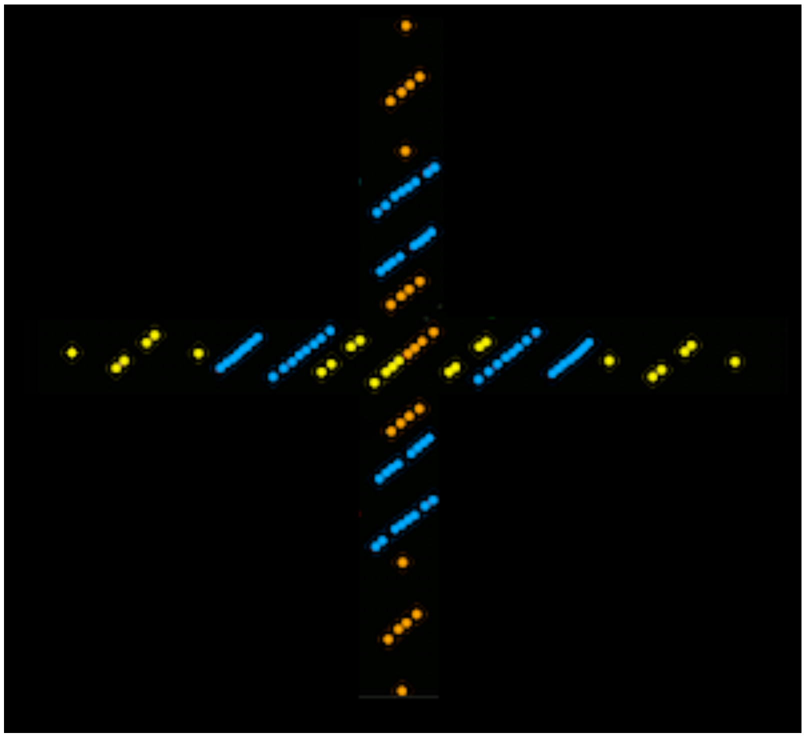
it is natural to consider the symmetric space corresponding to the 112 D8 Root Vectors

E8 / E7 x SU(2) = 112-dim set of (QxO)P2 in (OxO)P2

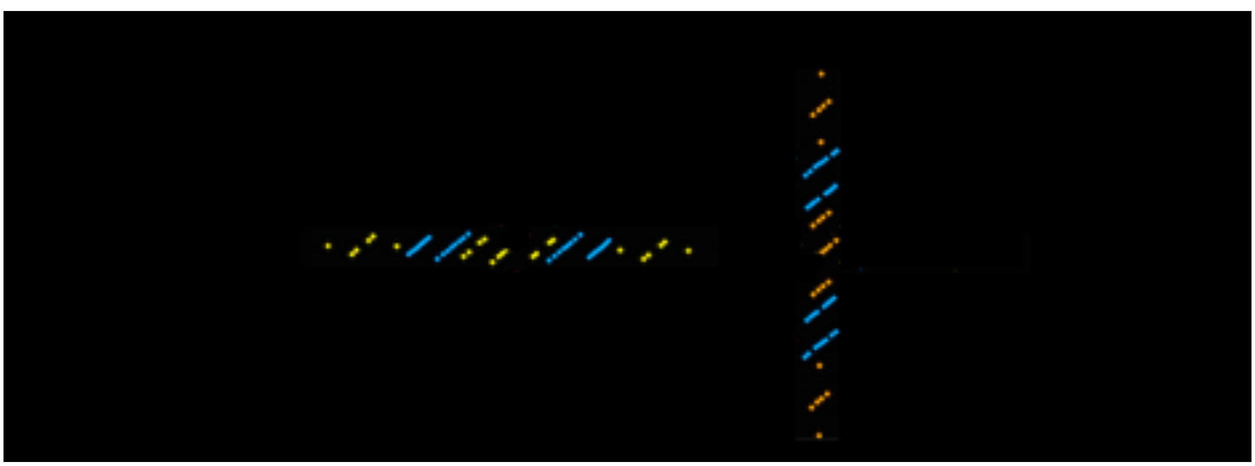
Since E7 x SU(2) is its local isotropy group

E8 Physics Structures of the 112 D8 Root Vectors have Quaternionic Symmetry

These are the 112 (QxO)P2 half-spinor E8 Root Vectors on a S7 in 8-dim space:

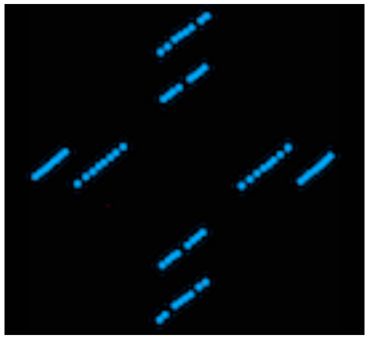


The 112 fall naturally into two sets of 56:



24 yellow / 32 blue horizontal 56 give D4 of Conformal Gravity / M4 of 4+4 Kaluza-Klein
and
24 orange / 32 blue vertical 56 give D4 of Standard Model / CP2 of 4+4 Kaluza-Klein

Spacetime, Unimodular Gravity, and Strong CP

The 32+32 = 64 blue  correspond to the
64-dim symmetric space D8 / D4 x D4 = Gr(8,16) Grassmannian = set of RP7 in RP15

Creation-Annihilation Operators for 8-dim spacetime x 8-dim momentum space are the 64-dim grade-0 part of the E8 Maximal Contraction generalized Heisenberg Algebra

h92 x A7 = 28 + 64 + **((SL(8,R)+1)** + 64 + 28

Unimodular SL(8,R) Gravity effectively describes a generalized checkerboard of 8-dim SpaceTime HyperVolume Elements and, with respect to Cl(16) = Cl(8)xCl(8), is the tensor product of the two 8v vector spaces of the two Cl(8) factors of Cl(16). If those two Cl(8) factors are regarded as Fourier Duals,
then **8v x 8v describes Position x Momentum in 8-dim SpaceTime**.

Bradonjic and Stachel in arXiv 1110.2159 said: "... in ... Unimodular relativity ... the metric tensor ... break[s up] ... into the conformal structure represented by a conformal metric ... with det = -1 and a four-volume element ... at each point of space-time ... [that]... may be the remnant, in the ... continuum limit, of a more fundamental discrete quantum structure of space-time itself ...".

Conformal Spin(2,4) = SU(2,2) Gravity and Unimodular SL(4,R) = Spin(3,3) Gravity seem to be effectively equivalent. Padilla and Saltas in arXiv 1409.3573 said:
"... classical unimodular gravity and classical GR are the same thing, and they can be extended into the UV such that the equivalence is maintained. ...
Classical unimodular gravity = classical GR. ...
Quantum unimodular gravity = quantum GR provided we make certain assumptions about how we extend into the UV. ...".

Frampton, Ng, and Van Dam in J. Math. Phys. 33 (1992) 3881-3882 said:
"... Because of the existence of topologically nontrivial solutions, instantons, of the classical field equations associated with quantum chromodynamics (QCD), the quantized theory contains a dimensionless parameter ø (0 < ø < 2π) not explicit in the classical lagrangian. Since ø multiplies an expression odd in CP, **QCD predicts violation of ... CP ... symmetry unless the phase ø takes one of the special values ... 0 (mod π) ... this fine tuning is the strong CP problem ... the quantum dynamics of ... unimodular gravity ... may lead to the relaxation of ø to ø = 0 (mod π) without the need ... for a new particle** ... such as the axion ...".

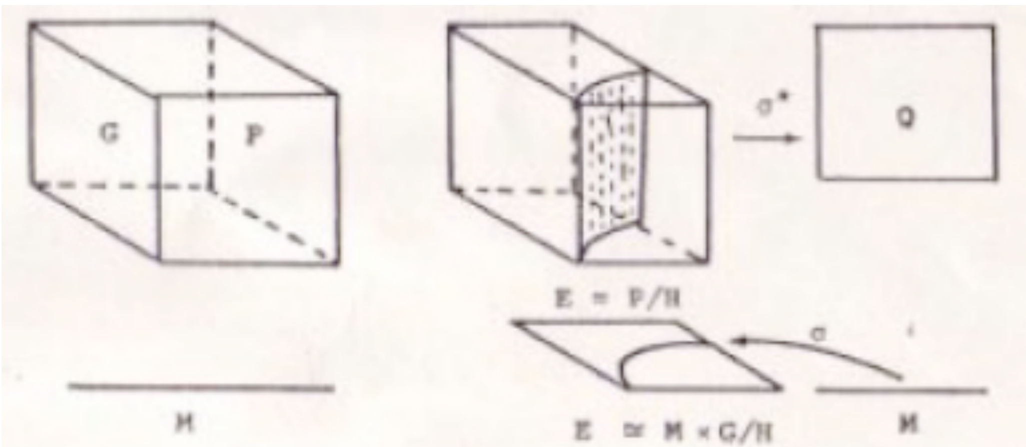
Higgs and 3-state Higgs-Tquark Sysytem

Quaternionic E7xSU(2) structure breaks 8-dim Spacetime Octonionic Symmetry to Quaternionic (4+4)-dim Associative x CoAssociative Kaluza-Klein Spacetime

(see Reese Harvey "Spinors and Calibrations" (Academic 1990))

where M4 = 4-dim Minkowski Physical Spacetime is Associative
and CP2 = SU(3) / SU(2) x U(1) Internal Symmetry Space is CoAssociative

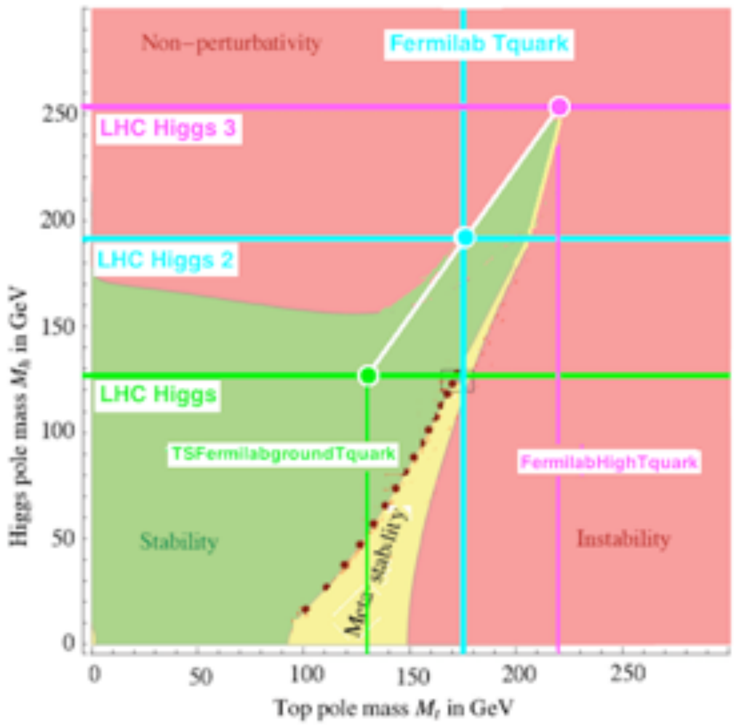
Meinhard Mayer said (Hadronic Journal 4 (1981) 108-152): "... each point of ... the ... fibre bundle ... E ...



... consists of
a four- dimensional spacetime point x [in M4]
to which is attached the homogeneous space G / H [SU(3) / U(2) = CP2]
...
the components of the curvature lying in the homogeneous space G / H could be reinterpreted as Higgs scalars (with respect to spacetime [M4])
...
the Yang-Mills action reduces to a Yang-Mills action for the h-components [U(2) components] of the curvature over M [M4] and a quartic functional for the "Higgs scalars", which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK ... Brout-Englert-Higgs-Kibble ... mechanism to work. ...".

(see Appendix - Mayer - Higgs)

The Cl(16)-E8 model identifies the Higgs with Primitive Idempotents of the Cl(8) real Clifford algebra, whereby the Higgs is not seen as a simple-minded single fundamental scalar particle, but rather the Higgs is seen as a quantum process that creates a fermionic condensate and effectively a 3-state Higgs-Tquark System.



The Green Dot where the White Line originates in our Ordinary Phase is the low-mass state of a 130 GeV Truth Quark and a 125 GeV Higgs**.**

The Cyan Dot where the White Line hits the Triviality Boundary leaving the Ordinary Phase is the middle-mass state of a 174 GeV Truth Quark and Higgs around 200 GeV. It corresponds to the Higgs mass calculated by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they say:
"... We perform the most attractive channel (MAC) analysis in the top mode standard model with TeV-scale extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in D(=6,8,10,...) dimensions. In such a model, bulk gauge couplings rapidly grow in the ultraviolet region. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. We then find that the top condensate can be the MAC for D=8 ... We predict masses of the top (m_t) and the Higgs (m_H) ... based on the renormalization group for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the scale where the bulk top condenses ... for ...[Kaluza-Klein type]... dimension... D=8 ...

m_t = 172-175 GeV and m_H=176-188 GeV ...".
As to composite Higgs and the Triviality boundary, Pierre Ramond says in his book Journeys Beyond the Standard Model (Perseus Books 1999) at pages 175-176: "... The Higgs quartic coupling has a complicated scale dependence. It evolves according to d lambda / d t = (1 / 16 pi^2) beta_lambda where the one loop contribution is given by beta_lambda = 12 lambda^2 - ... - 4 H ... The value of lambda at low energies is related [to] the physical value of the Higgs mass according to the tree level formula m_H = v sqrt(2 lambda) while the vacuum value is determined by the Fermi constant ... for a fixed vacuum value v, let us assume that the Higgs mass and therefore lambda is large. In that case, beta_lambda is dominated by the lambda^2 term, which drives the coupling towards its Landau pole at higher energies.
Hence the higher the Higgs mass, the higher lambda is and the close[r] the Landau pole to experimentally accessible regions.
This means that for a given (large) Higgs mass, we expect the standard model to enter a strong coupling regime at relatively low energies, losing in the process our ability to calculate. This does not necessarily mean that the theory is incomplete, only that we can no longer handle it ... it is natural to think that this effect is caused by new strong interactions, and that the Higgs actually is a composite ...
The resulting bound on lambda is sometimes called the triviality bound.
The reason for this unfortunate name (the theory is anything but trivial) stems from lattice studies where the coupling is assumed to be finite everywhere; in that case the coupling is driven to zero, yielding in fact a trivial theory.
In the standard model lambda is certainly not zero. ...".

Middle Mass State Cross Section:

In the Cl(16)-E8 model, the Middle-Mass Higgs has structure that is not restricted to Effective M4 Spacetime as is the case with the Low-Mass Higgs Ground State

but extends to the full 4+4 = 8-dim structure of M4xCP2 Kaluza-Klein.

```
T ----------- Tbar        in CP2 Internal Symmetry Space
 \           /
  \         /
   \       /
    \     /
     \   /
    Higgs                 in M4 Physical SpaceTime
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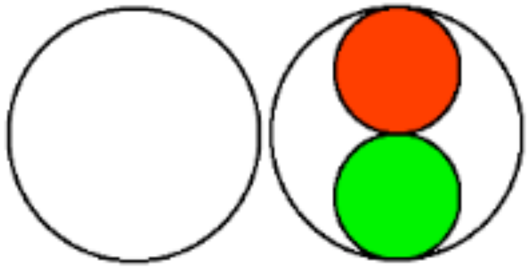
Therefore the Mid-Mass Higgs looks like a 3-particle system of Higgs + T + Tbar.

The T and Tbar form a Pion-like state.
Since Tquark Mid-Mass State is 174 GeV
the Middle-Mass T-Tbar that lives in the CP2 part of (4+4)-dim Kaluza-Klein has mass (174+174) x (135 / (312+312) = 75 GeV.

The Higgs that lives in the M4 part of (4+4)-dim Kaluza-Klein
has, by itself, its Low-Mass Ground State Effective Mass of 125 GeV.
So, the total Mid-Mass Higgs lives in full 8-dim Kaluza-Klein
with mass 75+125 = 200 GeV.
This is consistent with the Mid-Mass States of the Higgs and Tquark
being on the Triviality Boundary of the Higgs - Tquark System
and with the 8-dim Kaluza-Klein model in hep-ph/0311165 by Hashimoto, Tanabashi,
and Yamawaki.As to the cross-section of the Middle-Mass Higgs



consider that the entire Ground State cross-section lives only in 4-dim M4 spacetime
(left white circle)
while the Middle-Mass Higgs cross-section lives in full 4+4 = 8-dim Kaluza-Klein
(right circle with red area only in CP2 ISS and white area partly in CP2 ISS
with only green area effectively living in 4-dim M4 spacetime)
so that
our 4-dim M4 Physical Spacetime experiments only see for the Middle-Mass Higgs
a cross-section that is 25% of the full Ground State cross-section.

The 25% may also be visualized in terms of 8-dim coordinates {1,i,j,k,E,I,J,K}

	1	i	j	k	E	I	J	K
1	11	1i	1j	1k	1E	1I	1J	1K
i	i1	ii	ij	ik	iE	iI	iJ	iK
j	j1	ji	jj	jk	jE	jI	jJ	jK
k	k1	ki	kj	kk	kE	kI	kJ	kK
E	E1	Ei	Ej	Ek	EE	EI	EJ	EK
I	I1	Ii	Ij	Ik	IE	II	IJ	IK
J	J1	Ji	Jj	Jk	JE	JI	JJ	JK
K	K1	Ki	Kj	Kk	KE	KI	KJ	KK

in which {1,i,j,k} represent M4 and {E,I,J,K} represent CP2.

The Magenta Dot  at the end of the White Line is the high-mass state of a 220 GeV Truth Quark and a 240 GeV Higgs. It is at the critical point of the Higgs-Tquark System with respect to Vacuum Instability and Triviality. It corresponds to the description in hep-ph/9603293 by Koichi Yamawaki of the Bardeen-Hill-Lindner model:
"... the BHL formulation of the top quark condensate ... is based on the RG equation combined with the compositeness condition ... start[s] with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level ...
BHL is crucially based on the perturbative picture ...[which]... breaks down at high energy near the compositeness scale Λ ...[10^19 GeV]...
there must be a certain matching scale Λ_Matching such that
the perturbative picture (BHL) is valid for mu < Λ_Matching, while only the nonperturbative picture (MTY) becomes consistent for mu > Λ_Matching ...
However, thanks to the presence of a quasi-infrared fixed point,
BHL prediction is numerically quite stable against ambiguity at high energy region, namely, rather independent of whether this high energy region is replaced by MTY or something else. ... Then we expect mt = mt(BHL) = ... = 1/(sqrt(2)) ybart v within 1-2%, where ybart is the quasi-infrared fixed point given by Beta(ybart) = 0 in ... the one-loop RG equation ...
The composite Higgs loop changes ybart^2 by roughly the factor Nc/(Nc +3/2) = 2/3 compared with the MTY value, i.e., 250 GeV -> 250 x sqrt(2/3) = 204 GeV, while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. The BHL value is then given by mt = 218 +/- 3 GeV, at Λ = 10^19 GeV.
The Higgs boson was predicted as a tbar-t bound state
with a mass MH = 2mt based on the pure NJL model calculation.

Its mass was also calculated by BHL through the full RG equation ...
the result being ... MH / mt = 1.1) at /.\ = 10^19 GeV ...
... the top quark condensate proposed by Miransky, Tanabashi and Yamawaki (MTY) and by Nambu independently ... entirely replaces the standard Higgs doublet by a composite one formed by a strongly coupled short range dynamics (four-fermion interaction) which triggers the top quark condensate. The Higgs boson emerges as a tbar-t bound state and hence is deeply connected with the top quark itself. ... MTY introduced explicit four-fermion interactions responsible for the top quark condensate in addition to the standard gauge couplings. Based on the explicit solution of the ladder SD equation, MTY found that even if all the dimensionless four-fermion couplings are of O(1), only the coupling larger than the critical coupling yields non-zero (large) mass ... The model was further formulated in an elegant fashion by Bardeen, Hill and Lindner (BHL) in the SM language, based on the RG equation and the compositenes condition. BHL essentially incorporates 1/Nc sub-leading effects such as those of the composite Higgs loops and ... gauge boson loops which were disregarded by the MTY formulation. We can explicitly see that BHL is in fact equivalent to MTY at 1/Nc-leading order. Such effects turned out to reduce the above MTY value 250 GeV down to 220 GeV ...".

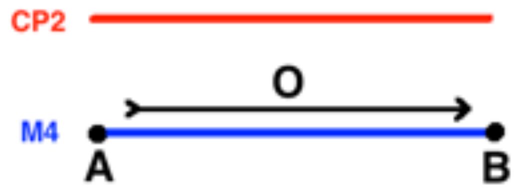
High Mass State Cross Section:

As with the Middle-Mass Higgs,
the High-Mass Higgs lives in all 4+4 = 8 Kaluza-Klein dimensions
so
its cross-section is also about 25% of the Higgs Ground State cross-section.

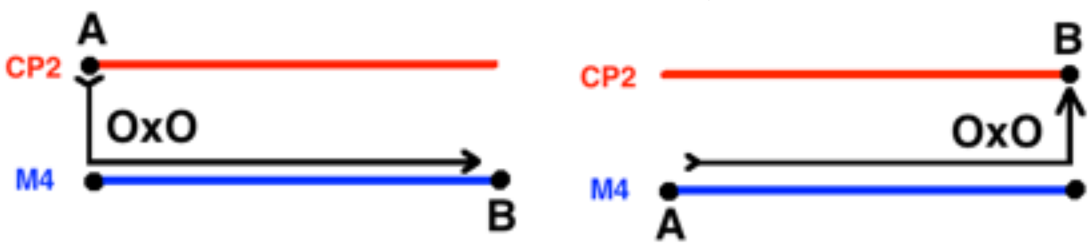
3 Generations of Fermions

In Kaluza-Klein M4 x CP2 there are 3 possibilities for a fermion represented by an Octonion O basis element to go from point A to point B:

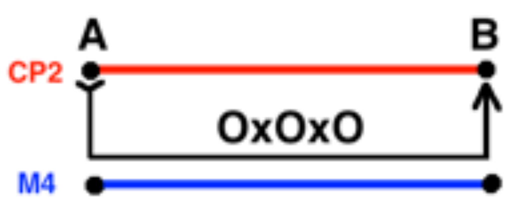
1 - A and B are both in M4: First Generation Fermion
whose path can be represented by the single O basis element
so that First Generation Fermions are represented by Octonions O.



2 - Either A or B, but not both, is in CP2: Second Generation Fermion
whose path must be augmented by one projection from CP2 to M4,
which projection can be represented by a second O basis element
so that Second Generation Fermions are represented by Octonion Pairs OxO.



3 - Both A and B are in CP2: Third Generation Fermion
whose path must be augmented by two projections from CP2 to M4,
which projections can be represented by a second O and a third O,
so that Third Generation Fermions are represented by Octonion Triples OxOxO.



D4 of Standard Model Gauge Bosons and Gravity Ghosts

The 24 orange  are Root Vectors of
the CP2-related D4 local isotropy group in the symmetric space D8 / D4 x D4
that acts on the CP2 Internal Symmetry Space of Kaluza-Klein M4 x CP2

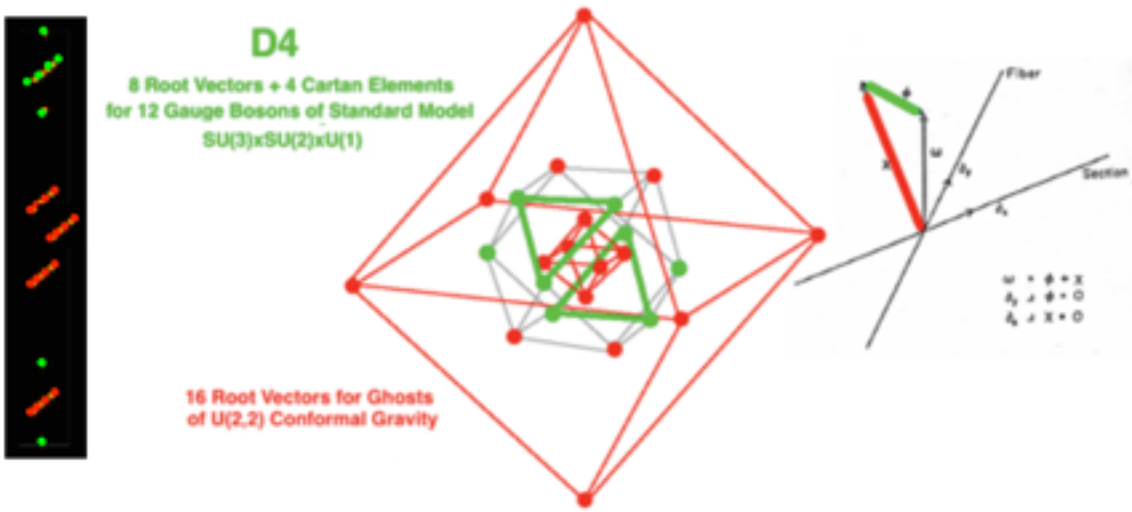
8 orange are Root Vectors for Standard Model SU(3) x SU(2) x U(1)
which have 2+1+1 = 4 Cartan SubAlgebra dimensions.

Standard Model Gauge groups come from CP2 = SU(3) / SU(2) x U(1)
(as described by Batakis in Class. Quantum Grav. 3 (1986) L99-L105)

Electroweak SU(2) x U(1) is gauge group as isotropy group of CP2.

SU(3) is global symmetry group of CP2 but due to Kaluza-Klein structure of compact
CP2 at every M4 spacetime point, it acts as Color gauge group with respect to M4.

The 24-8 = 16 D4 of CP2 Root Vectors represent Ghosts of U(2,2) Conformal Gravity.



Jean Thierry-Mieg in J. Math. Phys. 21 (1980) 2834-2838 said:
"... The ghost and the gauge field:
The single lines represent a local coordinate system
of a principal fiber bundle of base space-time.
The double lines are 1 forms.
The connection of the principle bundle w is assumed to be vertical.
Its contravariant components PHI and X are recognized, respectively,
as the Yang-Mills gauge field and the Faddeev-Popov ghost form ...".

D4 of Conformal Gravity and Standard Model Ghosts

The 24 yellow  are Root Vectors of the M4-related D4 local isotropy group in the symmetric space D8 / D4 x D4 that acts on the M4 Internal Symmetry Space of Kaluza-Klein M4 x CP2

12 yellow are Root Vectors for Conformal Gravity U(2,2) which has 4 Cartan SubAlgebra dimensions.

Gravity and Dark Energy come from its Conformal Subgroup SU(2,2) = Spin(2,4)
(see Appendix - Conformal MacDowell-Mansouri Gravity)

SU(2,2) = Spin(2,4) has 15 generators:

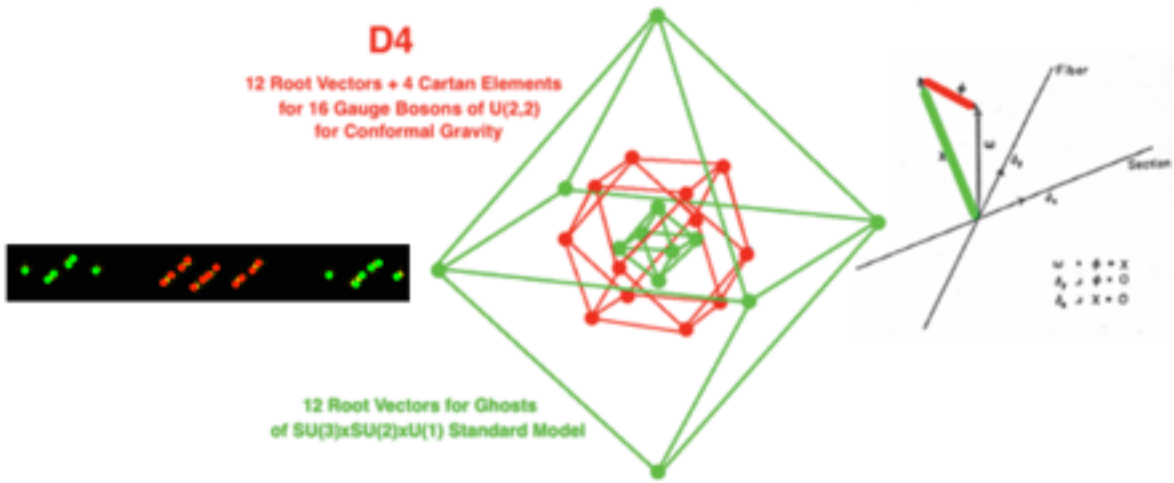
1 Dilation representing Higgs Ordinary Matter

4 Translations representing Primordial Black Hole Dark Matter

10 = 4 Special Conformal + 6 Lorentz representing Dark Energy
(see Irving Ezra Segal, “Mathematical Cosmology and Extragalactic Astronomy” (Academic 1976))

The basic ratio Dark Energy : Dark Matter : Ordinary Matter = 10:4:1 = 0.67 : 0.27 : 0.06
When the dynamics of our expanding universe are taken into account, the ratio is calculated to be **0.75 : 0.21 : 0.04**

The 24-12 = 12 D4 of M4 Root Vectors represent Standard Model Ghosts



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The connection of the principle bundle w is assumed to be vertical.
Its contravariant components PHI and X are recognized, respectively,
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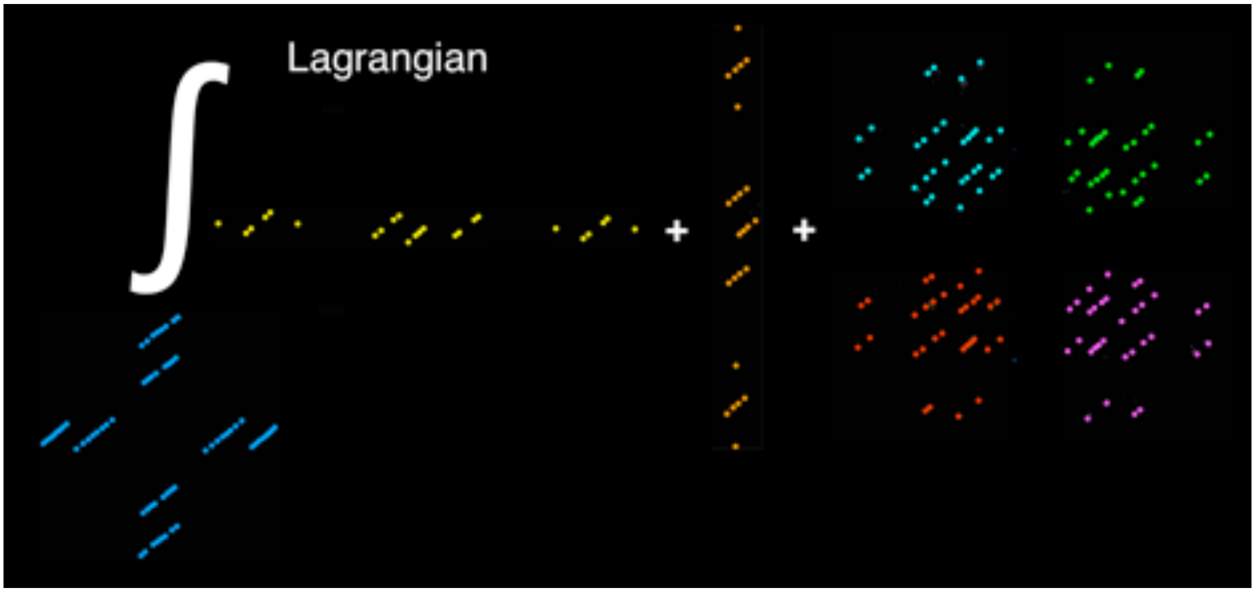
E8 Physics Lagrangian

Using the E8 Root Vector structure as described above,
it is natural to construct a Lagrangian density



having terms for
E8 / D8 Fermions (with Fermion Generations 2 and 3 from Quaternionic structure)
and
D4 Standard Model Gauge Bosons and Gravity Ghosts
(see Appendix - Lagrangian Terms)

and
D4 Conformal Gravity Gauge Bosons and Standard Model Ghosts
(see Appendix - Conformal MacDowell-Mansouri Gravity)

that is integrated over
D8 / D4xD4 (4+4)-dim M4 x CP2 Kaluza-Klein base manifold
with Higgs from the Mayer mechanism (see Appendix - Mayer - Higgs)

The E8 Lagrangian is Chiral because
E8 contains Cl(16) half-spinors (64+64) for a Fermion Generation
but does not contain Cl(16) Fermion AntiGeneration half-spinors (64+64).
Fermion +half-spinor Particles with high enough velocity are seen as left-handed.
Fermion -half-spinor AntiParticles with high enough velocity are seen as right-handed.

The E8 Lagrangian obeys Spin-Statistics because
the CP2 part of M4xCP2 Kaluza-Klein has index structure Euler number 2+1 = 3 and Atiyah-Singer index -1/8 which is not the net number of generations because
CP2 has no spin structure but you can use a generalized spin structure
(Hawking and Pope (Phys. Lett. 73B (1978) 42-44))
to get (for integral m) the generalized CP2 index n_R - n_L = (1/2) m (m+1)

Prior to Dimensional Reduction: m = 1, n_R - n_L = (1/2)x1x2 = 1 for 1 generation

After Reduction to 4+4 Kaluza-Klein: m = 2, n_R - n_L = (1/2)x2x3 = 1 for 3 generations
(see chapter “3 Generations of Fermions”)

Hawking and Pope say: "Generalized Spin Structures in Quantum Gravity ...what happens in CP2 … is a two-surface K which cannot be shrunk to zero. ... However, one could replace the electromagnetic field by
a Yang-Mills field whose group G had a double covering G~.
The fermion field would have to occur in representations which changed sign under the non-trivial element of the kernel of the projection … G~ -> G while the bosons would have to occur in representations which did not change sign …".

For Cl(16)-E8 model gauge bosons are in the 28+28=56-dim D4 + D4 subalgebra of E8.
D4 = SO(8) is the Hawking-Pope G which has double covering G~ = Spin(8).

The 8 fermion particles / antiparticles are D4 half-spinors represented within E8
by anti-commutators and so do change sign
while
the 28 gauge bosons are D4 adjoint represented within E8 by commutators
and so do not change sign.

E8 inherits from F4 the property whereby
its Spinor Part need not be written as Commutators
but can also be written in terms of Fermionic AntiCommutators.
(see Appendix - E8 Fermionic AntiCommutators)

However, something fundamental remains missing at this stage:

The above Lagrangian is mostly Classical.

E8 Quantum Theory

A natural way to make a Quantum Theory is to consider E8 to be Local Classical
and
to embed E8 into the real Clifford Algebra Cl(16)
and
use 8-Periodicity to form the Completion of the Union of all Tensor Products of Cl(16)
which produces a natural realistic Algebraic Quantum Field Theory (AQFT).

That is useful but it would be more useful to connect it directly to the Root Vector picture
so
consider that **there is redundancy in the E8 description of Quantum States:**

Fermion components carry Spacetime information
so E8 / D8 = 8x8 + 8x8 can be reduced to 8+8

Spacetime position and momentum are redundant
so D8 / D4 x D4 = 8x8 can be reduced to 8

Gauge Bosons and Ghosts are redundant
so D4 x D4 = 28+28 can be reduced to 28 = 16 for Gravity + 12 for Standard Model

Elimination of Redundancy gives 8+8 + 8 + 28 = 52-dim F4 with 48 Root Vectors
forming a 24-cell plus its dual

248-dim E8 is unusual in that its smallest non-trivial representation is also 248-dim.
52-dim F4 has 26-dim smallest non-trivial representation
which has structure of
J(3,O)o = traceless part of 27-dim exceptional Jordan Algebra J(3,O)
and is
the minimal structure containing the basic information of E8 Physics.
so
E8 Physics Quantum Theory can be formulated in terms of 26-dim J(3,O)o.

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

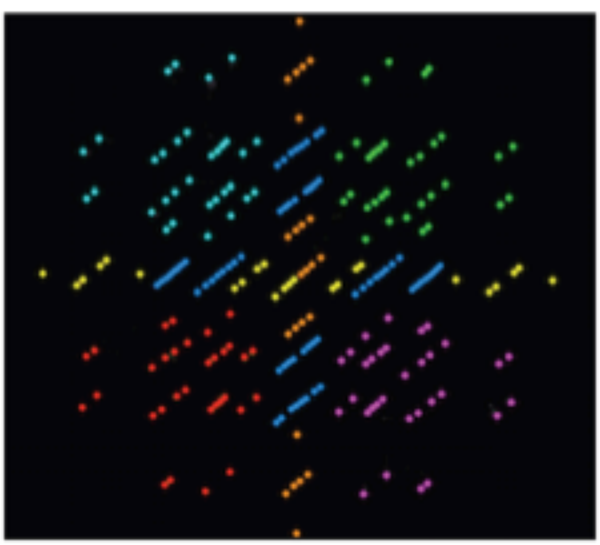
$\int$ Gauge Gravity + Standard Model + Fermion Particle-AntiParticle
8-dim SpaceTime .

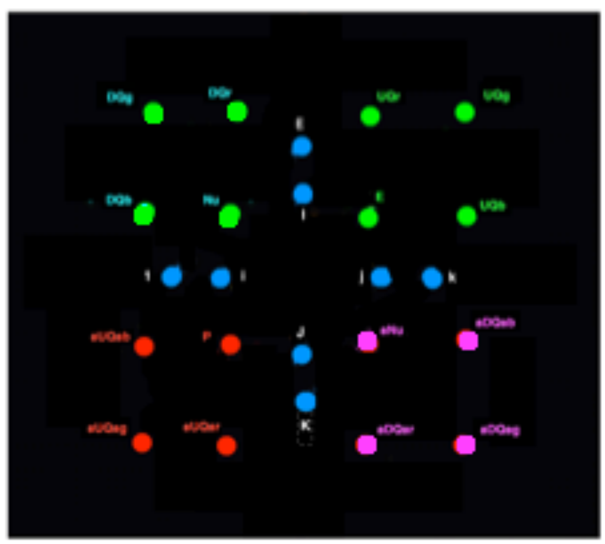
whereby World-Lines of Particles are represented by Strings moving in a space whose dimensionality includes 8v = 8-dim SpaceTime Dimensions +
+ 8s+ = 8 Fermion Particle Types + 8s- = 8 Fermion AntiParticle Types
combined in the traceless part J(3,O)o of the 3x3 Octonion Hermitian Jordan Algebra

a	8s+	8v
8s+*	b	8s-
8v*	8s-*	-a-b

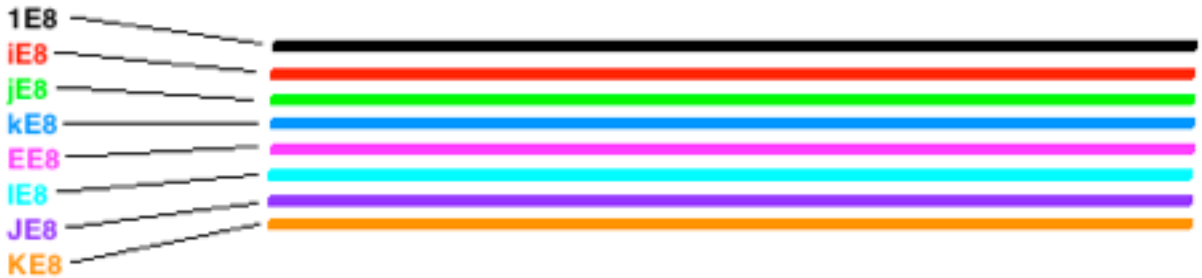
which has total dimension 8v + 8s+ + 8s- + 2 = 26 and is the space of a
26D String Theory with Strings seen as World-Lines.

24 = 8v + 8s+ + 8s- of the 26 dimensions of 26D String Theory correspond to 24x8 = 192 of the 240 E8 Root Vectors by representing the 8v + 8s+ + 8s- as superpositions of their respective 8 components

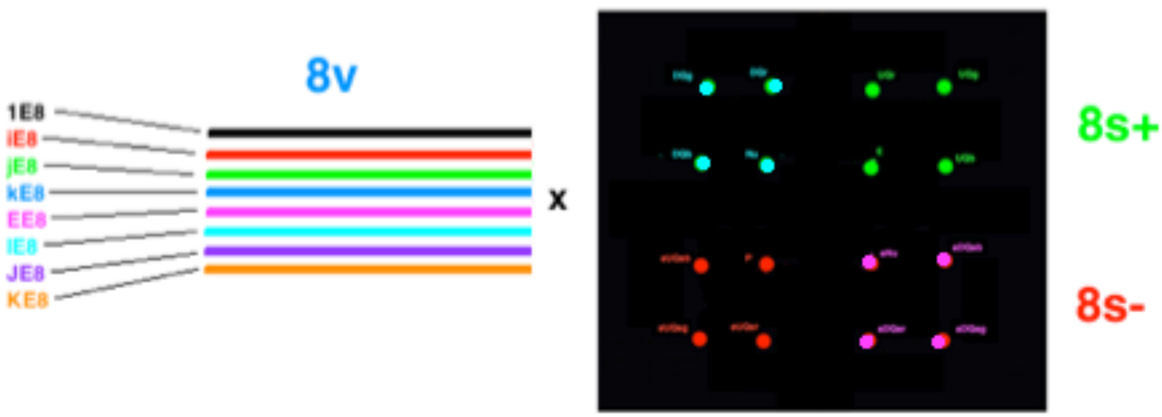




8v SpaceTime is represented by D8 branes. A D8 brane has
Planck-Scale Lattice Structure superpositions of 8 types of E8 Lattice
denoted by 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8

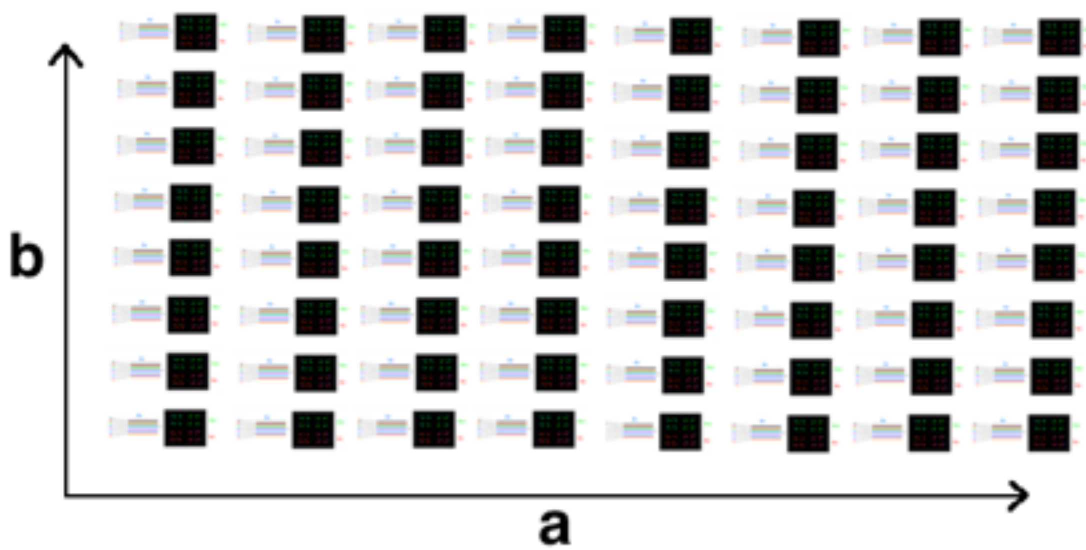


A single Snapshot of SpaceTime is represented by a D8 brane at each point of which is placed Fermion Particles or AntiParticles represented by 8+8 = 16 orbifolded dimensions of the 26 dimensions of 26D String Theory.



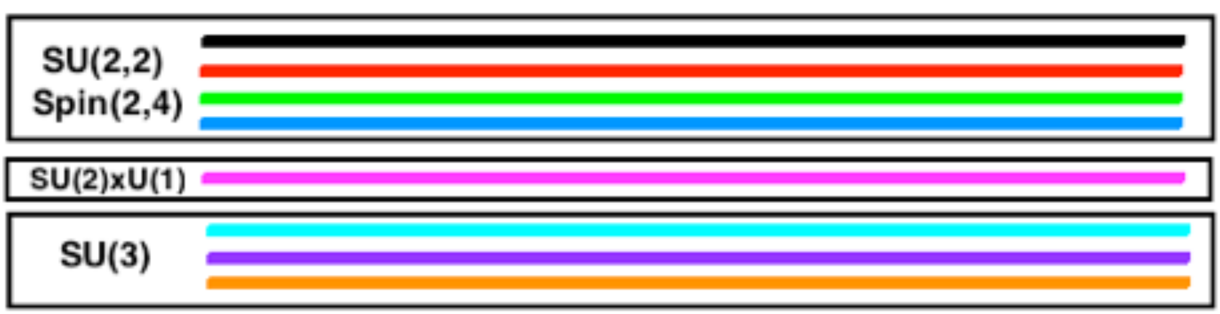
It is necessary to patch together SpaceTime Snapshots to form a Global Structure describing a Many-Worlds Global Algebraic Quantum Field Theory (AQFT)
whose structure is described by Deutsch in "The Fabric of Reality" (Penguin 1997 pp. 276-283):
"… there is no fundamental demarcation between snapshots of other times and snapshots of other universes ... Other times are just special cases of other universes ... Suppose ... we toss a coin ... Each point in the diagram represents one snapshot
... in the multiverse there are far too many snapshots for clock readings alone to locate a snapshot relative to the others. To do that, we need to consider the intricate detail of which snapshots determine which others. …
in some regions of the multiverse, and in some places in space,
the snapshots of some physical objects do fall, for a period, into chains,
each of whose members determines all the others to a good approximation …".
The Many-Worlds Snapshots are structured as a 26-dim Lorentz Leech Lattice

of 26D String Theory parameterized by the a and b of J(3,O)o
as indicated in this 64-element subset of Snapshots



The 240 - 192 = 48 = 24+24 Root Vector Vertices of E8 that do not represent the 8-dim D8 brane or the 8+8 = 16 dim of Orbifolds for Fermions do represent the **Gauge Bosons (and their Ghosts) of E8 Physics**:

Gauge Bosons from 1E8, iE8, jE8, and kE8 parts of a D8 give U(2,2) Conformal Gravity
Gauge Bosons from EE8 part of a D8 give U(2) Electroweak Force
Gauge Bosons from IE8, JE8, and KE8 parts of a D8 give SU(3) Color Force



Each Deutsch chain of determination represents a World-Line of Particles / AntiParticles corresponding to a String of 26D String Theory
such as the red line in this 64-element subset of Snapshots

26D String Theory is the Theory of Interactions of Strings = World-Lines.

Interactions of World-Lines can describe Quantum Theory
according to Andrew Gray (arXiv quant-ph/9712037):
"... probabilites are ... assigned to entire fine-grained histories ...
base[d] ... on the Feynman path integral formulation ...
The formulation is fully relativistic and applicable to multi-particle systems.
It ... makes the same experimental predictions as quantum field theory …".
Green, Schwartz, and Witten say in their book "Superstring Theory" vol. 1 (Cambridge 1986)
"... For the ... closed ... bosonic string [**26D String Theory**] .... The first excited level ... consists of ... the ground state ... tachyon ... and ... a scalar ... 'dilaton' ... and ... SO(24) ... little group of a ...[26-dim]... massless particle ... and ...
a ... massless ... spin two state ...".

Closed string tachyons localized at orbifolds of fermions produce virtual clouds of particles / antiparticles that dress fermions.

Dilatons are Goldstone bosons of spontaneously broken scale invariance that (analagous to Higgs) go from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The SO(24) little group is related to the Monster automorphism group that is the symmetry of each cell of Planck-scale local lattice structure.

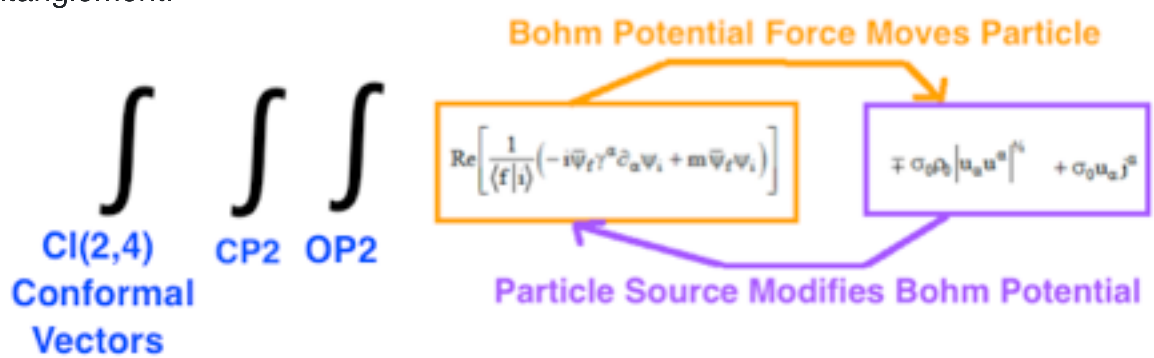
The massless spin 2 state = Bohmion = Carrier of the Bohm Force of the Bohm Quantum Potential.

Roderick Sutherland (arXiv 1509.02442) gave a Lagrangian for the Bohm Potential saying: "... This paper focuses on interpretations of QM in which the underlying reality is taken to consist of particles have definite trajectories at all times ... An example ... is the Bohm model ... This paper ... provid[es]... a Lagrangian ...[for]... the unfolding events ...
... describing more than one particle while maintaining a relativistic description requires the introduction of final boundary conditions as well as initial,
thereby entailing retrocausality ...
In addition ... the Lagrangian approach pursued here to describe particle trajectories also entails the natural inclusion of an accompanying field to influence the particle's motion away from classical mechanics and reproduce the correct quantum predictions. In so doing, it is ... providing a physical explanation for why quantum phenomena exist at all ... the particle is seen to be
the source of a field which alters the particle's trajectory via self-interaction ...
The Dirac case ... each particle in an entangled many-particle state will be described by an individual Lagrangian density ... of the form:

$$\mathfrak{L} = \mathrm{Re}\left[\frac{1}{\langle f|i\rangle}\left(-i\bar{\psi}_f\gamma^\alpha\partial_\alpha\psi_i + m\bar{\psi}_f\psi_i\right)\right] \mp \sigma_0\rho_0\left|u_\alpha u^\alpha\right|^{1/2} + \sigma_0 u_\alpha j^\alpha$$

... the ...[first]... term ...[is]... the ... Lagrangian densities for the PSI field alone ...
... sigma_o is the rest density distribution of the particle through space ... j is the current density ...
... rho_o and u are the rest density and 4-velocity of the probability flow ...".

Jack Sarfatti extended the Sutherland Lagrangian to include Back-Reaction entanglement.



where a, b and VM4 form Cl(2,4) vectors and VCP2 forms CP2
and S+ and S- form OP2 so that
26D = 16D orbifolded fermions + 10D and 10D = 6D Conformal Space + 4D CP2 ISS
(ISS = Internal Symmetry Space and 6D Conformal contains 4D M4 of Kaluza-Klein M4xCP2)
saying (linkedin.com Pulse 13 January 2016): "... the reason entanglement cannot be used as a direct messaging channel between subsystems of an entangled complex quantum system, is the lack of direct back-reaction of the classical particles and classical local gauge fields on their shared entangled Bohmian quantum information pilot wave ... Roderick. I. Sutherland ... using Lagrangian field theory, shows how to make the original 1952 Bohm pilot-wave theory completely relativistic,

and how to avoid the need for configuration space for many-particle entanglement.
The trick is that final boundary conditions on the action
as well as initial boundary conditions influence what happens in the present.
The general theory is "post-quantum" ... and it is non-statistical ...
There is complete two-way action-reaction between quantum pilot waves
and the classical particles and classical local gauge fields ...
orthodox statistical quantum theory, with no-signaling ...[is derived]... in two steps,
first arbitrarily set the back-reaction (of particles and classical gauge field on their pilot waves) to zero. This is analogous to setting the curvature equal to zero in general relativity, or more precisely in setting G to zero.
Second, integrate out the final boundary information, thereby adding the statistical Born rule to the mix. ...
the mathematical condition for zero post-quantum back-reaction of particles and classical fields (aka "beables" J.S. Bell's term) is exactly de Broglie's guidance constraint. That is, in the simplest case, the classical particle velocity is proportional to the gradient of the phase of the quantum pilot wave. It is for this reason, that the independent existence of the classical beables can be ignored in most quantum calculations.
However, orthodox quantum theory assumes that the quantum system is thermodynamically closed between strong von Neumann projection measurements that obey the Born probability rule.
The new post-quantum theory in the equations of Sutherland, prior to taking the limit of orthodox quantum theory, should apply to pumped open dissipative structures. Living matter is the prime example. This is a clue that should not be ignored. ...".

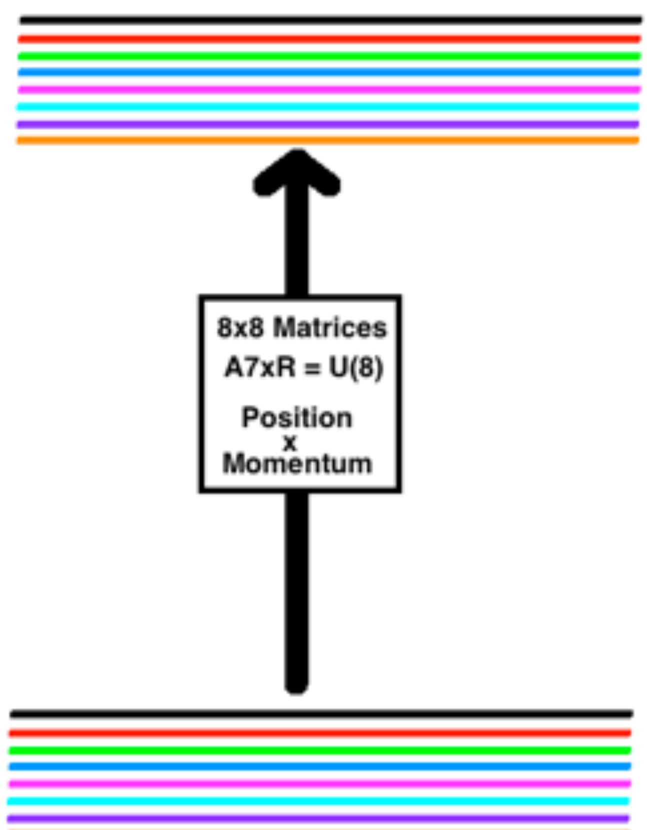
Jack Sarfatti (email 31 January 2016) said: "... Sabine [Hossenfelder]'s argument ...
"... two types of fundamental laws ... appear in contemporary theories.
One type is deterministic, which means that the past entirely predicts the future.
There is no free will in such a fundamental law because there is no freedom.
The other type of law we know appears in quantum mechanics and has an indeterministic component which is random. This randomness cannot be influenced by anything, and in particular it cannot be influenced by you, whatever you think "you" are.
There is no free will in such a fundamental law because there is no "will" - there is just some randomness sprinkled over the determinism.
In neither case do you have free will in any meaningful way."
... However ...[There is a Third Way]...
post-quantum theory with action-reaction between
quantum information pilot wave and its be-able is compatible with free will. ...".

The Creation-Annihilation Operator structure of the Bohm Quantum Potential of 26D String Theory is given by the

Maximal Contraction of E8 = semidirect product A7 x h92
where h92 = 92+1+92 = 185-dim Heisenberg algebra and A7 = 63-dim SL(8)

The Maximal E8 Contraction A7 x h92 can be written as a 5-Graded Lie Algebra
28 + 64 + (SL(8,R) + 1) + 64 + 28
Central Even Grade 0 = SL(8,R) + 1
The 1 is a scalar and SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices,
so SL(8,R) represents a local 8-dim SpaceTime in Polar Coordinates.
Odd Grades -1 and +1 = 64 + 64
Each = 64 = 8x8 = Creation/Annihilation Operators for 8 components of 8 Fundamental Fermions.
Even Grades -2 and +2 = 28 + 28
Each = Creation/Annihilation Operators for 28 Gauge Bosons of Gravity + Standard Model.

The 8x8 matrices linking one D8 to the next D8 of a World-Line String
give A7xR = U(8) representing Position x Momentum



The Algebraic Quantum Field Theory (AQFT) structure of the Bohm Quantum Potential of 26D String Theory is given by the Cl(16)-E8 Local Lagrangian

$\int$ Gauge Gravity + Standard Model + Fermion Particle-AntiParticle
8-dim SpaceTime .

living in Cl(16) and by 8-Periodicity of Real Clifford Algebras,
as the Completion of the Union of all Tensor Products of the form

Cl(16) x ...(N times tensor product)... x Cl(16) = Cl(16N)

For N = 2^8 = 256 the copies of Cl(16) are on the 256 vertices of the 8-dim HyperCube

For N = 2^16 = 65,536 = 4^8 the copies of Cl(16) fill in the 8-dim HyperCube as described by William Gilbert's web page: "... **The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube**. ...".

The vertices of the Hilbert curve are at the centers of the 2^8 sub-8-HyperCubes whose edge lengths are 1/2 of the edge lengths of the original 8-dim HyperCube

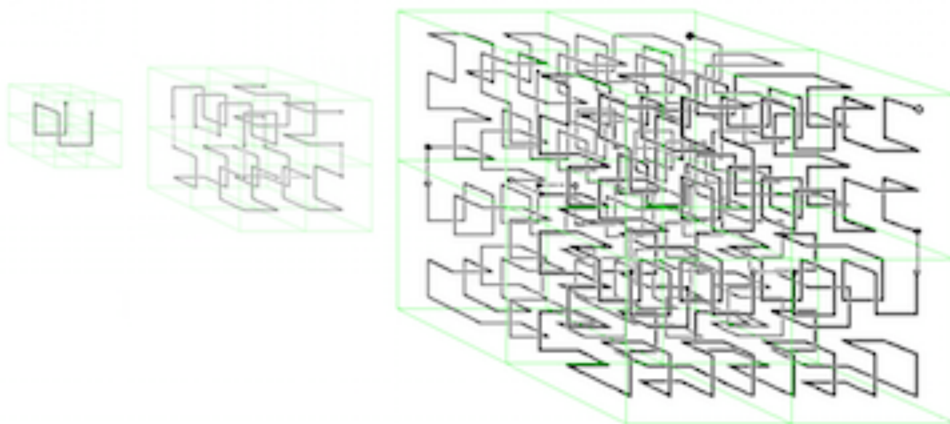
As N grows, the copies of Cl(16) continue to fill the 8-dim HyperCube of E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes.

If edges of sub-HyperCubes, equal to the distance between adjacent copies of Cl(16), remain constantly at the Planck Length, then the

full 8-dim HyperCube of our Universe expands as N grows to 2^16 and beyond

similarly to the way shown by this 3-HyperCube example for N = 2^3, 4^3, 8^3

from Wiliam Gilbert's web page:



The Union of all Cl(16) tensor products is the Union of all subdivided 8-HyperCubes and their Completion is a huge superposition of 8-HyperCube Continuous Volumes which Completion belongs to the Third Grothendieck Universe.

AQFT Quantum Code

Cerf and Adami in quantum-ph/9512022 describe virtual qubit-anti-qubit pairs (they call them ebit-anti-ebitpairs) that are related to negative conditional entropies for quantum entangled systems and are similar to fermion particle-antiparticle pairs. Therefore quantum information processes can be described by particle-antiparticle diagrams much like particle physics diagrams and **the Algebraic Quantum Field Theory of the E8 - Cl(16) = Cl(8)xCl(8) Physics Model should be equivalent to a Quantum Code Information System.**

Quantum Reed-Muller code [[256 , 0 , 24]]
corresponds to
Real Clifford Algebra Cl(8)

Tensor Product Quantum Reed-Muller code [[256 , 0 , 24]] x [[256 , 0 , 24]]
corresponds to
Real Clifford Algebra Cl(8) x Cl(8) = Cl(16) containing E8

Completion of the Union of All Tensor Products of [[256 , 0 , 24]] x [[256 , 0 , 24]]
corresponds to
AQFT (Algebraic Quantum Field Theory) hyperfinite von Neumann factor algebra that is Completion of the Union of All Tensor Products of Cl(16)

Schwinger Sources, Hua Geometry, and Wyler Calculations

Fock “Fundamental of Quantum Mechanics” (1931) showed that it requires
Linear Operators “... represented by a definite integral [of a]... kernel ... function ...”.

Hua “Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains” (1958) showed Kernel Functions for Complex Classical Domains.

Schwinger (1951 - see Schweber, PNAS 102, 7783-7788) “... introduced a description in terms of Green’s functions, what Feynman had called propagators ... The Green’s functions are vacuum expectation values of time-ordered Heisenberg operators, and the field theory can be defined non-perturbatively in terms of these functions ...[which]... gave deep structural insights into QFTs; in particular ... the structure of the Green's functions when their variables are analytically continued to complex values ...”.

Wolf (J. Math. Mech 14 (1965) 1033-1047) showed that the Classical Domains
(complete simply connected Riemannian symmetric spaces)
representing 4-dim Spacetime with Quaternionic Structure are:

S1 x S1 x S1 x S1 = 4 copies of U(1)
S2 x S2 = 2 copies of SU(2)
CP2 = SU(3) / SU(2)xU(1)
S4 = Spin(5) / Spin(4) = Euclidean version of Spin(2,3) / Spin(1,3)

Armand Wyler (1971 - C. R. Acad. Sc. Paris, t. 271, 186-188) showed how to use **Green’s Functions = Kernel Functions** of Classical Domain structures characterizing **Sources = Leptons, Quarks, and Gauge Bosons,
to calculate Particle Masses and Force Strengths**

Schwinger (1969 - see physics/0610054) said: “... operator field theory ... replace[s] the particle with ... properties ... distributed througout ... small volumes of three-dimensional space ... particles ... must be created ... even though we vary a number of experimental parameters ... The properties of the particle ... remain the same ... We introduce a quantitative description of the particle source in terms of a source function ...
we do not have to claim that we can make the source arbitrarily small ...
the experimeter... must detect the particles ...[by]... collision that annihilates the particle ... the source ... can be ... an abstraction of an annilhilation collision, with the source acting negatively, as a sink ... The basic things are ... the source functions ... describing the intermediate propagation of the particle ...”.

Creation and Annihilation operators indicate a Clifford Algebra, and 8-Periodicity shows that the basic Clifford Algebra is formed by tensor products of 256-dim Cl(8) such as Cl(8) x Cl(8) = Cl(16) containing 248-dim E8 = 120-dim D8 + 128-dim D8 half-spinor whose maximal contraction is a realistic generalized Heisenberg Algebra

h92 x A7 = 5-graded 28 + 64 + ((SL(8,R)+1) + 64 + 28

The Cl(16)-E8 model Lagrangian over 4-dim Minkowski SpaceTime M4 is

$\int_{4\text{-dim } M4}$ GG + SM + Fermion Particle-AntiParticle + Higgs .

Consider the Fermion Term.
In the conventional picture, the spinor fermion term is of the form m S S* where m is the fermion mass and S and S* represent the given fermion.
The Higgs coupling constants are, in the conventional picture, ad hoc parameters, so that effectively the mass term is, in the conventional picture, an ad hoc inclusion.

The Cl(16)-E8 model constructs the Lagrangian integral such that the mass m emerges as the integral over the Schwinger Source spacetime region of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion so that the volume of the Schwinger Source fermion defines its mass, which, being dressed with the particle/antiparticle pair cloud, gives quark mass as constituent mass.

Fermion Schwinger Sources correspond to the Lie Sphere Symmetric space

Spin(10) / Spin(8)xU(1)

which has
local symmetry of the Spin(8) gauge group from which the first generation spinor fermions are formed as +half-spinor and -half-spinor spaces
and
Bounded Complex Domain D8 of type IV8 and Shilov Boundary Q8 = RP1 x S7

Consider the GG + SM term from Gauge Gravity and Standard Model Gauge Bosons.
The process of breaking Octonionic 8-dim SpaceTime down to Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein creates differences in the way gauge bosons "see" 4-dim Physical SpaceTime. There 4 equivalence classes of 4-dimensional Riemannian Symmetric Spaces with Quaternionic structure consistent with 4-dim Physical SpaceTime:

S4 = 4-sphere = Spin(5) / Spin(4) where Spin(5) = Schwinger-Euclidean version of the Anti-DeSitter subgroup of the Conformal Group that gives MacDowell-Mansouiri Gravity

CP2 = complex projective 2-space = SU(3) / U(2) with the SU(3) of the Color Force

S2 x S2 = SU(2)/U(1) x SU(2)/U(1) with two copies of the SU(2) of the Weak Force

S1 x S1 x S1 x S1 = U(1) x U(1) x U(1) x U(1) = 4 copies of the U(1) of the EM Photon
(1 copy for each of the 4 covariant components of the Photon)

The Gravity Gauge Bosons (Schwinger-Euclidean versions) live in
a Spin(5) subalgebra of the Spin(6) Conformal subalgebra of D4 = Spin(8).
They "see" M4 Physical spacetime as the 4-sphere S4
so that their part of the Physical Lagrangian is

$$\int_{S4} \text{Gravity Gauge Boson Term} \; .$$

an integral over SpaceTime S4.
The Schwinger Sources for GRb bosons are the Complex Bounded Domains and Shilov Boundaries for Spin(5) MacDowell-Mansouri Gravity bosons.
However, due to Stabilization of Condensate SpaceTime
by virtual Planck Mass Gravitational Black Holes,
for Gravity, the effective force strength that we see in our experiments
is not just composed of the S4 volume and the Spin(5) Schwinger Source volume,
but is suppressed by the square of the Planck Mass.
The unsuppressed Gravity force strength is the Geometric Part of the force strength.

The Standard Model SU(3) Color Force bosons live in
a SU(3) subalgebra of the SU(4) subalgebra of D4 = Spin(8).
They "see" M4 Physical spacetime as the complex projective plane CP2
so that their part of the Physical Lagrangian is

$$\int_{CP2} \text{SU(3) Color Force Gauge Boson Term} \; .$$

an integral over SpaceTime CP2.
The Schwinger Sources for SU(3) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(3) Color Force bosons.
The Color Force Strength is given by
the SpaceTime CP2 volume and the SU(3) Schwinger Source volume.
Note that since the Schwinger Source volume is dressed with the particle/antiparticle pair cloud, the calculated force strength is
for the characteristic energy level of the Color Force (about 245 MeV).

The Standard Model SU(2) Weak Force bosons live in
a SU(2) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as two 2-spheres S2 x S2
so that their part of the Physical Lagrangian is

$$\int_{S2xS2} \text{SU(2) Weak Force Gauge Boson Term} \; .$$

an integral over SpaceTime S2xS2.
The Schwinger Sources for SU(2) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(2) Weak Force bosons.
However, due to the action of the Higgs mechanism,
for the Weak Force, the effective force strength that we see in our experiments
is not just composed of the S2xS2 volume and the SU(2) Schwinger Source volume,
but is suppressed by the square of the Weak Boson masses.
The unsuppressed Weak Force strength is the Geometric Part of the force strength.

The Standard Model U(1) Electromagnetic Force bosons (photons) live in
a U(1) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as four 1-sphere circles S1xS1xS1xS1 = T4
(T4 = 4-torus) so that their part of the Physical Lagrangian is

$$\int_{T4} \text{(U(1) Electromagnetism Gauge Boson Term} \; .$$

an integral over SpaceTime T4.
The Schwinger Sources for U(1) photons
are the Complex Bounded Domains and Shilov Boundaries for U(1) photons.
The Electromagnetic Force Strength is given by
the SpaceTime T4 volume and the U(1) Schwinger Source volume.

Schwinger Sources as described above are continuous manifold structures
of Bounded Complex Domains and their Shilov Boundaries
but
the Cl(16)-E8 model at the Planck Scale has spacetime condensing out of Clifford structures forming a Leech lattice underlying 26-dim String Theory of World-Lines with 8 + 8 + 8 = 24-dim of fermion particles and antiparticles and of spacetime.

The automorphism group of a single 26-dim String Theory cell modulo the Leech lattice is the Monster Group of order about 8 x 10^53.

When a fermion particle/antiparticle appears in E8 spacetime it does not remain a single Planck-scale entity becauseTachyons create a cloud of particles/antiparticles.

The cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively neutral cloud of particle/antiparticle pairs forming a Kerr-Newman black hole.
That cloud constitutes the Schwinger Source.

Its structure comes from the 24-dim Leech lattice part of the Monster Group which is 2^(1+24) times the double cover of Co1, for a total order of about 10^26.

(Since a Leech lattice is based on copies of an E8 lattice and since there are 7 distinct E8 integral domain lattices there are 7 (or 8 if you include a non-integral domain E8 lattice) distinct Leech lattices. The physical Leech lattice is a superposition of them, effectively adding a factor of 8 to the order.)

The volume of the Kerr-Newman Cloud is on the order of 10^27 x Planck scale,
so the Kerr-Newman Cloud should contain about 10^27 particle/antiparticle pairs
and its size should be about 10^(27/3) x 1.6 x 10^(-33) cm = roughly 10^(-24) cm.

Force Strength and Boson Mass Calculation

Cl(8) bivector Spin(8) is the D4 Lie algebra two copies of which are in the Cl(16)-E8 model Lagrangian (as the D4xD4 subalgebra of the D8 subalgebra of E8)

$\int$ GG + SM + Fermion Particle-AntiParticle + Higgs
4-dim M4 .
with the Higgs term coming from integrating over the CP2 Internal Symmetry Space of M4 x CP2 Kaluza-Klein by the Mayer-Trautman Mechanism

This shows that the Force Strength is made up of two parts:
the relevant spacetime manifold of gauge group global action
and
the relevant symmetric space manifold of gauge group local action.

The 4-dim spacetime Lagrangian GG SM gauge boson term is:
the integral over spacetime as seen by gauge boson acting globally
of the gauge force term of the gauge boson acting locally
for the gauge bosons of each of the four forces:
U(1) for electromagnetism
SU(2) for weak force
SU(3) for color force
Spin(5) - compact version of antiDeSitter Spin(2,3) subgroup of Conformal Spin(2,4) for gravity by the MacDowell-Mansouri mechanism.

In the conventional picture,
for each gauge force the gauge boson force term contains the force strength,
which in Feynman's picture is the amplitude to emit a gauge boson,
and can also be thought of as the probability = square of amplitude,
in an explicit (like g IFI^2) or an implicit (incorporated into the IFI^2) form.
Either way, the conventional picture is that the force strength g is an ad hoc inclusion.

The Cl(16)-E8 model does not put in force strength g ad hoc,
but constructs the integral such that
the force strength emerges naturally from the geometry of each gauge force.

To do that, for each gauge force:

1 - make the spacetime over which the integral is taken be spacetime as it is seen by that gauge boson, that is, in terms of the symmetric space with global symmetry of the gauge boson:

the U(1) photon sees 4-dim spacetime as T^4 = S1 x S1 X S1 x S1
the SU(2) weak boson sees 4-dim spacetime as S2 x S2
the SU(3) weak boson sees 4-dim spacetime as CP2
the Spin(5) of gravity sees 4-dim spacetime as S4

2 - make the gauge boson force term have the volume of the Shilov boundary corresponding to the symmetric space with local symmetry of the gauge boson.
The nontrivial Shilov boundaries are:

for SU(2) Shilov = RP^1xS^2
for SU(3) Shilov = S^5
for Spin(5) Shilov = RP^1xS^4

The result is (ignoring technicalities for exposition) the geometric factor for force strengths.

Each gauge group is the global symmetry of a symmetric space

S1 for U(1)
S2 = SU(2)/U(1) = Spin(3)/Spin(2) for SU(2)
CP2 = SU(3)/SU(2)xU(1) for SU(3)
S4 = Spin(5)/Spin(4) for Spin(5)

Each gauge group is the local symmetry of a symmetric space

U(1) for itself
SU(2) for Spin(5) / SU(2)xU(1)
SU(3) for SU(4) / SU(3)xU(1)
Spin(5) for Spin(7) / Spin(5)xU(1)

The nontrivial local symmetry symmetric spaces correspond to bounded complex domains

SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball)
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5

The nontrivial bounded complex domains have Shilov boundaries

SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3 Shilov = RP^1xS^2
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball) Shilov = S^5
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5 Shilov = RP^1xS^4

Very roughly, think of the force strength as
integral over global symmetry space of physical (ie Shilov Boundary) volume =
= strength of the force.

That is:
the geometric strength of the force is given by the product of
the volume of a 4-dim thing with global symmetry of the force and
the volume of the Shilov Boundary for the local symmetry of the force.

When you calculate the product volumes (using some tricky normalization stuff),
you see that roughly:

Volume product for gravity is the largest volume
so since (as Feynman says) force strength = probability to emit a gauge boson means
that the highest force strength or probability should be 1
the gravity Volume product is normalized to be 1, and so (approximately):

Volume product for gravity = 1
Volume product for color = 2/3
Volume product for weak = 1/4
Volume product for electromagnetism = 1/137

There are two further main components of a force strength:

1 - for massive gauge bosons, a suppression by a factor of 1 / M^2
2 - renormalization running (important for color force)

Consider Massive Gauge Bosons:
Gravity as curvature deformation of SpaceTime, with SpaceTime as a condensate of
Planck-Mass Black Holes, must be carried by virtual Planck-mass black holes,
so that the geometric strength of gravity should be reduced by 1/Mp^2

The weak force is carried by weak bosons,
so that the geometric strength of the weak force should be reduced by 1/MW^2
That gives the result (approximate):

gravity strength = G (Newton's G)
color strength = 2/3
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

Consider Renormalization Running for the Color Force:: That gives the result:

gravity strength = G (Newton's G)
color strength = 1/10 at weak boson mass scale
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

The use of compact volumes is itself a calculational device,
because it would be more nearly correct,
instead of the integral over the compact global symmetry space of
the compact physical (ie Shilov Boundary) volume=strength of the force
to use
the integral over the hyperbolic spacetime global symmetry space
of the noncompact invariant measure of the gauge force term.

However, since the strongest (gravitation) geometric force strength is to be normalized to 1, the only thing that matters is ratios,
and the compact volumes (finite and easy to look up in the book by Hua)
have the same ratios as the noncompact invariant measures.

In fact, I should go on to say that continuous spacetime and gauge force geometric objects are themselves also calculational devices,
and
that it would be even more nearly correct to do the calculations with respect to a discrete generalized hyperdiamond Feynman checkerboard.

Fermion Mass Calculations

In the Cl(16)-E8 model, the first generation spinor fermions are
seen as +half-spinor and -half-spinor spaces of Cl(1,7) = Cl(8).
Due to Triality, Spin(8) can act on those 8-dimensional half-spinor spaces
similarly to the way it acts on 8-dimensional vector spacetime.

Take the the spinor fermion volume to be the Shilov boundary corresponding
to the same symmetric space on which Spin(8) acts as a local gauge group
that is used to construct 8-dimensional vector spacetime:

the symmetric space Spin(10) / Spin(8)xU(1)
corresponding to a bounded domain of type IV8
whose Shilov boundary is RP^1 x S^7

Since all first generation fermions see the spacetime over which the integral is
taken in the same way (unlike what happens for the force strength calculation),
the only geometric volume factor relevant for calculating first generation fermion
mass ratios is in the spinor fermion volume term.

Cl(16)-E8 model fermions correspond to Schwinger Source Kerr-Newman Black Holes,
so the quark mass in the Cl(16)-E8 model is a constituent mass.

Fermion masses are calculated as a product of four factors:

V(Qfermion) x N(Graviton) x N(octonion) x Sym

V(Qfermion) is the volume of the part of the half-spinor fermion particle manifold
S^7 x RP^1 related to the fermion particle by photon, weak boson, or gluon interactions.

N(Graviton) is the number of types of Spin(0,5) graviton related to the fermion.
The 10 gravitons correspond to the 10 infinitesimal generators of Spin(0,5) = Sp(2).
2 of them are in the Cartan subalgebra.
6 of them carry color charge, and therefore correspond to quarks.
The remaining 2 carry no color charge, but may carry electric charge and so
may be considered as corresponding to electrons. One graviton takes the electron into
itself, and the other can only take the first generation electron into the massless electron
neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron.
The graviton number ratio of the down quark to the first-generation electron is therefore
6/1 = 6.

N(octonion) is an octonion number factor relating up-type quark masses to
down-type quark masses in each generation.

Sym is an internal symmetry factor, relating 2nd and 3rd generation massive
leptons to first generation fermions. It is not used in first-generation calculations.

The first generation down quark constituent mass : electron mass ratio is:

The electron, E, can only be taken into the tree-level-massless neutrino, 1, by photon, weak boson, and gluon interactions.

The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks.

The neutrino, being massless at tree level, does not add anything to the mass formula for the electron.

Since the electron cannot be related to any other massive Dirac fermion,
its volume V(Qelectron) is taken to be 1.

Next consider a red down quark i.
By gluon interactions, i can be taken into j and k, the blue and green down quarks.
By also using weak boson interactions,
it can also be taken into I, J, and K, the red, blue, and green up quarks.
Given the up and down quarks, pions can be formed from quark-antiquark pairs,
and the pions can decay to produce electrons and neutrinos.
Therefore the red down quark (similarly, any down quark)
is related to all parts of S^7 x RP^1,
the compact manifold corresponding to { 1, i, j, k, E, I, J, K }
and therefore a down quark should have
a spinor manifold volume factor V(Qdown quark) of the volume of S^7 x RP^1.

The ratio of the down quark spinor manifold volume factor
to the electron spinor manifold volume factor is
V(Qdown quark) / V(Qelectron) = V(S^7x RP^1)/1 = pi^5 / 3.

Since the first generation graviton factor is 6,
md / me = 6 V(S^7 x RP^1) = 2 pi^5 = 612.03937

As the up quarks correspond to I, J, and K, which are the octonion transforms under E of i, j, and k of the down quarks, the up quarks and down quarks have the same constituent mass

mu = md.

Antiparticles have the same mass as the corresponding particles.
Since the model only gives ratios of masses,
the mass scale is fixed so that the electron mass me = 0.5110 MeV.
Then, the constituent mass of the down quark is md = 312.75 MeV,
and the constituent mass for the up quark is mu = 312.75 MeV.
These results when added up give a total mass of first generation fermion particles:

Sigmaf1 = 1.877 GeV

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks
mproton = mu + mu + md = 938.25 MeV
which is close to the experimental value of 938.27 MeV.

The third generation fermion particles correspond to triples of octonions.
There are 8^3 = 512 such triples.
The triple { 1,1,1 } corresponds to the tau-neutrino.
The other 7 triples involving only 1 and E correspond to the tauon:
{ E, E, E }
{ E, E, 1 }
{ E, 1, E }
{ 1, E, E }
{ 1, 1, E }
{ 1, E, 1 }
{ E, 1, 1 }
The symmetry of the 7 tauon triples is the same
as the symmetry of the first generation tree-level-massive fermions,
3 down, quarks, the 3 up quarks, and the electron,
so by the Sym factor the tauon mass should be the same as
the sum of the masses of the first generation massive fermion particles.
Therefore the tauon mass is calculated at tree level as 1.877 GeV.
The calculated tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.
However, as the tauon mass is about 2 GeV,
the effective tauon mass should be renormalized
from the energy level of 1 GeV at which the mass is 1.88 GeV
to the energy level of 2 GeV.
Such a renormalization should reduce the mass.
If the renormalization reduction were about 5 percent,
the effective tauon mass at 2 GeV would be about 1.78 GeV.
The 1996 Particle Data Group Review of Particle Physics gives a tauon mass of 1.777 GeV.
All triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples.
They are triples of the same form as the 7 tauon triples involving 1 and E,
but for 1 and I, 1 and J, and 1 and K,
which correspond to the red, green, and blue beauty quarks,
respectively.
The seven red beauty quark triples correspond to the seven tauon triples,
except that
the beauty quark interacts with 6 Spin(0,5) gravitons
while the tauon interacts with only two.

The red beauty quark constituent mass should be the tauon mass times
the third generation graviton factor 6/2 = 3,
so the red beauty quark mass is mb = 5.63111 GeV.

The blue and green beauty quarks are similarly determined to also be 5.63111 GeV.

The theoretical model calculated Beauty Quark mass of 5.63 GeV
corresponds to a pole mass of 5.32 GeV,
which is somewhat higher than the conventional value of 5.0 GeV.
However, the theoretical model calculated value
of the color force strength constant alpha_s at about 5 GeV is about 0.166,
while the conventional value
of the color force strength constant alpha_s at about 5 GeV is about 0.216,
and
the theoretical model calculated value
of the color force strength constant alpha_s at about 90 GeV is about 0.106,
while the conventional value
of the color force strength constant alpha_s at about 90 GeV is about 0.118.

Triples of the type { 1, I, J } , { I, J, K }, etc.,
do not correspond to the beauty quark, but to the truth quark.
The truth quark corresponds to those 512 - 1 - 7 - 21 = 483 triples,
so the constituent mass of the red truth quark
is 161 / 7 = 23 times the red beauty quark mass,
and the red T-quark mass is
mt = 129.5155 GeV
The blue and green truth quarks are similarly determined to also be 129.5155 GeV.
This is the value of the Low Mass State of the Truth calculated in the Cl(16)_E8 model.
The Middle Mass State of the Truth Quark has been observed by Fermilab since 1994.
The Low and High Mass States of the Truth Quark have, in my opinion, also been
observed by Fermilab but the Fermilab and CERN establishments disagree.

These results when added up give a total mass of third generation fermion
particles:

Sigmaf3 = 1,629 GeV

E8 Physics Calculation Results

Here is a summary of E8 Physics model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed. Quark masses are constituent masses. Most of the calculations are tree-level, so more detailed calculations might be even closer to observations.

```
Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04

Fermions as Schwinger Sources have geometry of Complex Bounded Domains
with Kerr-Newman Black Hole structure size about 10^(-24) cm.

Particle/Force            Tree-Level        Higher-Order
e-neutrino                    0                0 for nu_1
mu-neutrino                   0           9 x 10^(-3) eV for nu_2
tau-neutrino                  0          5.4 x 10^(-2) eV for nu_3

electron                 0.5110 MeV
down quark                312.8 MeV      charged pion = 139 MeV
up quark                  312.8 MeV        proton = 938.25 MeV
                                       neutron - proton = 1.1 MeV
muon                      104.8 MeV            106.2 MeV
strange quark               625 MeV
charm quark                2090 MeV

tauon                      1.88 GeV
beauty quark               5.63 GeV
truth quark (low state)     130 GeV      (middle state) 174 GeV
                                           (high state) 218 GeV

W+                       80.326 GeV
W-                       80.326 GeV
W0                       98.379 GeV           Z0 = 91.862 GeV

Mplanck             1.217x10^19 GeV

Higgs VEV (assumed)       252.5 GeV
Higgs (low state)           126 GeV      (middle state) 182 GeV
                                          (high state) 239 GeV

Gravity Gg (assumed)          1
(Gg)(Mproton^2 / Mplanck^2)                     5 x 10^(-39)
EM fine structure         1/137.03608
Weak Gw                     0.2535
Gw(Mproton^2 / (Mw+^2 + Mw-^2 + Mz0^2))      1.05 x 10^(-5)
Color Force at 0.245 GeV    0.6286           0.106 at 91 GeV

Kobayashi-Maskawa parameters for W+ and W- processes are:
      d                    s                   b
u   0.975                0.222               0.00249 -0.00388i
c  -0.222 -0.000161i     0.974 -0.0000365i   0.0423
t   0.00698 -0.00378i   -0.0418 -0.00086i    0.999
The phase angle d13 is taken to be 1 radian.
```

Appendix - E8 Physics and 256 Cellular Automata

Raymond Aschheim (email May 2015) said:
"... An elementary CA is defined by the next value (either 0 or 1) for a cell, depending on its ... value, and the ... value of it[s] left and of it[s] right neighbor cell (it is one dimensional, and involve only the first neighbors, and the cell itself) ... So the next value depends [on] 3 bits ... eight possible combination of three bits, and for each ... combination... the next value is either zero or one. So the[re] are 256 ... CAs ...".

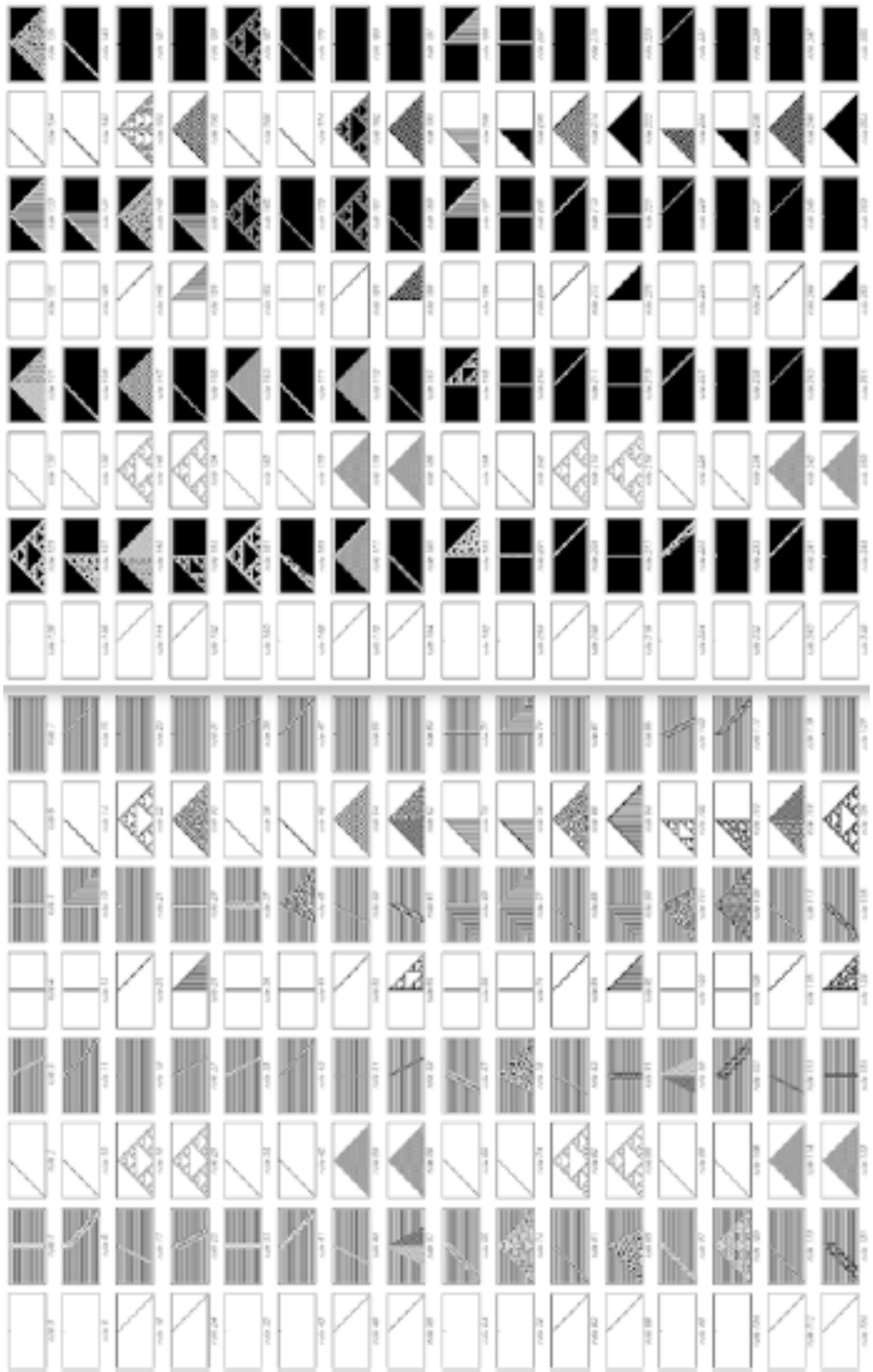
Since due to Real Clifford 8-periodicity any Real Clifford Algebra Cl(8N)) can be seen as tthe tensor product of N copies of Cl(8), any Real Clifford Algebra has fundamental structure of Cl(8) = Cl(1,7) = 16x16 real matrix algebra
so Cellular Automata correspondence with Cl(8) means
that any Real Clifford Algebra can be described by Cellular Automata.
Therefore Clifford Algebra E8 physics can also be seen in terms of Cellular Automata.

Each initial state for a CA rule for 1-dim nearest neighbor automata is a triple
* * * in which each of the 3 * (left, middle, right) can be either 0 or 1.
Each CA rule gives one of 2 outcomes 0 or 1 for each of the 8 states

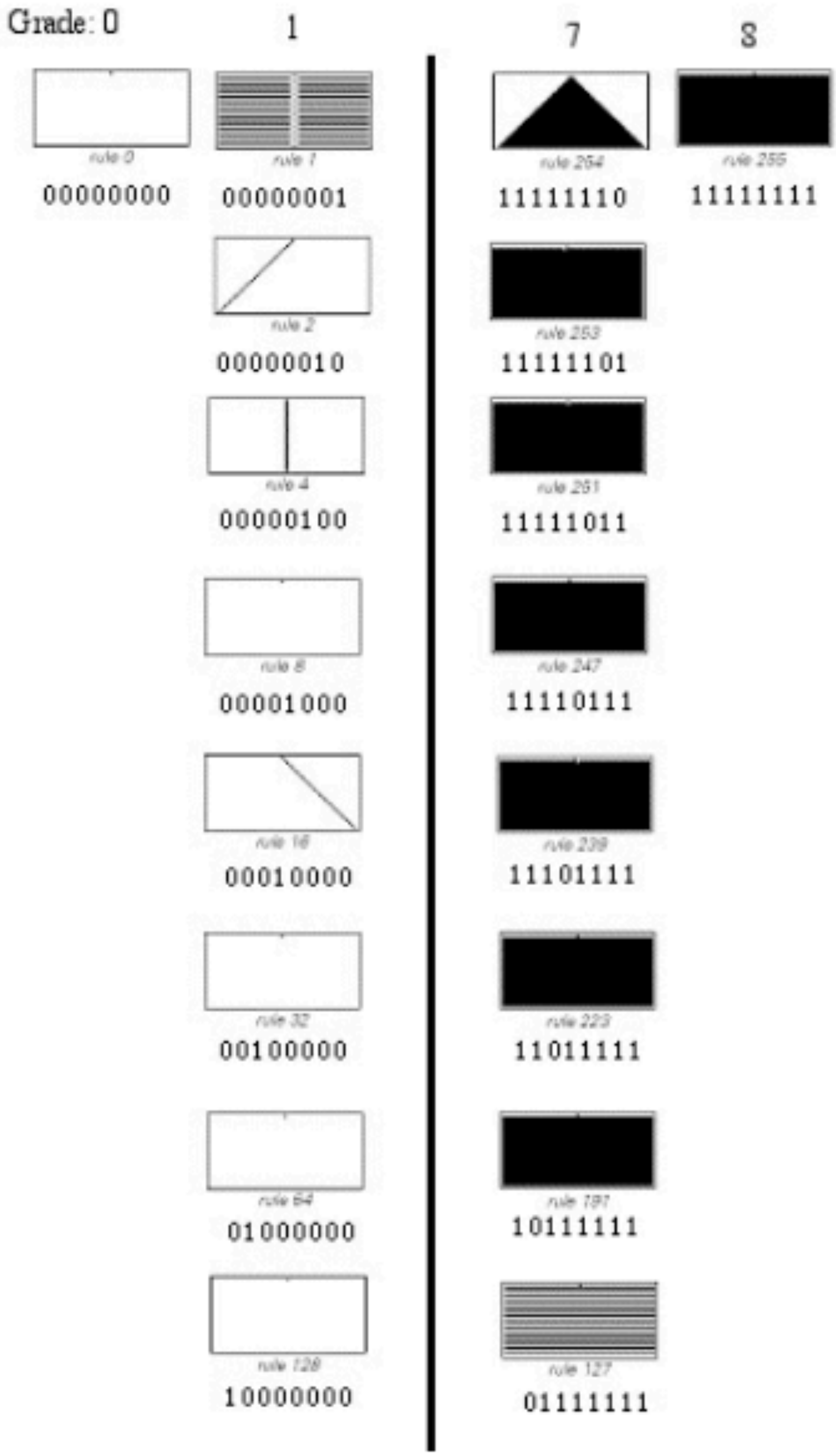
1 1 1	0 1 1	0 0 1	0 0 0
	1 0 1	0 1 0	
	1 1 0	1 0 0	

so there are 2^8 = 256 possible CA rules.
The 8 states correspond to the 8 vectors of the Clifford Algebra Cl(8)

The CA rule that gives 0 for all 8 states corresponds to the 1 scalar 0-vector of Cl(8)
There are 8 CA rules that give 1 for one of the 8 states and 0 for the other 7
and they correspond to the 8 vectors of Cl(8)
There are 28 CA rules that give 1 for 2 of the 8 states and 0 for the other 6
and they correspond to the 28 bivectors of Cl(8)
There are 56 CA rules that give 1 for 3 of the 8 states and 0 for the other 5
and they correspond to the 56 3-vectors of Cl(8)
There are 70 CA rules that give 1 for 4 of the 8 states and 0 for the other 4
and they correspond to the 70 4-vectors of Cl(8)
There are 56 CA rules that give 1 for 5 of the 8 states and 0 for the other 3
and they correspond to the 56 5-vectors of Cl(8)
There are 28 CA rules that give 1 for 6 of the 8 states and 0 for the other 2
and they correspond to the 28 6-vectors of Cl(8)
There are 8 CA rules that give 1 for 7 of the 8 states and 0 for the other 1
and they correspond to the 8 7-vectors of Cl(8)
There is 1 CA rule that gives 1 for all 8 states
and it corresponds to the 1 pseudo-scalar 8-vector of Cl(8)



256 Cellular Automata
1 8 28 56 70 56 28 8 1
(images from "A New Kind of Science" by Stephen Wolfram (Wolfram 2002))



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8
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00000001
rule 2
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rule 4
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rule 8
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00100000
rule 64
01000000
rule 128
10000000
rule 254
11111110
rule 255
11111111
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rule 127
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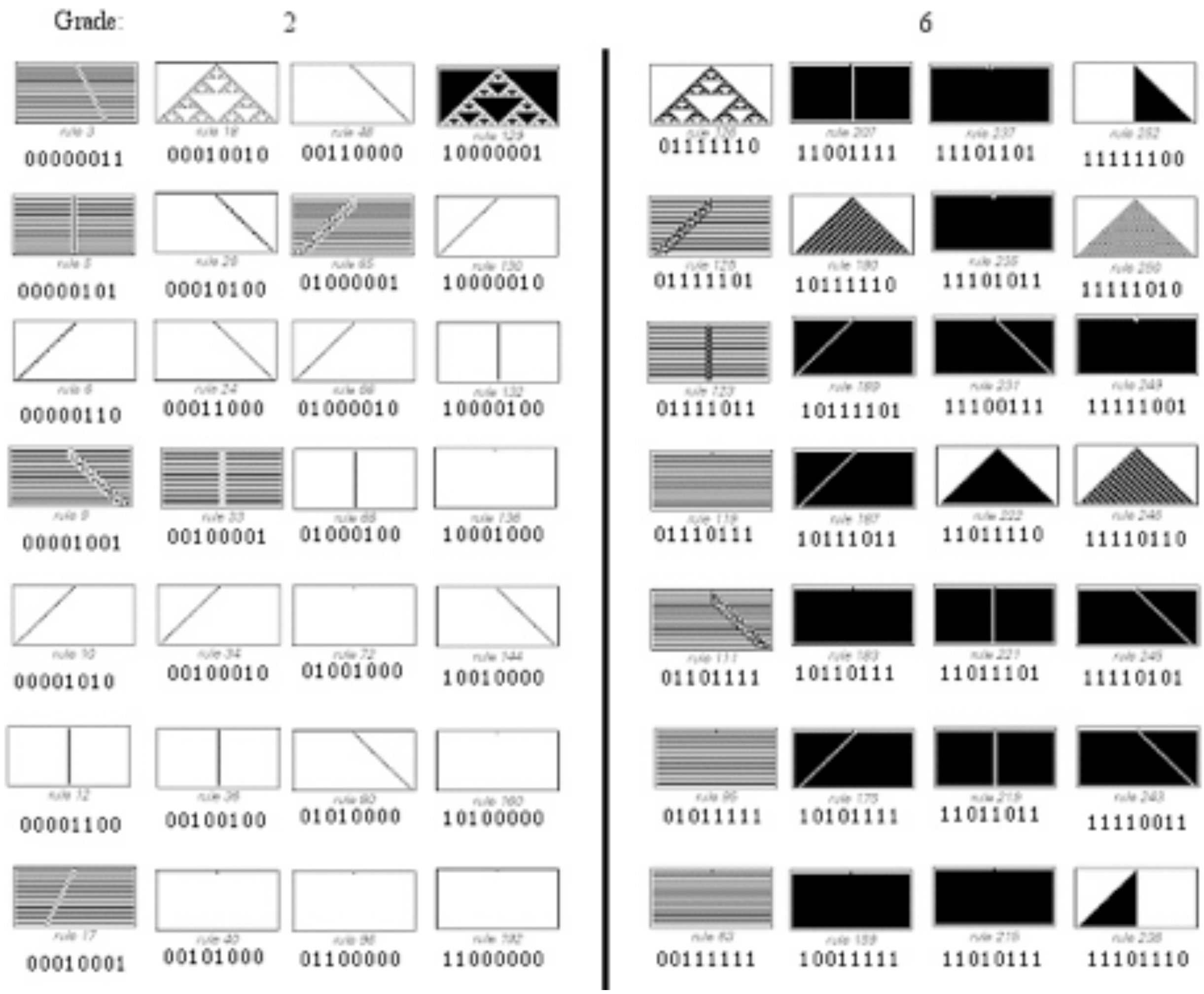
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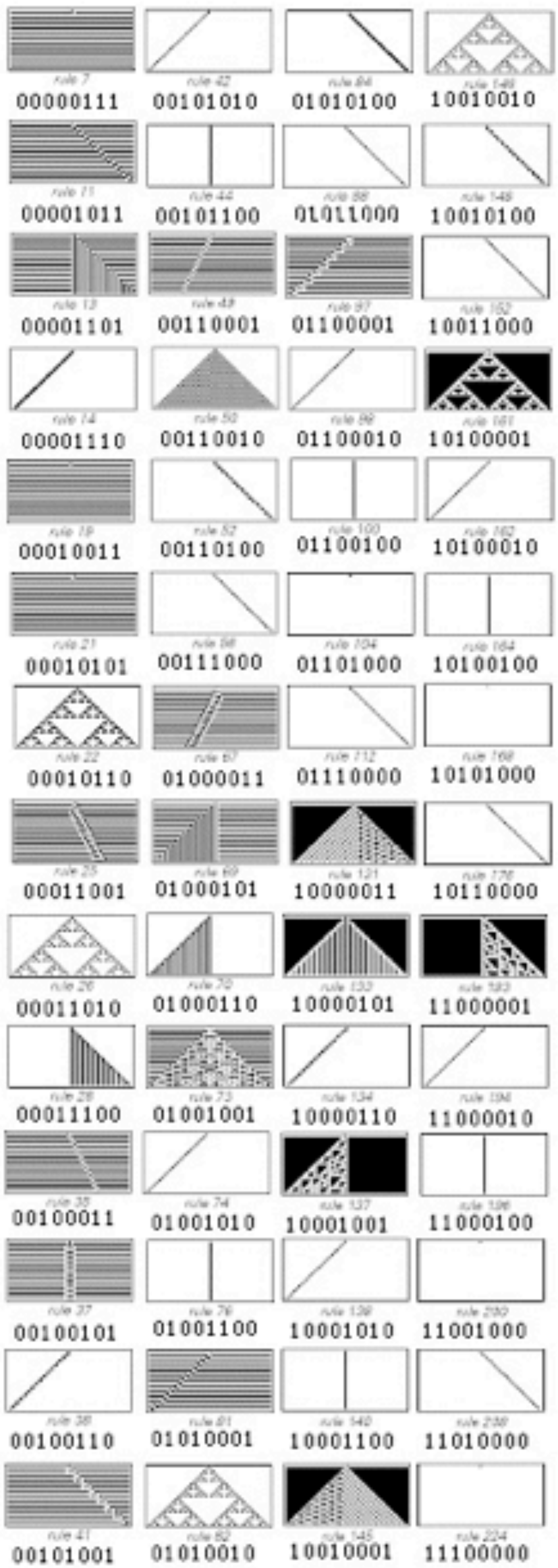

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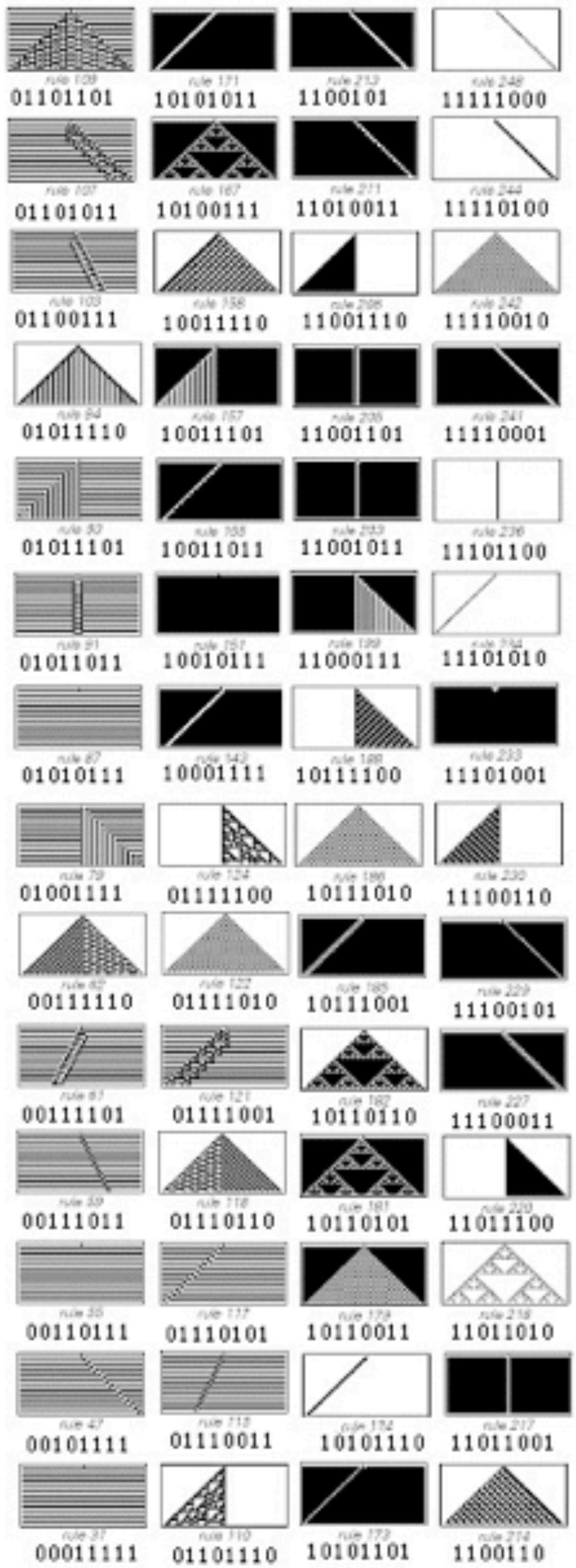

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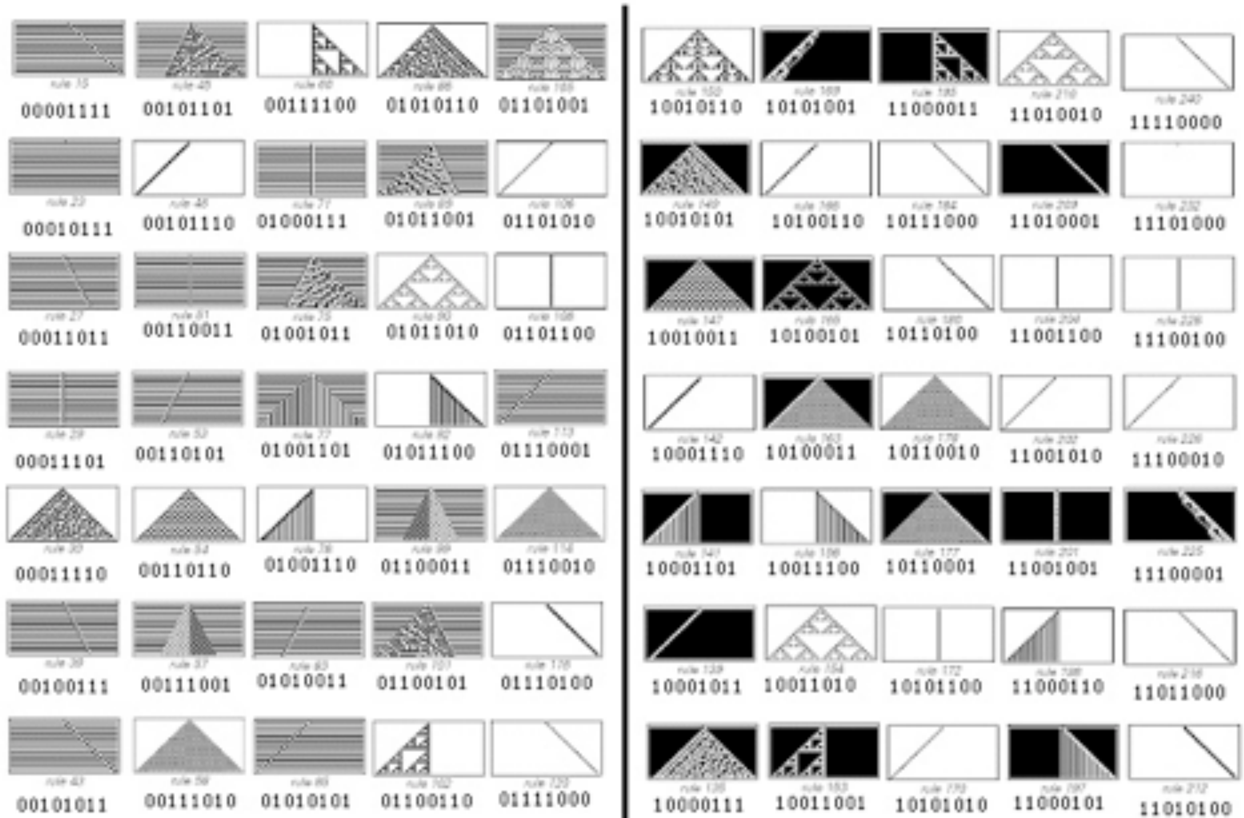
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10011010
10101100
11000110
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10000111
10011001
10101010
11000101
11010100

the 16 terms in the Cl(8) primitive idempotent

f = (1/2)(1 + e_1248) (1/2)(1 + e_2358) (1/2)(1 + e_3468) (1/2)(1 + e_4578) =

=(1/16)(1 + e_1248 + e_2358 + e_3468 + e_4578 + e_5618 + e_6728 + e_7138 - e_3567 - e_4671 - e_5712 - e_6123 - e_7234 - e_1345 - e_2456 + e_J)

correspond to 16 of the 256 Cellular Automata

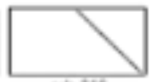
- + e_12345678


rule 255
11111111

- + e_6728 + e_3468 + e_4578 to


rule 226
11100010


rule 172
10101100


rule 216
11011000

- + e_2358 + e_1248 + e_5618 + e_7138

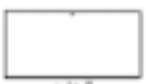

rule 150
10010110


rule 139
10001011


rule 177
10110001


rule 197
11000101

- + 1 to


rule 0
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- - e_5712 - e_1345 - e_6123


rule 83
01010011


rule 29
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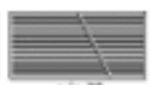
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- - e_4671 - e_7234 - e_2456 - e_3567


rule 105
01101001


rule 78
01001110


rule 58
00111010


rule 116
01110100

Note the Cl(0,8) = Cl(1,7) triality correspondences among:

- the 8 +half-spinors

00000000 01010011 00011101 00100111 01101001 01001110 00111010 01110100

- the 8 -half-spinors

11111111 11100010 10101100 11011000 10010110 10001011 10110001 11000101

- the 8 vectors

00000001 00000010 00000100 00010000 00001000 00100000 01000000 10000000

Note that:

the grade-0 scalars

are related to the Spinors and Primitive Idempotents of Cl(0,8);

the grade-1 vectors 1, 2, 4, 16 (the subset sequence 2^0 = 1, 2^1 = 2, 2^2 = 4, 2^4 = 16 related to Fermat primes)

correspond to the 4 dimensions of physical spacetime;

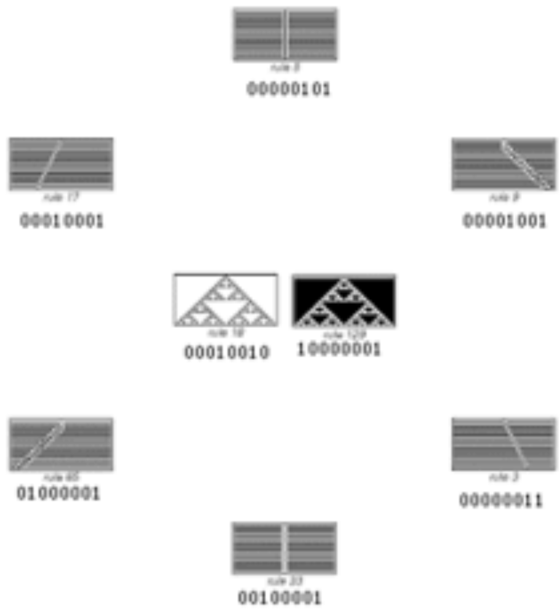
- 1 gives a succession of bands, the procession of time;
- 2 gives a slope to the left, one of three space dimensions;
- 4 gives a vertical slope, a second of three space dimensions;
- 16 gives a slope to the right, the third of three space dimensions;

the grade-1 vectors 8, 32, 64, 128 (all giving all white)

correspond to the 4 dimensions of internal symmetry space;

- rule 18 = 00010010 is the first rule to include both 16 = 00010000 with right slope and 2 = 00000010 with left slope and is the first rule with traingular self-similar fractal structure;
- rule 30 = 00011110 is the first rule to include 16, 8, 4, and 2 and is in the self-dual grade-4 and is the first rule with triangular chaotic behavior.

8 of the grade-2 bivectors,



after dimensional reduction to 4-dimensional physical spacetime, correspond to the 8 generators of color force SU(3), whose root vector diagram is illustrated above;

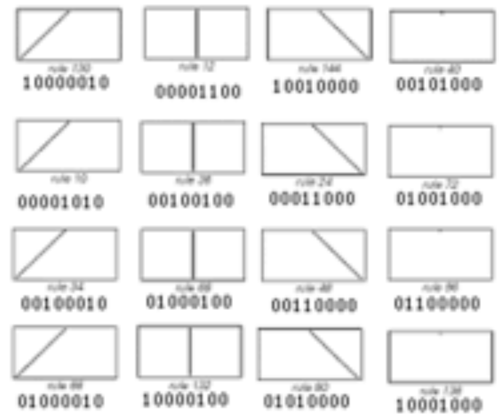
3 of the grade-2 bivectors,

after dimensional reduction to 4-dimensional physical spacetime, correspond to the 3 generators of weak force SU(2);

1 of the grade-2 bivectors,

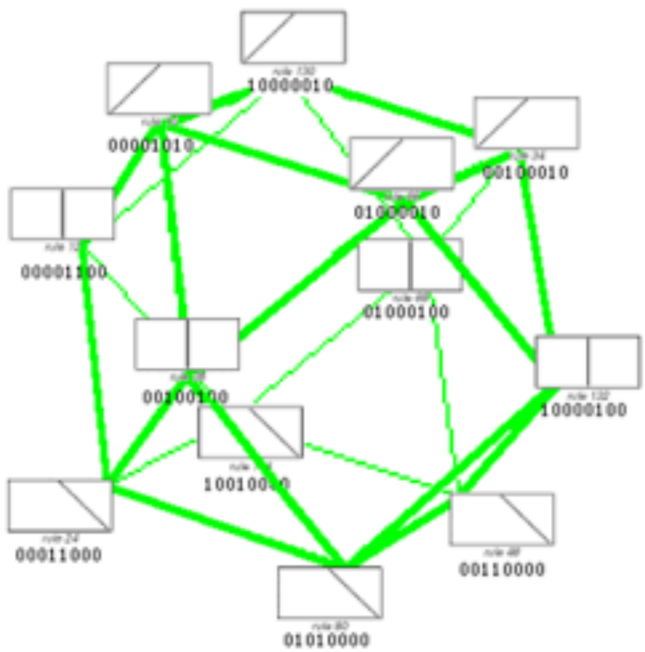
after dimensional reduction to 4-dimensional physical spacetime, correspond to the 1 generator of electromagnetic U(1);

16 of the grade-2 bivectors,



after dimensional reduction to 4-dimensional physical spacetime, correspond to the 16 generators of Gravity/Higgs/phase U(2,2). One of them

corresponds to the propagator phase U(1) while the other 15 correspond to the Conformal Group SU(2,2) = Spin(2,4) whose root vector diagram



is a 12-vertex cuboctahedron (the other 3 bivectors corresponding to the 3 generators of the Cartan Subalgebra).

Appendix - 4-dim M4 Spacetime Feynman Checkerboard

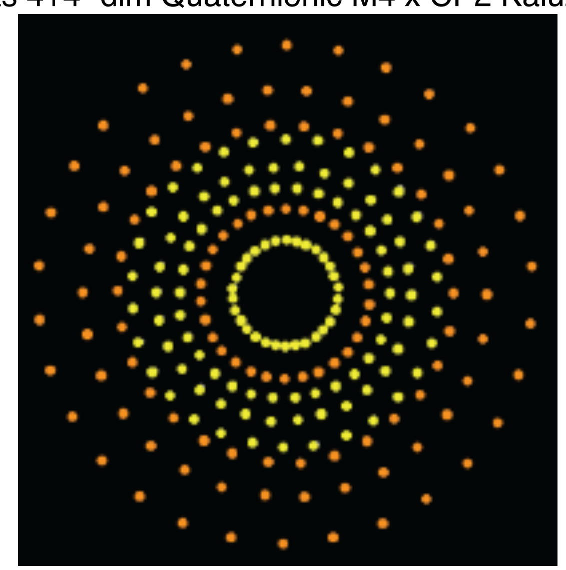
The main body of this paper discusses E8 Root Vectors and their relationship with continuous structures such as symmetric spaces E8 / D8 and D8 / D4xD4 etc useful in describing E8 Physics and doing E8 Physics calculations.

However, from a fundamental point of view, it is useful to describe E8 Physics in terms of discrete structures such as E8 Lattices and Gossett Polytopes in 8-dim and D4 Lattices and 600-cells and 24-cells in 4-dim Kaluza-Klein subspaces which leads to construction of 4-dim M4 Feynman Checkerboards with Planck-scale Lattice Spacings.

The 240 vertices of the E8 Gosset polytope in 8-dim have physical interpretations that produce a Local Classical Lagrangian for Gravity and the Standard Model. Embedding E8 in the Real Clifford Algebra Cl(16) = Cl(8)xCl(8) and taking the completion of the union of all tensor products of Cl(16) gives a realistic Algebraic Quantum Field Theory (AQFT).

An equivalent Quantum Field Theory can be constructed using Tetrahedra, 57G, 600-cells, and the E8 Gossett polytope along with a generalized Feynman Checkerboard in 4 SpaceTime dimensions.

To begin, consider the 240 Root Vectors, based on 8-dim Octonionic spacetime being seen as 4+4 -dim Quaternionic M4 x CP2 Kaluza-Klein Spacetime:

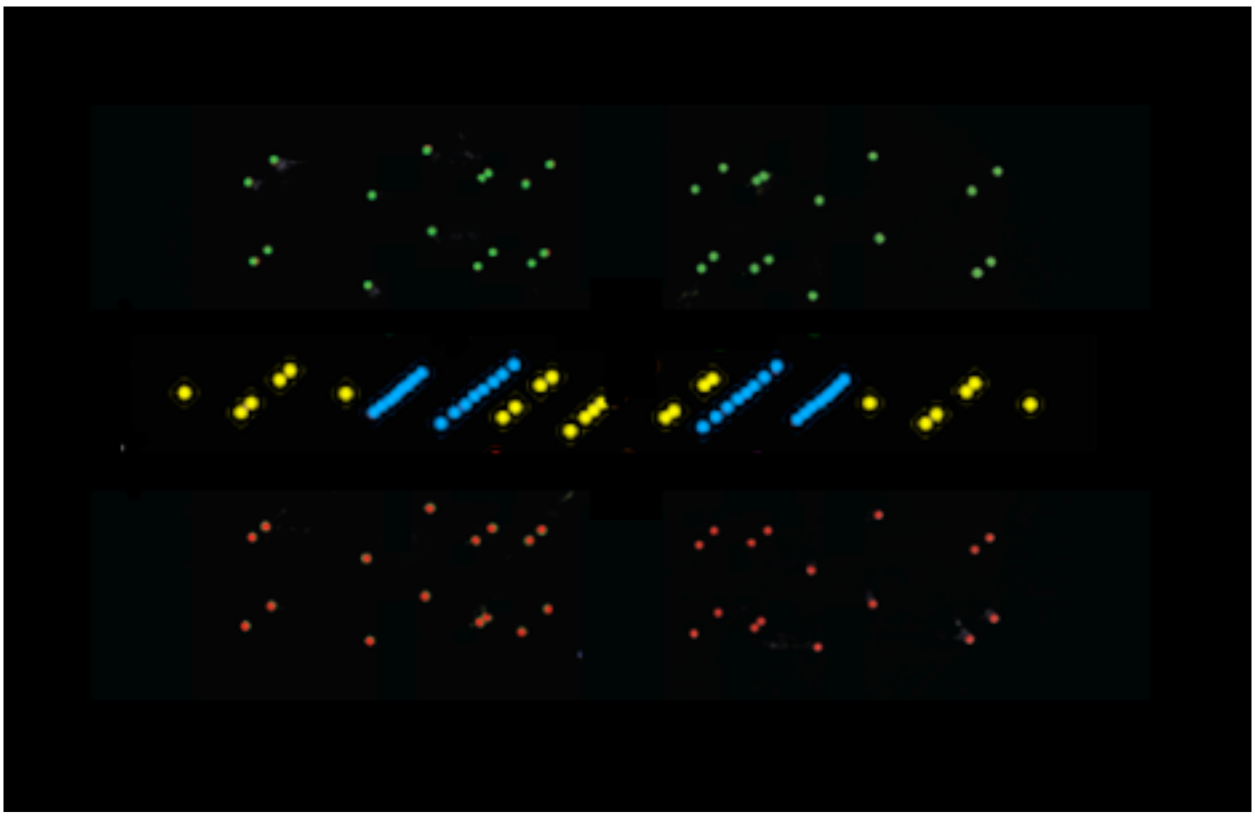


120 of the 240 (yellow dots) represent aspects of First-Generation Fermions, Gauge Bosons and Ghosts, and Position and Momentum related to M4 Physical Spacetime. 120 of the 240 (orange dots) represent aspects of First-Generation Fermions, Gauge Bosons and Ghosts, and Position and Momentum related to CP2 = SU(3) / SU(2)xU(1) Internal Symmetry Space. In the above 2-dim projection the CP2 120 have larger radii from the center than the M4 120 by a factor of the Golden Ratio.

Now go to my preferred represention of the 240 E8 Root Vectors in 2-dim / 3-dim space in a square / cube configuration.

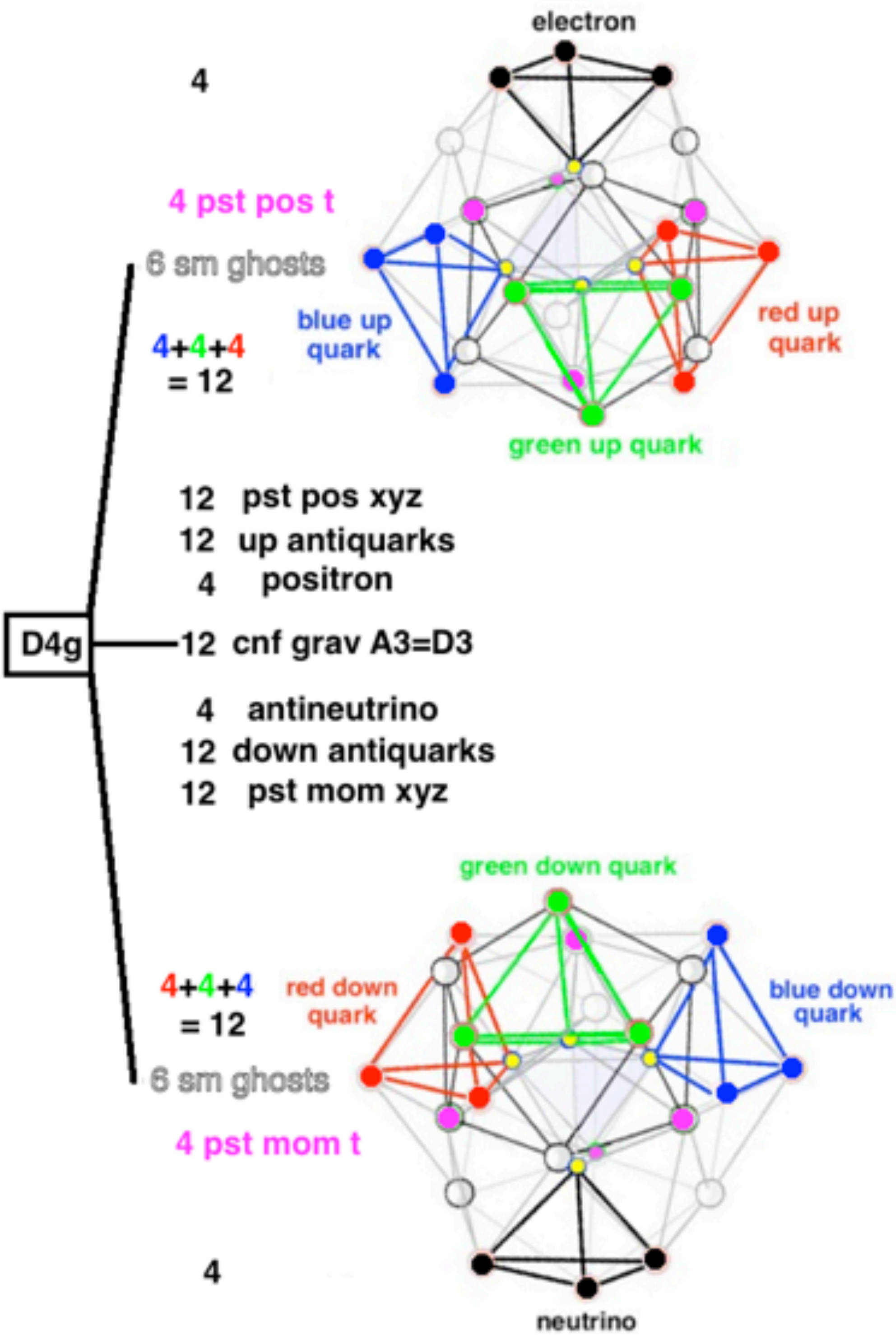
Split 8-dim Kaluza-Klein E8 SpaceTime into its two 4-dimensional components:
M4 Physical SpaceTime and CP2 = SU(3 / SU(2)xU(1) Internal Symmetry Space

Let one 600-cell represent Gravity and physics of Physical SpaceTime.
Here is a projection of its 120 vertices whose physical interpretations are:
red and green = M4 Components of Fermions, blue = M4 Physical SpaceTime,
yellow = D4g of Conformal Gravity and Standard Model Ghosts

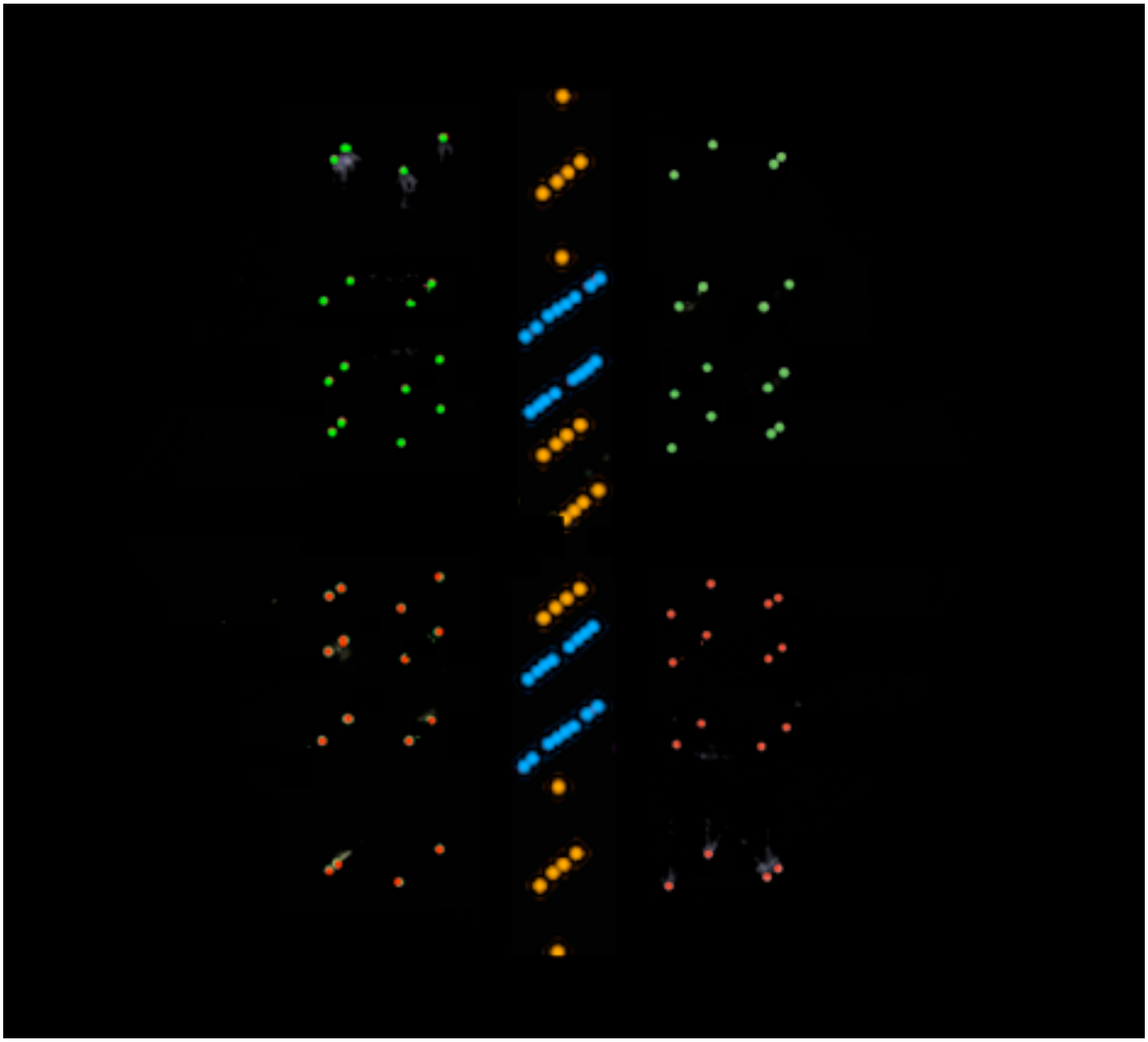


Here is how those 120 vertices appear in cell-centered sections of the D4g 600-cell:

Conformal Gravity 600-Cell
with
M4 Physical SpaceTime

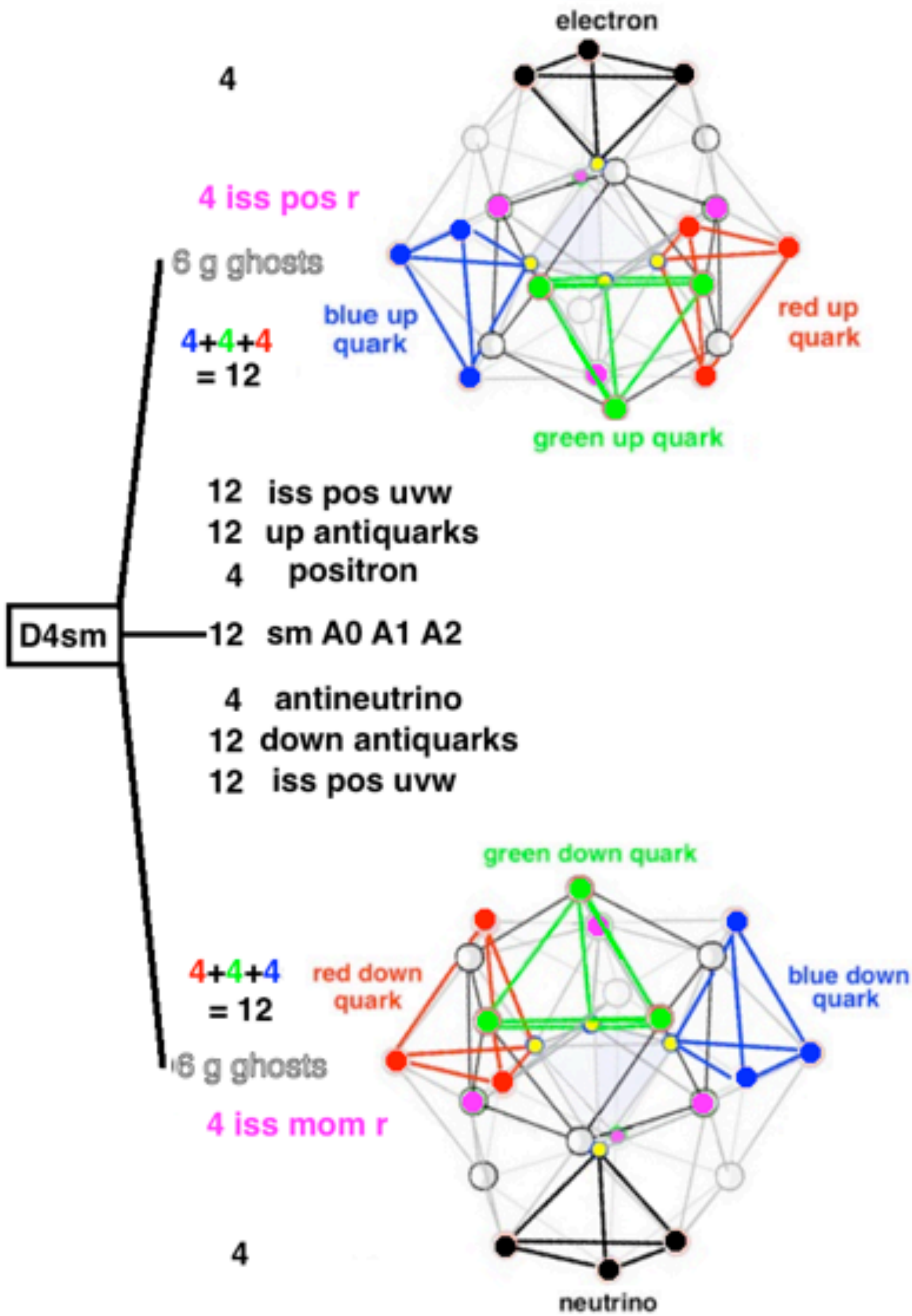


Let the other 600-cell represent the Standard Model and its Internal Symmetry Space.
Here is a projection of its 120 vertices whose physical interpretations are:
red and green = CP2 Components of Fermions, blue = CP2 Internal Symmetry Space,
orange = D4sm of the Standard Model and Conformal Gravity Ghosts

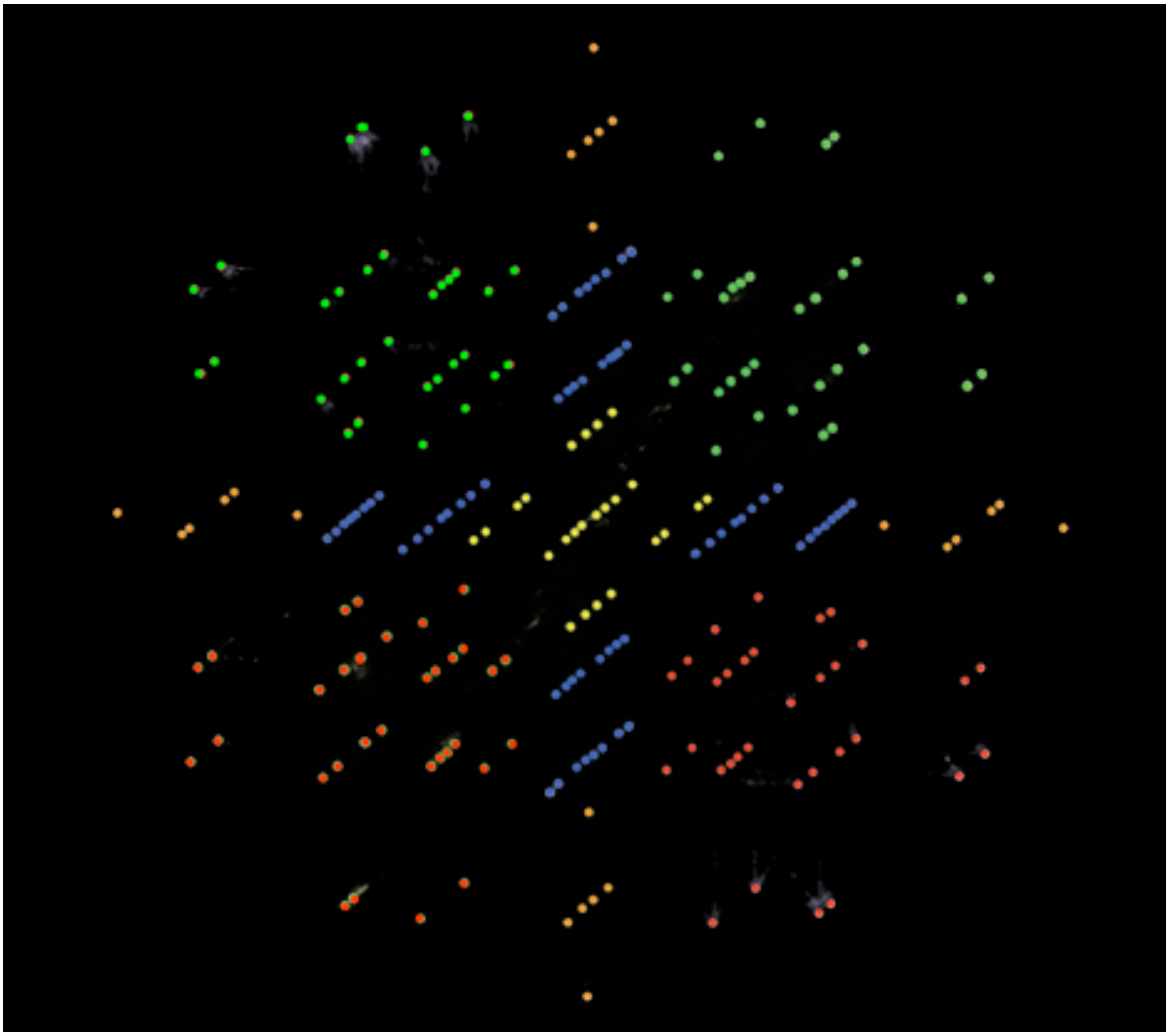


Here is how those 120 vertices appear in cell-centered sections of the D4sm 600-cell:

Standard Model 600-Cell
with
CP2 Internal Symmetry Space

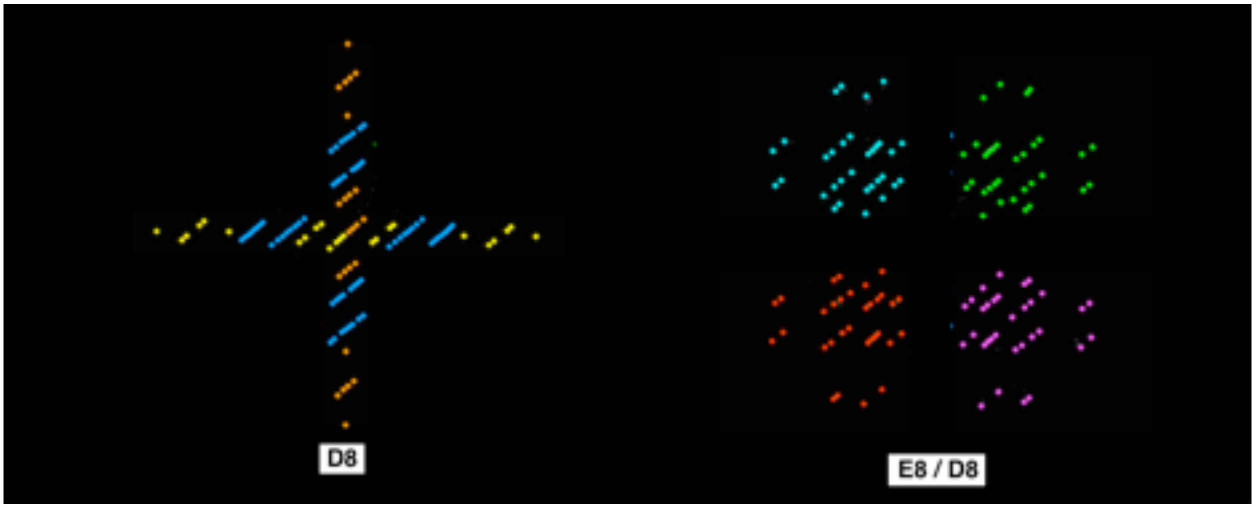


The 120 vertices of the D4g 600-cell and the 120 vertices of the D4sm 600-cell combined form the 240 vertices of the E8 Root Vectors of E8 Physics:



E8 lives inside the Real Clifford Algebra Cl(16) as E8 = D8 + Cl(16) half-spinors
so

240 E8 Root Vectors = 112 D8 Root Vectors + 128 Cl(16) half-spinors



E8 Lattice = D8 Lattice + ([1] + D8 Lattice)
where the lattice shifting glue vector [1] = (1/2, ... , 1/2)

Feynman Checkerboard Quantum Theory

Conway and Sloane, in their book Sphere Packings, Lattices, and Groups (3rd edition, Springer, 1999), in chapter 4, section 7.3, pages 119-120) define a packing [where the glue vector [1] = (1/2, ... , 1/2)]

D+n = Dn u ([1] + Dn)

and say:

"... D+n is a lattice packing if and only if n is even.
D+3 is the tetrahedral or diamond packing ... and
D+4 = Z4.
When n = 8 this construction is especially important,
the lattice **D+8 being known as E8** ...".

Therefore

E8 Lattice = D8 Lattice + ([1] + D8 Lattice)

There are 7 independent E8 Integral Domain Lattices.

Physically, the D8 Lattice represents SpaceTime and Gauge Bosons
while the ([1] + D8 Lattice) represents Fermions.

At high energies (for example, during Inflation) E8 Physics is Octonionic and there is only one generation of fermions, so the first generation is the only generation. Therefore, each charged Dirac fermion particle, and its antiparticle, correspond to one imaginary Octonion, to one associative triangle, and to one E8 lattice so each Fermion propagates in its own **E8 8D Feynman Checkerboard Lattice**:

```
red Down Quark                     red Up Quark           .
    green Down Quark    Electron     green Up Quark
             blue Down Quark                blue Up Quark


          rD      gD      bD        E      rU      gU      bU


          I       J       K        E       i       j       k


                                   j
                                  / \
                                 i---k

          J       j       J                I       J       K
         / \     / \     / \              / \     / \     / \
        i---K   I---K   I---k            E---i   E---j   E---k


         3E8     6E8     4E8      7E8     1E8     2E8     5E8
```

Since all the E8 lattices have in common the vertices { ±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke }, all the charged Dirac fermions can interact with each other. Composite particles, such as Quark-AntiQuark mesons and 3-Quark hadrons, propagate on the common parts of the E8 lattices involved. The uncharged neutrino fermion, which corresponds to the Octonion real axis with basis {1}, propagates on the 8th Kirmse E8 Lattice that is not an independent Octonion Integral Domain.

If a preferred Quaternionic Structure is introduced into an Octonionic E8 Lattice then the Octonionic E8 Lattice is transformed into Quaternionic Lattice structure. The Quaternionic Integral Domain Lattice is the D4 Lattice.

D8 Lattice is transformed to D4g + D4sm

([1] + D8 Lattice) is transformed to ([1] + D4g) + ([1] + D4sm)

so

E8 is transformed to { D4g + ([1] + D4g) } + { D4 sm + ([1] + D4sm) }

E8 = D+4g + D+4sm

D+4g corresponds to the 600-cell containing D4g

D+4sm corresponds to the 600-cell containing D4sm

Conway and Sloane (Sphere Packings, Lattices, and Groups - Springer) (Chapter 4, eq. 49)
give equations for the number of vertices N(m) in the m-th layer
of the D+4 HyperDiamond lattice where d is a divisor (including 1 and m) of m:
for m odd: N(m) = 8 SUM(d|m) d   for m even: N(m) = 24 SUM(d|m, d odd) d

```
Here are the numbers of vertices in some of the layers of the D4+ lattice.
The even-numbered layers correspond to the even D4 sublattice:
m=norm of layer            N(m)=no. vert.
   0                                1
   1                                8  =    1 x 8
   2                               24  =    1 x 24
   3                               32  =  ( 1 + 3 ) x 8
   4                               24  =    1 x 24
   5                               48  =  ( 1 + 5 ) x 8
   6                               96  =  ( 1 + 3 ) x 24
   7                               64  =  ( 1 + 7 ) x 8
   8                               24  =    1 x 24
   9                              104  =  ( 1 + 3 + 9 ) x 8
  10                              144  =  ( 1 + 5 ) x 24
  11                               96  =  ( 1 + 11 ) x 8
  12                               96  =  ( 1 + 3 ) x 24
  13                              112  =  ( 1 + 13 ) x 8
  14                              192  =  ( 1 + 7 ) x 24
  15                              192  =  ( 1 + 3 + 5 + 15 ) x 8
  16                               24  =    1 x 24
  17                              144  =  ( 1 + 17 ) x 8
```

First Stage of 4D Feynman Checkerboard:
D+4g vertices have HyperOctahedron 8 nearest-neighbors {+/-1,+/-i,+/-j,+/-k}
where 4-dim 1,i,j,k are descendants of 8-dim 1,i,j,k
to be used as 4D Feynman Checkerboard Primary Links representing the
4-dim M4 Physical SpaceTime of the Kaluza-Klein of E8 Physics whose
4 basis elements are {1,i,j,k} each of which has 8 momentum components
with respect to 8-dim SpaceTime to represent 4x8 = 32 of 600-cell vertices.

D+4g vertices have 24-cell 24 next-nearest neighbors representing the
12 Conformal Gravitons (Root Vectors of U(2,2) and
12 Ghosts of Standard Model Gauge Bosons
that live on the nearest-neighbor links and represent 24 of 600-cell vertices.

D+4g vertices have 6-semi-HyperCube 32 next-next-nearest neighbors representing
4 M4 Physical SpaceTime components of 8 First-Generation Fermion Particles.
Fermion AntiParticles are represented by Particles moving backward in Time
for representation of 2x32 = 64 of 600-cell vertices.

D+4g odd (1 and 3) layers correspond to Vectors and Fermion Spinors which are related by Triality.
D+4g even (2) layers correspond to BiVectors.

From each vertex of the 4D Feynman Checkerboard the First Stage uses a Triad of Quantum Choice Vectors.

Second Stage of 4D Feynman Checkerboard:

D+4sm vertices have HyperOctahedron 8 nearest-neighbors {+/-1,+/-i,+/-j,+/-k}
where 4-dim 1,i,j,k are descendants of 8-dim E,I,J,K
to be used as 4D Feynman Checkerboard Secondary Links representing the 4-dim CP2 Internal Symmetry Space of the Kaluza-Klein of E8 Physics whose 4 basis elements are {1,i,j,k} each of which has 8 momentum components with respect to 8-dim SpaceTime to represent 4x8 = 32 of 600-cell vertices.

D+4sm vertices have 24-cell 24 next-nearest neighbors representing the
12 Standard Model Gauge Bosons and
12 Ghosts of Conformal Gravitons (Root Vectors of U(2,2)
that live on the nearest-neighbor links and represent 24 of 600-cell vertices.

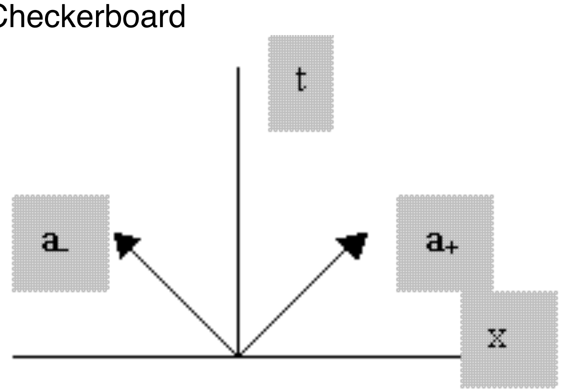
D+4sm vertices have 6-semi-HyperCube 32 next-next-nearest neighbors representing 4 CP2 Internal Symmetry Space components of 8 First-Generation Fermion Particles. Fermion AntiParticles are represented by Particles moving backward in Time for representation of 2x32 = 64 of 600-cell vertices.

D+4g odd (1 and 3) layers correspond to Vectors and Fermion Spinors which are related by Triality.
D+4g even (2) layers correspond to BiVectors.

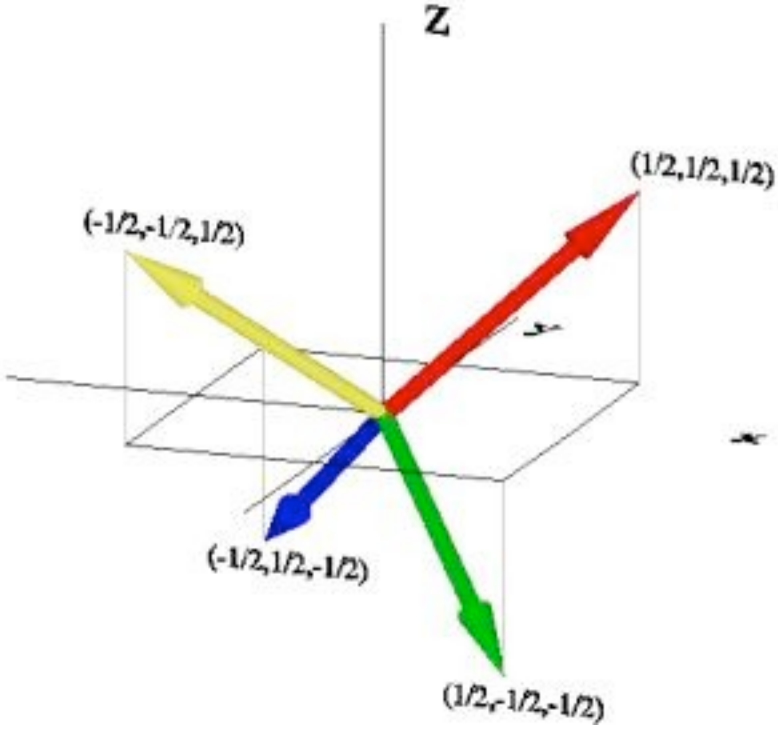
From each vertex of the 4D Feynman Checkerboard the Second Stage uses a second Triad of Quantum Choice Vectors.

A significant consequence of using two Triads of Quantum Choice Vectors is the emergence of Second and Third Generation Fermions.

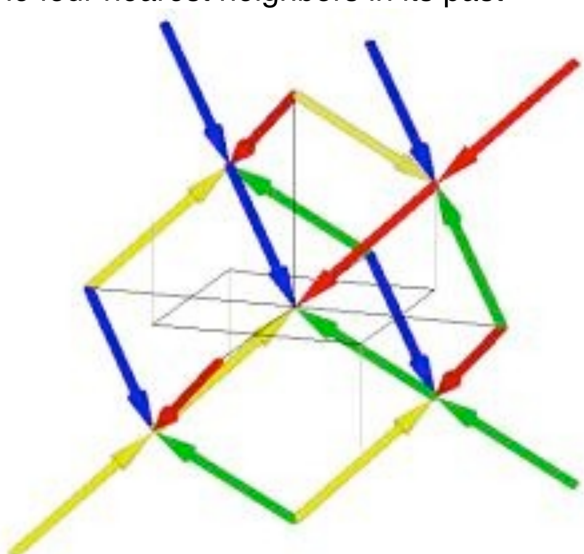
In my earlier paper (arXiv quant-ph/9503015) I used **a simpler version of 4D Feynman Checkerboard which is useful for showing consistency with the Dirac equation** using the following approach: The Feynman Checkerboard in 1+3 SpaceTime dimensions reproduces the Dirac equation, using work of Urs Schreiber and George Raetz. (See my paper at CERN-CDS-EXT-2004-030) A very nice feature of the George Raetz web site is its illustrations, which include an image of a vertex of a 1+1 dimensional Feynman Checkerboard



and an image of a projection into three dimensions of a vertex of a 1+3 dimensional Feynman Checkerboard



and an image of flow contributions to a vertex in a HyperDiamond Random Walk from the four nearest neighbors in its past



Urs Schreiber wrote on the subject:

Re: Physically understanding the Dirac equation and 4D

in the newsgroup sci.physics.research on 2002-04-03 19:44:31 PST (including an appended forwarded copy of an earlier post)
and again on 2002-04-10 19:03:09 PST as found on the web page
http://www-stud.uni-essen.de/~sb0264/spinors-Dirac-checkerboard.html

and the following are excerpts from those posts:

"... I know ... the ... lanl paper ...[http://xxx.lanl.gov/abs/quant-ph/9503015]...
and
I know that Tony Smith does give
a generalization of Feynman's summing prescription from 1+1 to 1+3 dimensions.

But I have to say that I fail to see that this generalization reproduces the Dirac propagator in 1+3 dimensions,
and that I did not find any proof that it does.

Actually, I seem to have convinced myself that it does not,
but
I may of course be quite wrong.

I therefore take this opportunity to state my understanding of these matters.

First, I very briefly summarize (my understanding of) Tony Smith's construction: The starting point is the observation that the left |-> and right |+> going states of the 1+1 dim checkerboard model can be labeled by complex numbers

|-> ---> (1 + i)
|+> ---> (1 - i)

(up to a factor) so that multiplication by the negative imaginary unit swaps components:

(-i) (1 + i)/2 = (1 - i)/2
(-i) (1 - i)/2 = (1 + i)/2 .

Since the path-sum of the 1+1 dim model reads
phi = sum over all possible paths of (-i eps m)^(number of bends of path) =
= sum over all possible paths of product over all steps of one path of -i eps m
(if change of direction after this step generated by i) 1 (otherwise)
this makes it look very natural to identify the imaginary unit appearing in the sum over paths with the "generator" of kinks in the path.
To generalize this to higher dimensions, more square roots of -1 are added,
which gives the quaternion algebra in 1+3 dimensions.
The two states |+> and |-> from above, which were identified with complex numbers, are now generalized to four states identified with the following quaternions
(which can be identified with vectors in M^4 indicating the direction in which a given path is heading at one instant of time):
(1 + i + j + k) (1 + i - j - k) (1 - i + j - k) (1 - i - j + k) ,
which again constitute a (minimal) left ideal of the algebra
(meaning that applying i,j, or k from the left on any linear combination of these four states gives another linear combination of these four states).
Hence,
now i,j,k are considered as "generators" of kinks in three spatial dimensions
and the above summing prescription naturally generalizes to
phi = sum over all possible paths of product over all steps of one path of
-i eps m (if change of direction after this step generated by i)
-j eps m (if change of direction after this step generated by j)
-k eps m (if change of direction after this step generated by k)
1 (otherwise)
The physical amplitude is taken to be
A * e^(i alpha)
where A is the norm of phi and alpha the angle it makes with the x0 axis.

As I said, this is merely my paraphrase of Tony Smith's proposal as I understand it.

I fully appreciate that the above construction is a nice (very "natural") generalization of the summing prescription of the 1+1 dim checkerboard model.

But if it is to describe real fermions propagating in physical spacetime, this generalized path-sum has to reproduce the propagator obtained from the Dirac equation in 1+3 dimensions, which we know to correctly describe these fermions. Does it do that?
...
Hence I have taken a look at the material [that] ... George Raetz ... present[s] ... titled "The HyperDiamond Random Walk", found at
http://www.pcisys.net/~bestwork.1/QRW/the_flow_quaternions.htm ,
which is mostly new to me. ...

I am posting this in order to make a suggestion for a more radical modification
...
[The]... equation ... DQ = (iE)Q ... is not covariant.
That is because of that quaternion E sitting on the left of the spinor Q
in the rhs of [the] equation ... .
The Dirac operator D is covariant,
but the unit quaternion E on the rhs refers to a specific frame.
Under a Lorentz transformation L one finds
L DQ = iE LQ = L E' Q <=> DQ = E'Q now with E' = L~ E L instead of E.
This problem disappears
when the unit quaternion E is brought to the *right* of the spinor Q.
What we would want is an equation of the form DQ = Q(iE) .
In fact, demanding that the spinor Q be an element of the minimal left ideal
generated by the primitive projector P = (1+y0)(1+E)/4 ,
so that Q = Q' P ,
one sees that DQ = Q(iE) almost looks like the the *Dirac-Lanczos equation*.
(See hep-ph/0112317, equation (5) or ... equation (9.36) [of]... W. Baylis, Clifford
(Geometric) Algebras, Birkhaeuser (1996) ...).
To be equivalent to the Dirac-Lanczos equation, and hence to be correct,
we need to require that D = y0 @0 + y1 @1 + y2 @2 + y3 @3
instead of ... = @0 + e1 @1 + e2 @2 + e3 @3 .
All this amounts to sorting out
in which particular representation we are actually working here.
In an attempt to address these issues, I now redo the steps presented on
http://www.pcisys.net/~bestwork.1/QRW/the_flow_quaternions.htm
with some suitable modifications to arrive at the correct Dirac-Lanczos equation
(this is supposed to be a suggestion subjected to discussion):

So consider a lattice in Minkoswki space
generated by a unit cell spanned by the four (Clifford) vectors

r = (y0 + y1 + y2 + y3)/2 g = (y0 + y1 - y2 - y3)/2 b =
= (y0 - y1 + y2 - y3)/2 y = (y0 - y1 - y2 + y3)/2 .

(yi are the generators of the Dirac algebra {yi,yj} = diag(+1,-1,-1,-1)_ij.)

This is Tony Smith's "hyper diamond".

(Note that I use Clifford vectors instead of quaternions.)
Now consider a "Clifford algebra-weighted" random walk along the edges of this lattice,
which is described by four Clifford valued "amplitudes": Kr, Kg, Kb, Ky and
such that
@r Kr = k (Kg y2 y3 + Kb y3 y1 + Ky y1 y2)
@b Kb = k (Ky y2 y3 + Kr y3 y1 + Kg y1 y2) @g Kg = k (Kr y2 y3 + Ky y3 y1 + Kb y1 y2)
@y Ky = k (Kb y2 y3 + Kg y3 y1 + Kr y1 y2) .

(This is geometrically motivated. The generators on the rhs are those that rotate the unit vectors corresponding to the amplitudes into each other. "k" is some constant.)

Note that I multiply the amplitudes from the *right* by the generators of rotation, instead of multiplying them from the left.

Next, assume that this coupled system of differential equations is solved by a spinor Q
Q = Q' (1+y0)(1+iE)/4
E = (y2 y3 + y3 y1 + y1 y2)/sqrt(3) with
Kr = r Q Kg = g Q Kb = b Q Ky = y Q .

This ansatz for solving the above system by means of a single spinor Q is,
as I understand it, the central idea.
But note that I have here modified it on the technical side:

Q is explicitly an algebraic Clifford spinor in a definite minial left ideal,

E squares to -1, not to +1,

and the Ki are obtained from Q by premultiplying with the Clifford basis vectors defined above.

Substituting this ansatz into the above coupled system of differential equations one can form one covariant expression by summing up all four equations:

(r @r + g @g + b @b + y @y) Q = k sqrt(3) Q E
The left hand side is immediate.
To see that the right hand side comes out as indicated
simply note that r + g + b + y = y0 and that Q y0 = Q by construction.

The above equation is the Dirac-Lanczos-Hestenes-Guersey equation,
the algebraic version of the equation describing the free relativistic electron.

The left hand side is the flat Dirac operator r @r + g @g + b @b + y @y = ym @m
and
the right hand side, with k = mc / (hbar sqrt(3)) ,
is equal to the mass term i mc / hbar Q.
As usual, there are a multitude of ways to rewrite this.
If one wants to emphasize biquaternions then
premultiplying everything with y0 and
splitting off the projector P on the right of Q to express everything in terms of the,
then also biquaternionic, Q' (compare the definitions given above)
gives Lanczos' version (also used by Baylis and others).

I think this presentation improves a little on that given on George Raetz's web site:

The factor E on the right hand side of the equation is no longer a nuisance
but a necessity.

Everything is manifestly covariant (if one recalls that algebraic spinors are manifestly covariant when nothing non-covariand stands on their *left* side). The role of the quaternionic structure is clarified,
the construction itself does not depend on it.
Also, it is obvious how to generalize to arbitrary dimensions.
In fact, one may easily check that for 1+1 dimensions the above scheme reproduces the Feynman model.

While I enjoy this, there is still some scepticism in order
as long as a central questions remains to be clarified:

How much of the Ansatz K(r,g,b,y) = (r,g,b,y) Q is whishful thinking?

For sure,
every Q that solves the system of coupled differential equations that describe the amplitude of the random walk on the hyper diamond lattice
also solves the Dirac equation.

But what about the other way round?
Does every Q that solves the Dirac equation also describe such a random walk. ...".

My proposal to answer the question raised by Urs Schreiber

Does every solution of the Dirac equation
also describe a HyperDiamond Feynman Checkerboard random walk?

uses symmetry.

The hyperdiamond random walk transformations
include the transformations of the Conformal Group:

rotations and boosts (to the accuracy of lattice spacing);
translations (to the accuracy of lattice spacing);
scale dilatations (to the accuracy of lattice spacing): and
special conformal transformations (to the accuracy of lattice spacing).

Therefore, to the accuracy of lattice spacing,
the hyperdiamond random walks give you all the conformal group Dirac solutions,
and since the full symmetry group of the Dirac equation is the conformal group,
the answer to the question is "Yes".

Thanks to the work of Urs Schreiber:

The HyperDiamond Feynman Checkerboard in 1+3 dimensions does reproduce the correct Dirac equation.

Here are some references to the **conformal symmetry of the Dirac equation**:

R. S. Krausshar and John Ryan in their paper Some Conformally Flat Spin Manifolds, Dirac Operators and Automorphic Forms at math.AP/022086 say:
"... In this paper we study Clifford and harmonic analysis on some conformal flat spin manifolds. ... manifolds treated here include RPn and S1 x S(n-1).
Special kinds of Clifford-analytic automorphic forms associated to the different choices of are used to construct Cauchy kernels, Cauchy Integral formulas, Green's kernels and formulas together with Hardy spaces and Plemelj projection operators for Lp spaces of hypersurfaces lying in these manifolds. ...
Solutions to the Dirac equation are called Clifford holomorphic functions or monogenic functions.
Such functions are covariant under ... conformal or .... Mobius transformations acting over Rn u {oo}. ...".

Barut and Raczka, in their book Theory of Group Representations and Applications (World 1986), say, in section 21.3.E, at pages 616-617:
"... E. The Dynamical Group Interpretation of Wave Equations.
... Example 1. Let G = O(4,2).
Take U to be the 4-dimensional non-unitary representation in which the generators of G are given in terms of the 16 elements of the algebra of Dirac matrices as in exercise 13.6.4.1.
Because (1/2)L_56 = gamma_0 has eigenvalues n = +/-1,
taking the simplest mass relation mn = K, we can write
(m gamma_0 - K) PSI(dotp) = 0, where K is a fixed constant.
Transforming this equation with the Lorentz transformation of parameter E
PSI(p) = exp(i E N) PSI(p)
N = (1/2) gamma_0 gamma
gives
(gamma^u p_u - K) PSI(p) = 0
which is the Dirac equation ...".

P. A. M. Dirac, in his paper Wave Equations in Conformal Space, Ann. Math. 37 (1936) 429-442, reprinted in The Collected Works of P. A. M. Dirac: Volume 1: 1924-1948, by P. A. M. Dirac (author), Richard Henry Dalitz (editor), Cambridge University Press (1995), at pages 823-836, said:
"... by passing to a four-dimensional conformal space ...
a ... greater symmetry of ... equations of physics ... is shown up, and their invariance under a wider group is demonstrated. ...
The spin wave equation ... seems to be the only
simple conformally invariant wave equation involving the spin matrices. ... This equation is equivalent to the usual wave equation for the electron, except ...[that it is multiplied by]... the factor (1 + alpha_5) ,
which introduces a degeneracy. ...".

Here are some comments on **Lorentz Invariance based on D4 Lattice** properties:

The D4 lattice nearest neighbor vertex figure, the 24-cell,
is the 4HD HyperDiamond lattice next-to-nearest neighbor vertex figure.
Fermions move from vertex to vertex along links.
Gauge bosons are on links between two vertices, and so can also be considered as moving from vertex to vertex along links.
The only way a translation or rotation can be physically defined is by a series of movements of a particle along links.
A TRANSLATION is defined as a series of movements of a particle along links, each of which is
the CONTINUATION of the immediately preceding link IN THE SAME DIRECTION.
An APPROXIMATE rotation, within an APPROXIMATION LEVEL D,
is defined with respect to a given origin as a series of movements of a particle along links among vertices ALL of which
are in the SET OF LAYERS LYING WITHIN D of norm (distance^2) R from the origin, that is,
the SET OF LAYERS LYING BETWEEN norm R-D and norm R+D from the origin.
Conway and Sloane (Sphere Packings, Lattices, and Groups - Springer) pp. 118-119 and 108, is the reference that I have most used for studying lattices in detail.
(Conway and Sloane define the norm of a vector x to be its squared length xx.)
In the D4 lattice of integral quaternions,
layer 2 has the same number of vertices as layer 1, N(1) = N(2) = 24.
Also (this only holds for real, complex, quaternionic, or octonionic lattices),
K(m) = N(m)/24 is multiplicative,
meaning that, if p and q are relatively prime, K(pq) = K(p)K(q).
The multiplicative property implies that:
K(2^a) = K(2) = 1 (for a greater than 0) and
K(p^a) = 1 + p + p^2 + ... + p^a (for a greater than or equal to 0).
So,
for the D4 lattice,
there is always an arbitrarily large layer (norm xx = 2^a, for some large a)
with exactly 24 vertices, and
there is always an arbitrarily large layer(norm xx = P, for some large prime P)
with 24(P+1) vertices (note that Mersenne primes are adjacent to powers of 2), and
given a prime number P whose layer is within D of the origin,
which layer has N vertices,
there is a layer kP with at least N vertices within D of any other given layer in D4.
Some examples I have used are chosen so that
the 2^a layer adjoins the prime 2^a +/- 1 layer.

The notation in the following table is based on the minimal norm of the D4 Lattice being 1, in which case **the D4 lattice is the lattice of integral quaternions**.
This is the second definition (equation 90) of the D4 Lattice in
Chapter 4 of Sphere Packings, Lattices, and Groups, 3rd ed., by Conway and Sloane (Springer 1999)
who note that the Dn lattice is the checkerboard lattice in n dimensions.

m=norm of layer	N(m)=no. vert.	K(m)=N(m)/24
1	24	1
2	24	1
3	96	4
4	24	1
5	144	6
6	96	4
7	192	8
8	24	1
9	312	13
10	144	6
11	288	12
12	96	4
13	336	14
14	192	8
15	576	24
16	24	1
17	432	18
18	312	13
19	480	20
20	144	6
127	3,072	128
128	24	1
65,536=2^16	24	1
65,537	1,572,912	65,538
2,147,483,647	51,539,607,552	2,147,483,648
2,147,483,648=2^31	24	1

Appendix - NCG and 130 GeV Tquark mass state

Connes has constructed a realistic physics model in 4-dim spacetime based on **NonCommutative Geometry (NCG) of M x F** where **M = 4-dim spacetime and F = C x H x M3(C)** and C = Complex Numbers, H = Quaternions, and M3(C) = 3x3 Complex Matrices.

E8 has been used as a basis for physics models such as those by Lisi (arXiv 1506.08073) and Smith (viXra 1508.0157) so the purpose of this paper is to show **a connection between Connes NCG Physics and E8**.

Connes NCG is described by van den Dungen and van Suijlekom in arXiv 1204.0328 where they say: "... this review article is to present
the applications of Connes' noncommutative geometry to elementary particle physics.
...
the noncommutative description of the Standard Model does not require the introduction of extra spacetime dimensions,
its construction is very much like the original Kaluza-Klein theories.
In fact, **one starts with a product M x F of ordinary four-dimensional spacetime M with an internal space F** which is to describe the gauge content of the theory.
Of course, spacetime itself still describesmthe gravitational part.
The main difference with Kaluza-Klein theories is that the additional space is a discrete ... space whose structure is described by a ... noncommutative algebra ...
This is very much like the description of spacetime M
by its coordinate functions as usual in General Relativity,
which form an algebra under pointwise multiplication:

(x^mu x^nu)(p) = x^mu(p) x^nu(p)

Such commutative relations are secretly used in any physics textbook.
However, for a discrete space, ... propose to describe F by matrices ...
yielding a much richer internal (algebraic) structure ...
one can also describe a metric on F in terms of algebraic data,
so that we can fully describe the geometrical structure of M x F .
This type of noncommutative manifolds are called almost-commutative (AC)
...
Given an AC manifold M x F ... the group of diffeomorphisms ... generalized to such noncommutative spaces combines ordinary diffeomorphisms of M with gauge symmetries ... we obtain a combination of general coordinate transformations on M with the respective groups ... U(1) x SU(2) x SU(3) ...[whose]... finite space is ...

internal space F ...[=]... C x H x M3(C)

... to construct a Lagrangian from the geometry of M x F . This is accomplished by ... a simple counting of the eigenvalues of a Dirac operator on M x F
which are lower than a cutoff Λ ... we derive local formulas (integrals of Lagrangians) ... using heat kernel methods ...
The fermionic action is given as usual by an inner product.
The Lagrangians that one obtains in this way ... are the right ones,
and in addition minimally coupled to gravity.

This is unification with gravity of ... the full Standard Model. ...
We study conformal invariance ... with particular emphasis on the Higgs mechanism coupled to the gravitational background
...
the Lagrangian derived ... from the relevant noncommutative space is not just the Standard Model Lagrangian, but it implies that there are relations between some of the Standard Model couplings and masses
...
If we would assume that the mass of the top quark is much larger than all other fermion masses, we may neglect the other fermion masses. In that case ...

m_top ≤ sqrt(8/3) Mw [= sqrt(8/3) 80 = 130 GeV]

...
we shall evaluate the renormalization group equations (RGEs) for the Standard Model from ordinary energies up to the ... GUT ... unification scale ...

The scale Λ12 ... is given by ... 1.03 x 10^13 GeV ...

The [scale] Λ23 is given by ... 9.92 x 10^16 GeV ...

we have ... included the simple case wherewe ignore the Yukawa coupling of the tau-neutrino
[as is realistic with no neutrino see-saw mechanism] ... Numerical results [are]...

Λgut (10^16 GeV) ... m_top (GeV) 186.0 ... m_h (GeV) 188.1 ...

Λgut (10^13 GeV) ... m_top (GeV) 183.2 ... m_h (GeV) 188.3 ...".

If you do a naive extrapolation down to the Higgs VeV 250 GeV energy scale where the compositeness of a Higgs as Tquark condensate system might become evident (the Non-perturbativity Boundary)

Λcomp (250 GeV) ... m_top (GeV) 173.2 ... m_h (GeV) 189

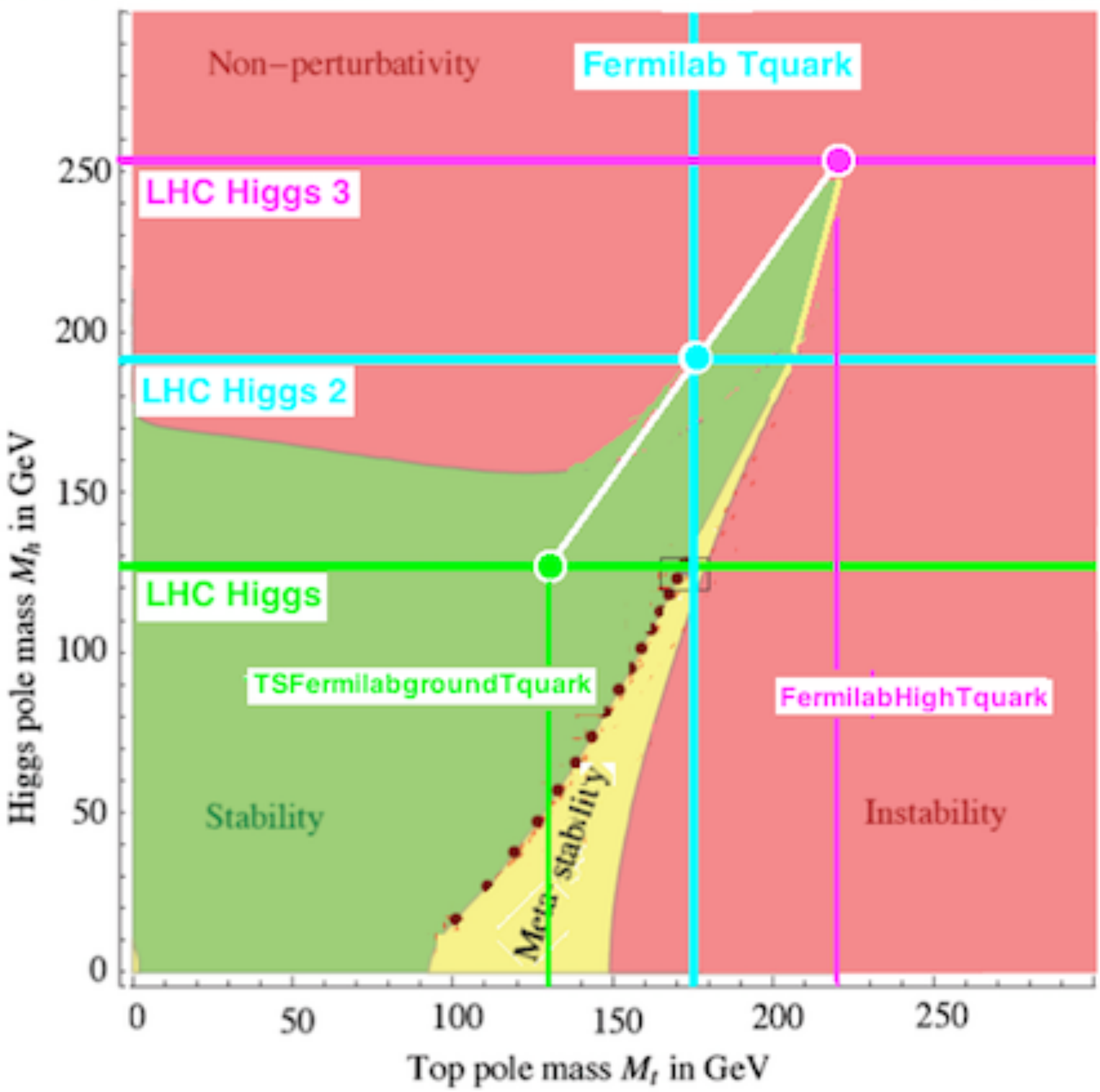
so the naively extrapolated

NCG masses for the Tquark-Higgs Middle Mass States are consistent with those of the E8 model of Smith (viXra 1508.0157)

Further,

the Basic Ground State NCG Tquark mass of 130 GeV is consistent with that of the E8 model of Smith (viXra 1508.0157)

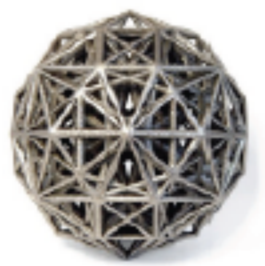
Here is a chart showing the 3 Mass States of the Smith E8 model (viXra 1508.0157):
the green dot in the Stable region (green) has the 130 GeV Tquark mass state
that is also calculated by NCG; the cyan dot on the Non-perturbativity Boundary has
the 173 GeV Tquark and 189 GeV Higgs mass states that are also calculated by NCG;
I have not seen where NCG may or may not calculate High-Mass (220 and 250 GeV)
Tquark and Higgs mass states indicated by the magenta dot at the Critical Point.



Structure of M and F of NCG

The **M of NCG** is 4-dim Spacetime, a discrete version of which is
the Integral Domain of Integral Quaternions whose vertex figure (nearest neighbors to the origin) is the 24-cell Root Vector Polytope of the 28-dim D4 Lie Algebra which contains as a subalgebra the 15-dim D3 Lie Algebra of the Conformal Group Spin(2,4) = SU(2,2) for MacDowell-Mansouri Gravity plus Conformal Dark Energy.

4-dim Riemannian Spacetime can be Wick Rotated to 4-dim Euclidean Space which can be compactified to the 4-sphere S4 which can be discretized as the 600-cell



so **the M of NCG can be locally represented as a 600-cell which has 120 vertices**.

F of NCG is the 24-dim algebra C + H + M3(C).
Identify the 24 generators of F with the 24 elements of the Binary Tetrahedral Group and therefore **identify F with the Tetrahedron** of which it is the symmetry group.
NCG, by using M x F as its basic structure, puts a copy of F at each point of M.

Consider a flat 2-dim subspace of M, and add to it F Tetrahedra following this construction recipe from a Don Davis 8 Sep 1999 sci math post:
"... build ... a hollow torus of 300 cells ... as follows:
lay out a 5x10 grid of unit edges. omit the lefthand and lower boundaries' edges, because we're going to roll this grid into a torus later.
thus, the grid contains 100 edges: 50 running N-S, and 50 running EW.
attach one tetrahedron to each edge from above the grid.
the opposite edges of these tetrahedra will form a new 5x10 grid,
whose vertices overlie the centers of the squares in the lower grid.
thus, these 100 tetrahedra now form an egg-carton shape,
with 50 squarepyramid cups on each side.
divide each cup into two non-unit tetrahedra,
by erecting a right-triangular wall across the cup, corner-to-corner.
make the upper cups' dividers run NE/SW,
and make the upside-down lower cups' dividers run NW/SE.
note that the egg-carton is now a solid flat layer, one tetrahedron deep,
containing 100 unit tetra- hedra and 200 non-unit tetrahedra.

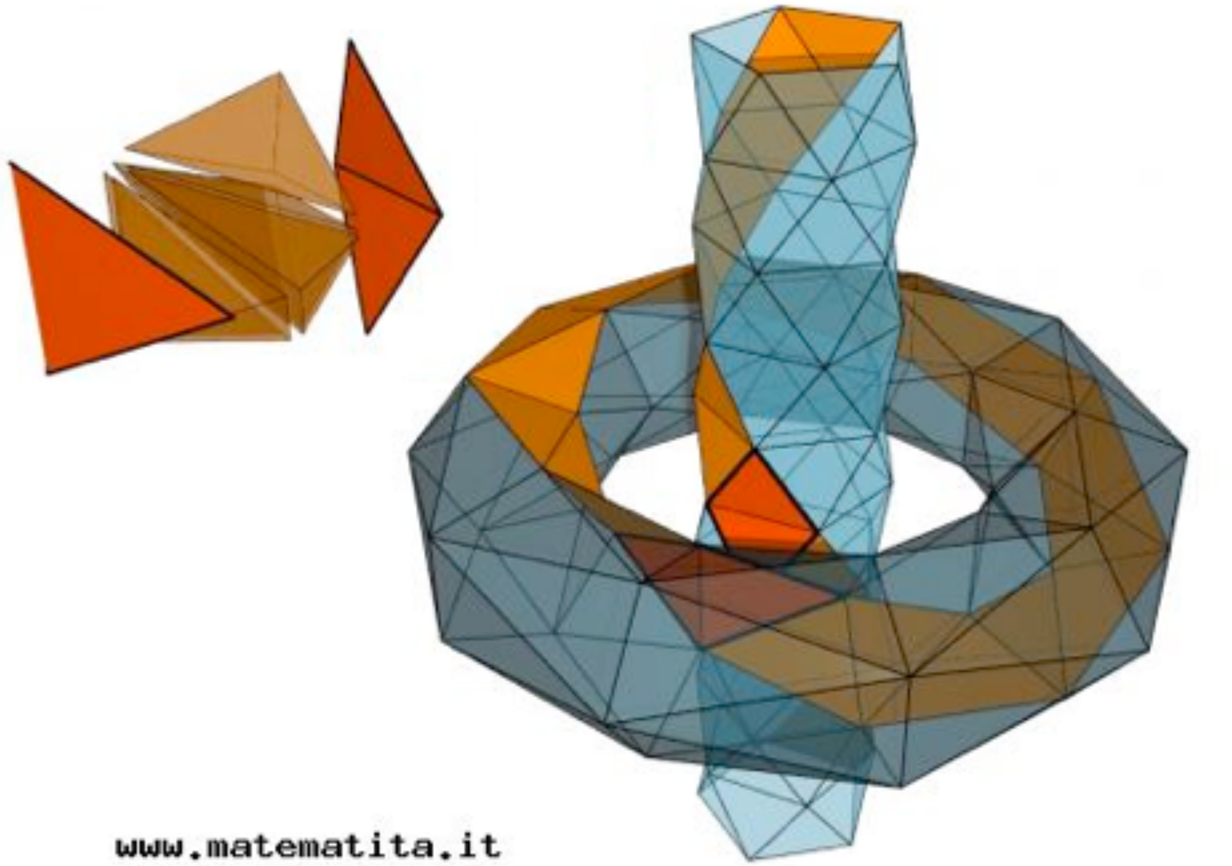
when we shrink the right-triangular dividing walls into equilateral triangles,
we distort each egg-cup into a pair of unit-tetrahedra.

at the same time,
the opening of each egg-cup changes from a square to a bent rhombus.
as the square openings bend,
the flat sheet of 300 tethrahedra is forced to wrap around into a hollow torus
with a one-unit- thick shell.

surprisingly,
this bends each 5x10 grid into a toroidal sheet of 100 equilateral triangles.
each grid's short edge is now a pentagon that threads through the donut hole.
the grid's long edge is now a decagon that wraps around both holes in its donut.
the two grids' long edges are now linked decagons.

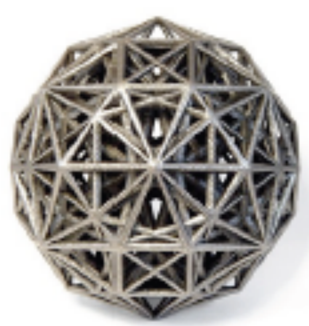
this wrapping cannot occur in R3, but it works fine in R4.
I admit that this part of my presentation is not easy to visualize.
perhaps a localized visualization image will help:
as an upper egg-cup is squeezed in one direction,
the edge-tetrahedra around it rotate,
squeezing the nearby lower egg-cups in the other direction.
this forces the flat sheet into a saddle-shape.
in R4, when this saddle-bending happens across the whole egg-carton at once,
the carton's edges can meet to make the toroidal sheet.
...
build each solid torus ....[of]... two solid tori of 150 cells each ... as follows:
using 100 tetrahedra, assemble 5 solid icosahedra (this is possible in R4).
daisy-chain five such icosahedra pole-to-pole ... between every pair of adjacent
icosahedra, surround the common vertex with 10 tetrahedra.
each solid torus has a decagonal "axis" running through the centers
and poles of the icosahedra. each solid torus contains 5*20 + 5*10 = 150
tetrahedra, and its surface is tiled with 100 equilateral triangles.
on this surface, six triangles meet at every vertex.
...

we will link these solid tori, like two links of a chain. with the hollow torus acting as a glue layer between them ...[



]...

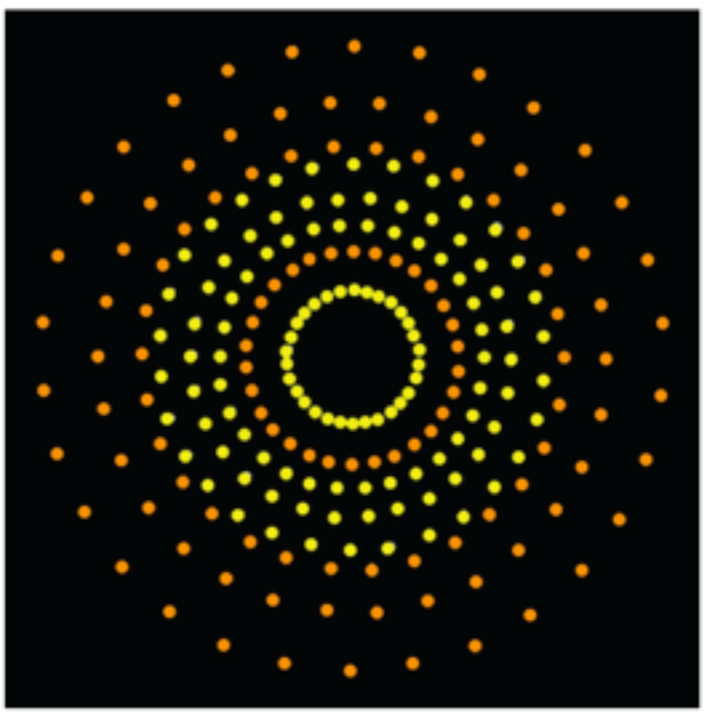
finally,
put one solid torus inside the hollow toroidal sheet,
attaching the 100 triangular faces of the solid
to the 100 triangles of the sheet's inner surface.
this gives us a fat solid torus,
10 units around and 4 units thick, containing 450 tetrahedral cells.
nevertheless, its surface has only 100 triangular faces.
thread the second 150-cell solid torus through this fat torus,
and attach the two solids' triangular faces. **this is the 600-cell polytope** ...”.



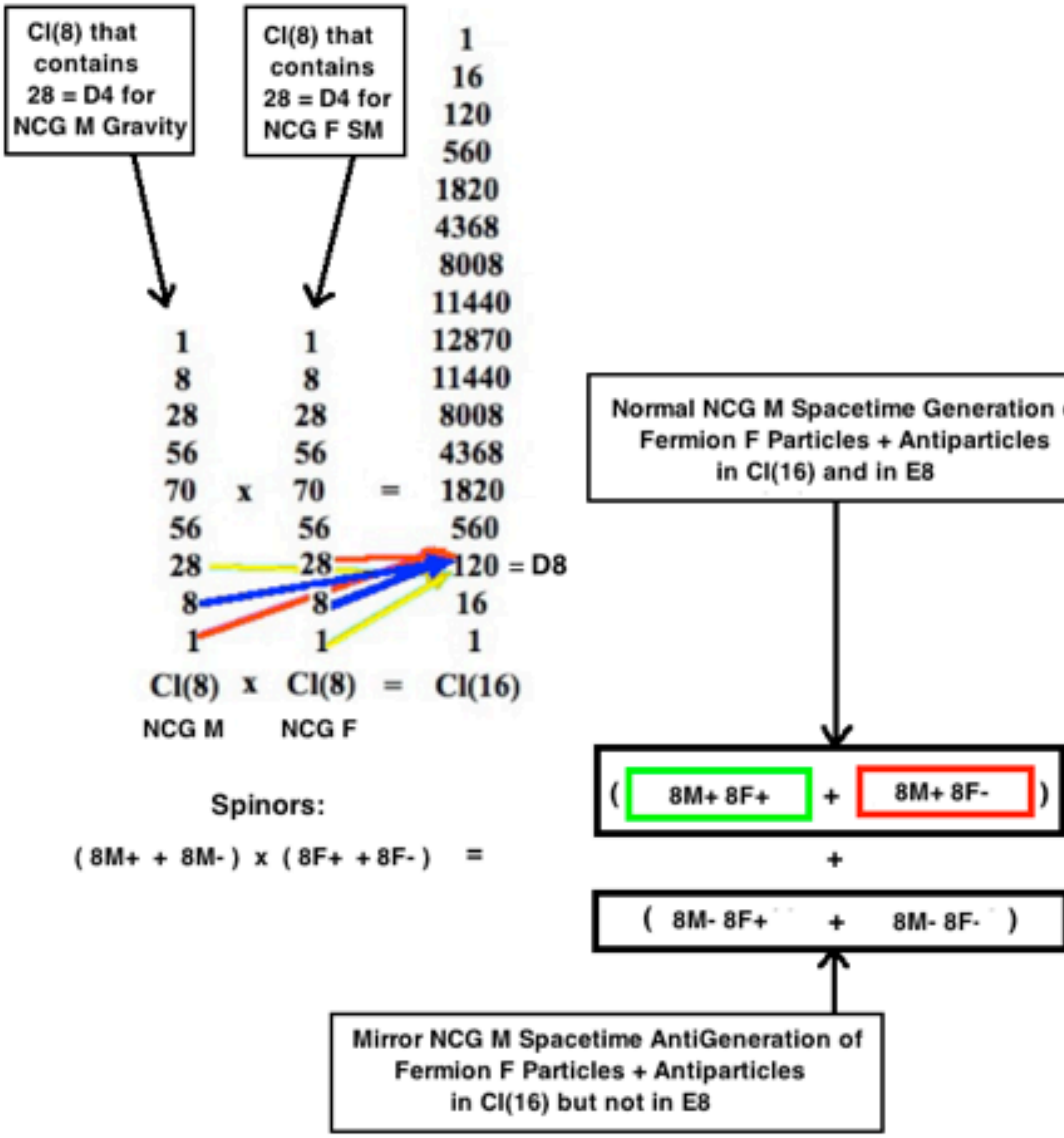
Combine the M 600-cell (yellow)
with the F 600-cell expanded by the Golden Ratio (orange)

to get the 120+120 = 240-vertex 8-dim E8 polytope
which is the Root Vector Polytope of the Lie Algebra E8

In this way the 8-dim space of E8 Root Vectors is seen as being made up of two independent 4-dim spaces:
a Rational Number 4-dim space of yellow M dots
and
an Algebraic Extension by the Golden Ratio 4-dim space of orange F dots



The Lie Algebra E8 lives in the Clifford Algebra Cl(16) = Cl(8) x Cl(8)

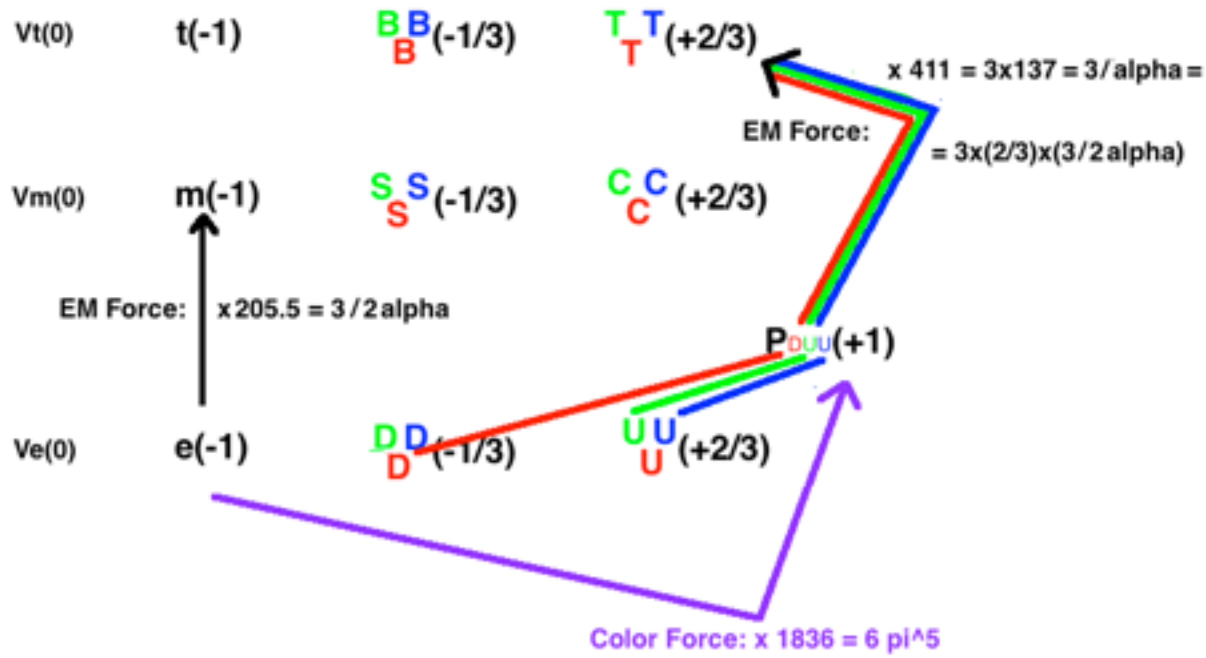


This is the basic structure of the E8 = Cl(16) Physics Model

described in viXra 1508.0157

Appendix - Mendel Sachs and Particle Masses

Mendel Sachs, in his books "General Relativity and Matter" (1982) and "Quantum Mechanics from General Relativity" (1986) calculated electron / muon and Proton / Tquark mass ratios substantially consistent with



Cl(16)-E8 Physics masses e = 0.511 MeV, m = 106 MeV, P = 938 MeV, T = 128.5 GeV saying (my comments set off by brackets [[ ]]):

"... the inertial mass of an elementary (spinor) particle [i]s determined by the curvature of space-time in its vicinity, representing the coupling of this particle to its environment of particle-antiparticle pairs ...

[[In Cl(16)-E8 Physics the particle-antiparticle pairs form a Schwinger Source Kerr-Newman Black Hole]]

Because the coupling of the observed electron to the pairs ... is electromagnetic, the electron's mass is proportional to the fine structure constant ...

[[In Cl(16)-E8 Physics the gauge symmetry of the force determines the geometry of the Schwinger Source and its Green's Function.]]

The electron mass is one member of a mass doublet, predicted by this theory. The other member, the muon, arises because occasionally the observed electron can excite a pair of the background, which in turn changes the features of the geometry of space-time in the vicinity of the electron. ...

[[In Cl(16)-E8 Physics the "excite" producing second and third generations is due to World-Lines traversing CP2 Internal Symmetry Space as well as M4 Physical Spacetime of M4xCP2 Kaluza-Klein]]

Because the excitation of the pair is due to an electromagnetic force, the new mass ... is 3 / 2 alpha = 206 times greater than the old mass. ...

This theory also predicts that the proton should have a sister member of a doublet ...

To compute the inertial mass of the electron, consider first
the frame of reference whose spatial origin is at the site of the observed electron,
with the pairs of the background in motion relative to this point
...
Using the method of Green's functions ... we see that the quaternion metrical field ... in the linear approximation, reduce to an integral equation with ... solutions ...[that]... are the linear approximation ... to the spin-affine connection field ...
the solutions ... of the integral Equation ... lead directly to the (squared) mass eigenvalues ... the eigenvalues of the mass operator are the absolute values of the squares of the matrix elements above
...
The pairs interact with each other in a way that makes them appear to some 'observed' constituent electron as 'photons'. ... Nevertheless,the pairs do have 'inertia' by virtue of their bound electrons and positrons that are not , in fact, annihilated. ...
From a distance greater than a 'first Bohr orbit' of one of the particle components of a pair, it appears, as a unit , to be an electrically neutral object. But as the (observed) electron comes sufficiently close to the pair so as to interact with its separate components, energy is used up in exciting the pair, thereby decreasing the relative speed between the pair and the observed electron.
If the primary excitation of a pair (as 'seen' by the observed electron) is quadrupolar, and if the ground state of the pair corresponds to n = 1; then the
first excited Bohr orbital with a quadrupolar component is the state with n' = 3.

[[Quadrupolar implies 4+4 Kaluza-Klein of Cl(16)-E8 Physics]]

With these values ... it follows that the ratio of mass eigenvalues is ... 3 / 2 alpha = 206 ... The reason for this is that the curvature of space-time, in the vicinity of the observed electron, that gives rise to its inertia, is a consequence of the electromagnetic coupling between the matter components of the system. ...
Summing up, the inertial mass of an elementary (spinor) particle was determined by the curvature of space-time in its vicinity, representing the coupling of this particle to its environment of particle-antiparticle pairs.

[[Green's functions for each force imply geometric structure of Schwinger Sources]]

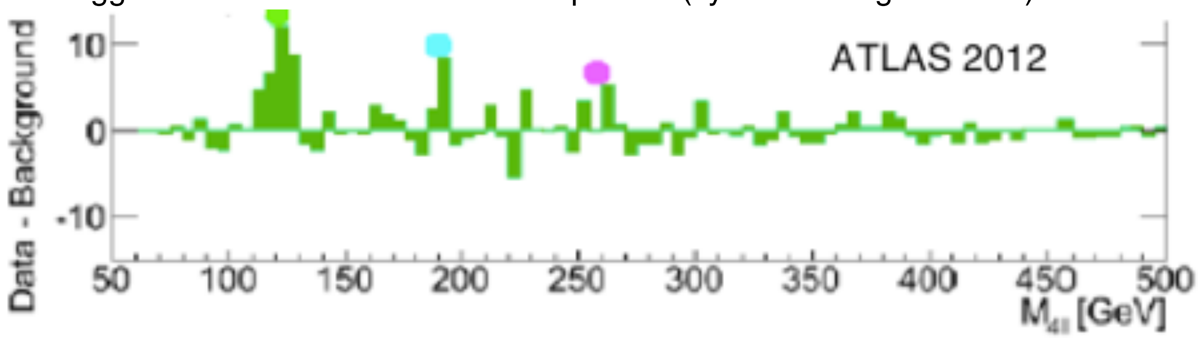
The significant domain of space populated by pairs that contributes to the electron mass is the order of 10^(-15) cm ,,,

[[Schwinger Source size in Cl(16)-E8 Physics is much smaller, about 10^(-24) cm]]

Because the coupling of the observed electron to the pairs - that gives it inertia - is electromagnetic, the electron's mass is proportional to the fine structure constant - which is a measure of the strength of this coupling. ...".

Appendix - Experiments Observing Higgs-Tquark 3-state System
LHC Run-1 (2012) and Run-2 (2015-16) and Higgs mass states:

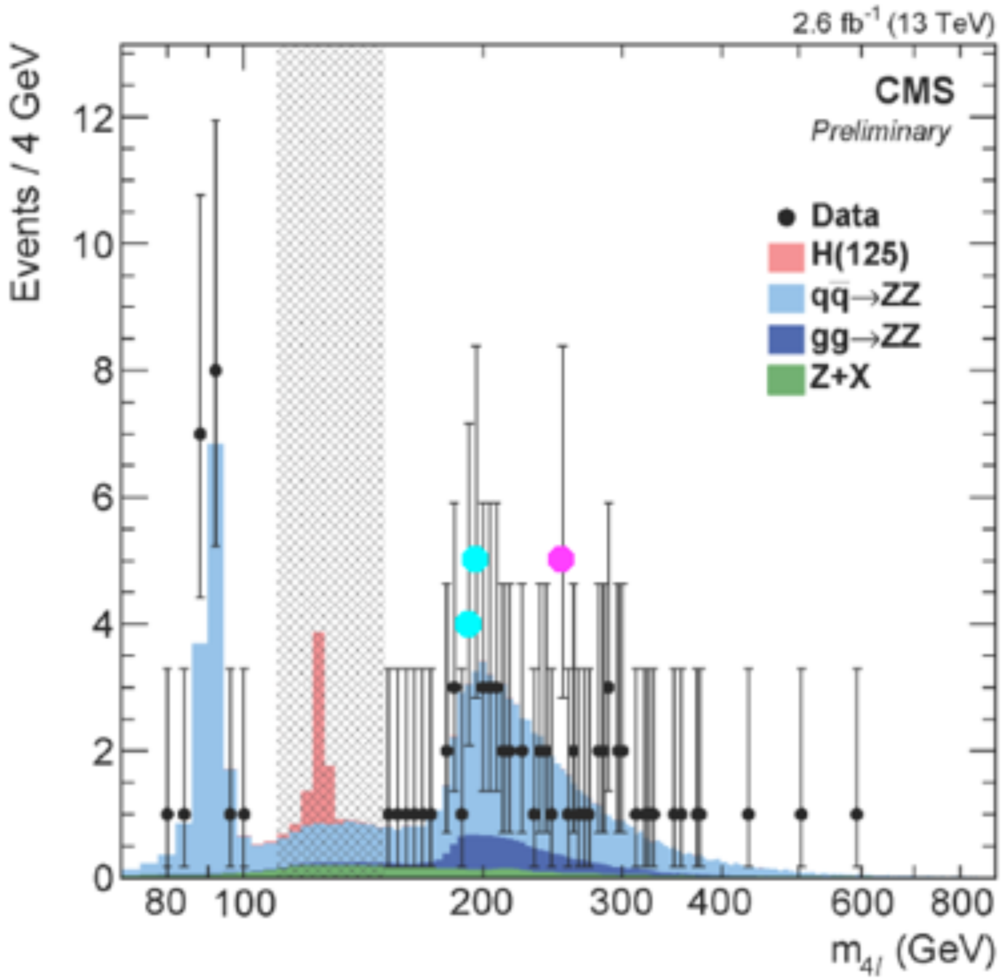
By the end of Run-1 in 2012 the LHC had seen clear evidence for a Higgs (green dot) with mass around 125 GeV and the expected Standard Model cross section. It also saw in the Higgs -> ZZ -> 4l channel two more peakss (cyan and magenta dots)



In 2015 Run-2 CMS also saw indications of the 200 and 250 GeV Higgs mass states

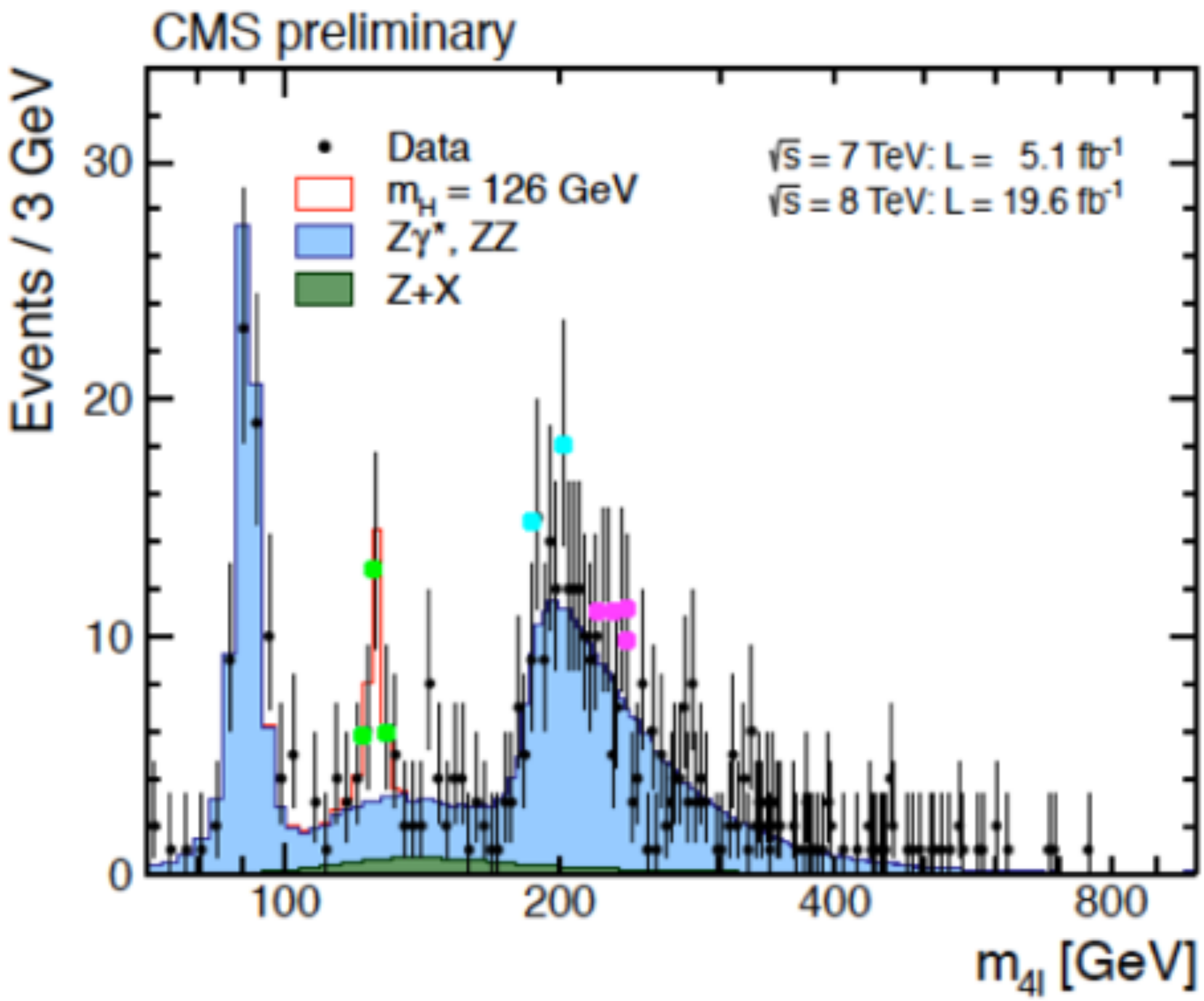
CMS Collaboration - 13 TeV Results

m_{4l} mass with Higgs region blinded

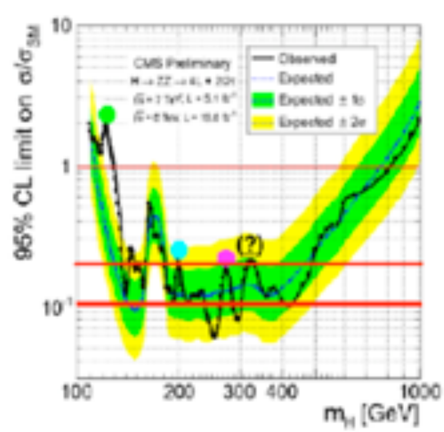


(from slide 28 by Jim Olsen for 15 Dec 2015 LPCC Special Seminar)

In Run-1 CMS had also seen indications of Higgs mass states around 200 and 250 GeV whose cyan and magenta dots coincide with their 2015 Run-2 positions

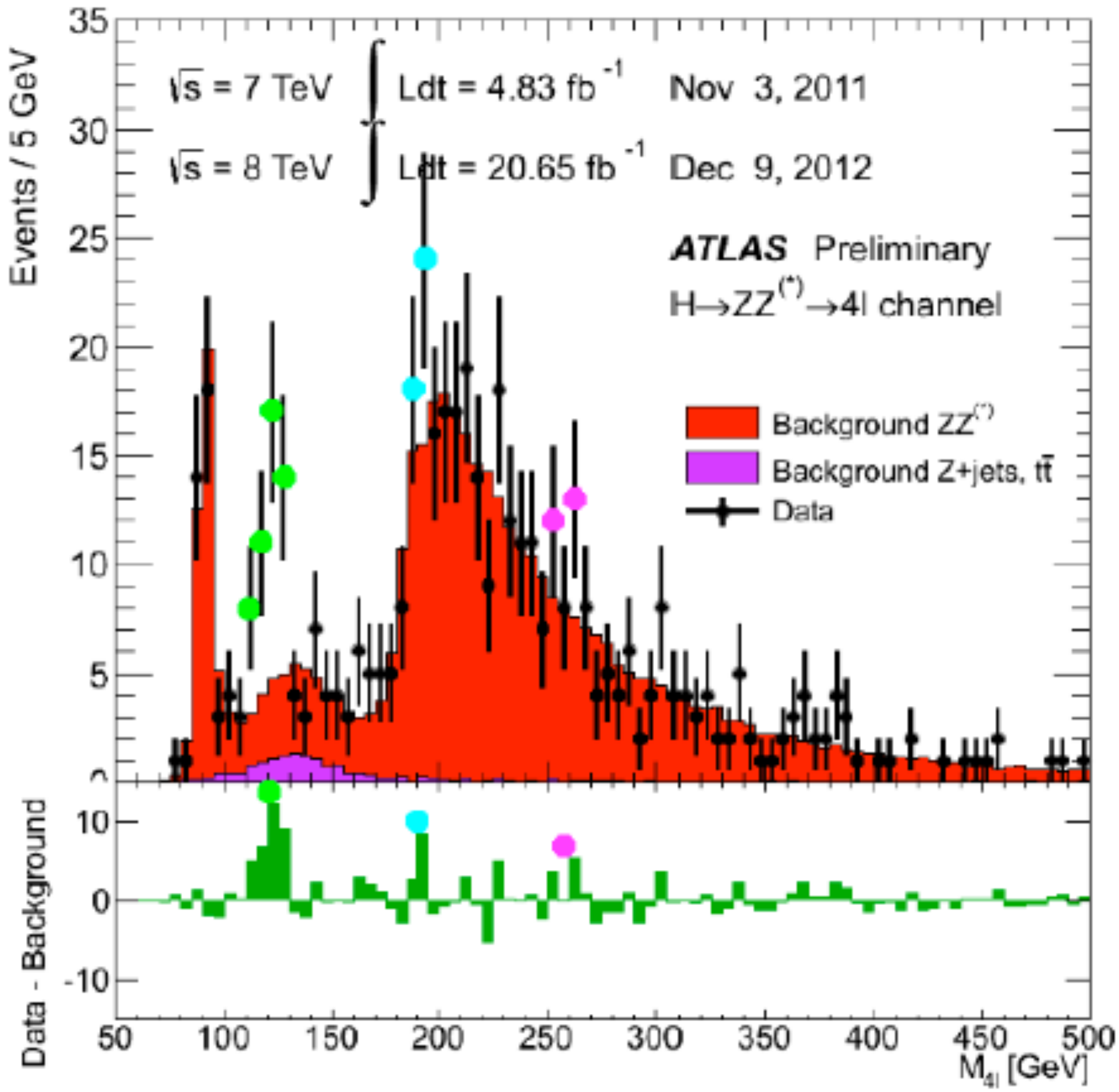


and with cross sections around 25% of SM expectation



CMS Run-1 also saw a (?) peak around 320 GeV that I expect to go away with 2016 Run-2 data.
The two unmarked peaks around 160 and 180 GeV are probably due to WW and ZZ.

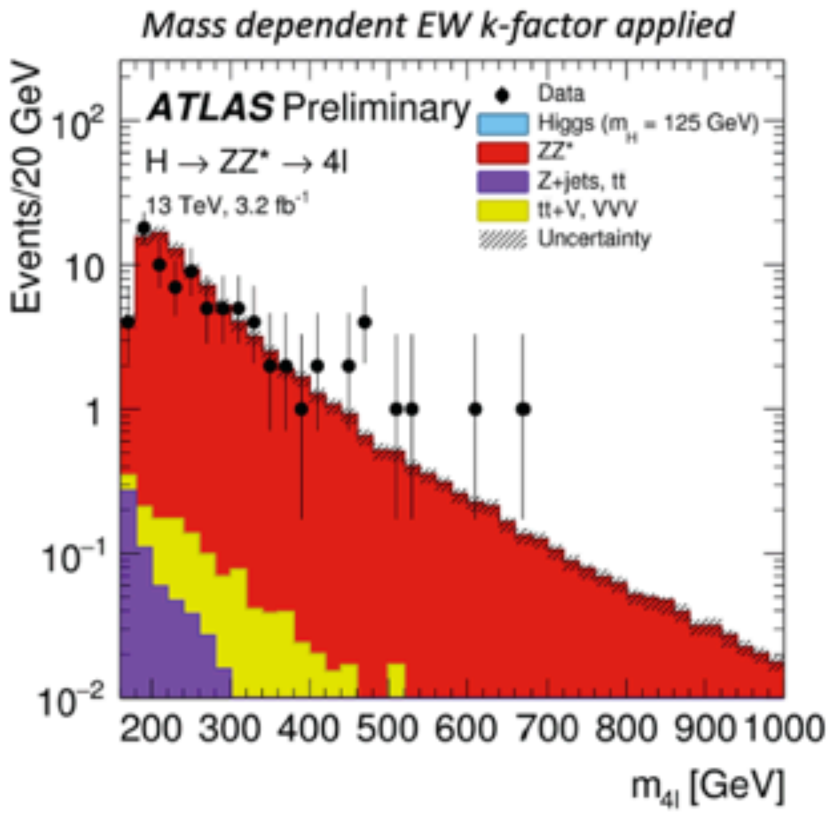
Further,
in Run-1 ATLAS had seen indications of Higgs mass states around 200 and 250 GeV whose cyan and magenta dots coincide with the CMS 2015 Run-2 positions



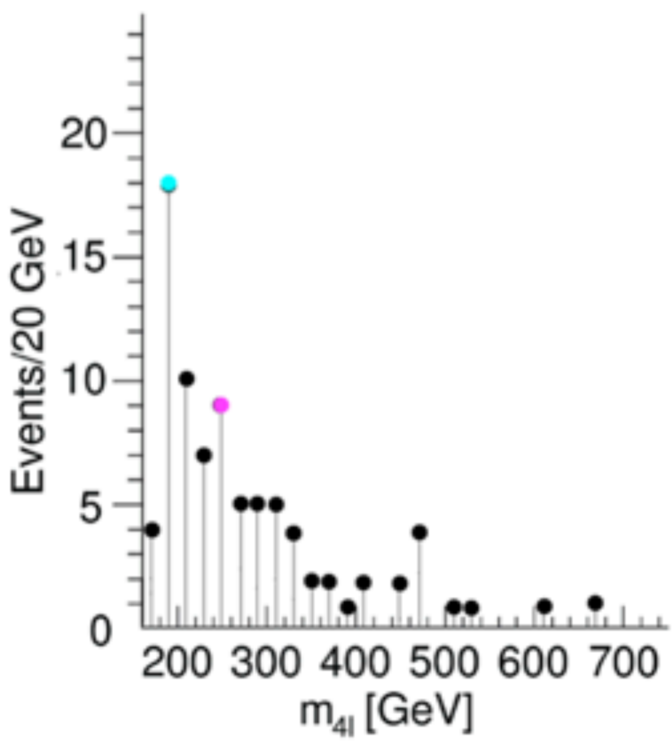
ATLAS Run-1 did not see the (?) CMS Run-1 peak around 320 GeV
as ATLAS saw an excess bin adjacent to two deficient bins.

In 2015 Run-2 did ATLAS see indications of 200 and 250 GeV Higgs mass states ?

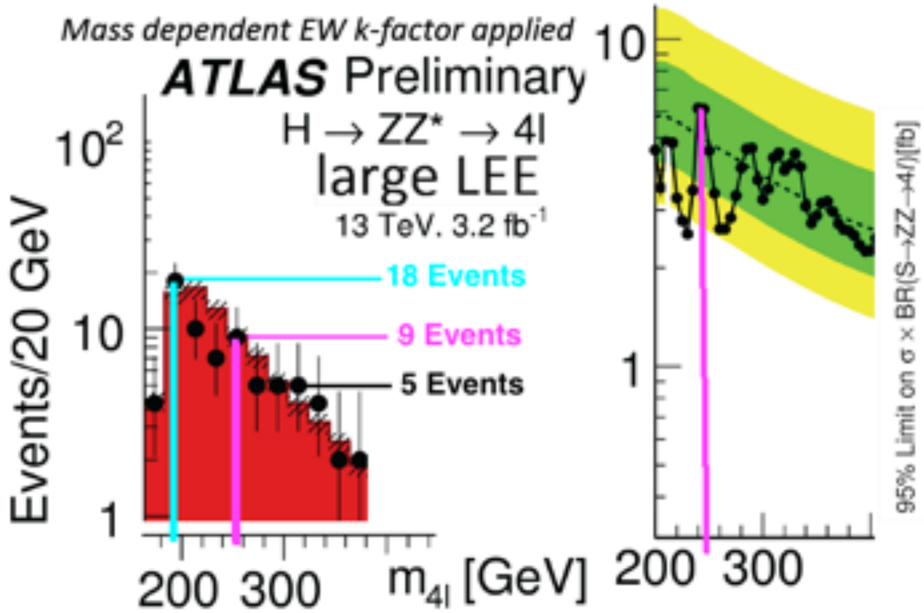
Here is what ATLAS reported (slide 22 by Marumi Kado) on 15 Dec 2015 LPCC Special Seminar:



Here is that ATLAS 2015 Higgs -> ZZ -> 4l histogram replotted with linear scale:



Here are some details (from slide 22 by Maarumi Kado for 15 Dec 2015 LPCC Special Seminar):



In my opinion the indications of 200 (cyan) and 250 (magenta) GeV Higgs mass states are there, but are obscured by:

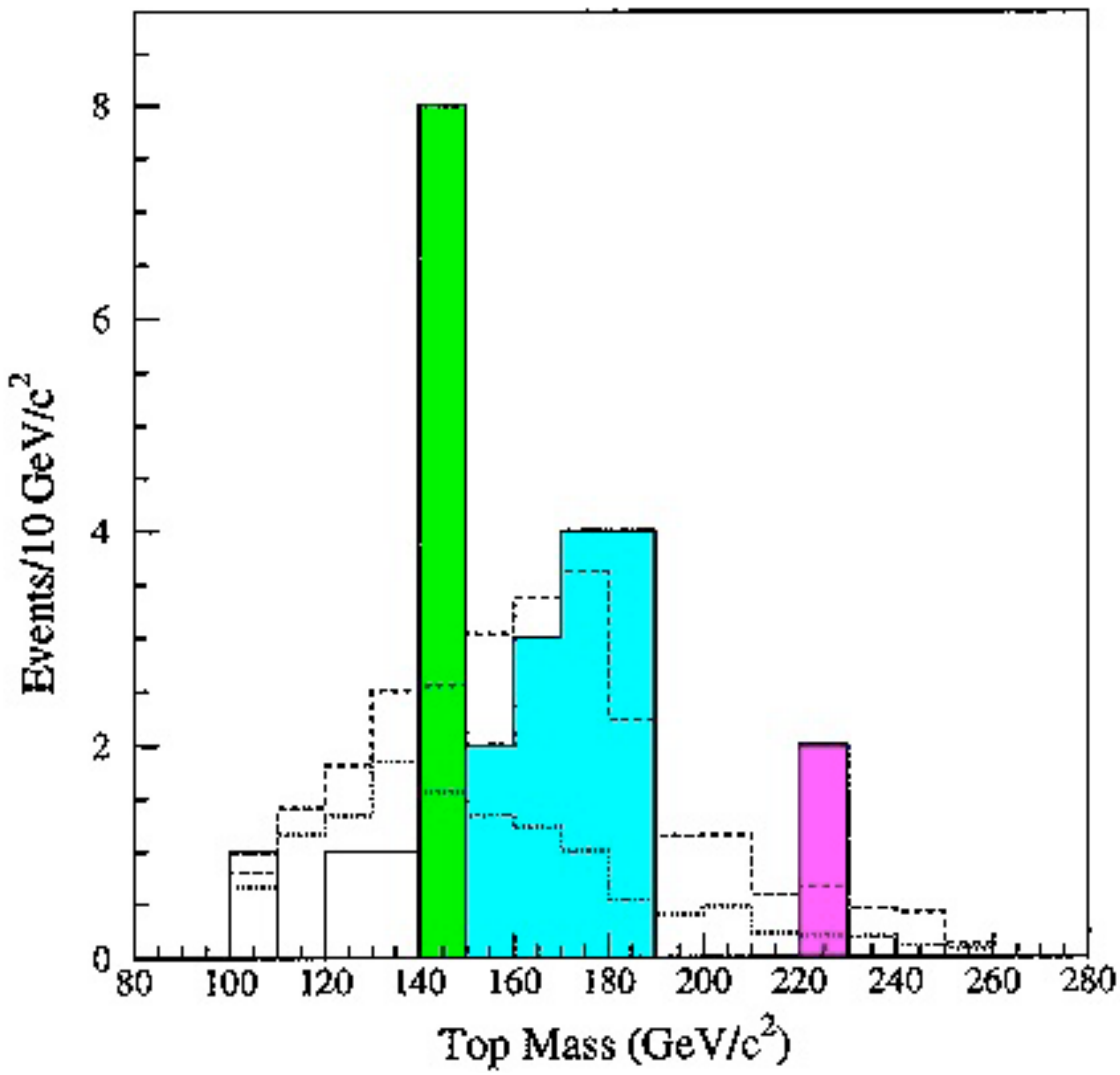
1 - a large LEE effect that is NOT appropriate for the 200 and 250 GeV Higgs mass states that were predicted by my E8 Physics model and indicated by prior Run-1 data

2 - the Brazil Band plot does NOT show the peak just below the 200 GeV line

2 - use of a log scale for the histogram of Events/20 GeV makes it hard to see the details of the Events around 200 and 250 GeV.
It seems clear to me that the linear plot indicates that the 200 GeV (cyan) peak and the 250 GeV (magenta) peak are serious candidates with over 5 Events that might well be confirmed by 2016 data as real Higgs mass states.

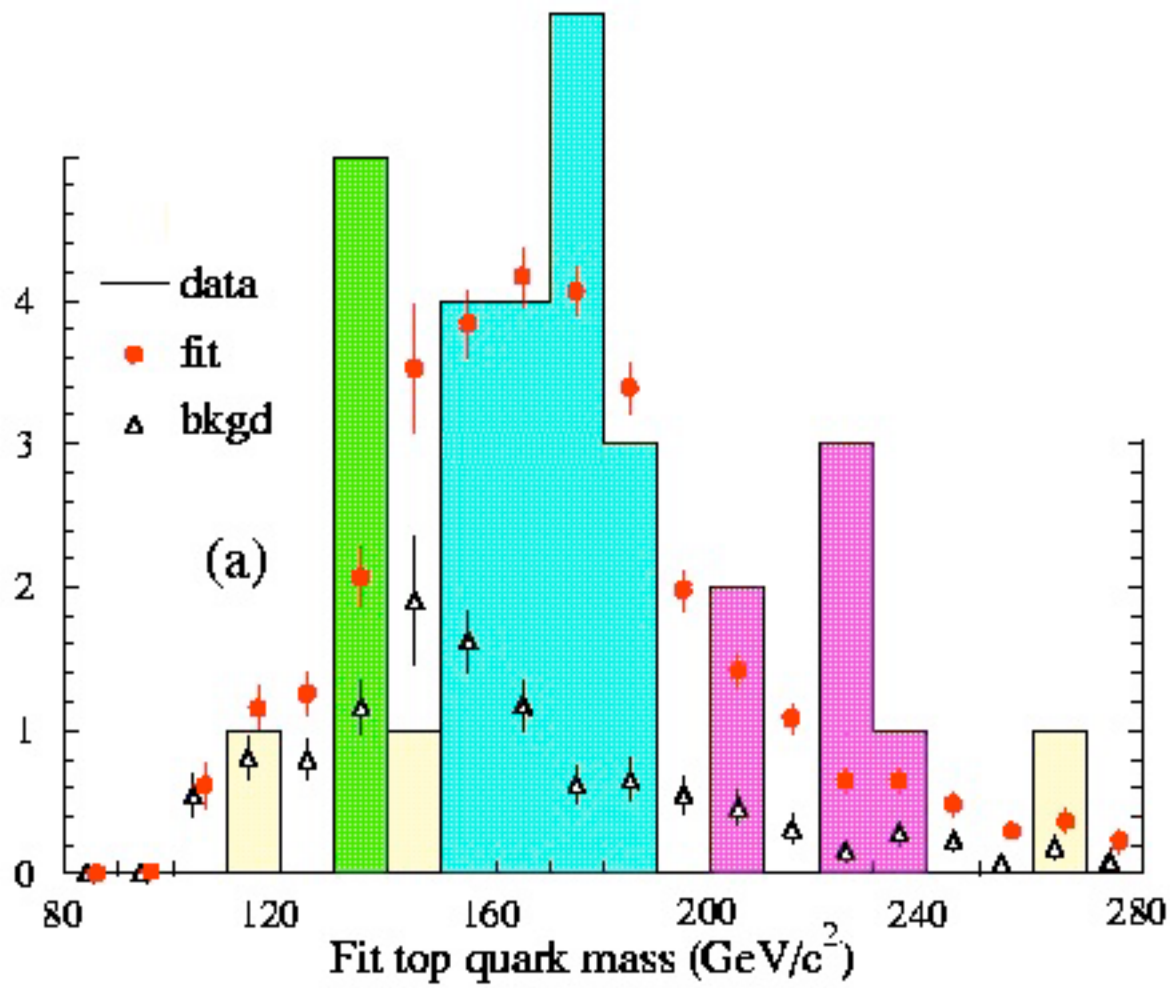
Three T-quark mass states

The 174 GeV Tquark mass state (cyan dot) is not controversial. It has been observed at Fermilab since 1994, when a semileptonic histogram from CDF (FERMILAB-PUB-94/097-E)



showed all three states of the T-quark.
In particular, the green bar represents a bin in the 140-150 GeV range containing Semileptonic events considered by me to represent the Truth Quark, but as to which CDF said "... We assume the mass combinations in the 140 to 150 GeV/c^2 bin represent a statistical fluctuation since their width is narrower than expected for a top signal. ...". I strongly disagree with CDF's "statistical fluctuation" interpretation, based on my interpretations of much Fermilab T-quark data.

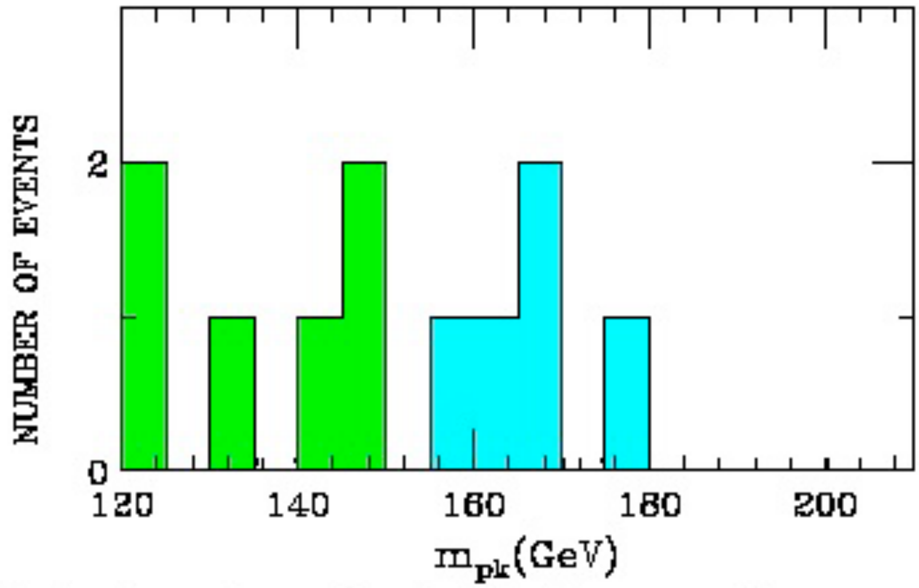
The same three Tquark mass states were seen in 1997 by D0 (hep-ex/9703008) in this semileptonic histogram:



The fact that the low (green) state showed up in both independent detectors indicates a significance of 4 sigma.

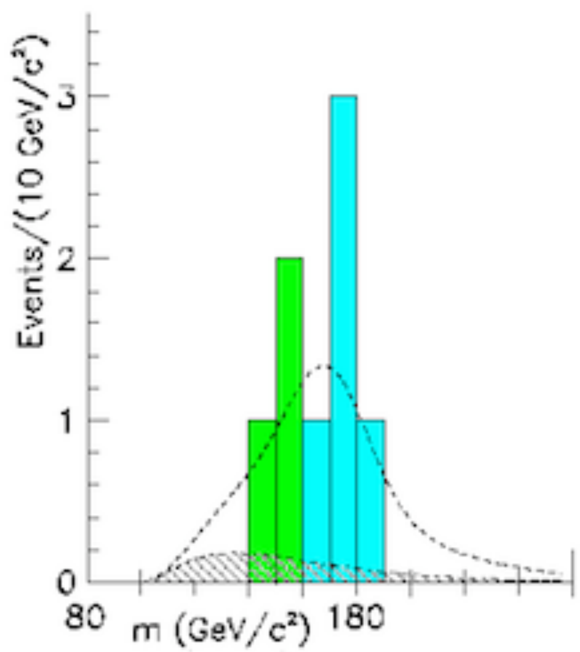
My opinion is that the middle (cyan) state is wide because
it is on the Triviality boundary
where the composite nature of the Higgs as T-Tbar condensate
becomes manifest
and
the low (cyan) state is narrow because it is in the usual non-trivial region
where the T-quark acts more nearly as a single individual particle.

Further, in February 1998 a dilepton histogram of 11 events from CDF (hep-ex/9802017)



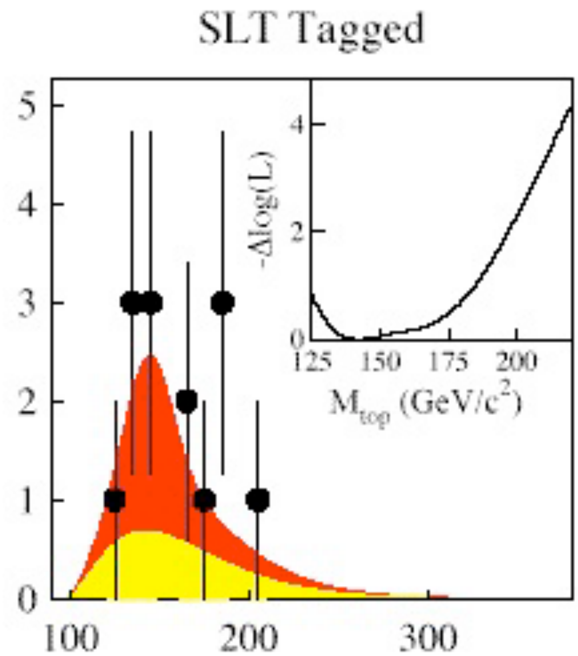
The distribution of m_{pk} values determined from 11 CDF dilepton events available empirically.

shows both the low (green) state and the middle (cyan) T-quark state
but
in October 1998 CDF revised their analysis using 8 Dilepton CDF events (hep-ex/9810029)



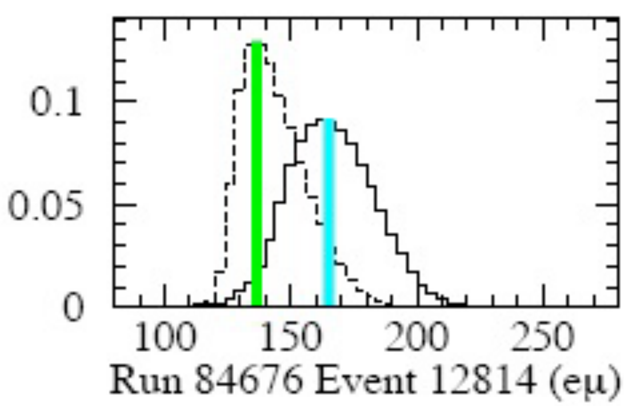
shows that CDF kept the 8 highest-mass dilepton events, and threw away the 3 lowest-mass dilepton events that were indicated to be in the 120-135 GeV range, and shifted the mass scale upward by about 10 GeV, indicating to me tthat Fermilab was attempting to discredit the low-mass T-quark state by use of cuts etc on its T-quark data.

In 1998 an analysis of 14 SLT tagged lepton + 4 jet events by CDF (hep-ex/9801014)



showed a T-quark mass of 142 GeV (+33,-14) that seems to me to be consistent with the low (green) state of the T-quark.

In his 1997 Ph.D. thesis Erich Ward Varnes (Varnes-fermilab-thesis-1997-28 at page 159) said: "... distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest ET jets in the reconstruction ...



...” (colored bars added by me)

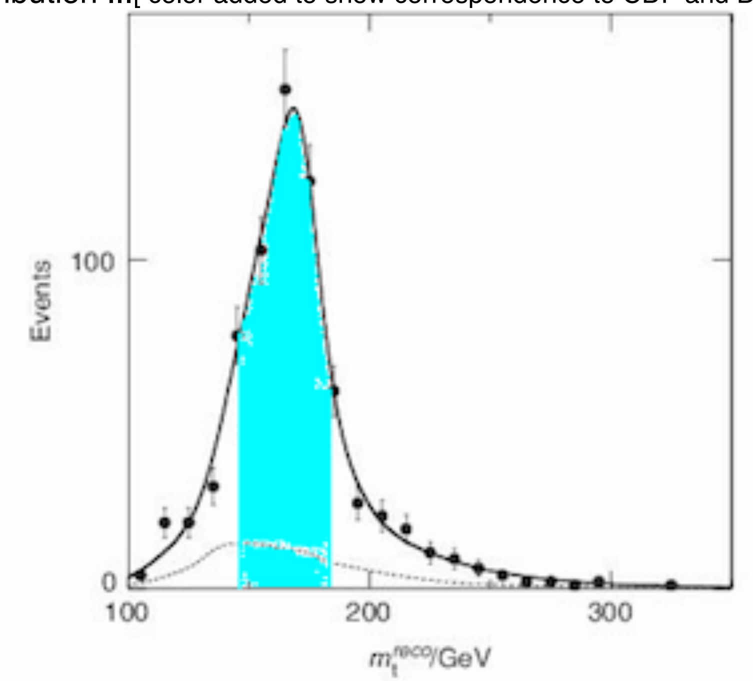
The event for all 3 jets (solid curve) seens to me to correspond to
decay of a middle (cyan) T-quark state
with one of the 3 jets corresponding to
decay from the Triviality boundary down to the low (green) T-quark state,
whose immediately subsequent decay corresponds to the 2-jet (dashed curve) event at
the low (green) energy level.

As to the T-quark width for the 174 GeV mass state, which appears
in the 1994 CDF and 1997 D0 semileptonic histograms to be about 40 GeV,
which is 4 of the 10 GeV histogram bins,
Mark Thomson, in “Modern Particle Physics” (Cambridge 2013) says:
“... Decay of the top quark ... The total decay rate is ...

$$\Gamma(t \to bW^+) = \frac{G_F m_t^3}{8\sqrt{2}\pi}\left(1 - \frac{m_W^2}{m_t^2}\right)^2\left(1 + \frac{2m_W^2}{m_t^2}\right),$$

... For ... mt = 173 GeV ... the lowest-order calculation of the total decay width of the top quark gives Γ t = 1.5 GeV ...

The total width of the top quark is measured to be Γ t = 2.0 +/- 0.6 GeV.
The top width is determined much less precisely than the top quark mass because the width of the distribution ...[color added to show correspondence to CDF and D0 histograms]...



... is dominated by the experimental resolution. ...”.

The T-quark total width Γ t = 2 GeV is much smaller than the 40 GeV width
experimentally observed at Fermilab and would, except for experimental resolution,
fit well within one single bin in the 1994 CDF and 1997 D0 semileptonic histograms.

As to the T-quark width for the 130 GeV mass state,
which appears in the 1994 CDF and 1997 D0 semileptonic histograms
to be less than the 10 GeV histogram bin width,
using the total width formula from Mark Thomson's book and paraphrasing:

"... For mt = 130 GeV ... the lowest-order calculation
of the total decay width of the top quark gives Γt = about 0.5 GeV ...".

I think that the CDF explanation
for the low mass T-quark peak in a single 10 GeV bin

"... We assume the mass combinations in the ... bin represent a statistical fluctuation since their width is narrower than expected for a top signal. ...".

is highly unlikely since
a similar low mass single 10 GeV bin T-quark mass peak was observed by the independent D0 detector.

The mt = 130 GeV width of 0.5 GeV is only 1/20 of the 10 GeV bin width of that peak.
The 20:1 = 10 : 0.5 observed width : actual width ratio for mt = 130 GeV
is the same as
the 20:1 = 40 : 2.0 observed width : actual width ratio for mt = 173 GeV.

What differences between the mt = 130 GeV and mt = 173 GeV states
might affect their relative experimental resolutions ?

The mt = 130 GeV peak is in the normal Stable region
in which the T-quark is represented by a Schwinger Source in M4 Physical Spacetime
which Schwinger Source has Green's Function structure based on Kernel Functions
of Bounded Symmetric Domains whose symmetry is that of the T-quark.
Since it is a simple Schwinger Source it has simple W - b - 2 jet decay.

The mt = 173 GeV peak is on the boundary
of the Non-Perturbativity region where the composite nature of Higgs as T-quark Condensate becomes manifest, as does the 8-dim nature of Kaluza-Klein spacetime M4 x CP2 with M4 Physical Spacetime and CP2 Internal Symmetry Space where CP2 = SU(3) / SU(2)xU(1) has symmetries of the Standard Model Gauge Groups.
Its decay scheme is more complicated, with 2 stages:

175 to 130 GeV, a process of the Higgs - T-quark condensate system of E8 Physics
and
simple W - b - 2 jet decay of the 130 GeV intermediate state.

The wider width of the 173 GeV decay peak
is due to the Higgs - T-quark condensate process.

The 1997 UC Berkeley PhD thesis of Erich Ward Varnes gives details of some D0 events and analysis, based on the Standard Model view of one T-quark mass state:
“... the leptonic decays of the t tbar events are divided into two broad categories:
the lepton plus jets and dilepton channels.
The former has the advantage of a large branching ratio, accounting for about 30% of all t tbar decays, with the disadvantage that electroweak processes or detector misidentification of fina-state particle can mimic the t tbar signal relatiely frequently.
Conversely,
the dilepton channels have lower backgrounds, but account for only 5% of all decays.
...
The kinematic selection of dilepton events is summarized in Table 5.2 ...

	ee	$e\mu$	$\mu\mu$
Leptons	E_T> 20 GeV $\lvert\eta\rvert < 2.5$	$E_T(e) > 15$ Gev, $p_T(\mu) > 15$ GeV/c $\lvert\eta(e)\rvert < 2.5$	$p_T(\mu) > 15$ GeV/c
Jets	≥ 2 with E_T> 20 GeV and $\lvert\eta\rvert < 2.5$		
$\not{E}_T$	> 25 GeV	$\not{E}_T$> 20 GeV $\not{E}_T^{cal} > 10$ GeV	N/A
H_T^e	> 120 GeV	> 120 GeV	> 100 GeV

Table 5.2: Kinematic cuts for the dilepton event selection. The cut used in place of $\not{E}_T$ to reject $Z \to \mu\mu$ events is described in the text, as is the H_T^e variable. Also, the muon η cut is run-dependent, as detailed in Chapter 4.

...
In the dilepton channels, one expects the final state to consist of two charged leptons, two neutrinos, and two b jets (see Fig. 6.1)

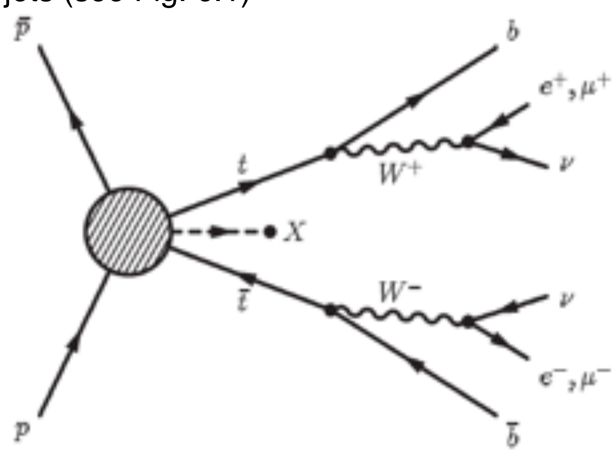


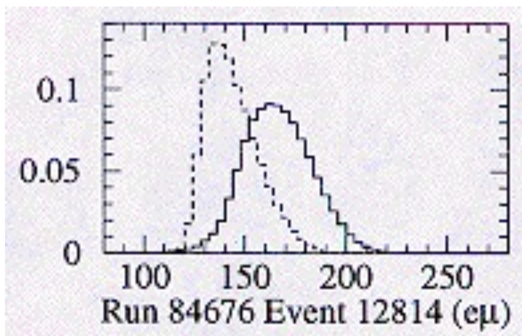
Figure 6.1: Schematic representation of $t\bar{t}$ production and decay in the dilepton channels.

so that the final state is completely specified by knowledge of the energy four-vectors of these six particles ... there are ... kinematic constraints:

The invariant mass of each lepton and neutrino pair is equal to the W mass.
The masses of the reconstructed t and tbar in the event are equal.

...
The result of reconstructing the top quark mass for a dilepton event is the distribution W(mt), which is evaluaed for 50 values of the top quark mass ... the intrinsic resolution of the dilepton mass reconstruction is much broader than the 4 GeV/c2 interval between assumed top quark masses ... the RMS of the typical W(mt) distribution ... typically lies between 35 and 40 GeV/c2 ...

...
Figure 8.1: W(mt) distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest ET jets in the reconstruction ...



...

Run 84676 Event 12814					z vertex: -6.17 cm		
Object	E	E_x	E_y	E_z	E_T	η	ϕ
Electron	81.3	-75.4	-1.1	-30.2	74.5	-0.39	3.16
Muon	30.2	-25.2	10.6	-12.8	27.4	-0.45	2.75
$\not{E}_T$	–	62.0	5.2	–	62.3	–	0.08
Jet 1	93.8	38.0	-83.7	-15.6	91.9	-0.17	5.14
	(95.9)	(38.9)	(-85.6)	(-16.0)	(94.0)		
Jet 2	37.8	13.9	32.3	-11.2	35.2	-0.31	1.17
	(38.8)	(14.2)	(33.1)	(-11.4)	(36.0)		
Jet 3	31.4	-1.6	28.6	11.6	28.7	0.39	1.63
	(32.2)	(-1.6)	(29.3)	(11.9)	(29.4)		

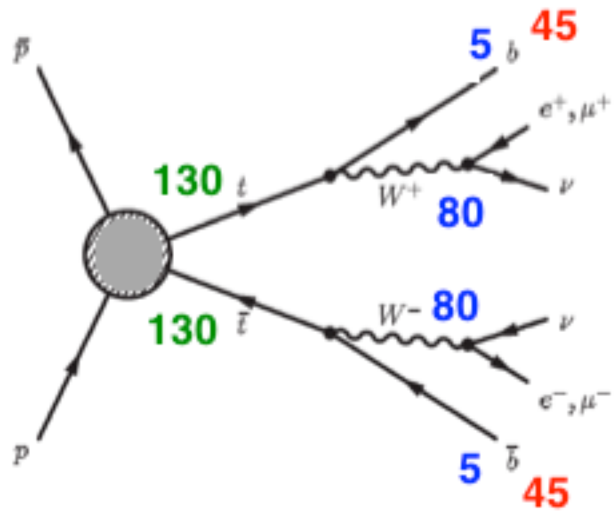
...".

In E8 Physics (viXra 1508.0157) there are, as stated above, three T-quark mass states, so in order to keep the kinematic constraint

"The masses of the reconstructed t and tbar in the event are equal"

the t and tbar must be in the same mass state, which is physically realistic because the t and tbar are created together in the same collider collision event.

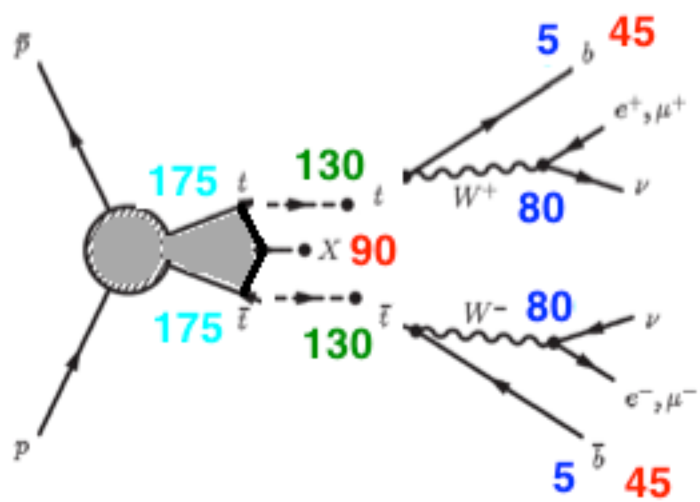
If the t and tbar are both in the 130 GeV mass state then the decay is simple with 2 jets:



and both jets are highly constrained as being related to the W - b decay process so it is reasonable to expect that the 130 GeV decay events would fall in the narrow width of a single 10 GeV histogram bin.

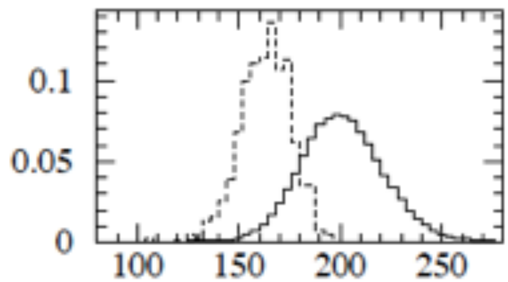
(In these two diagrams I have indicated energies only approximately for t and tbar mass states (cyan and green) and W and b-quark (blue) and jets (red). Actual kinematic data may vary from the idealized numbers on the diagrams, but they should give similar physics results.)

If the t and tbar are both in the 173 GeV mass state
(as, for example, in Run 84676 Event 12814 (e mu) described above)
the decay has two stages and 3 jets:



First, the 175 GeV t and tbar both decay to the 130 GeV state, emitting a jet.
Then, the 130 GeV t and tbar decay by the simple 2-jet process.
The first jet is a process of the Higgs - T-quark condensate system of E8 Physics and is not a W -b decay process so it is not so highly constrained and it is reasonable to expect that the 175 GeV decay events would appear to have a larger (on the order of 40 GeV) width.

As to t and tbar being the high T-quark mass state (around 225 GeV) there would be a third stage for decay from 225 GeV to 175 GeV with a fourth jet carrying around 100 GeV of decay energy.
In the Varnes thesis there is one dilepton event



Run 84395 Event 15530 (μμ)

Run 84395 Event 15530					z vertex: 5.9 cm		
Object	E	E_x	E_y	E_z	E_T	η	ϕ
Muon 1	68.6	-63.9	12.7	-21.4	65.1	-0.32	2.94
Muon 2	34.9	-16.0	31.0	1.9	34.9	0.05	2.05
$\not{E}_T$	–	71.2	53.2	–	88.9	–	0.64
Jet 1	146.1	32.1	-98.2	-102.4	103.3	-0.88	5.03
	(153.5)	(33.8)	(-103.1)	(-107.6)	(108.5)		
Jet 2	35.1	-8.6	21.4	26.2	23.1	0.97	1.95
	(37.2)	(-9.1)	(22.7)	(27.7)	(24.5)		
Jet 3	47.1	-7.6	-16.8	43.0	18.4	1.58	4.29
	(52.3)	(-8.4)	(-18.6)	(47.8)	(20.5)		

that seems me to represent that third stage of decay from 225 GeV to 175 GeV.
Since it is described as a 3-jet event and not a 4-jet event as I would have expected, my guess is that the third and fourth jets of my model were not distinguished by the experiment so that they appeared to be one third jet.

Appendix - Details of Force Strength and Boson Mass Calculations

Here are less approximate more detailed force strength calculations:

The force strength of a given force is

alphaforce = (1 / Mforce^2) (Vol(MISforce)) (Vol(Qforce) / Vol(Dforce)^(1 / mforce))

where:

alphaforce represents the force strength;

Mforce represents the effective mass;

MISforce represents the relevant part of the target Internal Symmetry Space;

Vol(MISforce) stands for volume of MISforce and is sometimes also denoted by Vol(M);

Qforce represents the link from the origin to the relevant target for the gauge boson;

Vol(Qforce) stands for volume of Qforce;

Dforce represents the complex bounded homogeneous domain
of which Qforce is the Shilov boundary;

mforce is the dimensionality of Qforce, which is
4 for Gravity and the Color force,
2 for the Weak force (which therefore is considered to have two copies of QW for SpaceTime),
1 for Electromagnetism (which therefore is considered to have four copies of QE for SpaceTime)

Vol(Dforce)^(1 / mforce) stands for a dimensional normalization factor
(to reconcile the dimensionality of the Internal Symmetry Space of the target vertex
with the dimensionality of the link from the origin to the target vertex).

The Qforce, Hermitian symmetric space, and Dforce manifolds for the four forces are:

Spin(5)	Spin(7) / Spin(5)xU(1)	IV5	4	RP^1xS^4
SU(3)	SU(4) / SU(3)xU(1)	B^6(ball)	4	S^5
SU(2)	Spin(5) / SU(2)xU(1)	IV3	2	RP^1xS^2
U(1)	-	-	1	-

The geometric volumes needed for the calculations are mostly taken from the book Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains (AMS 1963, Moskva 1959, Science Press Peking 1958) by L. K. Hua [unit radius scale].

Force	M	Vol(M)
gravity	S^4	8pi^2/3 - S^4 is 4-dimensional
color	CP^2	8pi^2/3 - CP^2 is 4-dimensional
weak	S^2 x S^2	2 x 4pi - S^2 is a 2-dim boundary of 3-dim ball 4-dim S^2 x S^2 = topological boundary of 6-dim 2-polyball Shilov Boundary of 6-dim 2-polyball = S^2 + S^2 = = 2-dim surface frame of 4-dim S^2 x S^
e-mag	T^4	4 x 2pi - S^1 is 1-dim boundary of 2-dim disk

4-dim T^4 = S^1 x S^1 x S^1 x S^1 = topological boundary of 8-dim 4-polydisk
Shilov Boundary of 8-dim 4-polydisk = S^1 + S^1 + S^1 + S^1 =
= 1-dim wire frame of 4-dim T^4

Note (thanks to Carlos Castro for noticing this) also that the volume listed for CP2 is unconventional, but physically justified by noting that S4 and CP2 can be seen as having the same physical volume, with the only difference being structure at infinity.
Note that for U(1) electromagnetism, whose photon carries no charge, the factors Vol(Q) and Vol(D) do not apply and are set equal to 1, and from another point of view, the link manifold to the target vertex is trivial for the abelian neutral U(1) photons of Electromagnetism, so we take QE and DE to be equal to unity.

Force	M	Vol(M)	Q	Vol(Q)	D	Vol(D)
gravity	S^4	8pi^2/3	RP^1xS^4	8pi^3/3	IV5	pi^5/2^4 5!
color	CP^2	8pi^2/3	S^5	4pi^3	B^6(ball)	pi^3/6
Weak	S^2xS^2	2x4pi	RP^1xS^2	4pi^2	IV3	pi^3/24
e-mag	T^4	4x2pi	-	-	-	-

Note (thanks to Carlos Castro for noticing this) that the volume listed for S5 is for a squashed S5, a Shilov boundary of the complex domain corresponding to the symmetric space SU(4) / SU(3) x U(1).

Using the above numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

Spin(5)	gravity	approx 10^19 GeV	1	GGmproton^2 approx 5 x 10^-39
SU(3)	color	approx 245 MeV	0.6286	0.6286
SU(2)	weak	approx 100 GeV	0.2535	GWmproton^2 approx 1.05 x 10^-5
U(1)	e-mag	approx 4 KeV	1/137.03608	1/137.03608

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels.
The effect is particularly pronounced with the color force.
The color force strength was calculated using a simple perturbative QCD renormalization group equation at various energies, with the following results:

Energy Level	Color Force Strength
245 MeV	0.6286
5.3 GeV	0.166
34 GeV	0.121
91 GeV	0.106

Taking other effects, such as Nonperturbative QCD, into account, should give a Color Force Strength of about 0.125 at about 91 GeV

Higgs, W+, W-, Z0:

As with forces strengths, the calculations produce ratios of masses,
so that only one mass need be chosen to set the mass scale.

In the Cl(16)-E8 model,
the value of the fundamental mass scale vacuum expectation value v = <PHI>
of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons,
W+, W-, and Z0, whose tree-level masses will then be shown by ratio calculations to be
80.326 GeV, 80.326 GeV, and 91.862 GeV, respectively,
and therefore the electron mass will be 0.5110 MeV.

The relationship between the Higgs mass and v is given
by the Ginzburg-Landau term from the Mayer Mechanism as

(1/4) Tr ([PHI , PHI] - PHI)^2

or, i
n the notation of quant-ph/9806009 by Guang-jiong Ni

(1/4!) lambda PHI^4 - (1/2) sigma PHI^2

where the Higgs mass M_H = sqrt(2 sigma)

Ni says:
"... the invariant meaning of the constant lambda in the Lagrangian is not the coupling constant, the latter will change after quantization ... The invariant meaning of lambda is nothing but the ratio of two mass scales:

lambda = 3 (M_H / PHI)^2

which remains unchanged irrespective of the order ...".

Since <PHI>^2 = v^2, and assuming that lambda = (cos(pi / 6))^2 = 0.866^2
(a value consistent with the Higgs-Tquark condensate model of Michio Hashimoto, Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165)
we have

M_H^2 / v^2 = (cos(pi / 6))^2 / 3

In the Cl(16)-E8 model, the fundamental mass scale vacuum expectation value v of the Higgs scalar field is the fundamental mass parameter that is to be set to define all other masses by the mass ratio formulas of the model and v is set to be 252.514 GeV
so that

M_H = v cos(pi / 6) / sqrt(1 / 3) = 126.257 GeV

This is the value of the Low Mass State of the Higgs observed by the LHC.
MIddle and High Mass States come from a Higgs-Tquark Condensate System.
The Middle and High Mass States may have been observed by the LHC at 20% of the Low Mass State cross section, and that may be confirmed by the LHC 2015-1016 run.

A Non-Condensate Higgs is represented by a Higgs at a point in M4
that is connected to a Higgs representation in CP2 ISS by a line whose length
represents the Higgs mass

```
Higgs      Higgs in CP2 Internal Symmetry Space
  |          |
  |          |
  |          |
  | mass = 145 | Non-Condensate Higgs Mass = 145
  |          |
  |          |
  |          |
Higgs      Higgs in M4 spacetime
```

and the value of lambda is 1 = 1^2
so that the Higgs mass would be M_H = v /sqrt(3) = 145.789 GeV

However, in the Cl(16)-E8 model, the Higgs has structure of a Tquark condensate

```
mass = 145
T ---------Tbar     Effective  Higgs  in CP2 Internal Symmetry Space
 \    |    /                 |
  \   |   /                  |
   \  |  /mass = 145         | Higgs Effective Mass =
    \ | /                    | = 145 x cos(pi/6)= 145 x 0.866 = 126
     \|/                     |
   Higgs                Higgs in M4 spacetime
```

in which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS
so that the vertices of the Higgs-T-Tbar system are connected
by lines forming an equilateral triangle composed of 2 right triangles
(one from the CP2 origin to the T and to the M4 Higgs and
another from the CP2 origin to the Tbar and to the M4 Higgs).
In the T-quark condensate picture
lambda = 1^2 = lambda(T) + lambda(H) = (sin(pi / 6))^2 + (cos(pi / 6))^2
and
lambda(H) = (cos(pi / 6))^2

Therefore the Effective Higgs mass observed by LHC is:

Higgs Mass = 145.789 x cos(pi/6) = 126.257 GeV.

To get W-boson masses,
denote the 3 SU(2) high-energy weak bosons
(massless at energies higher than the electroweak unification)
by W+, W-, and W0,
corresponding to the massive physical weak bosons W+, W-, and Z0.

The triplet { W+, W-, W0 } couples directly with the T - Tbar quark-antiquark pair,
so that the total mass of the triplet { W+, W-, W0 } at the electroweak unification
is equal to the total mass of a T - Tbar pair, 259.031 GeV.

The triplet { W+, W-, Z0 } couples directly with the Higgs scalar,
which carries the Higgs mechanism by which the W0 becomes the physical Z0,
so that the total mass of the triplet { W+, W-, Z0 }
is equal to the vacuum expectation value v of the Higgs scalar field, v = 252.514 GeV.

What are individual masses of members of the triplet { W+, W-, Z0 } ?

First, look at the triplet { W+, W-, W0 } which can be represented by the 3-sphere S^3.
The Hopf fibration of S^3 as

S^1 --> S^3 --> S^2

gives a decomposition of the W bosons into the neutral W0 corresponding to S^1
and the charged pair W+ and W- corresponding to S^2.

The mass ratio of the sum of the masses of W+ and W- to the mass of W0
should be the volume ratio of the S^2 in S^3 to the S^1 in S3.
The unit sphere S^3 in R^4 is normalized by 1 / 2.
The unit sphere S^2 in R^3 is normalized by 1 / sqrt(3).
The unit sphere S^1 in R^2 is normalized by 1 / sqrt(2).
The ratio of the sum of the W+ and W- masses to the W0 mass should then be
(2 / sqrt3) V(S^2) / (2 / sqrt2) V(S^1) = 1.632993

Since the total mass of the triplet { W+, W-, W0 } is 259.031 GeV,
the total mass of a T - Tbar pair, and the charged weak bosons have equal mass,
we have

M_W+ = M_W- = 80.326 GeV and M_W0 = 98.379 GeV.

The charged W+/- neutrino-electron interchange must be symmetric
with the electron-neutrino interchange, so that the tree-level absence
of right-handed neutrino particles requires that

the charged W+/- SU(2) weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector,
so that the
charged W+/- SU(2) weak bosons act only on left-handed fermion particles of all types.

The neutral W0 weak boson does not interchange Weyl neutrinos with Dirac fermions, and so is not restricted to left-handed fermions, but also has a component that acts on both types of fermions, both left-handed and right-handed, conserving parity.

However, the neutral W0 weak bosons are related to the charged W+/- weak bosons by custodial SU(2) symmetry, so that the left-handed component of the neutral W0 must be equal to the left-handed (entire) component of the charged W+/-.

Since the mass of the W0 is greater than the mass of the W+/-,
there remains for the W0 a component acting on both types of fermions.

Therefore the full W0 neutral weak boson interaction is proportional to
(M_W+/-^2 / M_W0^2) acting on left-handed fermions
and
(1 - (M_W+/-^2 / M_W0^2)) acting on both types of fermions.

If (1 - (M_W+/-2 / M_W0^2)) is defined to be sin(theta_w)^2 and denoted by K,
and if the strength of the W+/- charged weak force
(and of the custodial SU(2) symmetry) is denoted by T,
then the W0 neutral weak interaction can be written as W0L = T + K and W0LR = K.

Since the W0 acts as W0L with respect to the parity violating SU(2) weak force
and as W0LR with respect to the parity conserving U(1) electromagnetic force,
the W0 mass mW0 has two components:
the parity violating SU(2) part mW0L that is equal to M_W+/-
and the parity conserving part M_W0LR that acts like a heavy photon.

As M_W0 = 98.379 GeV = M_W0L + M_W0LR,
and as M_W0L = M_W+/- = 80.326 GeV, we have M_W0LR = 18.053 GeV.

Denote by *alphaE = *e^2 the force strength of the weak parity conserving U(1) electromagnetic type force that acts through the U(1) subgroup of SU(2).

The electromagnetic force strength alphaE = e^2 = 1 / 137.03608 was calculated above using the volume V(S^1) of an S^1 in R^2, normalized by 1 / sqrt(2).

The *alphaE force is part of the SU(2) weak force whose strength alphaW = w^2 was calculated above using the volume V(S^2) of an S^2 \subset R^3,
normalized by 1 / sqrt(3).

Also, the electromagnetic force strength alphaE = e^2 was calculated above using a 4-dimensional spacetime with global structure of the 4-torus T^4 made up of four S^1 1-spheres,
while the SU(2) weak force strength alphaW = w^2 was calculated above using two 2-spheres S^2 x S^2,
each of which contains one 1-sphere of the *alphaE force.

Therefore

*alphaE = alphaE (sqrt(2) / sqrt(3))(2 / 4) = alphaE / sqrt(6),

*e = e / (4th root of 6) = e / 1.565 ,

and
the mass mW0LR must be reduced to an effective value

M_W0LReff = M_W0LR / 1.565 = 18.053/1.565 = 11.536 GeV

for the *alphaE force to act like an electromagnetic force in the E8 model:

*e M_W0LR = e (1/5.65) M_W0LR = e M_Z0,

where the physical effective neutral weak boson is denoted by Z0.

Therefore, the correct Cl(16)-E8 model values for weak boson masses
and the Weinberg angle theta_w are:

M_W+ = M_W- = 80.326 GeV;

M_Z0 = 80.326 + 11.536 = 91.862 GeV;

Sin(theta_w)^2 = 1 - (M_W+/- / M_Z0)^2 = 1 - (6452.2663 / 8438.6270) = 0.235.

Radiative corrections are not taken into account here, and may change these tree- level values somewhat.

Appendix - Details of Fermion Mass Calculations

In the Cl(16)-E8 model, the first generation spinor fermions are
seen as +half-spinor and -half-spinor spaces of Cl(1,7) = Cl(8).
Due to Triality,
Spin(8) can act on those 8-dimensional half-spinor spaces
similarly to the way it acts on 8-dimensional vector spacetime.

Take the the spinor fermion volume to be the Shilov boundary corresponding
to the same symmetric space on which Spin(8) acts as a local gauge group
that is used to construct 8-dimensional vector spacetime:
the symmetric space Spin(10) / Spin(8)xU(1)
corresponding to a bounded domain of type IV8
whose Shilov boundary is RP^1 x S^7

Since all first generation fermions see the spacetime over which the integral is
taken in the same way (unlike what happens for the force strength calculation),
the only geometric volume factor relevant for calculating first generation fermion
mass ratios is in the spinor fermion volume term.
Cl(16)-E8 model fermions correspond to Schwinger Source Kerr-Newman Black Holes,
so the quark mass in the Cl(16)-E8 model is a constituent mass.

Fermion masses are calculated as a product of four factors:

V(Qfermion) x N(Graviton) x N(octonion) x Sym

V(Qfermion) is the volume of the part of the half-spinor fermion particle manifold
S^7 x RP^1 related to the fermion particle by photon, weak boson, or gluon interactions.

N(Graviton) is the number of types of Spin(0,5) graviton related to the fermion.
The 10 gravitons correspond to the 10 infinitesimal generators of Spin(0,5) = Sp(2).
2 of them are in the Cartan subalgebra.
6 of them carry color charge, and therefore correspond to quarks.
The remaining 2 carry no color charge, but may carry electric charge and so
may be considered as corresponding to electrons.
One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron. The graviton number ratio of the down quark to the first-generation electron is therefore 6/1 = 6.

N(octonion) is an octonion number factor relating up-type quark masses to
down-type quark masses in each generation.

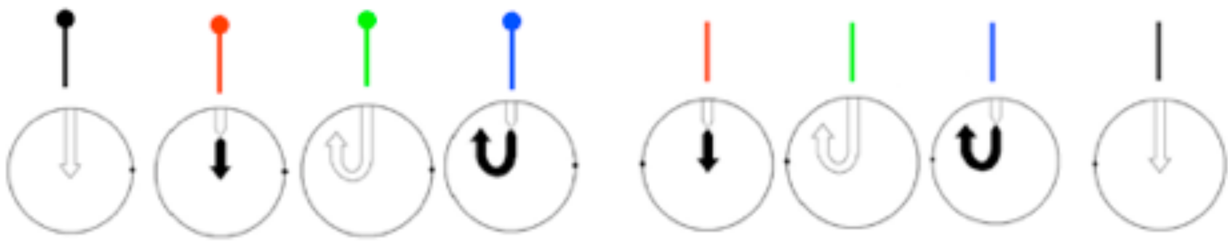
Sym is an internal symmetry factor, relating 2nd and 3rd generation massive
leptons to first generation fermions. It is not used in first-generation calculations.

3 Generation Fermion Combinatorics

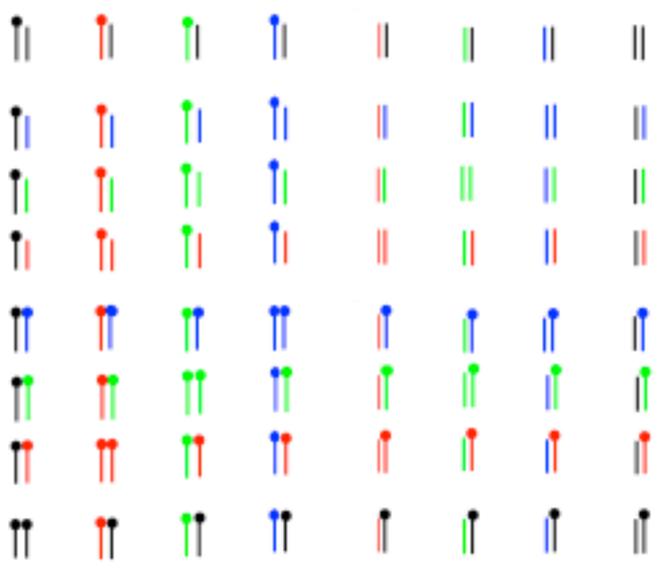
First Generation (8)

(geometric representation of Octonions is from arXiv 1010.2979)

electron	red up quark	green up quark	blue up quark	red down quark	green down quark	blue down quark	neutrino
E	I	J	K	i	j	k	1



Second Generation (64)

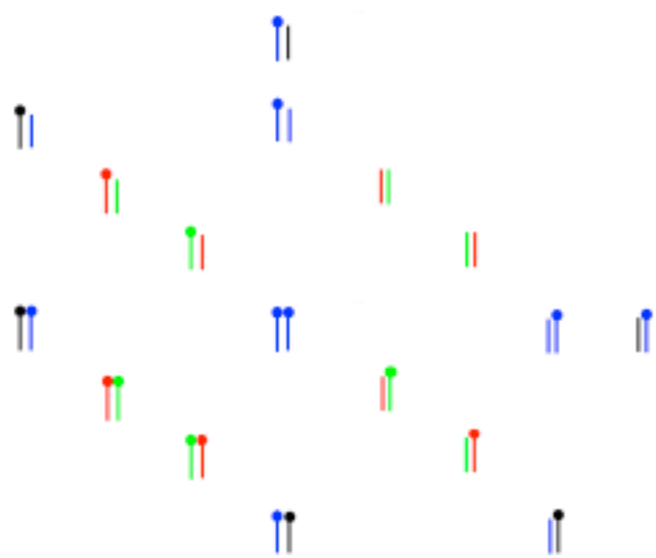


Mu Neutrino (1)
Rule: a Pair belongs to the Mu Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element) .

Muon (3)
Rule: a Pair belongs to the Muon if:
All elements are Colorless (black)
and at least one element is NonAssociative
(that is, is E which is the only Colorless NonAssociative element).

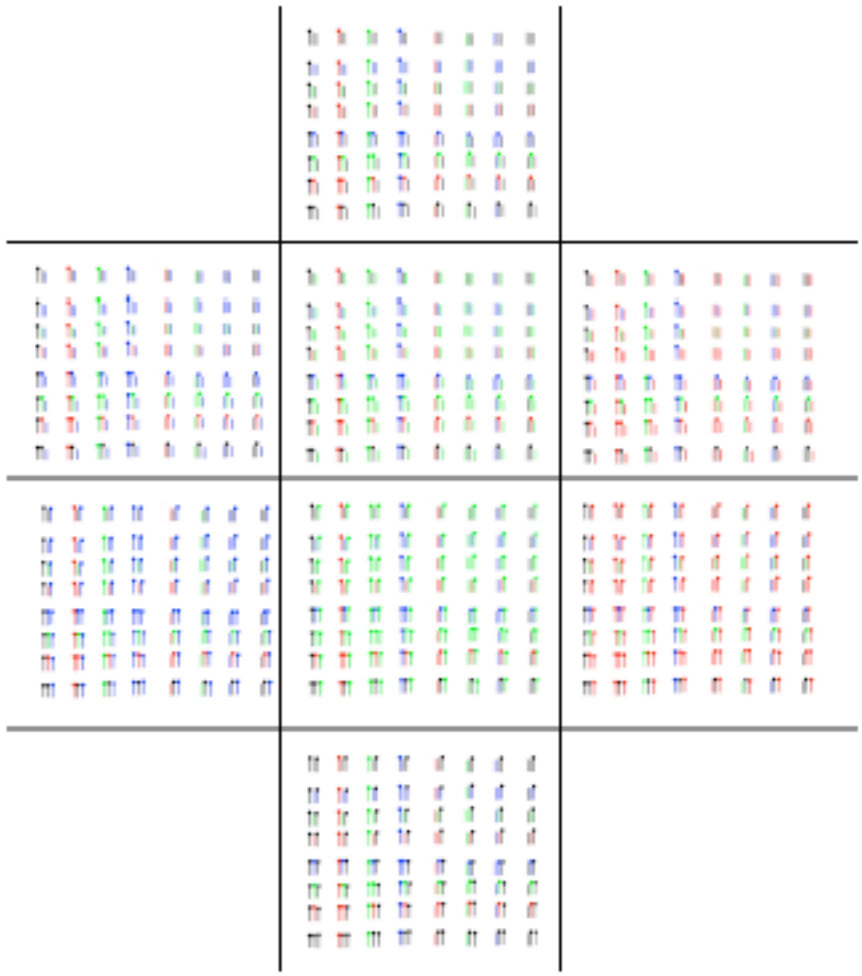
Blue Strange Quark **(3)**
Rule: a Pair belongs to the Blue Strange Quark if:
There is at least one Blue element and the other element is Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Charm Quark **(17)**
Rules: a Pair belongs to the Blue Charm Quark if:
1 - There is at least one Blue element and the other element is Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element (Red x Green = Blue).



(Red and Green Strange and Charm Quarks follow similar rules)

Third Generation (512)

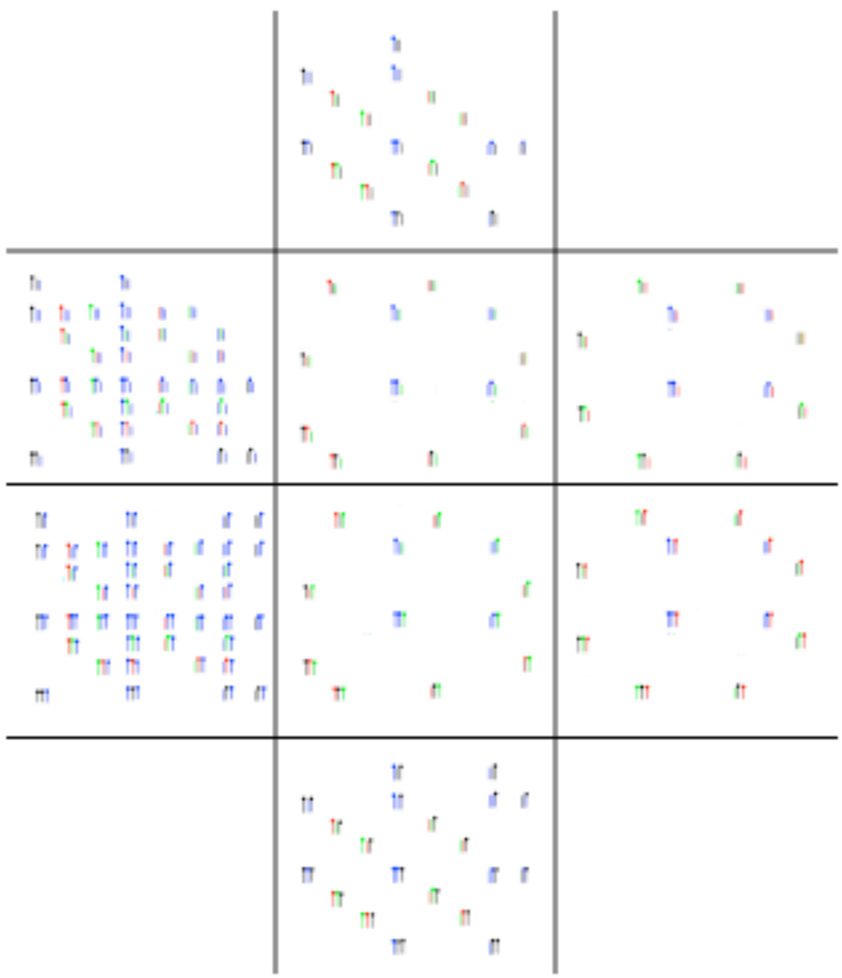


Tau Neutrino (1)
Rule: a Triple belongs to the Tau Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element)

Tauon (7)
Rule: a Triple belongs to the Tauon if:
All elements are Colorless (black)
and at least one element is NonAssociative (that is, is E which is the only Colorless NonAssociative element)

Blue Beauty Quark (7)
Rule: a Triple belongs to the Blue Beauty Quark if:
There is at least one Blue element and all other elements are Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Truth Quark (161)
Rules: a Triple belongs to the Blue Truth Quark if:
1 - There is at least one Blue element and all other elements are Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element and the other element is Colorless (Red x Green = Blue)
3 - The Triple has one element each that is Red, Green, or Blue,
in which case the color of the Third element (for Third Generation) is determinative and must be Blue.



(Red and Green Beauty and Truth Quarks follow similar rules)

The first generation down quark constituent mass : electron mass ratio is:

The electron, E, can only be taken into the tree-level-massless neutrino, 1, by photon, weak boson, and gluon interactions.
The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks.
The neutrino, being massless at tree level, does not add anything to the mass formula for the electron.
Since the electron cannot be related to any other massive Dirac fermion,
its volume V(Qelectron) is taken to be 1.

Next consider a red down quark i.
By gluon interactions, i can be taken into j and k, the blue and green down quarks.
By also using weak boson interactions,
it can also be taken into I, J, and K, the red, blue, and green up quarks.
Given the up and down quarks, pions can be formed from quark-antiquark pairs,
and the pions can decay to produce electrons and neutrinos.
Therefore the red down quark (similarly, any down quark)
is related to all parts of S^7 x RP^1,
the compact manifold corresponding to { 1, i, j, k, E, I, J, K }
and therefore
a down quark should have
a spinor manifold volume factor V(Qdown quark) of the volume of S^7 x RP^1.

The ratio of the down quark spinor manifold volume factor
to the electron spinor manifold volume factor is

V(Qdown quark) / V(Qelectron) = V(S^7x RP^1)/1 = pi^5 / 3.

Since the first generation graviton factor is 6,

md / me = 6 V(S^7 x RP^1) = 2 pi^5 = 612.03937

As the up quarks correspond to I, J, and K, which are the octonion transforms under E of i, j, and k of the down quarks, the up quarks and down quarks have the same constituent mass

mu = md.

Antiparticles have the same mass as the corresponding particles.
Since the model only gives ratios of masses,
the mass scale is fixed so that the electron mass me = 0.5110 MeV.

Then, the constituent mass of the down quark is md = 312.75 MeV,
and the constituent mass for the up quark is mu = 312.75 MeV.

These results when added up give a total mass of first generation fermion particles:

Sigmaf1 = 1.877 GeV

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks

mproton = mu + mu + md = 938.25 MeV

which is close to the experimental value of 938.27 MeV.

The third generation fermion particles correspond to triples of octonions. There are 8^3 = 512 such triples.

The triple { 1,1,1 } corresponds to the tau-neutrino.

The other 7 triples involving only 1 and E correspond to the tauon:

{ E, E, E }
{ E, E, 1 }
{ E, 1, E }
{ 1, E, E }
{ 1, 1, E }
{ 1, E, 1 }
{ E, 1, 1 }

The symmetry of the 7 tauon triples is the same
as the symmetry of the first generation tree-level-massive fermions,
3 down, quarks, the 3 up quarks, and the electron,
so by the Sym factor the tauon mass should be the same as
the sum of the masses of the first generation massive fermion particles.

Therefore the tauon mass is calculated at tree level as 1.877 GeV.

The calculated tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the tauon mass is about 2 GeV,
the effective tauon mass should be renormalized
from the energy level of 1 GeV at which the mass is 1.88 GeV
to the energy level of 2 GeV.
Such a renormalization should reduce the mass.

If the renormalization reduction were about 5 percent,
the effective tauon mass at 2 GeV would be about 1.78 GeV.
The 1996 Particle Data Group Review of Particle Physics gives a tauon mass of 1.777 GeV.

All triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples.
They are triples of the same form as the 7 tauon triples involving 1 and E,
but for 1 and I, 1 and J, and 1 and K,
which correspond to the red, green, and blue beauty quarks,
respectively.

The seven red beauty quark triples correspond to the seven tauon triples,
except that
the beauty quark interacts with 6 Spin(0,5) gravitons
while the tauon interacts with only two.

The red beauty quark constituent mass should be the tauon mass times
the third generation graviton factor 6/2 = 3,
so the red beauty quark mass is mb = 5.63111 GeV.

The blue and green beauty quarks are similarly determined to also be 5.63111 GeV.

The calculated beauty quark mass of 5.63 GeV is a consitituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.
Therefore, the calculated beauty quark mass of 5.63 GeV
corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives
a lattice gauge theory beauty quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass
if the color force strength constant alpha_s is known.
The conventional value of alpha_s at about 5 GeV is about 0.22.

Using alpha_s (5 GeV) = 0.22, a pole mass of 5.0 GeV
gives an MSbar 1-loop beauty quark mass of 4.6 GeV,
and
an MSbar 1,2-loop beauty quark mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV,
the MSbar mass decreases by about 1.3 GeV,
giving an expected MSbar mass of about 3.0 GeV at 90 GeV.

DELPHI at LEP has observed the Beauty Quark
and found a 90 GeV MSbar beauty quark mass of about 2.67 GeV,
with error bars +/- 0.25 (stat) +/- 0.34 (frag) +/- 0.27 (theo).

The theoretical model calculated Beauty Quark mass of 5.63 GeV
corresponds to a pole mass of 5.32 GeV,
which is somewhat higher than the conventional value of 5.0 GeV.

However, the theoretical model calculated value
of the color force strength constant alpha_s at about 5 GeV is about 0.166,
while the conventional value
of the color force strength constant alpha_s at about 5 GeV is about 0.216,
and
the theoretical model calculated value
of the color force strength constant alpha_s at about 90 GeV is about 0.106,
while the conventional value
of the color force strength constant alpha_s at about 90 GeV is about 0.118.

The theoretical model calculations gives a Beauty Quark pole mass (5.3 GeV) that
is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV),
and a color force strength alpha_s at 5 GeV (0.166)
such that 1 + alpha_s = 1.166 is about 4 percent lower
than the conventional value of 1 + alpha_s = 1.216 at 5 GeV.

Triples of the type { 1, I, J } , { I, J, K }, etc.,
do not correspond to the beauty quark, but to the truth quark.
The truth quark corresponds to those 512 - 1 - 7 - 21 = 483 triples,
so the constituent mass of the red truth quark
is 161 / 7 = 23 times the red beauty quark mass,
and the red T-quark mass is
mt = 129.5155 GeV

The blue and green truth quarks are similarly determined to also be 129.5155 GeV.

This is the value of the Low Mass State of the Truth calculated in the Cl(16)_E8 model.
The Middle Mass State of the Truth Quark has been observed by Fermilab since 1994.
The Low and High Mass States of the Truth Quark have, in my opinion, also been
observed by Fermilab (see Chapter 17 of this paper) but the Fermilab and CERN
establishments disagree.

All other masses than the electron mass
(which is the basis of the assumption of the value of the Higgs scalar field vacuum
expectation value v = 252.514 GeV),
including the Higgs scalar mass and Truth quark mass,
are calculated (not assumed) masses in the Cl(16)-E8 model.
These results when added up give a total mass of third generation fermion
particles:

Sigmaf3 = 1,629 GeV

The second generation fermion particles correspond to pairs of octonions.
There are 8^2 = 64 such pairs.

The pair { 1,1 } corresponds to the mu-neutrino.

The pairs { 1, E }, { E, 1 }, and { E, E } correspond to the muon.

For the Sym factor, compare the symmetries of the muon pairs
to the symmetries of the first generation fermion particles:
The pair { E, E } should correspond to the E electron.
The other two muon pairs have a symmetry group S2,
which is 1/3 the size of the color symmetry group S3
which gives the up and down quarks their mass of 312.75 MeV.

Therefore the mass of the muon should be the sum of
the { E, E } electron mass and
the { 1, E }, { E, 1 } symmetry mass, which is 1/3 of the up or down quark mass.
Therefore, mmu = 104.76 MeV .

According to the 1998 Review of Particle Physics of the Particle Data Group, the experimental muon mass is about 105.66 MeV which may be consistent with radiative corrections for the calculated tree-level mmu = 104.76 MeV as Bailin and Love, in “Introduction to Gauge Field Theory”, IOP (rev ed 1993), say: "... considering the order alpha radiative corrections to muon decay ... Numerical details are contained in Sirlin ... 1980 Phys. Rev. D 22 971 ... who concludes that the order alpha corrections have the effect of increasing the decay rate about 7% compared with the tree graph prediction ...". Since the decay rate is proportional to mmu^5 the corresponding effective increase in muon mass would be about 1.36%, which would bring 104.8 MeV up to about 106.2 MeV.

All pairs corresponding to the muon and the mu-neutrino are colorless.

The red, blue and green strange quark each corresponds
to the 3 pairs involving 1 and i, j, or k.

The red strange quark is defined as the three pairs { 1, i }, { i, 1 }, { i, i }
because i is the red down quark.
Its mass should be the sum of two parts:
the { i, i } red down quark mass, 312.75 MeV, and
the product of the symmetry part of the muon mass, 104.25 MeV,
times the graviton factor.

Unlike the first generation situation,
massive second and third generation leptons can be taken,
by both of the colorless gravitons that may carry electric charge,
into massive particles.

Therefore the graviton factor for the second and third generations is 6/2 = 3.

So the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV, and
the red strange quark constituent mass is ms = 312.75 MeV + 312.75 MeV = 625.5 MeV

The blue strange quarks correspond to the three pairs involving j,
the green strange quarks correspond to the three pairs involving k,
and their masses are similarly determined to also be 625.5 MeV.
The charm quark corresponds to the remaining 64 - 1 - 3 - 9 = 51 pairs.

Therefore, the mass of the red charm quark should be the sum of two parts:
the { i, i }, red up quark mass, 312.75 MeV;
and
the product of the symmetry part of the strange quark mass, 312.75 MeV,
and the charm to strange octonion number factor 51 / 9,
which product is 1,772.25 MeV.

Therefore the red charm quark constituent mass is
mc = 312.75 MeV + 1,772.25 MeV = 2.085 GeV

The blue and green charm quarks are similarly determined to also be 2.085 GeV.

The calculated Charm Quark mass of 2.09 GeV is a consitituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a
conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the
Charm Quark pole mass from 1.2 to 1.9 GeV.

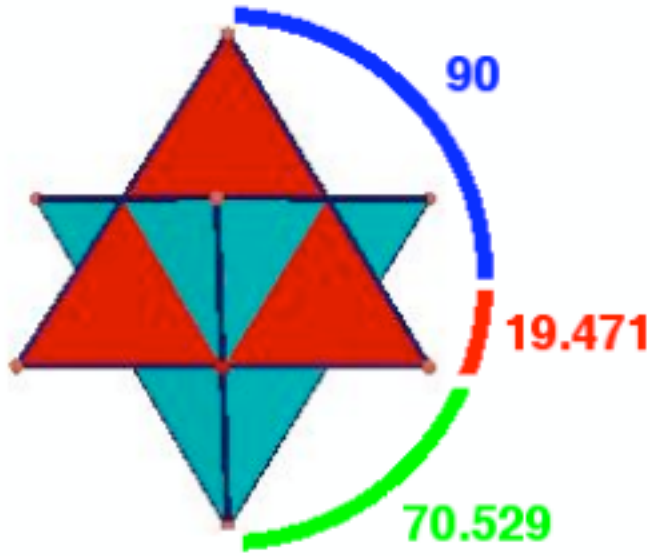
The pole mass can be converted to an MSbar mass if the color force strength constant alpha_s is known.
The conventional value of alpha_s at about 2 GeV is about 0.39,
which is somewhat lower than the theoretical model value.
Using alpha_s (2 GeV) = 0.39, a pole mass of 1.9 GeV
gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.

These results when added up give a total mass of second generation fermion particles:

Sigmaf2 = 32.9 GeV

Appendix - Kobayashi-Maskawa Parameters

In E8 Physics the KM Unitarity Triangle angles can be seen on the Stella Octangula



The Kobayashi-Maskawa parameters are determined in terms of
the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by

Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted by

Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The resulting KM matrix is:

	d	s	b
u	0.975	0.222 0.00249	-0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Below the energy level of ElectroWeak Symmetry Breaking the Higgs mechanism gives mass to particles.

According to a Review on the Kobayashi-Maskawa mixing matrix by Ceccucci, Ligeti, and Sakai in the 2010 Review of Particle Physics (note that I have changed their terminology of CKM matrix to the KM terminology that I prefer because I feel that it was Kobayashi and Maskawa, not Cabibbo, who saw that 3x3 was the proper matrix structure): "... the charged-current W± interactions couple to the ... quarks with couplings given by ...

Vud	Vus	Vub
Vcd	Vcs	Vcb
Vtd	Vts	Vtb

This Kobayashi-Maskawa (KM) matrix is a 3x3 unitary matrix.
It can be parameterized by three mixing angles and the CP-violating KM phase ...
The most commonly used unitarity triangle arises from
Vud Vub∗ + Vcd Vcb∗ + Vtd Vtb∗ = 0,
by dividing each side by the best-known one, Vcd Vcb∗
...
¯ρ + i¯η = −(Vud Vub∗)/(Vcd Vcb∗) is phase-convention- independent ...

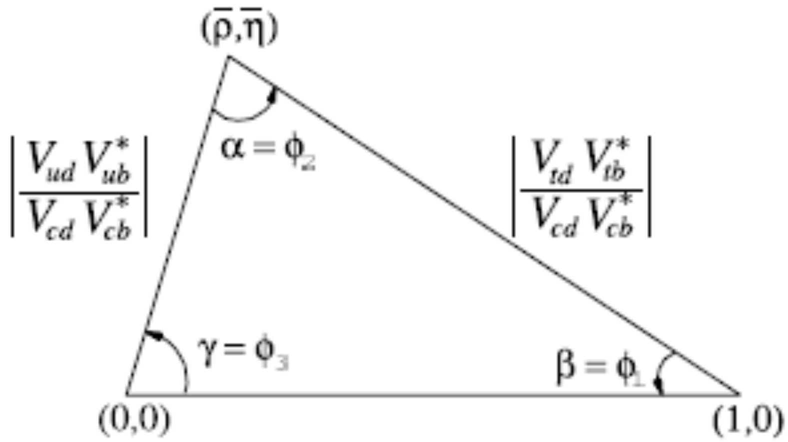


Figure 11.1: Sketch of the unitarity triangle.

... sin 2β = 0.673 ± 0.023 ... α = 89.0 +4.4 −4.2 degrees ... γ = 73 +22 −25 degrees ...
The sum of the three angles of the unitarity triangle, α + β + γ = (183 +22 −25) degrees, is ... consistent with the SM expectation. ...

The area... of ...[the]... triangle...[is]... half of the Jarlskog invariant, J, which is a phase-convention-independent measure of CP violation, defined by Im Vij Vkl Vil∗ Vkj∗ = J SUM(m,n) ε_ikm ε_jln

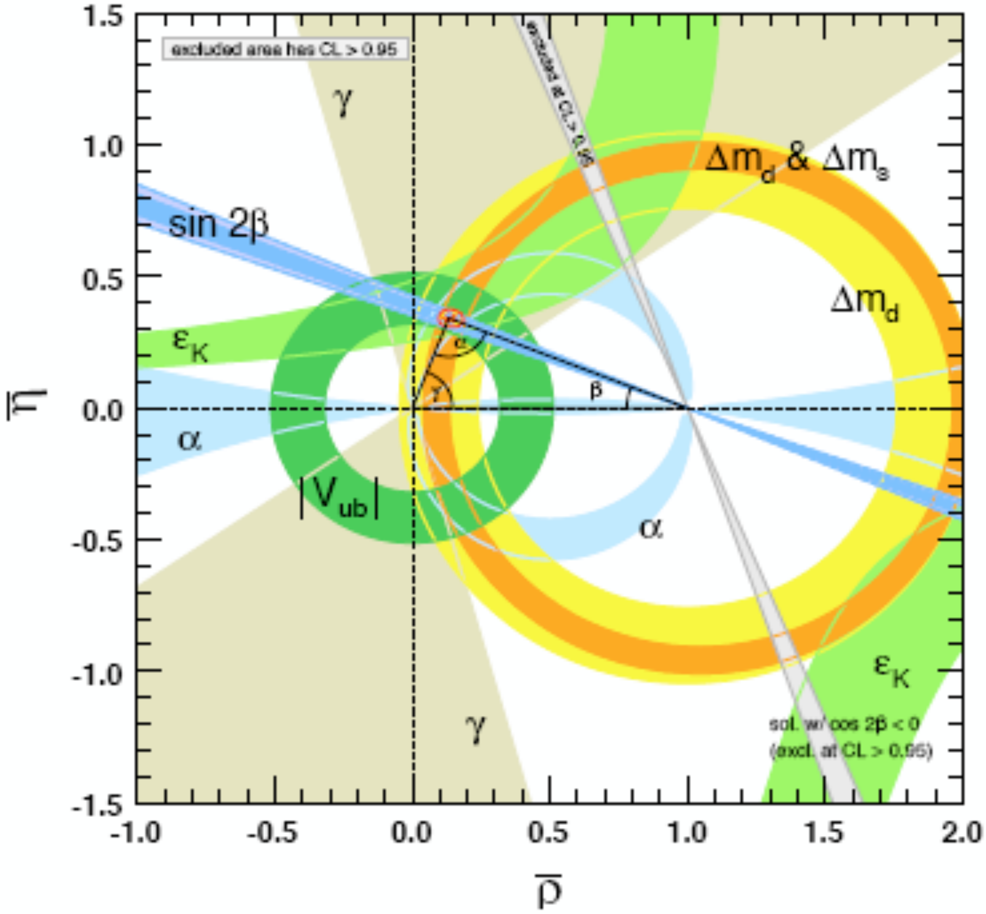


Figure 11.2: Constraints on the $\bar{\rho}, \eta$ plane.
The shaded areas have 95% CL.

The fit results for the magnitudes of all nine KM elements are ...

0.97428 ± 0.00015	0.2253 ± 0.0007	0.00347 +0.00016 −0.00012
0.2252 ± 0.0007	0.97345 +0.00015 −0.00016	0.0410 +0.0011 −0.0007
0.00862 +0.00026 −0.00020	0.0403 +0.0011−0.0007	0.999152 +0.000030−0.000045

and the Jarlskog invariant is J = (2.91 +0.19-0.11)x10−5. ...".

Above the energy level of ElectroWeak Symmetry Breaking particles are massless.

Kea (Marni Sheppeard) proposed
that in the Massless Realm the mixing matrix might be democratic.
In Z. Phys. C - Particles and Fields 45, 39-41 (1989) Koide said: "...
the mass matrix ... MD ... of the type ... 1/3 x m x

1	1	1
1	1	1
1	1	1

... has name... "democratic" family mixing ...
the ... democratic ... mass matrix can be diagonalized by the transformation matrix A ...

1/sqrt(2)	-1/sqrt(2)	0
1/sqrt(6)	1/sqrt(6)	-2/sqrt(6)
1/sqrt(3)	1/sqrt(3)	1/sqrt(3)

as A MD At =

0	0	0
0	0	0
0	0	m

...".

Up in the Massless Realm you might just say that there is no mass matrix,
just a democratic mixing matrix of the form 1/3 x

1	1	1
1	1	1
1	1	1

with no complex stuff and no CP violation in the Massless Realm.

When go down to our Massive Realm by ElectroWeak Symmetry Breaking
then you might as a first approximation use m = 1
so that all the mass first goes to the third generation as

0	0	0
0	0	0
0	0	1

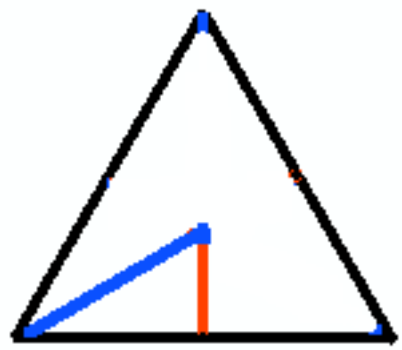
which is physically like the Higgs being a T-Tbar quark condensate.

Consider a 3-dim Euclidean space of generations:

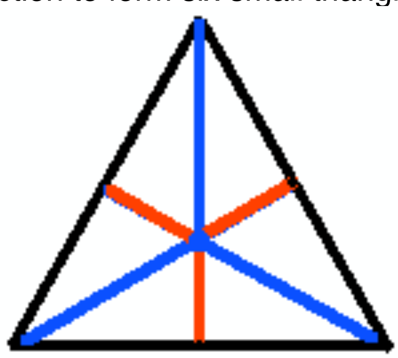
The case of mass only going to one generation
can be represented as a line or 1-dimensional simplex

in which the blue mass-line covers the entire black simplex line.

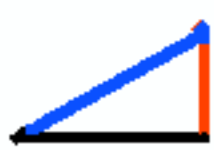
If mass only goes to one other generation
that can be represented by a red line extending to a second dimension
forming a small blue-red-black triangle



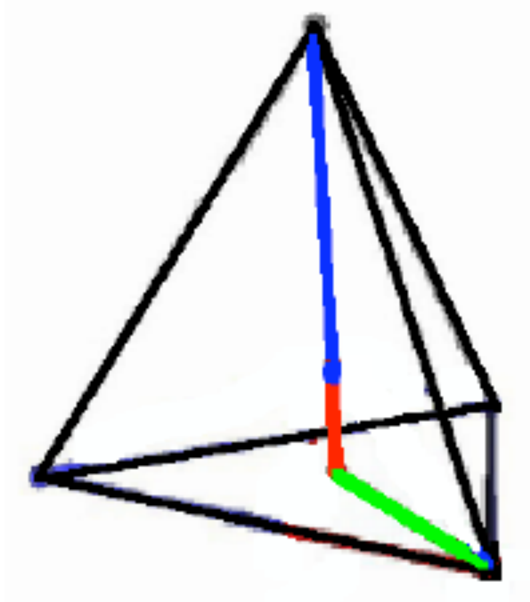
that can be extended by reflection to form six small triangles making up a large triangle



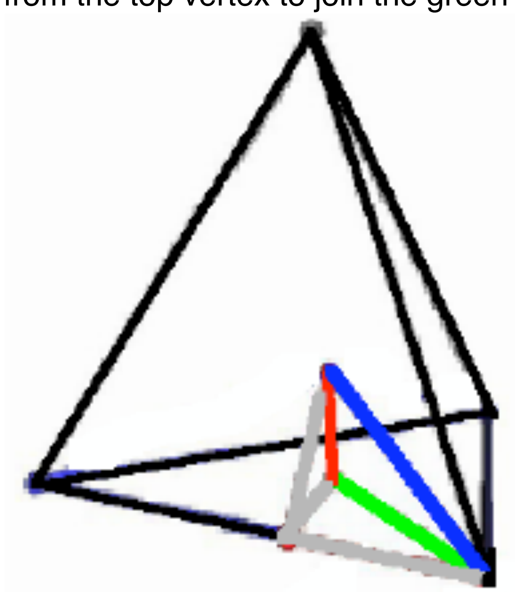
Each of the six component triangles has 30-60-90 angle structure:



If mass goes on further to all three generations
that can be represented by a green line extending to a third dimension



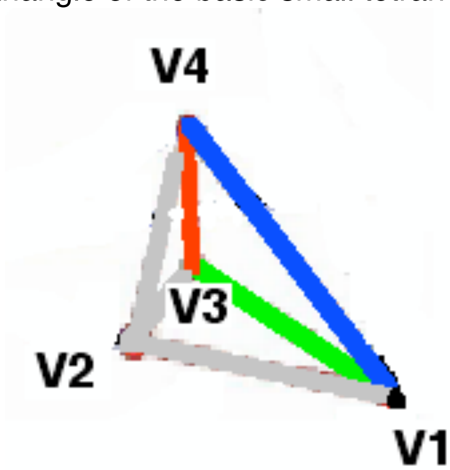
If you move the blue line from the top vertex to join the green vertex



you get a small blue-red-green-gray-gray-gray tetrahedron
that can be extended by reflection to form 24 small tetrahedra
making up a large tetrahedron.

Reflection among the 24 small tetrahedra corresponds
to the 12+12 = 24 elements of the Binary Tetrahedral Group.

The basic blue-red-green triangle of the basic small tetrahedron



has the angle structure of the K-M Unitary Triangle.

Using data from R. W. Gray's "Encyclopedia Polyhedra: A Quantum Module"
with lengths

V1.V2 = (1/2) EL ≡ Half of the regular Tetrahedron's edge length.
V1.V3 = (1 / sqrt(3)) EL ≅ 0.577 350 269 EL
V1.V4 = 3 / (2 sqrt(6)) EL ≅ 0.612 372 436 EL
V2.V3 = 1 / (2 sqrt(3)) EL ≅ 0.288 675 135 EL
V2.V4 = 1 / (2 sqrt(2)) EL ≅ 0.353 553 391 EL
V3.V4 = 1 / (2 sqrt(6)) EL ≅ 0.204 124 145 EL

the Unitarity Triangle angles are:

β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285

α = V1.V3.V4 = 90 degrees

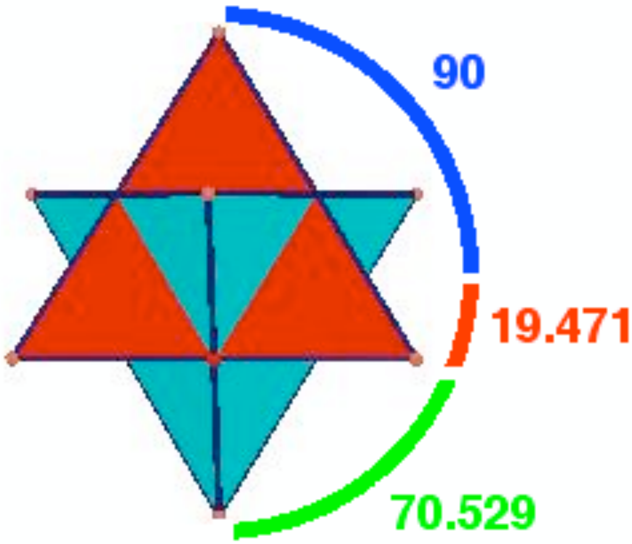
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees

which is substantially consistent with the 2010 Review of Particle Properties

sin $2\beta = 0.673 \pm 0.023$ so β = 21.1495 degrees
$\alpha = 89.0 {+4.4} {-4.2}$ degrees
$\gamma = 73 {+22} {-25}$ degrees

and so also consistent with the Standard Model expectation.

The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



In the Cl(16)-E8 model the Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by
Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted
by Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of Spin(8),
and the Spin(8) representations are considered to be fundamental.

The following formulas use the above masses
to calculate Kobayashi-Maskawa parameters:

phase angle d13 = gamma = 70.529 degrees

sin(theta12) = s12 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mmu^2+3ms^2+3mc^2]) = 0.222198

sin(theta13) = s13 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mtau^2+3mb^2+3mt^2]) = 0.004608

sin(*theta23 = [mmu+3ms+3mc]/sqrt([mtau^2+3mb^2+3mt^2]+
+ [mmu^2+3ms^2+3mc^2])

sin(theta23) = s23 = sin(*theta23) sqrt(Sigmaf2 / Sigmaf1) = 0.04234886

The factor sqrt(Smf2 /Smf1) appears in s23 because an s23 transition is to the second generation and not all the way to the first generation, so that the end product of an s23 transition has a greater available energy than s12 or s13 transitions by a factor of Smf2 / Smf1 .

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an s23 transition has greater available energy than the s12 or s13 transitions by a factor of Smf2 / Smf1
the effective magnitude of the s23 terms in the KM entries is increased by the factor sqrt(Smf2 /Smf1) .

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

1	0	0
0	cos(theta23)	sin(theta23)
0	-sin(theta23)	cos(theta23)

cos(theta13)	0	sin(theta13)exp(-i d13)
0	1	0
-sin(theta13)exp(i d13)	0	cos(theta13)

cos(theta12)	sin(theta12)	0
-sin(theta12)	cos(theta12)	0
0	0	1

The resulting Kobayashi-Maskawa parameters
for W+ and W- charged weak boson processes, are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The matrix is labelled by either (u c t) input and (d s b) output,
or, as above, (d s b) input and (u c t) output.

For Z0 neutral weak boson processes, which are suppressed by the GIM mechanism of cancellation of virtual subprocesses, the matrix is labelled by either (u c t) input and (u'c't') output, or, as below, (d s b) input and (d's'b') output:

	d	s	b
d'	0.975	0.222	0.00249 -0.00388i
s'	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
b'	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Since neutrinos of all three generations are massless at tree level,
the lepton sector has no tree-level K-M mixing.

In hep-ph/0208080, Yosef Nir says: "... Within the Standard Model,
the only source of CP violation is the Kobayashi-Maskawa (KM) phase ...
The study of CP violation is, at last, experiment driven. ...
The CKM matrix provides a consistent picture
of all the measured flavor and CP violating processes. ...
There is no signal of new flavor physics. ...
Very likely,
the KM mechanism is the dominant source of CP violation in flavor changing processes.
... The result is consistent with the SM predictions. ...".

Appendix - Neutrino Masses Beyond Tree Level

Consider the three generations of neutrinos:
nu_e (electron neutrino); nu_m (muon neutrino); nu_t
and three neutrino mass states: nu_1 ; nu_2 : nu_3
and
the division of 8-dimensional spacetime into
4-dimensional physical Minkowski spacetime
plus
4-dimensional CP2 internal symmetry space.

The heaviest mass state nu_3 corresponds to a neutrino
whose propagation begins and ends in CP2 internal symmetry
space,lying entirely therein. According to the Cl(16)-E8 model
the mass of nu_3 is zero at tree-level
but it picks up a first-order correction
propagating entirely through internal symmetry space by merging
with an electron through the weak and electromagnetic forces,
effectively acting not merely as a point
but
as a point plus an electron loop at beginning and ending points
so
the first-order corrected mass of nu_3 is given by
M_nu_3 x (1/sqrt(2)) = M_e x GW(mproton^2) x alpha_E
where the factor (1/sqrt(2)) comes from the Ut3 component
of the neutrino mixing matrix
so that

M_nu_3 = sqrt(2) x M_e x GW(mproton^2) x alpha_E =
= 1.4 x 5 x 10^5 x 1.05 x 10^(-5) x (1/137) eV =
= 7.35 / 137 = 5.4 x 10^(-2) eV.

The neutrino-plus-electron loop can be anchored by weak force
action through any of the 6 first-generation quarks
at each of the beginning and ending points, and that the
anchor quark at the beginning point can be different from
the anchor quark at the ending point,
so that there are 6x6 = 36 different possible anchorings.

The intermediate mass state nu_2 corresponds to a neutrino
whose propagation begins or ends in CP2 internal symmetry space
and ends or begins in M4 physical Minkowski spacetime,
thus having only one point (either beginning or ending) lying
in CP2 internal symmetry space where it can act not merely
as a point but as a point plus an electron loop.

According to the Cl(16)-E8 model the mass of nu_2 is zero at
tree-level but it picks up a first-order correction at only one
(but not both) of the beginning or ending points
so that so that there are 6 different possible anchorings
for nu_2 first-order corrections, as opposed to the 36 different
possible anchorings for nu_3 first-order corrections,
so that
the first-order corrected mass of nu_2 is less than
the first-order corrected mass of nu_3 by a factor of 6,
so

the first-order corrected mass of nu_2 is
M_nu_2 = M_nu_3 / Vol(CP2) = 5.4 x 10^(-2) / 6
= 9 x 10^(-3)eV.

The low mass state nu_1 corresponds to a neutrino
whose propagation begins and ends in physical Minkowski
spacetime.
thus having only one anchoring to CP2 interna symmetry space.

According to the Cl(16)-E8 model the mass of nu_1 is zero at
tree-level but it has only 1 possible anchoring to CP2
as opposed to the 36 different possible anchorings for nu_3
first-order corrections
or the 6 different possible anchorings for nu_2 first-order
corrections
so that
the first-order corrected mass of nu_1 is less than
the first-order corrected mass of nu_2 by a factor of 6,
so

the first-order corrected mass of nu_1 is
M_nu_1 = M_nu_2 / Vol(CP2) = 9 x 10^(-3) / 6
= 1.5 x 10^(-3)eV.

Therefore:

```
the mass-squared difference D(M23^2) = M_nu_3^2 - M_nu_2^2 =
                            = ( 2916 - 81 ) x 10^(-6) eV^2 =
                            = 2.8 x 10^(-3) eV^2
```

and

```
the mass-squared difference D(M12^2) = M_nu_2^2 - M_nu_1^2 =
                               = ( 81 - 2 ) x 10^(-6) eV^2 =
                               = 7.9 x 10^(-5) eV^2
```

The 3x3 unitary neutrino mixing matrix neutrino mixing matrix U

	nu_1	nu_2	nu_3
nu_e	Ue1	Ue2	Ue3
nu_m	Um1	Um2	Um3
nu_t	Ut1	Ut2	Ut3

can be parameterized (based on the 2010 Particle Data Book)
by 3 angles and 1 Dirac CP violation phase

```
     c12 c13                        s12 c13                        s13 e-id

U = - s12 c23 - c12 s23 s13 eid     c12 c23 - s12 s23 s13 eid      s23 c13

     s12 s23 - c12 c23 s13 eid    - c12 s23 - s12 c23 s13 eid      c23 c13
```

where cij = cos(theta_ij) , sij = sin(theta_ij)

The angles are

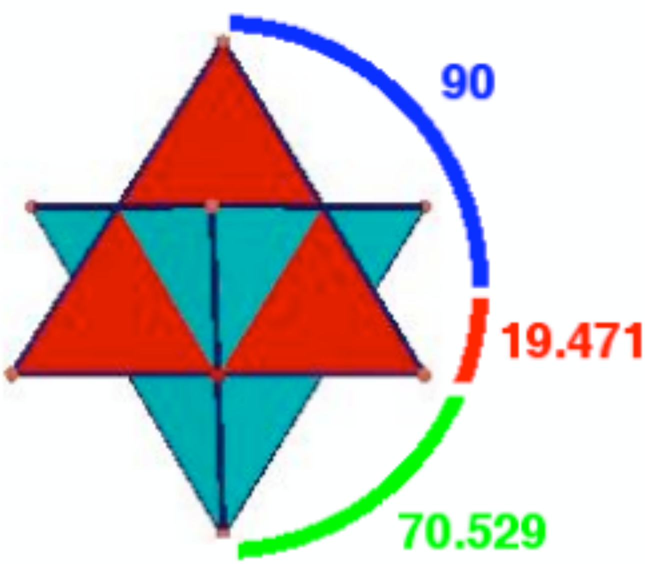
theta_23 = pi/4 = 45 degrees
because
nu_3 has equal components of nu_m and nu_t so
that Um3 = Ut3 = 1/sqrt(2) or, in conventional
notation, mixing angle theta_23 = pi/4
so that cos(theta_23) = 0.707 = sqrt(2)/2 = sin(theta_23)

theta_13 = 9.594 degrees = asin(1/6)
and cos(theta_13) = 0.986
because sin(theta_13) = 1/6 = 0.167 = |Ue3| = fraction of nu_3 that is nu_e

theta_12 = pi/6 = 30 degrees
because
sin(theta_12) = 0.5 = 1/2 = Ue2 = fraction of nu_2 begin/end points
that are in the physical spacetime where massless nu_e lives
so that cos(theta_12) = 0.866 = sqrt(3)/2

d = 70.529 degrees is the Dirac CP violation phase
ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
This is because the neutrino mixing matrix has 3-generation structure
and so has the same phase structure as the KM quark mixing matrix
in which the Unitarity Triangle angles are:
β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285
α = V1.V3.V4 = 90 degrees
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees

The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



Then we have for the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.866 x 0.986	0.50 x 0.986	0.167 x e-id
nu_m	-0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986
nu_t	0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	-0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.167 e-id
nu_m	-0.354 -0.102 eid	0.612 -0.059 eid	0.697
nu_t	0.354 -0.102 eid	-0.612 -0.059 eid	0.697

Since ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
and .333e-i(70.529) = cos(70.529) - i sin(70.529) = 0.333 - 0.943 i

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.354 -0.034 - 0.096 i	0.612 -0.020 - 0.056 i	0.697
nu_t	0.354 -0.034 - 0.096 i	-0.612 -0.020 - 0.056 i	0.697

for a result of

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.388 - 0.096 i	0.592 - 0.056 i	0.697
nu_t	0.320 - 0.096 i	0.632 - 0.056 i	0.697

which is consistent with the approximate experimental values of mixing angles shown in the Michaelmas Term 2010 Particle Physics handout of Prof Mark Thomson if the matrix is modified by taking into account the March 2012 results from Daya Bay observing non-zero theta_13 = 9.54 degrees.

Appendix - Proton-Neutron Mass Difference

An up valence quark, constituent mass 313 Mev,
does not often swap places with a 2.09 Gev charm sea quark,
but
a 313 Mev down valence quark
can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force
constituent mass of the down valence quark is heavier by about

$(ms - md)\ (md/ms)^2\ a(w)\ |Vds| = 312 \times 0.25 \times 0.253 \times 0.22$ Mev $= 4.3$ Mev,

(where $a(w) = 0.253$ is the geometric part of the weak force strength
and $|Vds| = 0.22$ is the magnitude
of the K-M parameter mixing first generation down and second generation strange)

so that the Quantum color force constituent mass Qmd of the down quark is

$$Qmd = 312.75 + 4.3 = 317.05 \text{ MeV}.$$

Similarly, the up quark Quantum color force mass increase is about

$(mc - mu)\ (mu/mc)^2\ a(w)\ |V(uc)| = 1777 \times 0.022 \times 0.253 \times 0.22$ Mev $= 2.2$ Mev,

(where $|Vuc| = 0.22$ is the magnitude
of the K-M parameter mixing first generation up and second generation charm)

so that the Quantum color force constituent mass Qmu of the up quark is

$$Qmu = 312.75 + 2.2 = 314.95 \text{ MeV}.$$

Therefore, the Quantum color force Neutron-Proton mass difference is

$mN - mP = Qmd - Qmu = 317.05$ Mev $- 314.95$ Mev $= 2.1$ Mev.

Since the electromagnetic Neutron-Proton mass difference is roughly

$$mN - mP = -1 \text{ MeV}$$

the total theoretical Neutron-Proton mass difference is

$$mN - mP = 2.1 \text{ Mev} - 1 \text{ Mev} = 1.1 \text{ Mev},$$

an estimate that is comparable to the experimental value of 1.3 Mev.

Appendix - Pion as Sine-Gordon Breather

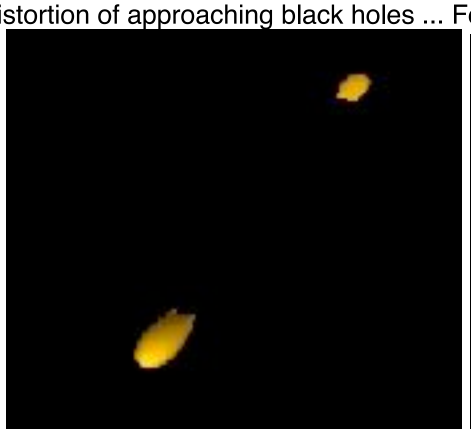
The quark content of a charged pion is a quark - antiquark pair: either Up plus antiDown or Down plus antiUp. Experimentally, its mass is about 139.57 MeV.

The quark is a Schwinger Source Kerr-Newman Black Hole
with constituent mass M 312 MeV.

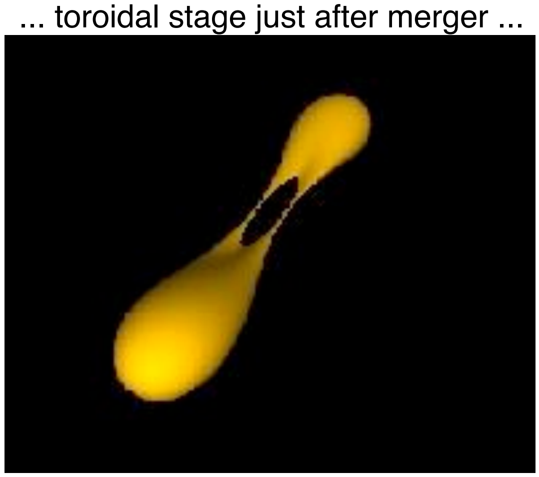
The antiquark is also a Schwinger Source Kerr-Newman Black Hole,
with constituent mass M 312 MeV.

According to section 3.6 of Jeffrey Winicour's 2001 Living Review of the Development of Numerical Evolution Codes for General Relativity (see also a 2005 update):
"... The black hole event horizon associated with ... slightly broken ... degeneracy [of the axisymmetric configuration]... reveals new features not seen in the degenerate case of the head-on collision ... If the degeneracy is slightly broken, the individual black holes form with spherical topology but as they approach, tidal distortion produces two sharp pincers on each black hole just prior to merger. ...

Tidal distortion of approaching black holes ... Formation of sharp pincers just prior to merger ..



... toroidal stage just after merger ...



At merger, the two pincers join to form a single ... toroidal black hole.

The inner hole of the torus subsequently [begins to] close... up (superluminally) ... [If the closing proceeds to completion, it]... produce[s] first a peanut shaped black hole and finally a spherical black hole. ...".

In the physical case of quark and antiquark forming a pion,
the toroidal black hole remains a torus.
The torus is an event horizon and therefore is not a 2-spacelike dimensional torus, but is a (1+1)-dimensional torus with a timelike dimension.

The effect is described in detail in Robert Wald's book General Relativity (Chicago 1984). It can be said to be due to extreme frame dragging, or to timelike translations becoming spacelike as though they had been Wick rotated in Complex SpaceTime.

As Hawking and Ellis say in The LargeScale Structure of Space-Time (Cambridge 1973):
"... The surface r = r+ is ... the event horizon ... and is a null surface ...

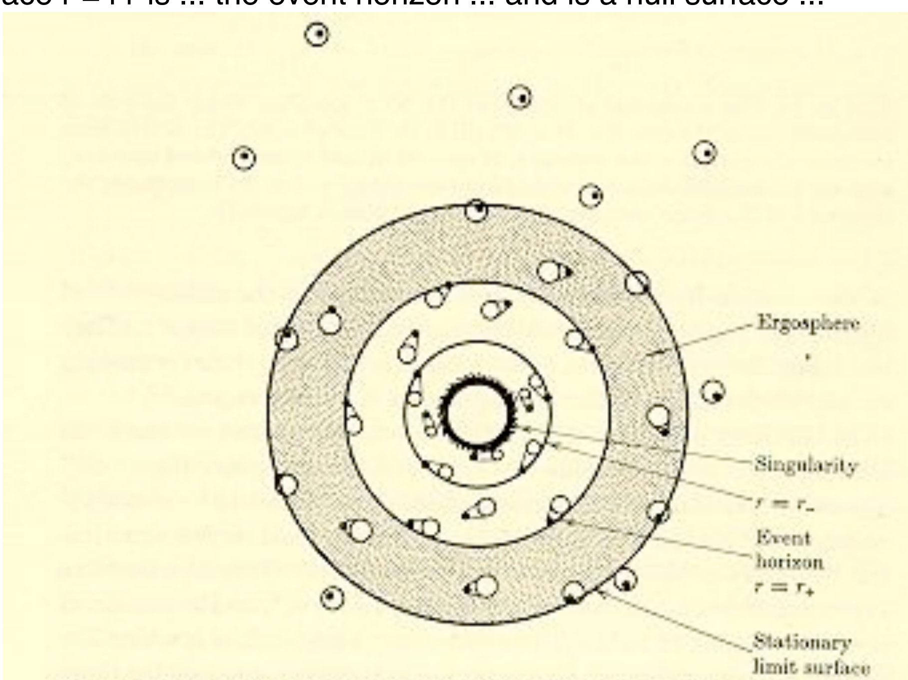


FIGURE 30. The equatorial plane of a Kerr solution with $m^2 > a^2$. The circles represent the position a short time later of flashes of light emitted by the points represented by heavy dots.

... On the surface r = r+ .... the wavefront corresponding to a point on this surface lies entirely within the surface. ...".

A (1+1)-dimensional torus with a timelike dimension can carry a Sine-Gordon Breather.
The soliton and antisoliton of a Sine-Gordon Breather correspond
to the quark and antiquark that make up the pion,
analagous to the Massive Thirring Model.

Sine-Gordon Breathers are described by Sidney Coleman in his Erica lecture paper Classical Lumps and their Quantum Descendants (1975), reprinted in his book Aspects of Symmetry (Cambridge 1985),
where he writes the Lagrangian for the Sine-Gordon equation as (Coleman's eq. 4.3):

$$L = (1 / B^2) ((1/2) (df)^2 + A (\cos(f) - 1))$$

Coleman says: “... We see that, in classical physics, B is an irrelevant parameter:
if we can solve the sine-Gordon equation for any non-zero B,
we can solve it for any other B.
The only effect of changing B is the trivial one of changing the energy and momentum assigned to a given solution of the equation. This is not true in quantum physics, because the relevant object for quantum physics is not L but [eq. 4.4]

$$L / \hbar = (1 / (B^2 \hbar)) ((1/2) (df)^2 + A (\cos(f) - 1))$$

An other way of saying the same thing is to say that in quantum physics we have one more dimensional constant of nature, Planck's constant, than in classical physics. ...
the classical limit, vanishing hbar, is exactly the same as the small-coupling limit, vanishing B ... from now on I will ... set hbar equal to one. ...
... the sine-Gordon equation ...[has]... an exact periodic solution ...[eq. 4.59]...

$$f(x, t) = (4 / B) \arctan((n \sin(w t) / \cosh(n w x))$$

where [eq. 4.60] $n = \sqrt{ A - w^2 } / w$ and w ranges from 0 to A.
This solution has a simple physical interpretation ... a soliton far to the left ...[and]... an antisoliton far to the right. As sin(w t) increases, the soliton and antisoliton move farther apart from each other. When sin(w t) passes through one,
they turn around and begin to approach one another. As sin(w t) comes down to zero ... the soliton and antisoliton are on top of each other ...
when sin(w t) becomes negative .. the soliton and antisoliton have passed each other.

... Thus, Eq. (4.59) can be thought of as a soliton and an antisoliton oscillation about their common center-of-mass. For this reason, it is called 'the doublet [or Breather] solution'. ... the energy of the doublet ...[eq. 4.64]

$$E = 2 M \sqrt{ 1 - (w^2 / A) }$$

where [eq. 4.65] $M = 8 \sqrt{ A } / B^2$ is the soliton mass.

Note that the mass of the doublet is always less than twice the soliton mass,
as we would expect from a soliton-antisoliton pair. ...

Dashen, Hasslacher, and Neveu ... Phys. Rev. D10, 4114; 4130; 4138 (1974).
...[found that]... there is only a single series of bound states, labeled by the integer N ...
The energies ... are ... [eq. 4.82]

E_N = 2 M sin(B'^2 N / 16)

where N = 0, 1, 2 ... < 8 pi / B'^2 , [eq. 4.83]
B'^2 = B^2 / (1 - (B^2 / 8 pi)) and M is the soliton mass.
M is not given by Eq. (4.65), but is the soliton mass corrected by the DHN formula,
or, equivalently, by the first-order weak coupling expansion. ...
I have written the equation in this form .. to eliminate A,
and thus avoid worries about renormalization conventions.
Note that the DHN formula is identical to the Bohr-Sommerfeld formula,
except that B is replaced by B'. ...
Bohr and Sommerfeld['s] ... quantization formula says that if we have a one-parameter
family of periodic motions, labeled by the period, T,
then an energy eigenstate occurs whenever [eq. 4.66]

[Integral from 0 to T](dt p qdot = 2 pi N,

where N is an integer. ... Eq.(4.66) is cruder than the WKB formula,
but it is much more general;
it is always the leading approximation for any dynamical system ...
Dashen et al speculate that Eq. (4.82) is exact. ...
the sine-Gordon equation is equivalent ... to the massive Thirring model.
This is surprising,
because the massive Thirring model is a canonical field theory
whose Hamiltonian is expressed in terms of fundamental Fermi fields only.
Even more surprising, when B^2 = 4 pi , that sine-Gordon equation is equivalent
to a free massive Dirac theory, in one spatial dimension. ...
Furthermore, we can identify the mass term in the Thirring model
with the sine-Gordon interaction, [eq. 5.13]

M = - (A / B^2) N_m cos(B f)

.. to do this consistently ... we must say [eq. 5.14]

B^2 / (4 pi) = 1 / (1 + g / pi)

....[where]... g is a free parameter, the coupling constant [for the Thirring model]...
Note that if B^2 = 4 pi , g = 0 ,
and the sine-Gordon equation is the theory of a free massive Dirac field. ...
It is a bit surprising to see a fermion appearing as a coherent state of a Bose field.
Certainly this could not happen in three dimensions,
where it would be forbidden by the spin-statistics theorem.
However, there is no spin-statistics theorem in one dimension,
for the excellent reason that there is no spin. ...
the lowest fermion-antifermion bound state of the massive Thirring model
is an obvious candidate for the fundamental meson of sine-Gordon theory. ...
equation (4.82) predicts that
all the doublet bound states disappear when B^2 exceeds 4 pi .

This is precisely the point where
the Thirring model interaction switches from attractive to repulsive. ...
these two theories ... the massive Thirring model .. and ... the sine-Gordon equation ...
define identical physics. ...
I have computed the predictions of ...[various]... approximation methods
for the ration of the soliton mass to the meson mass for three values of B^2 :
4 pi (where the qualitative picture of the soliton as a lump totally breaks down),
2 pi, and pi . At 4 pi we know the exact answer ...
I happen to know the exact answer for 2 pi, so I have included this in the table. ...

Method	B^2 = pi	B^2 = 2 pi	B^2 = 4 pi
Zeroth-order weak coupling expansion eq2.13b	2.55	1.27	0.64
Coherent-state variation	2.55	1.27	0.64
First-order weak coupling expansion	2.23	0.95	0.32
Bohr-Sommerfeld eq4.64	2.56	1.31	0.71
DHN formula eq4.82	2.25	1.00	0.50
Exact	?	1.00	0.50

...[eq. 2.13b]

E = 8 sqrt(A) / B^2

...[is the]... energy of the lump ... of sine-Gordon theory ...
frequently called 'soliton...' in the literature ...
[Zeroth-order is the classical case, or classical limit.] ...
... Coherent-state variation always gives
the same result as the ... Zeroth-order weak coupling expansion ... .
The ... First-order weak-coupling expansion ...
explicit formula ... is (8 / B^2) - (1 / pi). ...".

Using the Cl(16)-E8 model constituent mass of the Up and Down quarks and
antiquarks, about 312.75 MeV, as the soliton and antisoliton masses,
and setting B^2 = pi and using the DHN formula,
the mass of the charged pion is calculated to be (312.75 / 2.25) MeV = 139 MeV
which is close to the experimental value of about 139.57 MeV.

Why is the value B^2 = pi the special value that gives the pion mass ?

(or, using Coleman's eq. (5.14), the Thirring coupling constant g = 3 pi)

Because B^2 = pi is where the First-order weak coupling expansion substantially coincides with the (probably exact) DHN formula. In other words,

The physical quark - antiquark pion lives where the first-order weak coupling expansion is exact.

Appendix - Planck Mass as Superposition Fermion Condensate

At a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum sum of all possible virtual first-generation particle-antiparticle fermion pairs allowed by the Pauli exclusion principle to live on that vertex.

Once a Planck-mass black hole is formed, it is stable in the E8 model.
Less mass would not be gravitationally bound at the vertex.
More mass at the vertex would decay by Hawking radiation.

There are 8 fermion particles and 8 fermion antiparticles
for a total of 64 particle-antiparticle pairs.
Of the 64 particle-antiparticle pairs, 12 are bosonic pions.

A typical combination should have about 6 pions so
it should have a mass of about .14x6 GeV = 0.84 GeV.

Just as the pion mass of .14 GeV is less than the sum of the masses of a quark and an antiquark, pairs of oppositely charged pions may form a bound state of less mass than the sum of two pion masses.

If such a bound state of oppositely charged pions has a mass as small as .1 GeV,
and if the typical combination has one such pair and 4 other pions,
then the typical combination could have a mass in the range of 0.66 GeV.

Summing over all 2^64 combinations,
the total mass of a one-vertex universe should give a Planck mass roughly around
0.66 x 2^64 = 1.217 x 10^19 GeV.

The value for the Planck mass given in by the 1998 Particle Data Group is 1.221 x 10^19 GeV.

Appendix - Lagrangian Terms

Gauge Gravity and Standard Model terms of Lagrangian have total weight 28 x 1 = 28
12 generators for SU(3) and U(2) Standard Model +
+ 16 generators for U(2,2) of Conformal Gravity =
= 28 D4 Gauge Bosons each with 8-dim Lagrangian weight = 1

Fermion Particle-AntiParticle term also has total weight 8 x (7/2) = 28
8 Fermion Particle/Antiparticle types each with 8-dim Lagrangian weight = 7/2

Since Boson Weight 28 = Fermion Weight 28
the Cl(16)-E8 model has a Subtle SuperSymmetry and is UltraViolet Finite.

The 8 fermion particles each have dimension 7/2
because they are in the spinor fermion term of the Lagrangian.

In his part of the book
Elemetary Particles and the Laws of Physics:
The 1986 Dirac Memorial Lectures
Steven Weinberg described the Lagrangian in 4-dimensional spacetime
as in the attached pdf excerpt

WeinbergLag....pdf (732 KB)

in which he shows that in 4-dim spacetime (the base manifold over
which his Lagrangian is integrated)
the fermion particles each have dimension 3/2
because the fermion term is the product of two fermions and
a mass term which has dimension 1,
so that
3/2 + 3/2 + 1 = 4 = spacetime dimension.

In my model with 8-dim Kaluza-Klein spacetime,
my Lagrangian must be integrated over 8 dimensions
so the fermion term (also product of two fermions and one mass)
must have dimension 7/2
for the Lagrangian dimensionality formula
7/2 + 7/2 + 1 = 8 = spacetime dimension
to hold true.

Here is how the top of the last page of the Weinberg excerpt
should look for my 8-dimensional Kaluza-Klein spacetime:

All terms in the Lagrangian density must have units $[\text{mass}]^8$, because length and time have units of inverse mass and the Lagrangian density integrated over spacetime must have no units. From the $m\bar{\psi}\psi$ term, we see that the electron field must have units $[\text{mass}]^{7/2}$, because $\frac{7}{2} + \frac{7}{2} + 1 = 8$

$$\begin{aligned}\mathscr{L} = & -\bar{\psi}\left(\gamma^{\mu}\frac{\partial}{\partial x^{\mu}} + m\right)\psi \\ & -\frac{1}{4}\left(\frac{\partial A_{\nu}}{\partial x^{\mu}} - \frac{\partial A_{\mu}}{\partial x^{\nu}}\right)^{2} \\ & + \mathrm{i}eA_{\mu}\bar{\psi}\gamma^{\mu}\psi \\ & -\mu\left(\frac{\partial A_{\nu}}{\partial x^{\mu}} - \frac{\partial A_{\mu}}{\partial x_{\nu}}\right)\bar{\psi}\sigma^{\mu\nu}\psi \\ & -G\bar{\psi}\psi\bar{\psi}\psi + \cdots . \qquad (1)\end{aligned}$$

It may not mean very much to most of you; on the other hand it means a lot to some of you! Fortunately almost all of the details are irrelevant for the points that I want to make. Let me explain briefly what all the symbols mean. $\mathscr{L}$ stands for Lagrangian density; roughly speaking you can think of it as the density of energy. Energy is the quantity that determines how the state vector rotates with time, so this is the role that the Lagrangian density plays; it tells us how the system evolves. It's written as a sum of products of fields and their rates of change. ψ is the field of the electron (a function of the spacetime position x), and m is the mass of the electron. $\partial/\partial x^{\mu}$ means the rate of change of the field with position. γ^{μ} and $\sigma^{\mu\nu}$ are matrices about which I will say nothing, except that the γ^{μ} matrices are called Dirac matrices. A_{μ} is the field of the photon, called the electromagnetic field.

Looking in order at each term on the right-hand side of the equation, the first term involves the electron field twice, the next term involves the photon field twice because the bracket is squared, the third and fourth terms involve two electron fields and one photon field, the fifth term involves four electron fields, and so on. The symmetries of quantum electrodynamics give us well-defined rules for the construction of the terms in the Lagrangian, but there are an infinite number of terms allowed, with increasing numbers of fields, and also increasing numbers of derivative operators acting on them. Each term has an independent constant, called the coupling constant, that multiplies it. These are the quantities e, μ, $G, \ldots$ in (1). The coupling constant gives the strength with which the term affects the dynamics. No coupling constants appear in the first two terms simply because I have chosen to absorb them into the definition of the two fields ψ and A_μ. If there were a constant in front of the first term, for example, I would just redefine ψ to absorb it. But for all the other terms, infinity minus two of them, there is a constant in front of each term. In principle all these constants are there, and they are all unknown. How in the world can you make any money out of a theory like this?

In fact, it's not that bad. Experimentally we know that the formula consisting of just the first three terms, with all higher terms neglected, is adequate to describe electrons and photons to a fantastic level of accuracy. This theory is known as quantum electrodynamics, or QED.

it would be very easy to figure out what contribution an observable gets from its cloud of virtual photons and electron–positron pairs at very high energy E. Let's suppose an observable $\mathcal{O}$ has dimensions $[\text{mass}]^{-\alpha}$, where α is positive. (Of course, since the speed of light is one in these natural units, mass and energy are essentially the same quantity.) Now, at very high virtual-particle energy, E, much higher than any mass, or any energy of a particle in the initial or final state, there is nothing to fix a unit of energy. The contribution of high energy virtual particles to the observable $\mathcal{O}$ must then be given an integral like

$$\mathcal{O} = \int^{\infty} \frac{\mathrm{d}E}{E^{\alpha+1}} \tag{3}$$

because this is the only quantity which has the right dimensions, the right units, to give the observable $\mathcal{O}$. (The lower bound in the integral is some finite energy that marks the dividing line between what we call high and low energy.) This argument only works because there are no other quantities in the theory that have the units of mass or energy. All physicists use this sort of argument from time to time, especially when they can't think of anything else to do.

On the other hand, suppose that there are other constants around that have units of mass raised to a negative power. Then if you have an expression involving a constant C_1 with units $[\text{mass}]^{-\beta_1}$, and another constant C_2 with units $[\text{mass}]^{-\beta_2}$ and so on, then instead of the simple answer obtained above we get a sum of terms of the form

$$\mathcal{O} = C_1 C_2 \cdots \int^{\infty} \frac{E^{\beta_1 + \beta_2 + \cdots}}{E^{\alpha+1}} \, \mathrm{d}E \qquad (4)$$

again because these are the only quantities that have the right units for the observable $\mathcal{O}$. Expression (3) is perfectly well-defined, the integral converges (it doesn't add up to infinity), as long as the number α is greater than zero. However, if $\beta_1 + \beta_2 + \cdots$ is greater than α, then (4) will not be well-defined, because the numerator will have more powers of energy than the denominator and so the integral will diverge. The point is that no matter how many powers of energy you have in the denominator, i.e. no matter how large α is, (4) eventually will diverge when you get up to sufficiently high order in the coupling constants, C_1, C_2, etc., that have dimensions of negative powers of mass, because if you have enough of these constants, then eventually $\beta_1 + \cdots$ is greater than α.

Looking at the Lagrangian density in (1), we can easily work out what the units of the constant e, μ,G, etc., are.

All terms in the Lagrangian density must have units $[\text{mass}]^4$, because length and time have units of inverse mass and the Lagrangian density integrated over spacetime must have no units. From the $m\bar{\psi}\psi$ term, we see that the electron field must have units $[\text{mass}]^{3/2}$, because $\frac{3}{2} + \frac{3}{2} + 1 = 4$. The derivative operator (the rate of change operator) has units of $[\text{mass}]^1$, and so the photon field also has units $[\text{mass}]^1$. Now we can work out what the units of the coupling constants are. As I said before, the electric charge turns out to be a pure number, to have no units. But then as you add more and more powers of fields, more and more derivatives, you are adding more and more quantities that have units of positive powers of mass, and since the Lagrangian density has to have fixed units of $[\text{mass}]^4$, therefore the mass dimensions of the associated coupling constants must get lower and lower, until eventually you come to constants like μ and G which have negative units of mass. (Specifically, μ has the units of $[\text{mass}]^{-1}$, while G has the units $[\text{mass}]^{-2}$.) Such terms in (1) would completely spoil the agreement between theory and experiment for the magnetic moment of the electron, so experimentally we can say that they are not there to a fantastic order of precision, and for many years it seems that this could be explained by saying that such terms must be excluded because they would give infinite results, as in (4).

Appendix - E8 Fermionic AntiCommutators

Pierre Ramond has shown in hep-th/0112261 as shown that the exceptional Lie Algebra F4 can be described using anticommutators as well as commutators.
The periodicity property of Real Clifford Algebras shows that E8 Spinor Fermions can also be described using anticommutators as well as commutators
so that the E8 Physics model describes both Bosons and Fermions realistically.

Realistic Physics models must describe both
integer-spin Bosons whose statistics are described by commutators
(examples are Photons, W and Z bosons, Gluons, Gravitons, Higgs bosons)
and
half-integer-spin Fermions whose statistics are described by anticommutators.
(examples are 3 generations of Electrons, Neutrinos, Quarks and their antiparticles)

Lie Algebra elements are usually described by commutators of their elements
so
if a Physics model attempts to describe both Bosons and Fermions as elements of a single unifiying Lie Algebra (for example, Garrett Lisi's E8 TOE)
a common objection is:

since the Lie Algebra is described by commutators,
it can only describe Bosons and cannot describe Fermions
therefore
models (such as Garrett Lisi's) using E8 as a single unifying Lie Algebra
violate the consistency of spin and statistics
and are wrong.

However,
Pierre Ramond has shown in hep-th/0112261 as shown that the exceptional Lie Algebra F4 can be described using anticommutators as well as commutators.

The periodicity property of Real Clifford Algebras shows that E8 inherits from F4
a description using anticommutators as well as commutators so that
it may be possible to construct a realistic Physics model that uses
the exceptional Lie Algebra E8 to describe both Bosons and Fermions.

Here are relevant quotes from hep-th/0112261 by Pierre Ramond:
"... exceptional algebras relate tensor and spinor representations
of their orthogonal subgroups,
while Spin-Statistics requires them to be treated differently ...
all representations of the exceptional group F4 are generated by three sets
of oscillators transforming as 26. We label each copy of 26 oscillators as

Ak_0 ,   Ak_i , i = 1, ... , 9,   Bk_a , a = 1, ... , 16,

and their hermitian conjugates, and where k = 1, 2, 3.
...

One can ... use a coordinate representation of the oscillators by introducing real coordinates
...[for A_i]... which transform as transverse space vectors,
...[for A_0]... which transform ... as scalars,
and ...[for B_a]... which transform ... as space spinors which satisfy Bose commutation rules
...
Under SO(9), the Ak_i transform as 9, Bk_a transform as 16, and Ak_0 is a scalar.
They satisfy the commutation relations of ordinary harmonic oscillators ...
Note that the SO(9) spinor operators satisfy Bose-like commutation relations ...
both A_0 and B_a ... obey Bose commutation relations
...
Curiously,
if both ... A_0 and B_a ... are anticommuting, the F4 algebra is still satisfied ...".

To see how the anticommuting property of the 16 B_a elements of F4
can be inherited by some of the elements of E8,
consider that 52-dimensional F4 is made up of:

28-dimensional D4 Lie Algebra Spin(8) (in commutator part of F4)
8-dimensional D4 Vector Representation V8 (in commutator part of F4)
8-dimensional D4 +half-Spinor Representation S+8 (in anticommutator part of F4)
8-dimensional D4 -half-Spinor Representation S-8 (in anticommutator part of F4)

Since 28-dimensional D4 Spin(8) is the BiVector part BV28
of the Real Clifford Algebra Cl(8) with graded structure
Cl(8) = 1 + V8 + BV28 + 56 + 70 + 56 + 28 + 8 + 1
and with Spinor structure
Cl(8) = (S+8 + S-8) x (8 + 8)

F4 can be embedded in Cl(8) (blue commutator part, red anticommutator part):

F4 = V8 + BV28 + S+8 + S-8

Note that V8 and S+8 and S-8 are related by the Triality Automorphism.

Also consider the 8-periodicity of Real Clifford Algebras,
according to which for all N

Cl(8N) = Cl(8) x ...(N times tensor product)... Cl(8)

so that in particular Cl(16) = Cl(8) x Cl(8)
where Cl(16) graded structure is 1 + 16 + BV120 + 560 + ... + 16 + 1
and Cl(16) Spinor structure is ((S+64 + S-64) + (64 + 64)) x (128 + 128)
and Cl(16) contains 248-dimensional E8 as

E8 = BV120 + S+64 + S-64

where BV120 = 120-dimensional D8 Lie Algebra Spin(16)
and S+64 + S-64 = 128-dimensional D8 half-Spinor Representation

Consider two copies of F4 embedded into two copies of Cl(8).

For commutator structure:

The tensor product of the two copies of Cl(8) can be seen as

1 + V8 + BV28 + 56 + 70 + 56 + 28 + 8 + 1
x
1 + V8 + BV28 + 56 + 70 + 56 + 28 + 8 + 1

which produces the Real Clifford Algebra Cl(16) with graded structure

1 + 16 + BV120 + 560 + 1820 + ... + 16 + 1

where the Cl(16) BiVector BV120 is made up of 3 parts

BV120 = BV28x1 + 1xBV28 + V8xV8

that come from the V8 and BV28 commutator parts of the two copies of F4.

This gives the commutator part of E8 as BV120 inheriting commutator structure from the two copies of F4 embedded in two copies of Cl(8) whose tensor product produces Cl(16) containing E8.

For anticommutator structure:

The tensor product of the two copies of 256-dim Cl(8) can also be seen as

((S+8 + S-8) x (8 + 8))
x
((S+8 + S-8) x (8 + 8))

which produces the 2^16 = 65,536 = 256x256-dim Real Clifford Algebra Cl(16)

((S+8 + S-8) x (S+8 + S-8))
x
((8 + 8) x (8 + 8))

with 256-dimensional Spinor structure

((S+8 + S-8) x (S+8 + S-8)) =

= ((S+8 x S+8) + (S-8 x S-8)) + ((S+8 x S-8) + (S-8 x S+8))

that comes from the S+8 and S-8 anticommutator parts of the two copies of F4.

Since the (S+8 x S-8) and (S-8 x S+8) terms inherit mixed helicities from F4

only the (S+8 x S+8) and (S-8 x S-8) terms inherit consistent helicity from F4.

Therefore, define S+64 = (S+8 x S+8) and S-64 = (S-8 x S-8)
so that
(S+64 + S-64) = 128-dimensional D8 half-Spinor Representation

This gives the anticommutator part of E8 as S+64 + S-64 inheriting anticommutator structure from the two copies of F4 embedded in two copies of Cl(8) whose tensor product produces Cl(16) containing E8.

The result is that 248-dimensional E8 is made up of:

BV120 = 120-dimensional D8 Lie Algebra Spin(16) (commutator part of E8)

128-dimensional (S+64 + S-64) D8 half-Spinor (anticommutator part of E8)

Note that since the V8 and S+8 and S-8 components of F4 are related by Triality,
and since
the E8 component BV120 contains 64-dimensional V8xV8
and
the 64-dimensional E8 component S+64 = S+8 x S+8
and
the 64-dimensional E8 component S-64 = S-8 x S-8

E8 inherits from the two copies of F4 a Triality relation

V8xV8 = S+64 = S-64

The commutator - anticommutator structure of E8 allows construction of realistic Physics models that not only unify both Bosons and Fermions within E8
but
also contain Triality-based symmetries between Bosons and Fermions
that can give the useful results of SuperSymmetry
without requiring conventional SuperPartner particles that are unobserved by LHC.

CONCLUSION:

Unified E8 Physics models can be constructed without violating spin-statistics.

Appendix - Details of Coleman-Mandula

The Cl(16)-E8 model has 8-dim Lorentz structure satisfying Coleman-Mandula because its fermionic fundamental spinor representations are built with respect to spinor representations for 8-dim Spin(1,7) spacetime.

The Quantum Theory of Fields

Volume III
Supersymmetry

Steven Weinberg
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32

Supersymmetry Algebras in Higher Dimensions

Ever since the ground-breaking work of Kaluza[1] and Klein,[2] theorists have from time to time tried to formulate a more nearly fundamental physical theory in spacetimes of higher than four dimensions. This approach was revived in superstring theories, which take their simplest form in 10 spacetime dimensions.[3] More recently, it has been suggested that the various versions of string theory may be unified in a theory known as *M theory*, which in one limit is approximately described by supergravity in 11 spacetime dimensions.[4] In this chapter we shall catalog the different types of supersymmetry algebra possible in higher dimensions, and use them to classify supermultiplets of particles.

32.1 General Supersymmetry Algebras

Our analysis of the general supersymmetry algebra in higher dimensions will follow the same logical outline as the work of Haag, Lopuszanski, and Sohnius[5] on supersymmetry algebras in four spacetime dimensions, described in Section 25.2. The proof of the Coleman–Mandula theorem in the appendix of Chapter 24 makes it clear that the list of possible bosonic symmetry generators is essentially the same in $d > 2$ spacetime dimensions as in four spacetime dimensions: in an S-matrix theory of particles, there are only the momentum d-vector P^μ, a Lorentz generator $J^{\mu\nu} = -J^{\nu\mu}$ (with μ and ν here running over the values $1, 2, \ldots, d-1, 0$), and various Lorentz scalar 'charges.' (In some theories there are topologically stable extended objects such as closed strings, membranes, etc., in addition to particles, which make possible other conserved quantities, to which we will return in Section 32.3.) The anticommutators of the fermionic symmetry generators with each other are bosonic symmetry generators, and therefore must be a linear combination of P^μ, $J^{\mu\nu}$, and various conserved scalars. This puts severe limits on the Lorentz transformation properties of the fermionic generators, and on the superalgebra to which they belong.

We will first prove that the general fermionic symmetry generator must transform according to the fundamental spinor representations of the Lorentz group, which are reviewed in the appendix to this chapter, and not in higher spinor representations, such those obtained by adding vector indices to a spinor. As we saw in Section 25.2, the proof for $d = 4$ by Haag, Lopuszanski, and Sohnius made use of the isomorphism of $SO(4)$ to $SU(2) \times SU(2)$, which has no analog in higher dimensions. Here we will use an argument of Nahm,[6] which is actually somewhat simpler and applies in any number of dimensions.

Since the Lorentz transform of any fermionic symmetry generator is another fermionic symmetry generator, the fermionic symmetry generators furnish a representation of the homogeneous Lorentz group $O(d-1,d)$ (or, strictly speaking, of its covering group $Spin(d-1,1)$). Assuming that there are at most a finite number of fermionic symmetry generators, they must transform according to a finite-dimensional representation of the homogeneous Lorentz group. All of these representations can be obtained from the finite-dimensional *unitary* representations of the corresponding orthogonal group $O(d)$ (actually $Spin(d)$) by setting $x^d = ix^0$. So let us first consider the transformation of the fermionic generators under $O(d)$. For d even or odd, we can find $d/2$ or $(d-1)/2$ Lorentz generators J_{d1}, J_{23}, $J_{45}, \ldots$, which all commute with each other, and classify fermionic generators Q according to the values σ_{d1}, σ_{23}, ... that they destroy:

$$[J_{d1}\,, Q] = -\sigma_{d1} Q\,, \qquad [J_{23}\,, Q] = -\sigma_{23} Q\,, \qquad [J_{45}\,, Q] = -\sigma_{45} Q\,, \ldots\,. \tag{32.1.1}$$

Since the finite-dimensional representations of $O(d)$ are all unitary, the σs are all real.

Let us focus on one of these quantum numbers, $\sigma_{d1} \equiv w$ and refer to any fermionic or bosonic operator O as having *weight* w if

$$[J_{d1}\,, O] = -w\, O\,, \tag{32.1.2}$$

or, in terms of the Minkowski component $J_{01} = iJ_{d1}$,

$$[J_{01}\,, O] = -iw\, O\,. \tag{32.1.3}$$

The reason for concentrating on this particular quantum number is that it has the special property of being the same for an operator and its Hermitian adjoint. This is because J_{01} must be represented on Hilbert space (though not on field variables or symmetry generators) by a Hermitian operator, so that (remembering that w is real) the Hermitian adjoint of Eq. (32.1.3) is

$$-[J_{01}\,, O^*] = +iwO^*\,, \tag{32.1.4}$$

so O^* has the same weight as O.

Now consider the anticommutator $\{Q, Q^*\}$ of any fermionic symmetry generator Q with its Hermitian adjoint. According to the Coleman–Mandula theorem, it is at most a linear combination of P_μ, $J_{\mu\nu}$, and scalars. To calculate the weights of the components of P_μ, we recall the commutation relation (2.4.13)

$$i[P_\mu, J_{\rho\sigma}] = \eta_{\mu\rho}P_\sigma - \eta_{\mu\sigma}P_\rho \,,$$

which shows that $P_0 \pm P_1$ has weight $w = \pm 1$, while the other components $P_2, P_3, \ldots, P_{d-1}$ all have weight zero. In the same way, the commutation relation (2.4.12) of the $J_{\mu\nu}$ with each other show that $J_{0i} \pm J_{1i}$ with $i = 2, 3, \ldots d-1$ have weight $w = \pm 1$, the J_{ij} with both i and j between 2 and $d-1$ have weight zero, J_{10} has weight zero, and of course all scalars have weight zero. We conclude then that all bosonic symmetry generators have weight ± 1 or 0 and the anticommutator $\{Q, Q^*\}$ must be a linear combination of operators with such weights. If Q has weight w then $\{Q, Q^*\}$ has weight $2w$, and it is manifestly non-zero for any non-zero Q, so each fermionic generator can only have weight $\pm 1/2$. (Weight zero is excluded by the connection between spin and statistics — fermionic operators can only be constructed from odd numbers of operators with half-integer weights.) Going back to the Euclidean formalism, since the commutators of the particular $O(d)$ generator J_{01} with all generators Q in a representation of $O(d)$ are given by Eq. (32.1.2) with $w = \pm 1/2$, and there is nothing special about the 01 plane, $O(d)$ invariance requires that the same is true for all $O(d)$ generators J_{ij}, so that all the σs in Eq. (32.1.1) are $\pm 1/2$. The only irreducible representations of the homogeneous Lorentz group with all σs equal to $\pm 1/2$ are the fundamental spinor representations, so Q must belong to some direct sum of these representations.

We can also use this approach to show that the fermionic generators Q all commute with the d-momentum P_μ. For this purpose, note that the double commutator of a momentum operator $P_0 \pm P_1$ of weight ± 1 with any fermionic generator Q would have weight either $\pm 5/2$ if Q has weight $\pm 1/2$ or weight $\pm 3/2$ if Q has weight $\mp 1/2$, and since we have found that there are no fermionic symmetry generators of weight $\pm 3/2$ or $\pm 5/2$, these double commutators must all vanish:

$$[P_0 \pm P_1, [P_0 \pm P_1, Q]] = 0 \,.$$

It follows then that

$$[P_0 \pm P_1, [P_0 \pm P_1, \{Q, Q^*\}]] = -2\{Q_\pm, Q_\pm^*\} \,,$$

where

$$Q_\pm \equiv [P_0 \pm P_1, Q] \,.$$

Now, $\{Q, Q^*\}$ is at most a linear combination of Js, Ps, and scalar symmetry generators. The commutators of $P_0 \pm P_1$ with the Ps and scalar symmetry generators vanish, while the commutators of $P_0 \pm P_1$ with the Js are linear combinations of Ps, which commute with the other $P_0 \pm P_1$, so the double commutator $[P_0 \pm P_1, [P_0 \pm P_1, \{Q, Q^*\}]]$ must vanish and therefore $\{Q_\pm, Q_\pm^*\} = 0$, which implies that $Q_\pm = 0$. Since *all* members of the representation of the Lorentz group provided by the Qs thus commute with P_0 and P_1, Lorentz invariance implies that all Qs commute with all Ps, as was to be shown.

There is an important corollary that since the Lorentz generators $J_{\mu\nu}$ do not commute with the momentum operators, they cannot appear on the right-hand side of the anticommutation relations. For the moment let us label the Qs as Q_n, where n runs over the labels for the different (not necessarily inequivalent) irreducible spinor representations among the Qs, now *including* their adjoints Q^*, and also over the index labelling members of these representations. The general anticommutation relation is then of the form

$$\{Q_n\,, Q_m\} = \Gamma^\mu_{nm} P_\mu + Z_{nm}\,, \tag{32.1.5}$$

where the Γ^μ_{nm} are c-number coefficients and the Z_{nm} are conserved scalar symmetry generators, which commute with the P_μ and $J_{\mu\nu}$. We now want to show that the Z_{nm} are *central charges* of the supersymmetry algebra — that is, that they commute with the Q_ℓ and each other as well as with the P_μ and $J_{\mu\nu}$ and all other symmetry generators.

To prove this for $d \geq 4$, note that for a given Z_{nm} to be non-zero, since it is a scalar all of the σs in Eq. (32.1.1) must be opposite for Q_n and Q_m. Consider another fermionic symmetry generator Q_ℓ, for which the σs of Eq. (32.1.1) are not all the same as those of either Q_n or Q_m. (For $d \geq 4$ there is always such a Q_ℓ in each set of Qs forming an irreducible spinor representation of $O(d)$.) We apply the super-Jacobi identity

$$[Q_\ell\,, \{Q_m\,, Q_n\}] + [Q_m\,, \{Q_n\,, Q_\ell\}] + [Q_n\,, \{Q_\ell\,, Q_m\}] = 0\,. \tag{32.1.6}$$

The anticommutators $\{Q_n\,, Q_\ell\}$ and $\{Q_\ell\,, Q_m\}$ are operators that have some σs non-zero, so they can only be linear combinations of Ps rather than Zs, and so must commute with all Qs. This leaves just

$$0 = [Q_\ell\,, \{Q_m\,, Q_n\}] = [Q_\ell\,, Z_{mn}]\,. \tag{32.1.7}$$

Thus in each set of Qs forming an irreducible spinor representation of $O(d)$ there is at least one that commutes with the given Z_{mn}. But Z_{mn} is a Lorentz scalar, so it must then commute with all Qs. It follows then immediately from Eq. (32.1.5) that they also commute with each other.

The fermionic generators must form a representation (perhaps trivial) of the algebra $\mathscr{A}$ consisting of *all* scalar bosonic symmetry generators. It follows then by precisely the same argument used in Section 25.2 that the central charges Z_{mn} furnish an invariant Abelian subalgebra of $\mathscr{A}$. The Coleman–Mandula theorem tells us that $\mathscr{A}$ must be a direct sum of a compact semi-simple Lie algebra, which by definition contains no invariant Abelian subalgebras, together with $U(1)$ generators, so the Z_{mn} must be $U(1)$ generators, which commute with all other bosonic symmetry generators, not just with each other.

To obtain more detailed information about the structure of the anticommutation relations (32.1.5), we must be more specific about the Lorentz transformation and reality properties of the fermionic symmetry generators Q_n. These are very different for spacetimes of even and odd dimensionality.

Odd Dimensionality

The appendix to this chapter shows that for odd spacetime dimensions d there is just one fundamental spinor representation of the Lorentz algebra, by matrices $\mathscr{J}_{\mu\nu}$ given in terms of Dirac matrices by Eq. (32.A.2), so we must label the fermionic generators as $Q_{\alpha r}$, where α is a $2^{(d-1)/2}$-valued Dirac index, and $r = 1, 2, \ldots, N$ labels different spinors in the case of N-extended supersymmetry. With this notation, the Lorentz transformation properties of the Qs imply that

$$[J_{\mu\nu}, Q_{\alpha r}] = -\sum_{\beta}(\mathscr{J}_{\mu\nu})_{\alpha\beta}\, Q_{\beta r}\,, \qquad (32.1.8)$$

so that the anticommutators of these generators have the transformation rule

$$[J_{\mu\nu}, \{Q_{\alpha r}, Q_{\beta s}\}] = -\sum_{\bar\alpha}(\mathscr{J}_{\mu\nu})_{\alpha\bar\alpha}\{Q_{\bar\alpha r}, Q_{\beta s}\} - \sum_{\bar\beta}(\mathscr{J}_{\mu\nu})_{\beta\bar\beta}\{Q_{\alpha r}, Q_{\bar\beta s}\}\,.$$

Recalling the Lorentz transformation rule (2.4.13) for the momentum operator P_λ, we see that the matrix Γ^λ_{rs} and the operator Z_{rs} in Eq. (32.1.5) (with Dirac indices now suppressed) must satisfy the conditions

$$\mathscr{J}_{\mu\nu}(\Gamma_\lambda)_{rs} + (\Gamma_\lambda)_{rs}\mathscr{J}^{\mathrm{T}}_{\mu\nu} = -i\,(\Gamma_\mu)_{rs}\eta_{\nu\lambda} + i\,(\Gamma_\nu)_{rs}\eta_{\mu\lambda}\,, \qquad (32.1.9)$$

$$\mathscr{J}_{\mu\nu}Z_{rs} + Z_{rs}\mathscr{J}^{\mathrm{T}}_{\mu\nu} = 0\,. \qquad (32.1.10)$$

But Eq. (32.A.38) gives $\mathscr{J}^{\mathrm{T}}_{\mu\nu} = -\mathscr{C}^{-1}\mathscr{J}_{\mu\nu}\mathscr{C}$, so Eqs. (32.1.9) and (32.1.10) may be expressed as the requirement that $(\Gamma_\mu)_{rs}\mathscr{C}^{-1}$ satisfies the same commutation relation (32.A.32) with $\mathscr{J}_{\mu\nu}$ as γ_μ, while $Z_{rs}\mathscr{C}^{-1}$ commutes with $\mathscr{J}_{\mu\nu}$. For odd d the matrices satisfying these conditions are unique up to multiplication with constants, so we can conclude that

$$\Gamma^\lambda_{\alpha r\,\beta s} = i\,g_{rs}\,(\gamma^\lambda\mathscr{C})_{\alpha\beta} \qquad (32.1.11)$$

Appendix - Details of Mayer - Higgs

New Developments in Mathematical Physics

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FOREWORD

The papers contained in this volume are lectures and seminars presented at the 20th "Universitätswochen für Kernphysik" in Schladming in February 1981. The goal of this school was to review some rapidly developing branches in mathematical physics. Thanks to the generous support provided by the Austrian Federal Ministry of Science and Research, the Styrian Government and other sponsors, it has been possible to keep up with the - by now already traditional - standards of this school. The lecture notes have been reexamined by the authors after the school and are now published in their final form, so that a larger number of physicists may profit from them. Because of necessary limitations in space all details connected with the meeting have been omitted and only brief outlines of the seminars were included. It is a pleasure to thank all the lecturers for their efforts, which made it possible to speed up the publication. Thanks are also due to Mrs. Krenn for the careful typing of the notes.

H.Mitter L.Pittner

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A BRIEF INTRODUCTION TO THE GEOMETRY OF GAUGE FIELDS+

by

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1. INTRODUCTION

In view of the common background required for the understanding of the lectures of both authors, and in order to avoid unnecessary duplications, we have decided to present jointly this brief introduction to the language and properties of fiber bundles. By now the advantages of the fiber-bundle formulation of gauge field theories have led to a widespread acceptance of this language, and a number of reviews of the subject have appeared or are in course of publication. These, together with a number of standard textbooks are listed in the references to this

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introduction. Nevertheless, we felt that it would be convenient for the reader of these proceedings to have at his disposal a summary of the basic facts. We also tried to clarify a number of concepts and propose an acceptable terminology wherever a standard has not been established in the literature. This refers, in particular, to the terms gauge transformation , pure gauge transformation, and the related (infinite-dimensional) groups as well as to the concepts of extension, prolongation, restriction, and reduction of bundles, which are used with slightly varying meaning in different texts.

In the oral presentation most of the general background material was presented by Andrzej Trautman, and the material related to reduction and symmetry of connections was given in Meinhard Mayer's lectures. Little, if anything, in this introduction is original. The actual text has been written in California by the first author and slightly revised by the second during his stay in France after the Schladming meeting.

No detailed proofs are given here, but wherever possible illustrations and examples are used to make the concepts plausible to physicists. Many proofs are straightforward and can be carried out by introducting local coordinates and bases. However, we recommend to the reader who wants to become familiar with the spirit of modern, coordinate-free, differential geometry to try to stay away from bases and indices as much as possible.

It is easy to see that the orbit space of P under the action of the subgroup H of G, P/H, can be identified with the associated bundle E. Denoting by γ the canonical projection of G onto G/H, we can set for $p \in P, \delta(p) = p \cdot \gamma(e)$, where e is the identity of G. The mapping $\delta : P \to E$ is a projection for the new principal bundle (P, H, E, δ) over the larger base $E = P \times_G G/H$ which is canonically identified with the orbit space P/H (this is illustrated in Fig. 4, in the middle).

Let now $\sigma : M \to E$ denote a section of E and $\sigma^* : (P, H, E, \delta) = (Q, H, M, \rho)$ the pullback (induced bundle) of this map. It is obvious (cf. Fig. 4, right) that this is now a principal bundle with structure group H over M, and its extension to G is isomorphic to the original bundle P. Two different sections σ_1 and σ_2 of E will define isomorphic restrictions iff they are mapped into each other by a pure gauge transformation (G-M-automorphism) of P. Otherwise different sections of E determine different (nonisomorphic) restrictions.

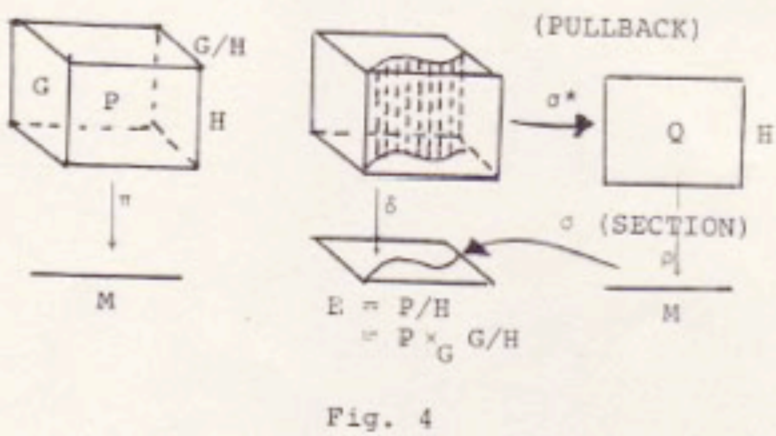

Fig. 4

6. INVARIANCE AND SYMMETRIES OF CONNECTIONS

In spite of the fact that conditions for the invariance of a connection have been discussed in the mathematical literature over twenty years ago, and Wang's theorem can be found in textbooks, physicists rediscovered them only in 1978-79. This section contains a brief survey of this topic, which has been discussed from a more physical point of view by Jackiw in last year's Schladming lectures.

The problem is quite simple when viewed globally, on the principal bundle; complications arise only when one tries to express the invariance conditions for the connection forms on local trivializations of P.

Before discussing connections we summarize the definitions of gauge transformations to be used. An isomorphism of a principal bundle onto itself is called an <u>automorphism</u> of the bundle. Such an automorphism consists of a pair of diffeomorphisms (u,v) of P and M such that $\pi \circ u = v \circ \pi$ (Eq.(3.1)), and $u(p\cdot g) = u(p)\cdot g$ for all $p \in P$, $g \in G$. An automorphism is called vertical if $v = Id_M$. If we denote the group of all automorphisms (an infinite-dimensional group) by Aut P, the subgroup of all vertical automorphisms $Aut_M P$ is a normal subgroup, the quotient being the group of all diffeomorphisms of M onto itself, i.e., we have the exact sequence of homomorphisms:

$$I \to Aut_M P \xrightarrow{\;i\;} Aut\, P \xrightarrow{\;j\;} Diff\, M \to I \,, \qquad (6.1)$$

where i is the canonical injection and $v = j(u)$. If $u \in Aut_M P$, its action is in the fiber and therefore can be implemented by an element U(p) of G such that for any p in P and g in G

$$u(p) = p\cdot U(p)\,,\; U(p\cdot g) = g^{-1}U(p)g \quad . \qquad (6.2)$$

Thus, there is a natural isomorphism of $\mathrm{Aut}_M P$ onto the multiplicative group of (smooth) maps $U : P \to G$, subject to the equivariance condition (6.2), or equivalently, to sections of the associated bundle $P \times_{\mathrm{Ad}G} G$ with fibers G, but the right action replaced by the adjoint action.

The group Aut P (as well as $\mathrm{Aut}_M P$) acts on (local) sections of P in the following manner: if $s : V \to P$ (V an open subset of M), then its transform is $s' = u \circ s \circ v^{-1}$. If $u \in \mathrm{Aut}_M P$, the subset V of M is left invariant and the section is subject to what a physicist would call a gauge transformation:

$$s'(x) = s(x)\cdot U(s(x)), \quad x \in V \subset M \quad . \tag{6.3}$$

If one deals only with Yang-Mills fields over a flat spacetime (or a Euclidean, compact version thereof) one is thus entitled to identify $\mathrm{Aut}_M P$ with the group of gauge transformations (this is the definition adopted by Atiyah, Singer, and many other mathematicians). However, in theories involving gravity, or other structures on spacetime, it is convenient to introduce a further differentiation.

Definition. The gauge group of a theory in which the bundle has some absolute elements, such as the metric tensor of special relativity, or some other structure element of P or M, is the subgroup $\mathcal{G}$ of Aut P such that the diffeomorphism v and the projection preserve the absolute elements of M. The group of pure gauge transformations consists of the vertical automorphisms in $\mathcal{G}$; this group will be denoted by $\mathcal{G}_0 = \mathcal{G} \cap \mathrm{Aut}_M P$, it is a normal subgroup of $\mathcal{G}$, and the quotient $\mathcal{G}/\mathcal{G}_0$ in the exact sequence

$$I \to \mathcal{G}_0 \xrightarrow{i} \mathcal{G} \xrightarrow{j} \mathcal{G}/\mathcal{G}_0 \to I \tag{6.4}$$

is the subgroup of Diff M leaving the absolute elements invariant (e.g., if M is Minkowski space, $\mathcal{G}/\mathcal{G}_o$ is the Poincaré group; this corresponds to the necessity of sometimes combining a gauge transformation with a change of Lorentz frame in some calculations).

Invariance of connections under automorphisms of the bundle P is simply expressed as the fact that the pullback of the connection form ω on P by the mapping $u \in \text{Aut } P$, $\omega' = u^{*}\omega$ is again a connection form on P. If u is a vertical automorphism (in particular, a pure gauge transformation), then

$$\omega' = \text{Ad}(U^{-1}(p))\omega + U^{-1}(p)dU(p) \quad , \tag{6.5}$$

where U(p) is the map defined in Eq.(6.2). We see that the form ω is subject to the usual gauge transformation of a gauge potential (albeit, on P rather than on M). The curvature form Ω' of the pullback $u^{*}\omega$ is given by the adjoint action of U(p) on the original curvature form:

$$\Omega' = \text{Ad}(U^{-1}(p))\Omega \quad . \tag{6.6}$$

The equations (6.5), (6.6) can easily be pulled down to the forms A, F on the base space given by a locally trivializing section s. Here one can either pull ω back to M by the transformed section, or pull ω' back by the original section, obtaining the usual gauge transformation formulas for A and F:

$$A' = \text{Ad}(S^{-1})A + S^{-1}dS \ , \quad F' = \text{Ad}(S^{-1})F \ , \tag{6.7}$$

where $S = U \circ s$.

Among the automorphisms of the principal bundle P with a connection ω and the associated bundles carrying the particle fields, symmetries are distinguished by the fact that they preserve the connection ω and the absolute elements of the theory (e.g., they preserve the action, or they modify the Lagrangian density by a divergence). In particular, a symmetry of a gauge theory is a gauge transformation (in the wider sense defined above) which leaves the connection form ω invariant (in addition to the other absolute elements):

$$u^{*}\omega = \omega, \; u^{*}\Omega = \Omega \quad ; \tag{6.8}$$

since a nonabelian gauge theory is not completely determined by the curvature, it is not sufficient to require invariance only of the curvature form.

When this condition is pulled back by a local trivialization to the base space, it will usually be formulated as the requirement that the one-form A be unchanged up to a pure gauge transformation, or in other words, a gauge field is invariant under a symmetry, if the symmetry transformation can be compensated by a gauge transformation of the locally trivializing section (this is the formulation given by Bergmann and Flaherty, Trautman, Jackiw, and other authors).

To write the invariance condition (6.8) for the physical fields A, F, we consider first a one-parameter group $u_t : R \to \text{Aut } P$ of automorphisms of P. Let Y denote the corresponding vector field on P, and X the projection of Y onto M:

$$X = \pi_{*}Y \quad . \tag{6.9}$$

The vector field X generates a one-parameter group $v_t = j(u_t)$ of transformations on M. Let ω be a u_t-invariant connection on P,

$$u_t^{*}\omega = \omega, \quad u_t^{*}\Omega = \Omega \quad . \tag{6.10}$$

For an arbitrary point p_o in P the groups u_t, v_t define curves in P, M, respectively:

$$p_t = u_t(p_o) \ , \quad x_t = v_t(\pi p_o) = \pi(p_t) \ . \tag{6.11}$$

The connection defines a horizontal lift of x_t which we denote by h_t. Then it is obvious that $p_t = h_t g_t$ for a suitable element g_t of G, and g_t is a one-parameter Lie subgroup of G, generated by the Lie algebra element $T = \omega_{p_o}(Y)$. The invariance of the connection and its curvature on P can be expressed infinitesimally as the vanishing of their Lie derivatives with respect to Y:

$$L_Y\omega = 0 \ , \qquad L_Y\Omega = 0 \ . \tag{6.12}$$

(Recall that for forms the Lie derivative is defined by $L_Y = d \circ Y \rfloor + Y \rfloor \circ d$, where denotes the interior product of Y with the differential form following the sign.)

The expressions (6.12) for the invariance of connections are identical to the usual conditions for the invariance of fields encountered in physics, but hidden behind the simple form is the gauge freedom inherent in the theory, particularly if one works in terms of the pullbacks A, F, to the base space. If we denote the value of the one-form ω_p (at the point p in P) on the vector field Y at p by $Z = \omega_p(Y)$, we obtain an equivariant map of P into the Lie algebra $Z : P \to G$, $Z \circ R_g = \mathrm{Ad}(g^{-1}) \circ Z$. Its covariant exterior differential

$$DZ = dZ + [\omega, Z] \tag{6.13}$$

is a horizontal one-form (with values in G) of type Ad, and the definition of the Lie derivative and Eq.(6.14) yield the detailed form of the invariance condition:

$$L_Y\omega = Y \lrcorner \Omega + DZ = 0, \quad L_Y\Omega = D(Y \lrcorner \Omega) + [\Omega, Z] = 0 \qquad (6.14)$$

(Trautman, 1979). If we use a local section s to pull back the connection and curvature to the gauge potential A, and the field strength F on M, the vector field Y is to be replaced by the generator X of the transformations in M, and the Lie-algebra-valued function on P, Z, defines a function on M, $\Phi = Z \circ s : M \to G$. Then the invariance conditions for A and F under the symmetry induced on M by the vector field X (such a vector field always has a horizontal lift under the given connection; adding an arbitrary vertical vector field of the type of Z to it, will give a field on P) can be written in the form

$$X \lrcorner F + D\Phi = 0 \qquad (6.15)$$

where $D\Phi = d\Phi + [A, \Phi]$, and

$$D(X \lrcorner F) + [F, \Phi] = 0 \;. \qquad (6.16)$$

In terms of the potential one-form A the invariance condition can be rewritten as $L_X A = DW(X)$, where $W(X)$ differs from Φ by the zero-form $-X \lrcorner A$. The right-hand side of the last equation has the infinitesimal form of a gauge transformation, and under a change of chart (gauge transformation) with transition functions g_{ij} the function W is subject to the transformation

$$W_j = \mathrm{Ad}(g_{ij}^{-1})\; W_i + g_{ij}^{-1}\, X \lrcorner dg_{ij} \;. \qquad (6.17)$$

If X_1 and X_2 denote two vector fields on M inducing symmetries of the connection A, then consistency requires that

$$2F(X_1, X_2) = \Phi([X_1, X_2]) - [\Phi(X_1), \Phi(X_2)] \;, \qquad (6.18)$$

where the left-hand side denotes the value of the two-form

F on the two vector fields X_1, X_2, and the right-hand side expresses the dependence of the G-valued 0-form on the vector field X_i (and implicitly, on the trivializing section s). The infinitesimal forms of the invariance conditions have been independently discovered by Forgacs and Manton, Harnad, Shnider and Vinet, and Jackiw (cf. the bibliography to Mayer's contribution for references), and the usefulness of Eqs.(6.15), (6.18) (with a difference in sign) has been discussed in Jackiw's 1980 Schladming lectures.

To end this section we give, for the convenience of the reader, a brief statement of Wang's theorems on invariant connections, in a notation which is close to the one used by Kobayashi and Nomizu, where the detailed proofs can be found.

Consider, as before, a principal bundle P(M,G), with a connection ω which is invariant with respect to a group of automorphisms K of P(M,G), assumed to be a connected Lie group with fiber-transitive action, i.e., for any two fibers there is an element of K which maps one into the other, hence K acts transitively on the base space M. We denote by u_o a reference point in P, chosen once and for all, and by x_o its projection in M, $x_o = \pi(u_o)$. Furthermore we denote by J the isotropy subgroup of K at x_o, i.e., the subgroup of all transformations in K which leave x_o invariant (it is clear that M can then be viewed as the homogeneous space K/J). We denote the Lie algebras of the groups G, K, J by $\mathfrak{g}$, $\mathfrak{k}$, $\mathfrak{j}$, respectively and, when it exists, the subspace of $\mathfrak{k}$ complementary to $\mathfrak{j}$ by $\mathfrak{m}$: $\mathfrak{k} = \mathfrak{j} \dotplus \mathfrak{m}$ (direct sum). Then we define a linear mapping $\Lambda : \mathfrak{k} \to \mathfrak{g}$ by $\Lambda(X) = \omega_{u_o}(\tilde{X})$, where $X \in \mathfrak{k}$ and $\tilde{X}$ is the vector field on P induced by X, which has the properties

(i) $\Lambda(X) = \lambda_{*}(X)$ for $X \in \mathfrak{j}$; here λ_{*} is the homomorphism $\lambda_{*} : \mathfrak{j} \to \mathfrak{g}$ defined as the differential of the homomorphism

$\lambda : J \to G$, which assigns an element $g \in G$ taking the point u_o into the same point as the left action of $j \in J$: $ju_o = u_o g$: $g = \lambda(j)$;

(ii) for $j \in J$ and $X \in k$, $\Lambda(Ad(j)(X)) = Ad(\lambda(j))(\Lambda(X))$, where $Ad(j)$ is the adjoint action of J on k and $Ad(\lambda(j))$ is that of G on g. The geometric meaning of these homomorphisms should be clear from our discussion of the lifting of the horizontal projection of any one-parameter group of automorphisms given by Eq.(6.11) and the discussion following it. Note that u_o denotes our previous p_o (and not the value of the automorphism at $t = 0$), and the vertical action $\lambda(j)$ is the same as the previous g_t.

It is easy to verify, by using the definition of curvature (the structure equation), that the curvature form Ω satisfies the condition (from which Eq.(6.18) follows by pullback to M):

$$2\Omega_{u_o}(\tilde{X},\tilde{Y}) = [\Lambda(X),\Lambda(Y)] - \Lambda([X,Y]), \text{ for } X, Y \in k . \quad (6.19)$$

What Wang's theorem asserts is the existence of a bijection between the set of K-invariant connections in P and the set of linear mappings $\Lambda : k \to g$ satisfying the conditions listed above, bijection which is given by

$$\Lambda(X) = \omega_{u_o}(X), \text{ for } X \in k \quad . \quad (6.20)$$

The proof is straightforward and can be found, e.g., in Kobayashi and Nomizu (p.107, with the same notations as here).

It also follows immediately that a K-invariant connection is flat (i.e., has vanishing curvature) iff $\Lambda : k \to g$ is a Lie algebra homomorphism (since then the right-hand side of Eq.~~(23)~~ 6.19 vanishes, and hence so does the left-hand side).

Moreover, if in addition the Lie subalgebra j admits a complementary subspace m in κ such that $Ad(J)(m) = (m)$, then there is a bijection between the set of K-invariant connections in P and the set of linear mappings $\Lambda_m : m \to g$, such that for $X \in m$, $j \in J$ we have $\Lambda_m(Ad(j)(X)) = Ad(\lambda(j))(\Lambda_m(X))$, with the bijection given in terms of the Λ defined above by $\Lambda(X) = \lambda(X)$ if $X \in j$, and $\Lambda(X) = \Lambda_m(X)$ if $X \in m$. The curvature form of the K-invariant connection defined by the linear mapping Λ_m satisfies the following condition:

$$2\Omega_{u_0}(X,Y) = [\Lambda_m(X),\Lambda_m(Y)] - \Lambda_m([X,Y]_m) - \lambda([X,Y]_j),$$

$$X,Y \in m,$$

where the subscripts on the brackets denote components in the corresponding subspaces of the algebra κ where the bracket is originally defined. If $\Lambda_m = 0$ then the corresponding invariant connection is called the <u>canonical invariant connection</u> with respect to the decomposition $\kappa = j \dotplus m$. Physically, this corresponds to choosing the gauge functions Z and the connection A in eqs.(6.13) - (6.18) so that the components of Φ in the subspace m, corresponding to the given decomposition, should vanish.

It is to be noted that the existence of a complementary subspace m invariant under the adjoint action of J is equivalent to the reductivity of the homogeneous space $K/J = M$, a rather restrictive condition on the base space M.

Finally, it should be noted that the Lie algebra of the holonomy group of a K-invariant connection at u_0 is defined by a sum of iterated brackets of $\Lambda(\kappa)$ with the subspace m_0 of g spanned by the right-hand side of eq. (6.19) (for details we refer the reader again to Kobayashi-Nomizu, p.110-111).

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THE GEOMETRY OF SYMMETRY BREAKING IN GAUGE THEORIES+

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ABSTRACT

This together with Sections 3 and 6 of the joint contribution with A. Trautman (this volume, pp. 433 to be referred to as Mayer-Trautman) constitutes a summary of the first two lectures. Much of the material is available elsewhere [1], so only results and some open questions are discussed. The subject matter of the second two lectures is treated in the following contribution (pp. 491).

1. INTRODUCTION

The motivation of these lectures is a search for an alternative to the traditional Brout-Englert-Higgs-Kibble (BEHK) method of symmetry breaking in gauge theories, based on the geometry of principal bundles with connections. In the BEHK approach the action of the classical theory of a Dirac or Weyl field interacting with a Klein-Yang-Mills field A, F is

+Lectures given at the XX. Internationale Universitätswochen für Kernphysik, Schladming, Austria, February 17-26, 1981.

$$S_{DYM} = \frac{1}{4} \int_M \mathrm{Tr} F \wedge {}^*F + \int_M {}^*\{\psi \not{D} \psi\} \quad , \tag{1.1}$$

where M denotes the spacetime base manifold of the bundles, F is the Yang-Mills field strength two-form on M (the pull-down of the curvature two-form to M, cf. Mayer-Trautman, Eq.(4.12)), * is the Hodge-dual on M, ψ denotes a Dirac or Weyl spinor which transforms under a representation of the structure group G (a local representation of a section in a tensor product of a spin bundle and a vector bundle associated to the gauge principal bundle P by that representation). $\not{D}$ denotes the "gauge-covariant Dirac-Weyl operator" (in coordinates, with e_a a basis for the representation of the Lie algebra G, in which I denotes the unit matrix, and γ^μ the usual Dirac matrices) $\not{D} = I\not{\partial} + A^a_\mu \gamma^\mu e_a$. In order to produce the symmetry breaking, leading to a restriction of the original bundle P to a subbundle Q(M,H), the BEHK model introduces "by hand" a scalar field ϕ which represents a section of an associated bundle (or a smooth function on P with values in some representation space V of G, cf. Mayer-Trautman, Eq.(2.5)), which is supposed to be an extremal of the Ginzburg-Landau action:

$$S_{GL} = \int D\phi \wedge {}^*D + V[\phi], \; V[\phi] = -\mu\|\phi\|^2 + \lambda\|\phi\|^4 \quad , \tag{1.2}$$

where the norms in the Ginzburg-Landau functional V are to be understood as the result of integration over M of the hermitian norm in V. For positive λ,μ the functional V has nontrivial critical points ϕ_o and the stabilizer subgroup of these is H, the symmetry group of the "vacuum" to which the bundle is then restricted. Reducing the connection form A and the curvature form F to the corresponding Lie algebra H, one can choose a gauge in which the terms involving the nonvanishing ϕ_o^2 in (1.2) appear like "mass terms" for the components of A in the complement $M = G - H$ of the Lie algebra of H in that of G, thus leading to a loss of conformal invariance for the appropriate Yang-Mills equations. The surviving Higgs fields and the fermions are also aquiring "masses" by this mechanism. For details of this and other aspects of traditional symmetry breaking the reader is referred to the review by O'Raifeartaigh [2] where further references can be found.

A closer look at the BEHK mechanism (and this will partially be true of the geometric models discussed in this lecture too) shows that the presence of the Ginzburg-Landau potential is not really at the heart of the matter. The scalar field and the quartic interaction were chosen because they lead to a renormalizable quantum theory and are the simplest combination which does the job. In effect, the reason why they do the job is revealed by a careful analysis of group actions on manifolds, particularly of pairs of groups such as G and H, and their homogeneous spaces G/H. Such an analysis was carried out in other contexts of symmetry breaking by Michel and Radicati [3] over a decade ago, and a good summary can be found in Ref.[2]. It turns out that if one is given G and H, there are relatively few invariants which lead to the desired physical results, among them the Ginzburg-Landau action.

The same remarks apply, mutatis mutandis, to the orbit structure of the associated bundles E(M,P,G/H,p) discussed below in symmetry breaking models. In fact, the symmetry breaking sections should appear from a detailed analysis,in the spirit of Michel and Radicati, of the orbits and strata of group actions in these bundles. There does not seem to exist in the literature an explicit discussion of this topic, and it was hoped that such a discussion could be included here. However, time pressure forced me to defer this to a future publication.

Until recently, Higgs fields were considered almost sacrosanct, but the view that they really exist (and the appropriate particles should indeed manifest themselves experimentally) is becoming less widely held. Many of us who were not satisfied by the artificial introduction of the Higgs fields have been searching for alternatives to the introduction of the Higgs bosons, yielding the same results which made this model so appealing and successful in the electroweak unification, and in all grand unified gauge theories. I will not discuss theories in which the Higgs scalars are treated as bound states of more elementary fermions, or metric theories of the Kaluza-Klein type (for an interesting recent attempt to obtain the putative $SU(3)\times SU(2)\times U(1)$ symmetry of strong and electroweak interactions, I refer the reader to a recent paper by Witten [4]) but will describe briefly two "geometric" approaches to the problem which can be described in the language of bundle restrictions (cf. Mayer-Trautman, Secs. 3 and 6).

The second is based on the introduction of hidden dimensions followed by a "dimensional reduction" [7,8,9] and makes use of symmetries of connections and curvatures discussed in Sec. 6 of Mayer-Trautman, and should be considered a direct application of the methods discussed there.

3. HIDDEN DIMENSIONS AND SYMMETRY BREAKING

Another approach to symmetry breaking (more correctly, a whole class of approaches) is based on the introduction of "hidden dimensions" into the principal bundle on which a gauge theory is based, with the result that certain components of the connection take over the role of the Higgs bosons and produce the required violation of conformal invariance of the Yang-Mills equations. There are essentially two ways of introducing hidden dimensions into a principal bundle, which I will call the Kaluza-Klein, and Weyl methods, respectively. In a Kaluza-Klein approach one starts from a Riemannian or pseudo-Riemannian manifold of dimension 4 + k, writes down the Einstein-Hilbert action (linear in the curvature) for the metric in this space, and treats the non-block-diagonal terms as a Yang-Mills connection. The appropriate terms in the action then yield, among others, terms which are quadratic in the Yang-Mills curvature and can thus be interpreted as a Yang-Mills action. The general theory of such models has been discussed in many places; cf., e.g., Trautman's lecture in this volume, the forthcoming book by Bleecker [12], and Witten's recent atempt [4] to obtain an $SU(3)\times SU(2)\times U(1)$ gauge theory from an eleven-dimensional Kaluza-Klein model.

The Weyl approach, also known as "dimensional reduction" or "fiberflipping", has been particularly popular among supersymmetrists (I will not discuss supersymmetric gauge models here), and has been successfully used by Manton [8] in a model which derives many of the features of the standard electroweak unification from a G_2-principal bundle by this method.

The general theory of such symmetry breaking can be formulated as follows. We start out from a principal bundle P(M,G) over four-dimensional M, with hidden symmetry group G as the structure group. The symmetry group of the vacuum H, a closed subgroup of G, is assumed known. A given symmetry breaking is then described, as already discussed, by a section of the associated bundle E(M,P,G/H,p). We now take E as the base space of a new bundle R(E,G) with structure group G (not H, as was the case for the restriction Q(M,H)) obtained as the pullback of P under the projection p of the associated bundle E: $R = p^{*}P$. (Pictorially, one can think of R as the bundle obtained from P by "reattaching, or flipping" part of the fiber, G/H, so that it appears both in the base space and in the fiber.)

The result is a larger principal bundle, where the "hidden dimensions" of the manifold G/H appear both in the base space E and in the fiber G. The group G acts both on the base space and the bundle space, and therefore the results obtained in Sec. 6 of the Mayer-Trautman article in this volume apply. The connection on P is pulled back into a connection on R, and both this connection and its curvature acquire extra components, since they are now defined on a larger base space. Let us denote by script letters the pullbacks to E of the connection and curvature on R in a local trivialization determined by a section $s: U \subset E \to R$:

$$\mathcal{A} = s^* p^* \omega \quad , \quad \mathcal{F} = s^* p^* \Omega \quad , \tag{3.1}$$

where ω and Ω are the connection and curvature on P, and s^*p^* denote their pullback to R pulled down to E by s (this symbolic notation can be interpreted easily in terms of local bases, which we leave as an exercise for the reader). If G/H is k-dimensional, then $\mathcal{A}$ is a G-valued one-form on the 4 + k-dimensional manifold E, and $\mathcal{F}$ is a G-valued two-form. Thus $\mathcal{A}$ can be thought of as a set of 4 + k matrices, and $\mathcal{F}$ as a set of $(4 + k)(3 + k)/2$ matrices. Moreover, one can choose in the Lie algebra $\mathcal{G}$ a basis adapted to the splitting $\mathcal{G} = \mathcal{H} + \mathcal{M}$, dim $\mathcal{G}$ = n, dim $\mathcal{H}$ = m, dim $\mathcal{M}$ = n - m = k. Clearly, the physical surviving components of $\mathcal{A}$ and $\mathcal{F}$, which we will denote by A and F, respectively, are a one-form and two form on M with values in $\mathcal{H}$, and the remaining components will be subjected to symmetry and gauge transformations, thus reducing the Yang-Mills action on E to a Yang-Mills-Ginzburg-Landau action on M!

Consider the Yang-Mills action on R (numerical factors are omitted)

$$S_{YM} = \int \mathrm{Tr}\ (\mathcal{F} \wedge *\mathcal{F}) \ , \tag{3.2}$$

where the trace is the Killing-Cartan trace on $\mathcal{G}$, and the Hodge-dual is taken on the oriented Riemannian manifold E. (This presupposes a fiber metric on G/H, which is the same as the h of the preceding section; it should be recalled that $\mathcal{F}$ is a two-form, hence $*\mathcal{F}$ is a $(2 + k)$-form, and the integrand is a $(4 + k)$-form, as it should be.) The connection and its curvature are clearly invariant under the action of G on the base space E, hence we can apply Eqs.(6.15) and (6.18) of Mayer-Trautman (p.433 this volume). We can obviously split the curvature F into components along M (spacetime) and those along directions tangent to G/H. We denote the former components by $F_{!!}$ and the latter by $F_{??}$, whereas the mixed components (one along M, the other along G/H) will be denoted by $F_{!?}$; the Hodge-dual can be reexpressed in terms of the corresponding contravariant components. Then the integrand of (3.2) becomes

$$\mathrm{Tr}\,(F_{!!}F^{!!} + 2F_{!?}F^{!?} + F_{??}F^{??}) \ . \tag{3.3}$$

Exploiting the invariance of the connection with respect to transformations in the ?-directions, i.e., assuming the vector fields X, Y in Eqs.(6.15) and (6.18) in Mayer-Trautman to be along G/H, the components $F_{!?}$ can be expressed as the $D_{!}\Phi(?)$, where $\Phi(?)$ is the Lie-algebra-valued 0-form corresponding to the invariance of A with respect to the vector field ?, in the G/H direction of E. Thus, the middle term in Eq.(3.3) becomes, symbolically,

$$\mathrm{Tr} \sum D_{!}\Phi(?)\, D^{!}\Phi(?) \ , \tag{3.4}$$

where the summation is over the repeated symbols !, ?. The first term in (3.3), after integration over the homogeneous spaces G/H and reduction to the Lie-algebra H, becomes the Yang-Mills action for the reduced Yang-Mills theory on Q. Finally, in order to handle the third term, which involves the contraction $F_{??}$ of F with two vector fields lying along G/H, we make use of the equation (6.18) in Mayer-Trautman, which becomes:

$$2F_{??} = [\Phi(?),\Phi(?)] - \Phi([?,?]) \ , \tag{3.5}$$

with the obvious meaning for the bracket of two ?. Thus, the third term in Eq.(3.3) reduces to what is essentially a Ginzburg-Landau polynomial in the components of Φ:

$$\mathrm{Tr}F_{??}F^{??} = \tfrac{1}{4}\mathrm{Tr}([\Phi,\Phi]-\Phi)^2 , \qquad (3.6)$$

where the square means contraction in the appropriate vector field directions with the metric h on G/H. As was pointed out by Professor O'Raifeartaigh, it is necessary to analyze the expression (3.6) more carefully, since the presence of the brackets may in some cases lead to instability problems. However, special cases which were considered show that Eq.(3.6) has indeed the properties required of a Ginzburg-Landau-Higgs potential, and moreover the relative signs of the quartic and quadratic terms are correct, and only one overall normalization constant (rather than the two which are usual in the expression (1.2)) is needed.

There remains, of course, the problem of how to introduce spinors into a model of this type, and how to couple the spinors to the new fields Φ which have been introduced. There are two obvious ways in which spinors can be handled in this context, neither of which leads to satisfactory results in physical contexts. ~~The first is to treat the spinors as tensor products of four-dimensional spinors with objects behaving trivially on G/H.~~ The second is to introduce spinors on E (i.e., objects transforming under the group Spin (4 + k)), and then carry out the reduction. k = 4 Spin (8)

FOUNDATIONS OF DIFFERENTIAL GEOMETRY

VOLUME I

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and

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1963

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we have $\dot{u}_t = \varphi_t(X_{u_0})$ and hence $\omega(\dot{u}_t) = \omega(X_{u_0}) = A$, since the connection form ω is invariant by φ_t. Thus we obtain $a_t^{-1}\dot{a}_t = A$. QED.

Let K be a Lie group acting on a principal fibre bundle $P(M, G)$ as a group of automorphisms. Let u_0 be an arbitrary point of P which we choose as a reference point. Every element of K induces a transformation of M in a natural manner. The set J of all elements of K which fix the point $x_0 = \pi(u_0)$ of M forms a closed subgroup of K, called the isotropy subgroup of K at x_0. We define a homomorphism $\lambda: J \to G$ as follows. For each $j \in J$, ju_0 is a point in the same fibre as u_0 and hence is of the form $ju_0 = u_0 a$ with some $a \in G$. We define $\lambda(j) = a$. If $j, j' \in J$, then

$$u_0\lambda(jj') = (jj')u_0 = j(u_0\lambda(j')) = (ju_0)\lambda(j')$$
$$= (u_0\lambda(j))\lambda(j') = u_0(\lambda(j)\lambda(j')).$$

Hence, $\lambda(jj') = \lambda(j)\lambda(j')$, which shows that $\lambda: J \to G$ is a homomorphism. It is also easy to check that λ is differentiable. The induced Lie algebra homomorphism $\mathfrak{j} \to \mathfrak{g}$ will be also denoted by the same λ. Note that λ depends on the choice of u_0; the reference point u_0 is chosen once for all and is fixed throughout this section.

Proposition 11.3. *Let K be a group of automorphisms of $P(M, G)$ and Γ a connection in P invariant by K. We define a linear mapping $\Lambda: \mathfrak{k} \to \mathfrak{g}$ by*

$$\Lambda(X) = \omega_{u_0}(\tilde{X}), \qquad X \in \mathfrak{k},$$

where $\tilde{X}$ is the vector field on P induced by X. Then

(1) $\Lambda(X) = \lambda(X)$ *for $X \in \mathfrak{j}$;*

(2) $\Lambda(\mathrm{ad}\,(j)(X)) = \mathrm{ad}\,(\lambda(j))(\Lambda(X))$ *for $j \in J$ and $X \in \mathfrak{k}$,*

where ad (j) *is the adjoint representation of J in $\mathfrak{k}$ and* ad $(\lambda(j))$ *is that of G in $\mathfrak{g}$.*

Note that the geometric meaning of $\Lambda(X)$ is given by Proposition 11.2.

Proof. (1) We apply Proposition 11.2 to the 1-parameter subgroup φ_t of K generated by X. If $X \in \mathfrak{j}$, then the curve $x_t = \pi(\varphi_t(u_0))$ reduces to a single point $x_0 = \pi(u_0)$. Hence we have $\varphi_t(u_0) = u_0\lambda(\varphi_t)$. Comparing the tangent vectors of the orbits $\varphi_t(u_0)$ and $u_0\lambda(\varphi_t)$ at u_0, we obtain $\Lambda(X) = \lambda(X)$.

(2) Let $X \in \mathfrak{k}$ and $j \in J$. We set $Y = \mathrm{ad}\,(j)(X)$. Then Y generates the 1-parameter subgroup $j\varphi_t j^{-1}$ which maps u_0 into $j\varphi_t j^{-1}(u_0) = j\varphi_t(u_0\lambda(j^{-1})) = j(R_{\lambda(j^{-1})}\varphi_t u_0)$. It follows that $\tilde{Y}_{u_0} = j(R_{\lambda(j^{-1})}\tilde{X}_{u_0})$. Since the connection form ω is invariant by j, we have

$$\begin{aligned}\omega_{u_0}(\tilde{Y}) = \omega_{u_0}(j(R_{\lambda(j^{-1})}\tilde{X}_{u_0})) &= \omega_{j^{-1}u_0}(R_{\lambda(j^{-1})}\tilde{X}_{u_0}) \\ &= \mathrm{ad}\,(\lambda(j))(\omega_{u_0}(\tilde{X}_{u_0})) = \mathrm{ad}\,(\lambda(j))(\Lambda(X)).\end{aligned}$$

QED.

Proposition 11.4. *With the notation of Proposition* 11.3, *the curvature form* Ω *of* Γ *satisfies the following condition:*

$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda(X), \Lambda(Y)] - \Lambda([X, Y]) \qquad \textit{for } X, Y \in \mathfrak{k}.$$

Proof. From the structure equation (Theorem 5.2) and Proposition 3.11 of Chapter I, we obtain

$$\begin{aligned}2\Omega(\tilde{X}, \tilde{Y}) &= 2d\omega(\tilde{X}, \tilde{Y}) + [\omega(\tilde{X}), \omega(\tilde{Y})] \\ &= \tilde{X}(\omega(\tilde{Y})) - \tilde{Y}(\omega(\tilde{X})) - \omega([\tilde{X}, \tilde{Y}]) + [\omega(\tilde{X}), \omega(\tilde{Y})].\end{aligned}$$

Since ω is invariant by K, we have by (c) of Proposition 3.2 of Chapter I (cf. also Proposition 3.5 of Chapter I)

$$\begin{aligned}\tilde{X}(\omega(\tilde{Y})) - \omega([\tilde{X}, \tilde{Y}]) &= (L_{\tilde{X}}\omega)(\tilde{Y}) = 0, \\ \tilde{Y}(\omega(\tilde{X})) - \omega([\tilde{Y}, \tilde{X}]) &= (L_{\tilde{Y}}\omega)(\tilde{X}) = 0.\end{aligned}$$

On the other hand, $X \to \tilde{X}$ being a Lie algebra homomorphism, we have

$$\omega_{u_0}([\tilde{X}, \tilde{Y}]) = \Lambda([X, Y]).$$

Thus we obtain

$$\begin{aligned}2\Omega_{u_0}(\tilde{X}, \tilde{Y}) &= [\omega_{u_0}(\tilde{X}), \omega_{u_0}(\tilde{Y})] - \Lambda([X, Y]) \\ &= [\Lambda(X), \Lambda(Y)] - \Lambda([X, Y]).\end{aligned}$$

QED.

We say that K acts *fibre-transitively* on P if, for any two fibres of P, there is an element of K which maps one fibre into the other, that is, if the action of K on the base M is transitive. If J is the isotropy subgroup of K at $x_0 = \pi(u_0)$ as above, then M is the homogeneous space K/J.

The following result is due to Wang [1].

Theorem 11.5. *If a connected Lie group* K *is a fibre-transitive automorphism group of a bundle* $P(M, G)$ *and if* J *is the isotropy subgroup of*

K at $x_0 = \pi(u_0)$, *then there is a* 1:1 *correspondence between the set of K-invariant connections in P and the set of linear mappings* $\Lambda: \mathfrak{k} \to \mathfrak{g}$ *which satisfies the two conditions in Proposition* 11.3; *the correspondence is given by*

$$\Lambda(X) = \omega_{u_0}(\tilde{X}) \qquad \textit{for } X \in \mathfrak{k},$$

where $\tilde{X}$ *is the vector field on P induced by X.*

Proof. In view of Proposition 11.3, it is sufficient to show that, for every $\Lambda: \mathfrak{k} \to \mathfrak{g}$ satisfying (1) and (2) of Proposition 11.3, there is a K-invariant connection form ω on P such that $\Lambda(X) = \omega_{u_0}(\tilde{X})$ for $X \in \mathfrak{k}$. Let $X^* \in T_u(P)$. Since K is fibre-transitive, we can write

$$u_0 = kua = k \circ R_a u$$

$$k \circ R_a X^* = \tilde{X}_{u_0} + A^*_{u_0},$$

where $k \in K$, $a \in G$, $X \in k$ and A^* is the fundamental vector field corresponding to $A \in \mathfrak{g}$. We then set

$$\omega(X^*) = \text{ad}\,(a)(\Lambda(X) + A).$$

We first prove that $\omega(X^*)$ is independent of the choice of X and A. Let

$$\tilde{X}_{u_0} + A^*_{u_0} = \tilde{Y}_{u_0} + B^*_{u_0}, \qquad \text{where } Y \in \mathfrak{k} \quad \text{and} \quad B \in \mathfrak{g},$$

so that $\tilde{X}_{u_0} - \tilde{Y}_{u_0} = B^*_{u_0} - A^*_{u_0}$. From the definition of $\lambda: \mathfrak{j} \to \mathfrak{g}$, it follows that $\lambda(X - Y) = B - A$. By condition (1) of Proposition 11.3, we have $\lambda(X - Y) = \Lambda(X - Y) = \Lambda(X) - \Lambda(Y)$. Hence, $\Lambda(X) + A = \Lambda(Y) + B$.

We next prove that $\omega(X^*)$ is independent of the choice of k and a. Let

$$u_0 = kua = k_1 u a_1 \qquad (k_1 \in K \quad \text{and} \quad a_1 \in G),$$

so that $k_1 k^{-1} u_0 = u_0 a_1^{-1} a$ and $k_1 k^{-1} \in J$. We set $j = k_1 k^{-1}$. Then $\lambda(j) = a_1^{-1} a$. We have

$$\begin{aligned} k_1 \circ R_{a_1} X^* &= jk \circ R_{a\lambda(j^{-1})} X^* \\ &= j \circ R_{\lambda(j^{-1})}(k \circ R_a X^*) = j \circ R_{\lambda(j^{-1})}(\tilde{X}_{u_0} + A^*_{u_0}). \end{aligned}$$

By Proposition 1.7 of Chapter I, we have

$$j \circ R_{\lambda(j^{-1})}(\tilde{X}_{u_0}) = j(\tilde{X}_{u_0\lambda(j^{-1})}) = \tilde{Z}_{u_0}, \qquad \text{where } Z = \text{ad}\,(j)(X).$$

By Proposition 5.1 of Chapter I, we have

$$j \circ R_{\lambda(j^{-1})}(A^*_{u_0}) = R_{\lambda(j^{-1})}(jA^*_{u_0}) = R_{\lambda(j^{-1})}A^*_{ju_0} = R_{\lambda(j^{-1})}A^*_{u_0\lambda(j)} = C^*_{u_0},$$

where $C = \text{ad}\,(\lambda(j))(A)$. Hence we have

$$k_1 \circ R_{a_1}X^* = \tilde{Z}_{u_0} + C^*_{u_0},$$

$$\begin{aligned}\text{ad}\,(a_1)(\Lambda(Z) + C) &= \text{ad}\,(a_1)(\Lambda(\text{ad}\,(j)(X)) + \text{ad}\,(\lambda(j))A) \\ &= \text{ad}\,(a_1)[\text{ad}\,(\lambda(j))(\Lambda(X) + A)] \\ &= \text{ad}\,(a)(\Lambda(X) + A).\end{aligned}$$

This proves our assertion that $\omega(X^*)$ depends only on X^*.

We now prove that ω is a connection form. Let $X^* \in T_u(P)$ and $u_0 = kua$ as above. Let b be an arbitrary element of G. We set

$$Y^* = R_bX^* \in T_v(P), \qquad \text{where } v = ub,$$

so that $u_0 = kub(b^{-1}a) = kv(b^{-1}a)$. We then have

$$k \circ R_{b^{-1}a}Y^* = k \circ R_{b^{-1}a}R_bX^* = k \circ R_aX^* = (\tilde{X}_{u_0} + A^*_{u_0})$$

and hence

$$\omega(R_bX^*) = \omega(Y^*) = \text{ad}\,(b^{-1}a)(\Lambda(X) + A) = \text{ad}\,(b^{-1})(\omega(X^*)),$$

which shows that ω satisfies condition (b′) of Proposition 1.1. Now, let A be any element of $\mathfrak{g}$ and let $u_0 = kua$. Then

$$k \circ R_a(A^*_u) = R_a \circ k(A^*_u) = R_a(A^*_{ku}) = B^*_{u_0}, \quad \text{where } B = \text{ad}\,(a^{-1})(A).$$

Hence we have

$$\omega(A^*_u) = \text{ad}\,(a)(B) = A,$$

which shows that ω satisfies condition (a′) of Proposition 1.1.

To prove that ω is differentiable, let u_1 be an arbitrary point of P and let $u_0 = k_1u_1a_1$. Consider the fibre bundle $K(M, J)$, where $M = K/J$. Let $\sigma\colon U \to K$ be a local cross section of this bundle defined in a neighborhood U of $\pi(u_1)$ such that $\sigma(\pi(u_1)) = k_1$. For each $u \in \pi^{-1}(U)$, we define $k \in K$ and $a \in G$ by

$$k = \sigma(\pi(u)) \quad \text{and} \quad u_0 = kua.$$

Then both k and a depend differentiably on u. We decompose the vector space $\mathfrak{k}$ into a direct sum of subspaces: $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$. For an arbitrary $X^* \in T_u(P)$, we set

$$k \circ R_a(X^*) = \tilde{X}_{u_0} + A^*_{u_0}, \qquad \text{where } X \in \mathfrak{m}.$$

Then both X and A are uniquely determined and depend differentiably on X^*. Thus $\omega(X^*) = \text{ad}\,(a)(\Lambda(X) + A)$ depends differentiably on X^*.

Finally, we prove that ω is invariant by K. Let $X^* \in T_u(P)$ and $u_0 = kua$. Let k_1 be an arbitrary element of K. Then $k_1X^* \in T_{k_1u}(P)$ and $u_0 = kk_1^{-1}(k_1u)a$. Hence,

$$kk_1^{-1} \circ R_a(k_1X^*) = k \circ R_a(X^*).$$

From the construction of ω, we see immediately that $\omega(k_1X^*) = \omega(X^*)$. QED.

In the case where K is fibre-transitive on P, the curvature form Ω, which is a tensorial form of type ad G (cf. §5) and is invariant by K, is completely determined by the values $\Omega_{u_0}(\tilde{X}, \tilde{Y})$, $X, Y \in \mathfrak{k}$. Proposition 11.4 expresses $\Omega_{u_0}(\tilde{X}, \tilde{Y})$ in terms of Λ. As a consequence of Proposition 11.4 and Theorem 11.5, we obtain

Corollary 11.6. *The K-invariant connection in P defined by Λ is flat if and only if $\Lambda: \mathfrak{k} \to \mathfrak{g}$ is a Lie algebra homomorphism.*

Proof. A connection is flat if and only if its curvature form vanishes identically (Theorem 9.1). QED.

Theorem 11.7. *Assume in Theorem 11.5 that $\mathfrak{k}$ admits a subspace $\mathfrak{m}$ such that $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$ (direct sum) and $\text{ad}\,(J)(\mathfrak{m}) = \mathfrak{m}$, where $\text{ad}\,(J)$ is the adjoint representation of J in $\mathfrak{k}$. Then*

(1) *There is a 1:1 correspondence between the set of K-invariant connections in P and the set of linear mappings $\Lambda_\mathfrak{m}: \mathfrak{m} \to \mathfrak{g}$ such that*

$$\Lambda_\mathfrak{m}(\text{ad}\,(j)(X)) = \text{ad}\,(\lambda(j))(\Lambda_\mathfrak{m}(X)) \qquad \text{for } X \in \mathfrak{m} \text{ and } j \in J;$$

the correspondence is given via Theorem 11.5 by

$$\Lambda(X) = \begin{cases} \lambda(X) & \text{if } X \in \mathfrak{j}, \\ \Lambda_\mathfrak{m}(X) & \text{if } X \in \mathfrak{m}. \end{cases}$$

(2) *The curvature form Ω of the K-invariant connection defined by $\Lambda_\mathfrak{m}$ satisfies the following condition:*

$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda_\mathfrak{m}(X), \Lambda_\mathfrak{m}(Y)] - \Lambda_\mathfrak{m}([X, Y]_\mathfrak{m}) - \lambda([X, Y]_\mathfrak{j}) \qquad \text{for } X, Y \in \mathfrak{m},$$

Appendix - Higgs as Primitive Idempotent

By identifying the Higgs with Primitive Idempotents of the Cl(8) real Clifford algebra, the Higgs is not seen as a simple-minded fundamental scalar particle, but rather the Higgs is seen as a quantum process that creates a fermionic condensate with which it interacts to make the fermions appear massive.
The Primitive Idempotent Higgs is part of my E8 Physics model in terms of which the Primitive Idempotent Higgs is seen to do all the nice things that the fundamental scalar particle Higgs needs to do, and to be effectively a Higgs-Tquark system with 3 mass states.

The conventional Standard Model has structure:
spacetime is a base manifold;
particles are representations of gauge groups
gauge bosons are in the adjoint representation
fermions are in other representations (analagous to spinor)
Higgs boson is in scalar representation.

E8 Physics (see vixra 1108.0027 and tony5m17h.net) has structure
(from 248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8):
spacetime is in the adjoint D8 part of E8 (64 of 120 D8 adjoints)
gauge bosons are in the adjoint D8 part of E8 (56 of the 120 D8 adjoints)
fermions are in the half-spinor D8 part of E8 (64+64 of the 128 D8 half-spinors.

There is no room for a fundamental Higgs in the E8 of E8 Physics.

However,
for E8 Physics to include the observed results of the Standard Model
it must have something that acts like the Standard Model Higgs
even though it will NOT be a fundamental particle.

To see how the E8 Physics Higgs works,
embed E8 into the 256-dimensional real Clifford algebra Cl(8):

Cl(8)	256 =	1	+ 8	+ 28	+ 56	+ 70	+ 56	+ 28	+ 8	+ 1
Primitive Idempotent	16 =	1				+ 6 + 8				+ 1
E8 Root Vectors	240 =		8	+ 28	+ 56	+ 56	+ 56	+ 28	+ 8	

The Cl(8) Primitive Idempotent is 16-dimensional and can be decomposed into two 8-dimensional half-spinor parts each of which is related by Triality to 8-dimensional spacetime and has Octonionic structure. In that decomposition: the 1+6+1 = (1+3)+(3+1) is related to two copies of a 4-dimensional Associative Quaternionic subspace of the Octonionic structure

and
the 8 = 4+4 is related to two copies of
a 4-dimensional Co-Associative subspace of the Octonionic structure
(see the book “Spinors and Calibrations” by F. Reese Harvey)

The 8 = 4+4 Co-Associative part of the Cl(8) Primitive Idempotent
when combined with the 240 E8 Root Vectors
forms the full 248-dimensional E8.

It represents a Cartan subalgebra of the E8 Lie algebra.

The (1+3)+(3+1) Associative part of the Cl(8) Primitive Idempotent
is the Higgs of E8 Physics.

The half-spinors generated by the E8 Higgs part of the Cl(8) Primitive Idempotent represent:
neutrino; red, green, blue down quarks; red, green, blue up quarks; electron
so
the E8 Higgs effectively creates/annihilates the fundamental fermions and
the E8 Higgs is effectively a condensate of fundamental fermions.

In E8 Physics the high-energy 8-dimensional Octonionic spacetime reduces,
by freezing out a preferred 4-dim Associative Quaternionic subspace,
to a 4+4 -dimensional Batakis Kaluza-Klein of the form M4 x CP2
with 4-dim M4 physical spacetime.

Since the (1+3)+(3+1) part of the Cl(8) Primitive Idempotent
includes the Cl(8) grade-0 scalar 1
and 3+3 = 6 of the Cl(8) grade-4 which act as pseudoscalars for 4-dim spacetime
and the Cl(8) grade-8 pseudoscalar 1
t**he E8 Higgs transforms with respect to 4-dim spacetime as a scalar**
(or pseudoscalar) and in that respect is similar to Standard Model Higgs.

Not only does the E8 Higgs fermion condensate transform with respect
to 4-dim physical spacetime like the Standard Model Higgs but
the geometry of the reduction from 8-dim Octonionic spacetime
to 4+4 -dimensional Batakis Kaluza-Klein, by the Mayer mechanism, gives
E8 Higgs the ElectroWeak Symmetry-Breaking Ginzburg-Landau structure.

Since the second and third fermion generations emerge dynamically from the
reduction from 8-dim to 4+4 -dim Kaluza-Klein, they are also created/annihilated
by the Primitive Idempotent E8 Higgs and are present in the fermion condensate.
Since the Truth Quark is so much more massive that the other fermions,
the E8 Higgs is effectively a Truth Quark condensate.
When Triviality and Vacuum Stability are taken into account,
the E8 Higgs and Truth Quark system has 3 mass states.

Appendix - Joy Christian Correlations

Bell's Theorem
on Quantum Correlations is based on the Hopf Fibration RP1 -> S1 -> S0 = {-1,+1}.

Joy Christian has shown that it is more realistic
to base Quantum Correlations on the Hopf Fibrations
S1 -> S3 -> S2 = CP1 and S3 -> S7 -> S4 = QP1 and S7 -> S15 -> S8 = OP1
where R, C, Q, and O are Real, Complex, Quaternion, and Octonion Division Algebras.

In his book "Disproof of Bell's Theorem" (BrownWalker Press, 2nd ed, 2014)
Joy Christian said:
"... Every quantum mechanical correlation can be understood as
a classical, local-realistic correlation among a set of points of a parallelized 7-sphere
...
physical space ... respects the symmetries and topologies of a parallelized 7-sphere
...
because 7-sphere ...[is]... homeomorphic to the ...[Octonion]... division algebra ...
it is the property of division that ....[is]... responsible for ... local causality in the world
...
To understand this reasoning better, recall that, just as a parallelized 3-sphere is a 2-sphere worth of 1-spheres but with a twist in the manifold S3 (=/= S2xS1) ,
a parallelized 7-sphere is a 4-sphere worth of 3-spheres
but with a twist in the manifold S7 (=/= S4xS3)
... just as S3 is
a nontrivial fiber bundle over S2 with Clifford parallels S1 as its linked fibers,
S7 is also
a nontrivial fiber bundle ... over S4 ... with ... spheres S3 as its linked fibers.
...
it is the twist in the bundle S3 that forces one
to forgo the commutativity of complex numbers (corresponding to the circles S1)
in favor of the non-commutativity of quaternions.
In other words, a 3-sphere is not parallelizable by the commuting complex numbers but only by the non-commuting quaternions. And it is this noncommutativity that gives rise to the non-vanishing of the torsion in our physical space.

In a similar vein, the twist in the bundle S7 (=/= S4xS3) forces one to forgo the associativity of quaternions (corresponding to the fibers) in favor of the non-associativity of octonions.
In other words, a 7-sphere is not parallelizable by the associative quaternions but only by the non-associative octonions.

... it can be parallelized ... because its tangent bundle happens to be trivial:
Once parallelized by a set of unit octonions,
both the 7-sphere and each of its 3-spherical fibers remain closed under multiplication.
This, in turn, means that

the factorizability or locality condition of Bell is ... satisfied within a parallelized 7-sphere. The lack of associativity of octonions, however, entails that, unlike the unit 3-sphere [which is homeomorphic to the ... group SU(2)], a 7-sphere is not a group manifold ... the torsion within the 7-sphere ... varies from one point to another of the manifold. It is this variability of the parallelizing torsion within that is ultimately responsible for the diversity and non-linearity of the quantum correlations we observe in nature ...”.

The 7-sphere S7 is the unit sphere in 8-dim space.
S7 is not a Lie algebra, but is a Malcev algebra
and is naturally embedded in the D4 Lie algebra Spin(8) which
is topologically composed of (but =/= the simple product S7 x S7 x G2)
2 copies of S7 and 14-dim Lie Algebra G2 of the Octonion Automorphism Group.

28-dim D4 Lie algebra Spin(8) can be represented by 8x8 antisymmetric real matrices
It is a subalgebra of 63-dim A7 Lie Algebra SL(8,R) of all 8x8 real matrices with det = 1.

Unimodular SL(8,R) is the non-compact Lie algebra corresponding to SU(8).
SL(8,R) effectively describes the 8-dim SpaceTime of E8 Physics
as a generalized checkerboard of SpaceTime HyperVolume Elements.
Anderson and Finkelstein in Am . J. Phys. 39 (1971) 901-904 said:
"... Unimodular relativity ... expresses the existence of a fundamental element of spacetime hypervolume at every point. ...".
From the Real Clifford Algebra Cl(16) and 8-Periodicity
64-dim R+SL(8,R) appears from factoring Cl(16) = tensor product Cl(8)xCl(8)
as the tensor product of the 8-dim vector spaces 8v of each of the Cl(8) factors
so that 64-dim R+SL(8,R) = 8v x 8v
If you regard the two Cl(8) as Fourier duals then
one 8v describes 8-dim Spacetime Position and the other 8v describes its Momentum.

David Brown, in May 2012 comments on scottaaronson.com blog, said:
“... Where did Bell go wrong? Bell used quantum SU(1) states
whereas Christian correctly uses quantum SU(8) states ...[from]...
Christian’s parallelized 7-sphere model. ...
Every quantum mechanical Christian SU(8) correlation can be understood
as a realistic, non-local Christian SU(8) correlations among a set of points
of a parallelized 7-sphere ... More importantly, if Christian’s theory of local realism is true
then SU(8) should be the gauge group for physical reality ...”.
SU(8) is the compact version of SL(8,R), so it seems to me that it is David Brown’s idea, possibly motivated by SU(8) and SL(8,R) in E7 of D = 4 N = 8 supergravity models, that **Joy Christian’s S7 Quantum Correlations have fundamental SL(8,R) structure**.

Rutwig Campoamor-Stursberg in Acta Physica Polonica B 41 (2010) 53-77 , “Contractions of Exceptional Lie Algebras and SemiDirect Products” , showed that SL(8,R) appears in the E8 Maximal Contraction = semi-direct product H92 x SL(8,R) where
H92 is (8+28+56 +1+ 56+28+8)-dim Heisenberg Creation/Annihilation Algebra

so that H92 x SL(8,R) has 7-graded structure:

grade -3 = Creation of 1 fermion (tree-level massless neutrino)
with 8 SpaceTime Components for a total of 8 fermion component creators
(related to SpaceTime by Triality)

grade -2 = Creation of 8+3+1 = 12 Bosons for Standard Model
and 16 Conformal U(2,2) Bosons for MacDowell-Mansouri Gravity
for a total of 28 Boson creators

grade -1 = Creation of 7 massive Dirac fermion
each with 8 SpaceTime Components for a total of 56 fermion component creators

grade 0 = 1 + SL(8) = 1+63 = 64-dim
representing 8-dim SpaceTime of HyperVolume Elements

grade 1 = Annihilation of 7 massive Dirac fermions
each with 8 SpaceTime Components for a total of 56 fermion component annihilators

grade 2 = Annihilation of 8+3+1 = 12 Bosons for Standard Model
and 16 Conformal U(2,2) Bosons for MacDowell-Mansouri Gravity
for a total of 28 Boson annihilators

grade 3 = Annilation of 1 fermion (tree-level massless neutrino)
with 8 SpaceTime Components for a total of 8 fermion component annihilators
(related to SpaceTime by Triality)

Here is how Physics Structures expand from Joy Christian’s S7 to E8 Physics:

7-dim S7 - Lie Algebra -> 28-dim Spin(8)

28-dim Spin(8) - Full 8x8 Matrix -> 63-dim SL(8,R)

63-dim SL(8,R) - Creation/Annihilation -> 248-dim H92xSL(8,R)

248-dim H92xSL(8,R) - Expansion -> 248-dim E8

The E8 expansion of H92 x SL(8,R) has physical interpretation
leading to a Local Classical Lagrangian with Base Manifold Spacetime,
Gravity + Standard Model Gauge Boson terms, and Fermion terms
for 8-dim spacetime and First-Generation Fermions (with 4+4 dim Kaluza-Klein and Second and Third Fermion Generations emerging with Octonionic Symmetry being broken to Quaternionic) :

248-dim E8 = 120-dim D8 + 128-dim half-spinors of D8

In Symmetric Space terms:

E8 / D8 = (64+64)-dim (OxO)P2 Octo-Octonionic Projective Plane
64 = 8 components of 8 fermion particles
64 = 8 components of 8 fermion antiparticles

D8 / D4xD4 = 64-dim = 8 position coordinates x 8 momentum coordinates

one D4 = 28 = 12 Standard Model Ghosts + 16 Conformal Gravity Gauge Bosons
(4 of the 16 are not in the 240 E8 root vectors, but are in its 8-dim Cartan subalgebra)

other D4 = 28 = 16 ConformalGravity Ghosts + 12 Standard Model Gauge Bosons
(4 of the 12 are not in the 240 E8 root vectors, but are in its 8-dim Cartan subalgebra)

My E8 Physics model (viXra 1405.0030 vG) was initially inspired back in the 1980s by D = 4, N = 8 supergravity models.

Yoshiaki Tanii in his book "Introduction to Supergravity" (Springer 2014) said:
"... Poincare supergravity constructed in the highest spacetime dimension
is D = 11, N = 1 theory ... the low energy effective theory of M theory ...
D = 11 supergravity has AdS4 x S7 spacetime ...
This ... corresponds to the AdS4 solution of D = 4, N = 8 gauged supergravity ...
D = 4, N = 8 gauged supergravity is ... related to
a compactification of D = 11 supergravity ... by a seven-dimensional sphere S7 ...
N = 8 supergravity ... the maximal supergravity ...[has]... multiplets ...
1 8 28 56 70 56 28 8 1
... D = 4, N = 8 Supergravity ... has global E7(+7) and local SU(8) symmetries. ...".

Supergravity itself did not quite work for me. In hindsight,
D = 4, N = 8 maximal global symmetry is only E7 with maximal compact SU(8)
(noncompact version of SU(8) is SL(8,R) which is only part of the maximal contraction of E8)
and the supergravity with maximal global symmetry E8 with maximal compact D8
is D = 3, N = 8 whose spacetime is only 3-dimensional. (Samtleben, arXiv 0808.4076).

The S7 led me to work with Spin(8) which is the bivector Lie algebra
of the Real Clifford Algebra Cl(8) with graded structure 1 8 28 56 70 56 28 8 1
When Spin(8) seemed too small, I went to F4 which contained
Spin(8) for Gauge Bosons, Spin(9) / Spin(8) for 8-dim SpaceTime,
and F4 / Spin(9) for 8 fermion particles + 8 fermion antiparticles.
When F4 failed to have desired complex structure, I went to E6.
When E6 failed to have all the necessary fermion components and gauge boson ghosts,
I went to E8 and found the E8 Physics model that as of now seems to be realistic.

How does Bell-Christian-Brown SL(8,R) Quantum Theory fit with the Bohm Quantum Potential of E8 Physics (http://vixra.org/pdf/1405.0030vG.pdf) ?

Comparison of Bohm's Quantum Potential hidden variable "lambdas" with Bell's "lambdas" and Joy Christian's (arxiv 0904.4259)"lambdas":
Peter Holland, in his book "The Quantum Theory of Motion, an Account of the de Broglie - Bohm Causal Interpretation of Quantum Mechanics" (Cambridge 1993) said:
"... 11.5.1 Bell's Inequality ... In discussing the EPR spin experiment Bell supposed that the results of the two spin measurements are determined completely by a set of hidden variables lambda and made two assumptions which he claimed should be satisfied by a local hidden-variables theory:
(i) The result A of measuring sigma1 . a on particle 1 is determined solely by a and lambda, and the result B of measuring sigma2 . b on particle 2 is determined solely by b and lambda , where a and b are unit vectors with a . b = cos(delta).
Thus A = A(a , lambda) = +/- 1 and B = B(b , lambda) = +/- 1
Possibilities such as A = A(a , b , lambda) and B = B(a , b , lambda) are excluded.
(ii) The normalized probability distribution of the hidden variables depends only on lambda : rho = rho(lambda).
Possibilities such as rho = rho(lambda , a , b) are excluded.
...
We now consider to what extent assumptions (i) and (ii) are valid in the causal [Bohm Potential] interpretation ... The hidden variables are then the particle positions x1 , x2 (the internal orientation spin vectors s1 , s2 along the trajectories are determined by the positions and the wavefunction ...) … the eventual results ... for each of sz1 and sz2 is determined by the intial positions of both particles and by delta , i.e., A = A (x1 , x2 , a . b) , B = B(x1 , x2 , a . b) Thus assumption (i) is not valid ...
Neither is assumption (ii) satisfied. ...
In reproducing ... the quantum mechanical correlation function ...
Ppsi(a , b) = ... = - cos(delta) ... the causal [Bohm Potential] interpretation disobeys both of Bell's basic assumptions. ...".

So, Bell's "lambdas" obey (i) and (ii) and so obey Bell's inequality
and
Bohm's "lambdas" violate (i) and (ii) and so violate Bell's Inequality but obey the quantum experimentally observed correlation function.

Joy Christian (see arxiv 0904.4259) explicitly violates (i) by replacing
A = A(a , lambda) = +/- 1 and B = B(b , lambda) = +/- 1
with
A = A(a , lambda) in S2 and B = B(b , lambda) in S2.
However, Joy does not violate (ii). Joy says: "... once the state lambda is specified and the two particles have separated, measurements of A can depend only on lambda and a, but not b, and likewise measurements of B can depend only on lambda and b, but not a ...[compare the (ii)-violation by Bohm's lamdbas as stated above]... Assuming ... that the distribution rho(lambda) is normalized on the space Λ, .we finally arrive at the inequalities ... exactly what is predicted by quantum mechanics ... we have been able to derive these results without specifying what the complete state lambda is or the distribution rho(lambda) is, and without employing any averaging procedure ... the correlations [for the examples of 0904.4259] … are simply the local, realistic, and deterministic correlations among certain points of ... S3 and S7 ... This implies that the violations of Bell inequalities ... have nothing to do with quantum mechanics per se ...".

So, even though Joy's lambdas do not violate (ii), when Joy "... derive[s] ... the exact quantum mechanical expectation value ... - a . b " Joy's result is consistent with that of Bohm's "lambdas".

Joy's "lambdas" are classical and local (in Joy's sense).

Bohm's "lambdas" are quantum and, since Joy does not change Bell’s (ii), nonlocal (in Joy's sense).

Joy's "lambdas" and Bohm's "lambdas" are consistent with each other with respect to their calculated quantum expectation values.

Could Joy's "lambdas" be considered as a Classical Limit of Bohm's "lambdas" ?

Consider again Peter Holland's book in which he says:
"... 6.9 Remarks on the path integral approach ... Feynman['s] ... route to quantum mechanics ... rests on the trajectory concept and so may be expected to have some connection with the causal [Bohm Potential] formulation. ... Feynman provides a technique for computing ... the transition amplitude (Green function or propagator) ... from the classical Lagrangian ... One considers all the paths ... and associates with each an amplitude ... These tracks are ... called 'classical paths' ... one sums (integrates) over all the paths ... the solution .. is given by ... Huygens' principle ... of all the paths ... one of them will be the actual trajectory pursued by the quantum particle according to the [Bohm Potential] guidance formula ... We shall refer to ... it ... as the 'quantum path' ... For an infinitesimal time interval ... the propagator is just the classical wavefunction ... a finite path may be decomposed into many such infinitesimal steps, the net propagator being obtained by successive applications of Huygens' construction ... We may view the Feynman procedure as a method of obtaining the quantum action from the set of all classical actions. ...".

If Joy Christian's classical "lambdas" are identified with Feynman path Lagrangian / Green function propagators, and if their Huygens' sums can be seen to produce the Bohm "lambdas",
then Joy's work will show a nice smooth classical limit (as opposed to Bell's discordant classical limit) for the Bohm Quantum Potential.

If the Bohm Quantum Potential can then be used as a basis for a construction of a realistic AQFT (Algebraic Quantum Field Theory)
then maybe Joy Christian's work will help show a useful connection (and philosphical reconciliation) between
the Classical Lagrangian physics so useful in detailed understanding of the Standard Model
and
of AQFT along the lines of
generalization of the Hyperfinite II1 von Neumann factor algebra.

Appendix - Details of Conformal Gravity and ratio DE : DM : OM

MacDowell-Mansouri Gravity is described by Rabindra Mohapatra
in section 14.6 of his book "Unification and Supersymmetry":

§14.6. Local Conformal Symmetry and Gravity

Before we study supergravity, with the new algebraic approach developed, we would like to discuss how gravitational theory can emerge from the gauging of conformal symmetry. For this purpose we briefly present the general notation for constructing gauge covariant fields. The general procedure is to start with the Lie algebra of generators X_A of a group

$$[X_A, X_B] = f_{AB}^{C} X_C, \qquad (14.6.1)$$

where f_{AB}^{C} are structure constants of the group. We can then introduce a gauge field connection h_μ^A as follows:

$$h_\mu \equiv h_\mu^A X_A. \qquad (14.6.2)$$

Let us denote the parameter associated with X_A by ε^A. The gauge transformations on the fields h_μ^A are given as follows:

$$\delta h_\mu^A = \partial_\mu \varepsilon^A + h_\mu^B \varepsilon^C f_{CB}^A \equiv (D_\mu \varepsilon)^A. \qquad (14.6.3)$$

We can then define a covariant curvature

$$R_{\mu\nu}^A = \partial_\nu h_\mu^A - \partial_\mu h_\nu^A + h_\nu^B h_\mu^C f_{CB}^A. \qquad (14.6.4)$$

Under a gauge transformation

$$\delta_{\text{gauge}} R_{\mu\nu}^A = R_{\mu\nu}^B \varepsilon^C f_{CB}^A. \qquad (14.6.5)$$

We can then write the general gauge invariant action as follows:

$$I = \int d^4x \, Q_{AB}^{\mu\nu\rho\sigma} R_{\mu\nu}^A R_{\rho\sigma}^B. \qquad (14.6.6)$$

Let us now apply this formalism to conformal gravity. In this case

$$h_\mu = P_m e_\mu^m + M_{mn} \omega_\mu^{mn} + K_m f_\mu^m + D b_\mu. \qquad (14.6.7)$$

The various $R_{\mu\nu}$ are

$$R_{\mu\nu}(P) = \partial_\nu e_\mu^m - \partial_\mu e_\nu^m + \omega_\mu^{mn} e_\nu^n - \omega_\nu^{mn} e_\mu^n - b_\mu e_\nu^m + b_\nu e_\mu^m, \qquad (14.6.8)$$

$$R_{\mu\nu}(M) = \partial_\nu \omega_\mu^{mn} - \partial_\mu \omega_\nu^{mn} - \omega_\nu^{mp} \omega_{\mu,p}^{n} - \omega_\mu^{mp} \omega_{\nu,p}^{n} - 4(e_\mu^m f_\nu^n - e_\nu^m f_\mu^n), \qquad (14.6.9)$$

$$R_{\mu\nu}(K) = \partial_\nu f_\mu^m - \partial_\mu f_\nu^m - b_\mu f_\nu^m + b_\nu f_\mu^m + \omega_\mu^{mn} f_\nu^n - \omega_\nu^{mn} f_\mu^n, \qquad (14.6.10)$$

$$R_{\mu\nu}(D) = \partial_\nu b_\mu - \partial_\mu b_\nu + 2e_\mu^m f_\nu^m - 2e_\nu^m f_\mu^m. \qquad (14.6.11)$$

The gauge invariant Lagrangian for the gravitational field can now be written down, using eqn. (14.6.6), as

$$S = \int d^4x \, \varepsilon_{mnrs} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu}^{mn}(M) R_{\rho\sigma}^{rs}(M). \qquad (14.6.12)$$

We also impose the constraint that

$$R_{\mu\nu}(P) = 0, \qquad (14.6.13)$$

which expresses ω_μ^{mn} as a function of (e, b). The reason for imposing this constraint has to do with the fact that P_m transformations must be eventually identified with coordinate transformation. To see this point more explicitly let us consider the vierbein e_μ^m. Under coordinate transformations

$$\delta_{GC}(\xi^\nu)e_\mu^m = \partial_\mu \xi^\lambda e_\lambda^m + \xi^\lambda \partial_\lambda e_\mu^m. \tag{14.6.14}$$

Using eqn. (14.6.8) we can rewrite

$$\delta_{GC}(\xi^\nu)e_\mu^m = \delta_P(\xi e^\nu)e_\mu^m + \delta_M(\xi\omega^{mn})e_\mu^m + \delta_D(\xi b)e_\mu^m + \xi^\nu R_{\mu\nu}^m(P),$$

where

$$\delta_P(\xi^n)e_\mu^m = \partial_\mu \xi^m + \xi^n \omega_\mu^{mn} + \xi^m b_\mu. \tag{14.6.15}$$

If $R^{\mu\nu}(P) = 0$, the general coordinate transformation becomes related to a set of gauge transformations via eqn. (14.6.15).

At this point we also wish to point out how we can define the covariant derivative. In the case of internal symmetries $D_\mu = \partial_\mu - iX_A h_\mu^A$; now since momentum is treated as an internal symmetry we have to give a rule. This follows from eqn. (14.6.15) by writing a redefined translation generator $\bar{P}$ such that

$$\delta_{\bar{P}}(\xi) = \delta_{GC}(\xi^\nu) - \sum_{A'} \delta_{A'}(\xi^m h_m^A), \tag{14.6.16}$$

where A' goes over all gauge transformations excluding translation. The rule is

$$\delta_{\bar{P}}(\xi^m)\phi = \xi^m D_m^C \phi. \tag{14.6.17}$$

We also wish to point out that for fields which carry spin or conformal charge, only the intrinsic parts contribute to D_m^C and the orbital parts do not play any rule.

Coming back to the constraints we can then vary the action with respect to f_μ^m to get an expression for it, i.e.,

$$e_\nu^m f_{\mu m} = -\tfrac{1}{4}[e_m^\lambda e_{n\nu} R_{\mu\lambda}^{mn} - \tfrac{1}{6} g_{\mu\nu} R], \tag{14.6.18}$$

where f_μ^m has been set to zero in R written in the right-hand side.

This eliminates (from the theory the degrees of freedom) ω_μ^{mn} and f_μ^m and we are left with e_μ^m and b_μ. Furthermore, these constraints will change the transformation laws for the dependent fields so that the constraints do not change.

Let us now look at the matter coupling to see how the familiar gravity theory emerges from this version. Consider a scalar field ϕ. It has conformal weight $\lambda = 1$. So we can write a convariant derivative for it, eqn. (14.6.17)

$$D_\mu^C \phi = \partial_\mu \phi - \phi b_\mu. \tag{14.6.19}$$

We note that the conformal charge of ϕ can be assumed to be zero since $K_m = x^2 \partial$ and is the dimension of inverse mass. In order to calculate $\Box^c \phi$ we start with the expression for d'Alambertian in general relativity

$$\frac{1}{e}\partial_\nu(g^{\mu\nu}eD_\mu^C\phi). \tag{14.6.20}$$

The only transformations we have to compensate for are the conformal transformations and the scale transformations. Since

$$\delta b_\mu = -2\xi_K^m e_{m\mu}, \qquad \delta(\phi b_\mu) = \phi\delta b_\mu = -2\phi f_\mu^m e_m^\mu = +\tfrac{2}{12}\phi R, \tag{14.6.21}$$

where, in the last step, we have used the constraint equation (14.6.18). Putting all these together we find

$$\Box^C\phi = \frac{1}{e}\partial_\nu(g^{\mu\nu}eD_\mu^c\phi) + b_\mu D_\mu^c\phi + \tfrac{2}{12}\phi R. \tag{14.6.22}$$

Thus, the Lagrangian for conformal gravity coupled to matter fields can be written as

$$S = \int e\, d^4x\, \tfrac{1}{2}\phi\Box^C\phi. \tag{14.6.23}$$

Now we can use conformal transformation to gauge $b_\mu = 0$ and local scale transformation to set $\phi = \kappa^{-1}$ leading to the usual Hilbert action for gravity. To summarize, we start with a Lagrangian invariant under full local conformal symmetry and fix conformal and scale gauge to obtain the usual action for gravity. We will adopt the same procedure for supergravity. An important technical point to remember is that, $\Box^C$, the conformal d'Alambertian contains R, which for constant ϕ, leads to gravity. We may call ϕ the auxiliary field.

After the scale and conformal gauges have been fixed,
the conformal Lagrangian becomes a de Sitter Lagrangian.

Einstein-Hilbert gravity can be derived from the de Sitter Lagrangian,
as was first shown by MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739).
(Frank Wilczek, in hep-th/9801184 says that the MacDowell-Mansouri "... approach
to casting gravity as a gauge theory was initiated by MacDowell and Mansouri ...
S. MacDowell and F. Mansouri, Phys. Rev. Lett. 38 739 (1977) ... ,
and independently Chamseddine and West ... A. Chamseddine and P. West Nucl. Phys. B 129, 39 (1977);
also quite relevant is A. Chamseddine, Ann. Phys. 113, 219 (1978). ...".)

The minimal group required to produce Gravity,
and therefore **the group that is used in calculating Force Strengths**,
is the [anti] de Sitter group, as is described by
Freund in chapter 21 of his book Supersymmetry (Cambridge 1986) (chapter 21 is a Non-Supersymmetry chapter leading up to a Supergravity description in the following chapter 22):
"... Einstein gravity as a gauge theory ... we expect a set of gauge fields w^ab_u for the Lorentz group and a further set e^a_u for the translations, ...
Everybody knows though, that Einstein's theory contains but one spin two field, originally chosen by Einstein as g_uv = e^a_u e^b_v n_ab
(n_ab = Minkowski metric).
What happened to the w^ab_u ?
The field equations obtained from the Hilbert-Einstein action by varying the w^ab_u are algebraic in the w^ab_u ... permitting us to express the w^ab_u in terms of the e^a_u ... The w do not propagate ...
We start from the four-dimensional de-Sitter algebra ... so(3,2).
Technically this is the anti-de-Sitter algebra ...
We envision space-time as a four-dimensional manifold M.
At each point of M we have a copy of SO(3,2) (a fibre ...) ...
and we introduce the gauge potentials (the connection) h^A_mu(x)
A = 1,..., 10 , mu = 1,...,4. Here x are local coordinates on M.
From these potentials h^A_mu we calculate the field-strengths
(curvature components) [let @ denote partial derivative]
R^A_munu = @_mu h^A_nu - @_nu h^A_mu + f^A_BC h^B_mu h^C_nu
...[where]... the structure constants f^C_AB ...[are for]... the anti-de-Sitter algebra ....
We now wish to write down the action S as an integral over
the four-manifold M ... S(Q) = INTEGRAL_M R^A /\ R^B Q_AB
where Q_AB are constants ... to be chosen ... we require
... the invariance of S(Q) under local Lorentz transformations
... the invariance of S(Q) under space inversions ...
...[AFTER A LOT OF ALGEBRA NOT SHOWN IN THIS QUOTE]...
we shall see ...[that]... the action becomes invariant
under all local [anti]de-Sitter transformations ...[and]... we recognize ... t
he familiar Hilbert-Einstein action with cosmological term in vierbein notation ...
Variation of the vierbein leads to the Einstein equations with cosmological term.
Variation of the spin-connection ... in turn ... yield the torsionless Christoffel connection ... the torsion components ... now vanish.
So at this level full sp(4) invariance has been checked.
... Were it not for the assumed space-inversion invariance ...
we could have had a parity violating gravity. ...
Unlike Einstein's theory ...[MacDowell-Mansouri].... does not require Riemannian invertibility of the metric. ... the solution has torsion ... produced by an interference between parity violating and parity conserving amplitudes.
Parity violation and torsion go hand-in-hand.
Independently of any more realistic parity violating solution of the gravity equations this raises the cosmological question whether
the universe as a whole is in a space-inversion symmetric configuration. ...".

According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:
"... If the fundamental spacetime symmetry of the laws of Physics is that given by the de Sitter instead of the Poincare group, the P-symmetry of the weak cosmological-constant limit and the Q-symmetry of the strong cosmological constant limit can be considered as limiting cases of the fundamental symmetry. ...
... N ...[is the space]... whose geometry is gravitationally related to an infinite cosmological constant ...[and]... is a 4-dimensional cone-space in which ds = 0, and whose group of motion is Q. Analogously to the Minkowski case, N is also a homogeneous space, but now under the kinematical group Q, that is, N = Q/L [where L is the Lorentz Group of Rotations and Boosts]. In other words, the point-set of N is the point-set of the special conformal transformations. Furthermore, the manifold of Q is a principal bundle P(Q/L,L), with Q/L = N as base space and L as the typical fiber. The kinematical group Q, like the Poincare group, has the Lorentz group L as the subgroup accounting for both the isotropy and the equivalence of inertial frames in this space. However, the special conformal transformations introduce a new kind of homogeneity. Instead of ordinary translations, all the points of N are equivalent through special conformal transformations. ...
... Minkowski and the cone-space can be considered as dual to each other, in the sense that their geometries are determined respectively by a vanishing and an infinite cosmological constants. The same can be said of their kinematical group of motions: P is associated to a vanishing cosmological constant and Q to an infinite cosmological constant.
The dual transformation connecting these two geometries is the spacetime inversion x^u -> x^u / sigma^2 . Under such a transformation, the Poincare group P is transformed into the group Q, and the Minkowski space M becomes the conespace N. The points at infinity of M are concentrated in the vertex of the conespace N, and those on the light-cone of M becomes the infinity of N. It is concepts of space isotropy and equivalence between inertial frames in the conespace N are those of special relativity. The difference lies in the concept of uniformity as it is the special conformal transformations, and not ordinary translations, which act transitively on N. ..."

Gravity and the Cosmological Constant come from the MacDowell-Mansouri Mechanism and the 15-dimensional Spin(2,4) = SU(2,2) Conformal Group, which is made up of:

3 Rotations
3 Boosts
4 Translations
4 Special Conformal transformations
1 Dilatation

The **Cosmological Constant / Dark Energy** comes from
the **10 Rotation, Boost, and Special Conformal generators**
of the Conformal Group Spin(2,4) = SU(2,2),
so the fractional part of our Universe of the Cosmological Constant
should be **about 10 / 15 = 67% for tree level**.

Black Holes, including **Dark Matter Primordial Black Holes**, are curvature singularities in our 4-dimensional physical spacetime,
and since Einstein-Hilbert curvature comes from the **4 Translations**
of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2)
through the MacDowell-Mansouri Mechanism (in which the generators corresponding to the 3 Rotations and 3 Boosts do not propagate),
the fractional part of our Universe of Dark Matter Primordial Black Holes
should be **about 4 / 15 = 27% at tree level**.

Since **Ordinary Matter** gets mass from the Higgs mechanism
which is related to the **1 Scale Dilatation**
of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2),
the fractional part of our universe of Ordinary Matter
should be **about 1 / 15 = 6% at tree level**.

However,
as Our Universe evolves the Dark Energy, Dark Matter, and Ordinary Matter densities evolve at different rates,
so that the differences in evolution must be taken into account
from the initial End of Inflation to the Present Time.

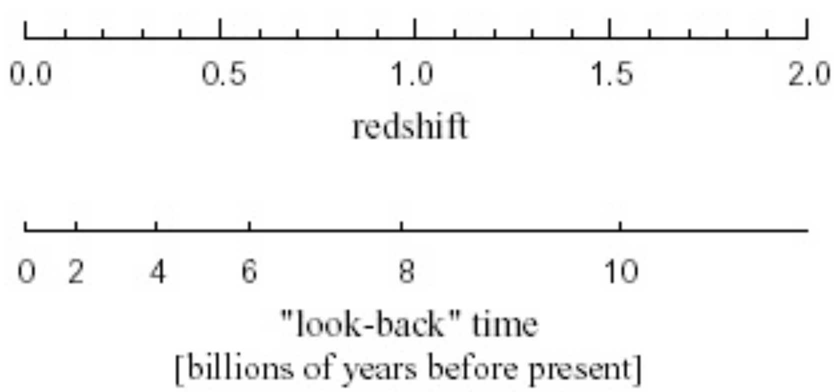
Without taking into account any evolutionary changes with time,
our Flat Expanding Universe should have roughly:

67% Cosmological Constant
27% Dark Matter - possilbly primordial stable Planck mass black holes
6% Ordinary Matter

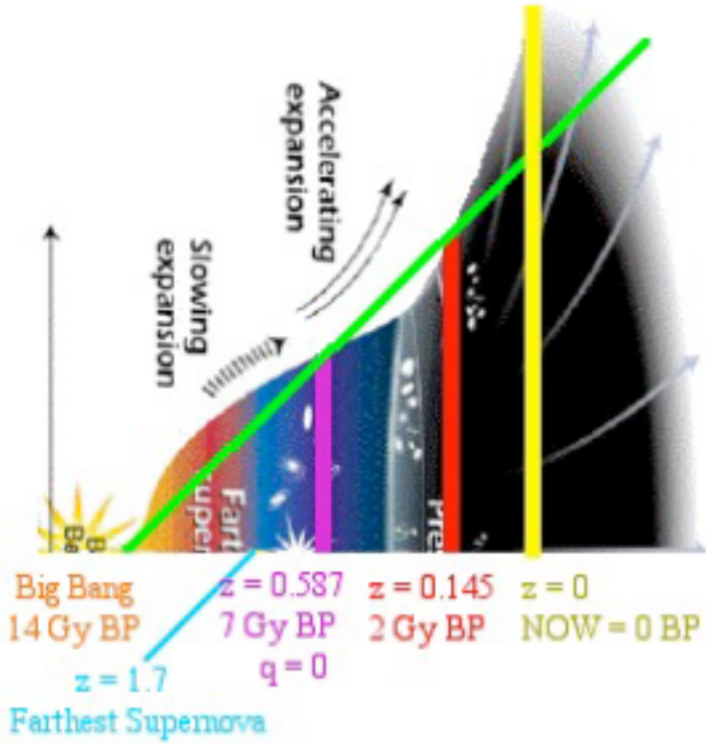
As Dennis Marks pointed out to me,
since density rho is proportional to (1+z)^3(1+w) for red-shift factor z
and a constant equation of state w:
w = -1 for Λ and the average overall density of Λ Dark Energy remains constant with time and the expansion of our Universe;
and
w = 0 for nonrelativistic matter so that the overall average density of Ordinary Matter declines as 1 / R^3 as our Universe expands;
and
w = 0 for primordial black hole dark matter - stable Planck mass black holes - so that Dark Matter also has density that declines as 1 / R^3 as our Universe expands;
so that the ratio of their overall average densities must vary with time, or scale factor R of our Universe, as it expands.
Therefore,
the above calculated ratio 0.67 : 0.27 : 0.06 is valid
only for a particular time, or scale factor, of our Universe.

When is that time ? Further, what is the value of the ratio now ?

Since WMAP observes Ordinary Matter at 4% NOW,
the time when Ordinary Matter was 6% would be
at redshift z such that
1 / (1+z)^3 = 0.04 / 0.06 = 2/3 , or (1+z)^3 = 1.5 , or 1+z = 1.145 , or z = 0.145.
To translate redshift into time,
in billions of years before present, or Gy BP, use this chart



from a www.supernova.lbl.gov file SNAPoverview.pdf to see that
the time when Ordinary Matter was 6%
would have been a bit over 2 billion years ago, or 2 Gy BP.



In the diagram, there are four Special Times in the history of our Universe:
the Big Bang Beginning of Inflation (about 13.7 Gy BP);

1 - the End of Inflation = Beginning of Decelerating Expansion
(beginning of green line also about 13.7 Gy BP);

2 - the End of Deceleration (q=0) = Inflection Point =
= Beginning of Accelerating Expansion
(purple vertical line at about z = 0.587 and about 7 Gy BP).
According to a hubblesite web page credited to Ann Feild, the above diagram "... reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart as a faster rate. ...".
According to a CERN Courier web page: "... Saul Perlmutter, who is head of the Supernova Cosmology Project ... and his team have studied altogether some 80 high red-shift type Ia supernovae. Their results imply that the universe was decelerating for the first half of its existence, and then began accelerating approximately 7 billion years ago. ...".
According to astro-ph/0106051 by Michael S. Turner and Adam G. Riess: "... current supernova data ... favor deceleration at z > 0.5 ... SN 1997ff at z = 1.7 provides direct evidence for an early phase of slowing expansion if the dark energy is a cosmological constant ...".

3 - the Last Intersection of the Accelerating Expansion of our Universe
of Linear Expansion (green line) with the Third Intersection
(at red vertical line at $z = 0.145$ and about 2 Gy BP),
which is also around the times of the beginning of the Proterozoic Era and Eukaryotic Life, Fe_2O_3 Hematite ferric iron Red Bed formations, a Snowball Earth, and the start of the Oklo fission reactor. 2 Gy is also about 10 Galactic Years for our Milky Way Galaxy and is on the order of the time for the process of a collision of galaxies.

4 - Now.
Those four Special Times define four Special Epochs:
The Inflation Epoch, beginning with the Big Bang and ending with the End of Inflation. The Inflation Epoch is described by Zizzi Quantum Inflation ending with Self-Decoherence of our Universe (see gr-qc/0007006).
The Decelerating Expansion Epoch, beginning with the Self-Decoherence of our Universe at the End of Inflation. During the Decelerating Expansion Epoch, the Radiation Era is succeeded by the Matter Era, and the Matter Components (Dark and Ordinary) remain more prominent than they would be under the "standard norm" conditions of Linear Expansion.
The Early Accelerating Expansion Epoch, beginning with the End of Deceleration and ending with the Last Intersection of Accelerating Expansion with Linear Expansion. During Accelerating Expansion, the prominence of Matter Components (Dark and Ordinary) declines, reaching the "standard norm" condition of Linear Expansion at the end of the Early Accelerating Expansion Epoch at the Last Intersection with the Line of Linear Expansion.
The Late Accelerating Expansion Epoch, beginning with the Last Intersection of Accelerating Expansion and continuing forever, with New Universe creation happening many times at Many Times. During the Late Accelerating Expansion Epoch, the Cosmological Constant Λ is more prominent than it would be under the "standard norm" conditions of Linear Expansion.
Now happens to be about 2 billion years into the Late Accelerating Expansion Epoch.

What about Dark Energy : Dark Matter : Ordinary Matter now ?

As to how the Dark Energy Λ and Cold Dark Matter terms have evolved
during the past 2 Gy, a rough estimate analysis would be:

Λ and CDM would be effectively created during expansion in their natural ratio
67 : 27 = 2.48 = 5 / 2, each having proportionate fraction 5 / 7 and 2 / 7, respectively;
CDM Black Hole decay would be ignored; and
pre-existing CDM Black Hole density would decline by the same $1 / R^3$ factor as Ordinary Matter, from 0.27 to 0.27 / 1.5 = 0.18.

The Ordinary Matter excess 0.06 - 0.04 = 0.02 plus the first-order CDM excess 0.27 - 0.18 = 0.09 should be summed to get a total first-order excess of 0.11, which in turn should be distributed to the Λ and CDM factors in their natural ratio 67 : 27, producing, for NOW after 2 Gy of expansion:

CDM Black Hole factor = 0.18 + 0.11 x 2/7 = 0.18 + 0.03 = 0.21
for a total calculated Dark Energy : Dark Matter : Ordinary Matter ratio for now of

0.75 : 0.21 : 0.04

so that the present ratio of 0.73 : 0.23 : 0.04 observed by WMAP seems to me to be substantially consistent with the cosmology of the E8 model.

2013 Planck Data (arxiv 1303.5062) showed "... anomalies ... previously observed in the WMAP data ... alignment between the quadrupole and octopole moments ... asymmetry of power between two ... hemispheres ... Cold Spot ... are now confirmed at ... 3 sigma ... but a higher level of confidence ...".

E8 model rough evolution calculation is: DE : DM : OM = 75 : 20 : 05
WMAP: DE : DM : OM = 73 : 23 : 04
Planck: DE : DM : OM = 69 : 26 : 05
basic unevolved E8 Conformal calculation: DE : DM : OM = 67 : 27 : 06

Since uncertainties are substantial, I think that there is reasonable consistency.

Appendix - Spinor Growth, Octonion Inflation ended by Quaternions

Where does the E8 of E8 Physics come from ?
Based on David Finkelstein's view of Fundamental Physics:

In the beginning there was Cl(0) spinor fermion void

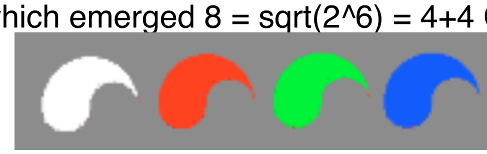
from which emerged 2 = sqrt(2^2) = 1+1 Cl(2) half-spinor fermions/antifermions

 and 

from which emerged 4 = sqrt(2^4) = 2+2 Cl(4) half-spinor fermions/antifermions

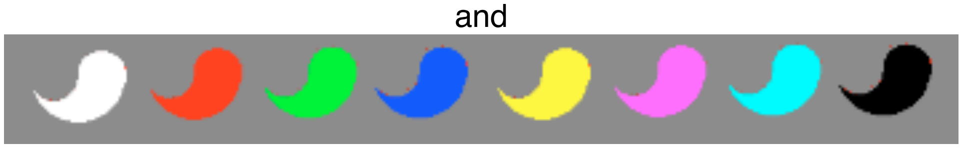
 and 

from which emerged 8 = sqrt(2^6) = 4+4 Cl(6) half-spinor fermions/antifermions

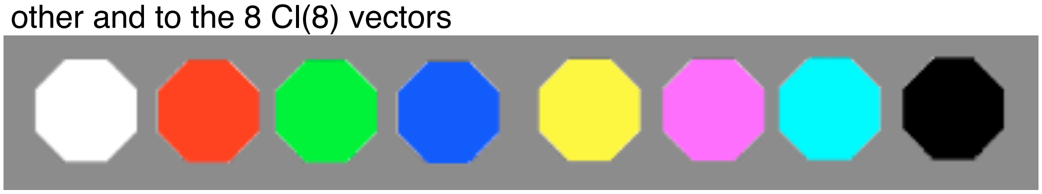
 and 

from which emerged 16 = sqrt(2^8) = 8+8 Cl(8) half-spinor fermions/antifermions

and

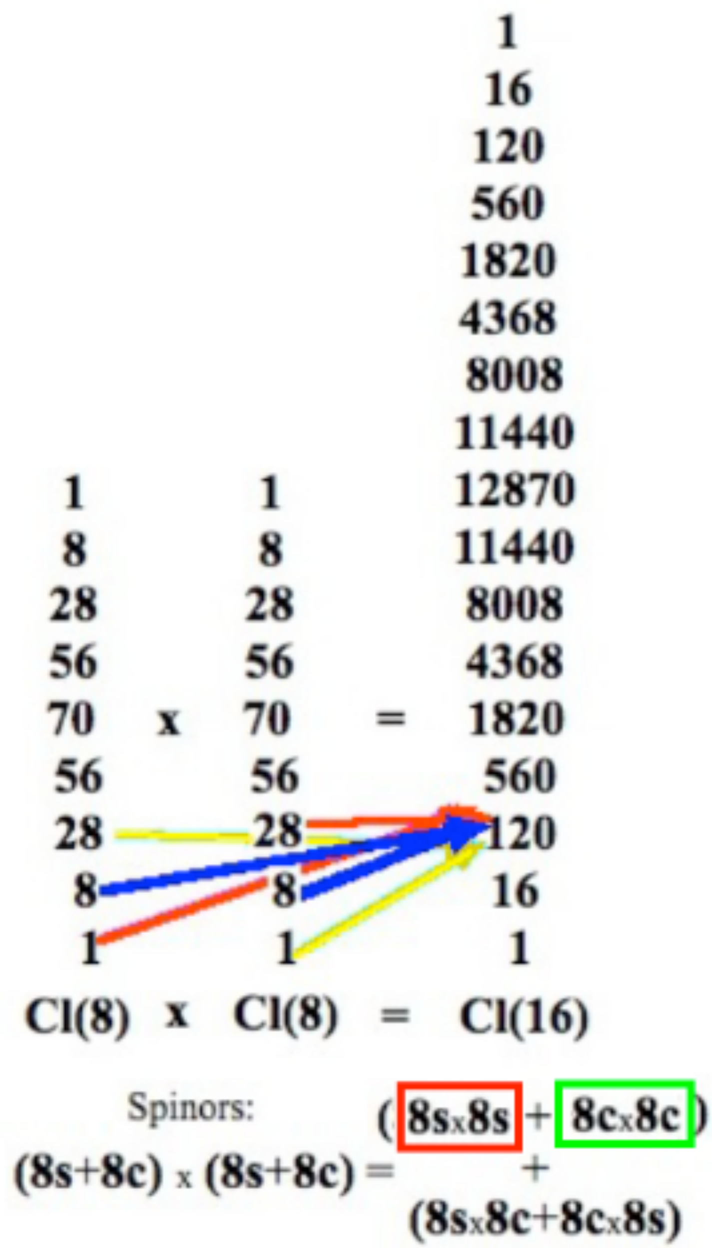


8 half-spinor fermions and 8 half-spinor antifermions are isomorphic by Cl(8) Triality to each other and to the 8 Cl(8) vectors



8-Periodicity of Real Clifford Algebras
Cl(8) x ...(N times tensor product)... x Cl(8) = Cl(8N)
shows that Cl(8) (or any tensor multiple it) is the basic building block
of ALL Real Clifford Algebras, no matter how large they may be.

In particular, the tensor product Cl(8) x Cl(8) = Cl(16)



256 = sqrt(2^16) = 128 + 128 Cl(16) spinors

128 Cl(16) half-spinors = 64 + 64 fermions + antifermions

120 = Cl(16) bivectors = D8 root vectors

120 + 64 + 64 = E8 root vectors

E8 / D8 = 128-dim (OxO)P2 OctoOctonionic Projective Plane

D8 / D4xD4 = Gr(8,16) = 64-dim Octonionic Subspaces of R16

(Gr = Grassmanian and R16 = Vectors of Clifford Cl(16) Matrix Algebra for D8)

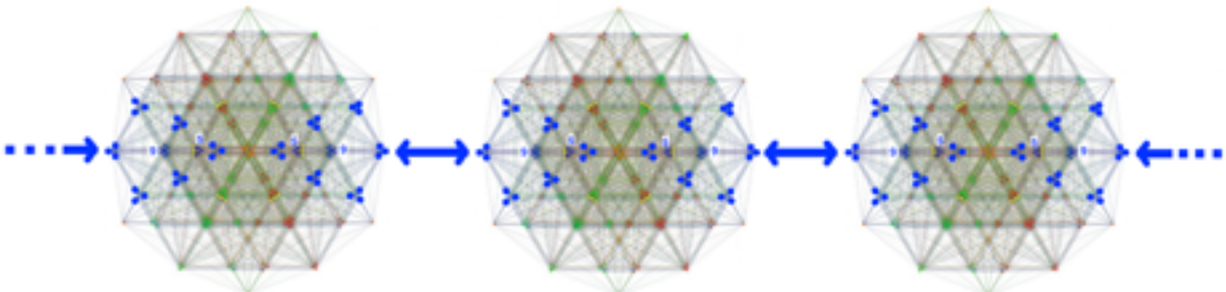
one D4 contains D3 of Conformal Gravity+Dark Energy

other D4 contains A3 of Standard Model Color Force SU(3)

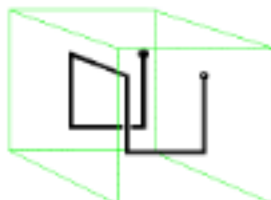
(CP2 = SU(3) / SU(2)xU(1) of Kaluza-Klein contains SU(2)xU(1) of Electroweak Forces)

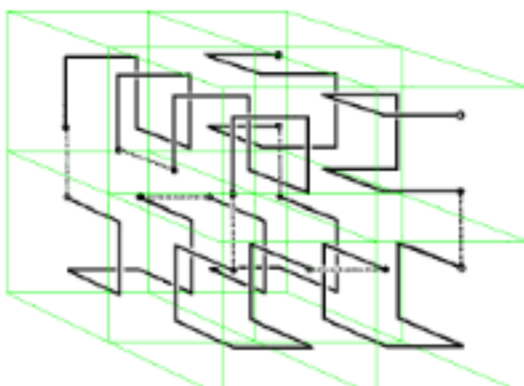
One Cl(16) containing one E8 gives a Lagrangian description of one local spacetime neighborhood. To get a realistic global spacetime structure, take the tensor product Cl(16) x ... x Cl(16) with all E8 local 8-dim Octonionic spacetimes consistently aligned as described by 64-dim D8 / D4xD4 (blue dots)

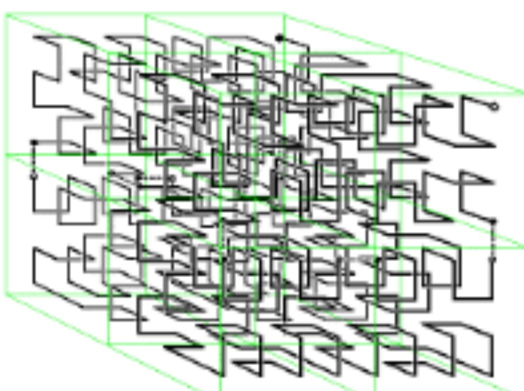
(this visualization uses a hexagonal type of projection of the 240 E8 root vectors to 2-dim)



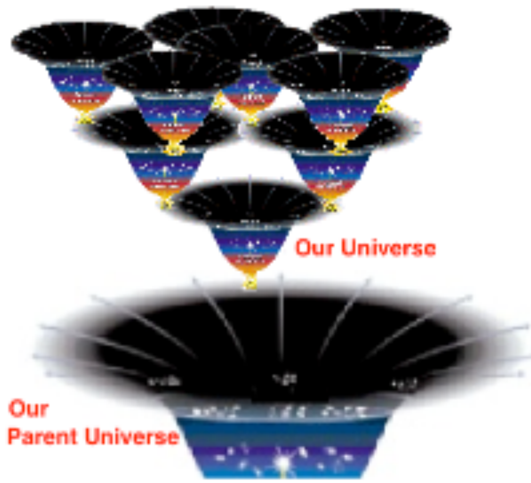
which then fill up spacetime according to Gray Code Hilbert's curves:







Our Universe emerged from its parent in Octonionic Inflation



As Our Parent Universe expanded to a Cold Thin State Quantum Fluctuations occurred.
Most of them just appeared and disappeared as Virtual Fluctuations,
but at least one Quantum Fluctuation had enough energy to produce 64 Unfoldings
and reach Paola Zizzi's State of Decoherence
thus making it a Real Fluctuation that became Our Universe.

As Our Universe expands to a Cold Thin State, it will probably give birth
to Our Child, GrandChild, etc, Universes.

Unlike "the inflationary multiverse" decribed by Andrei Linde in arXiv 1402.0526 as
"a scientific justification of the anthropic principle",
in the Cl(16)-E8 model ALL Universes (Ours, Ancestors, Descendants)
have the SAME Physics Structure as E8 Physics (viXra 1312.0036 and 1310.0182)

In the Cl(16)-E8 model,
our SpaceTime remains Octonionic 8-dimensional throughout inflation.

Stephen L. Adler in his book Quaternionic Quantum Mechanics and Quantum Fields (1995) said at pages 50-52, 561: "... If the multiplication is associative, as in the complex and quaternionic cases, we can remove parentheses in ... Schroedinger equation dynamics ... to conclude that ... the inner product < f(t) I g(t) > ... is invariant ... this proof fails in the octonionic case, and hence one cannot follow the standard procedure to get a unitary dynamics. ...[so there is a]...
failure of unitarity in octonionic quantum mechanics ...".

The NonAssociativity and Non-Unitarity of Octonions
accounts for particle creation without the need for a conventional inflaton field.

E8 Physics has Representation space for 8 Fermion Particles + 8 Fermion Antiparticles on the original Cl(16) E8 Local Lagrangian Region

_ _ _ _ _ _ _ _ + _ _ _ _ _ _ _ _

where a Fermion Representation slot _ of the 8+8 = 16 slots can be filled

by Real Fermion Particles  or Real Fermion Antiparticles 

IF the Quantum Fluctuation(QF) has enough Energy to produce them as Real
and
IF the Cl(16) E8 Local Lagrangian Region has an Effective Path from its QF Energy to that Particular slot.

Let Cl(16) = Cl(8) x Cl(8) where
the first Cl(8) contains the D4 of Conformal Gravity with actions on M4 physical spacetime whose CPT symmetry determines the property matter - antimatter.

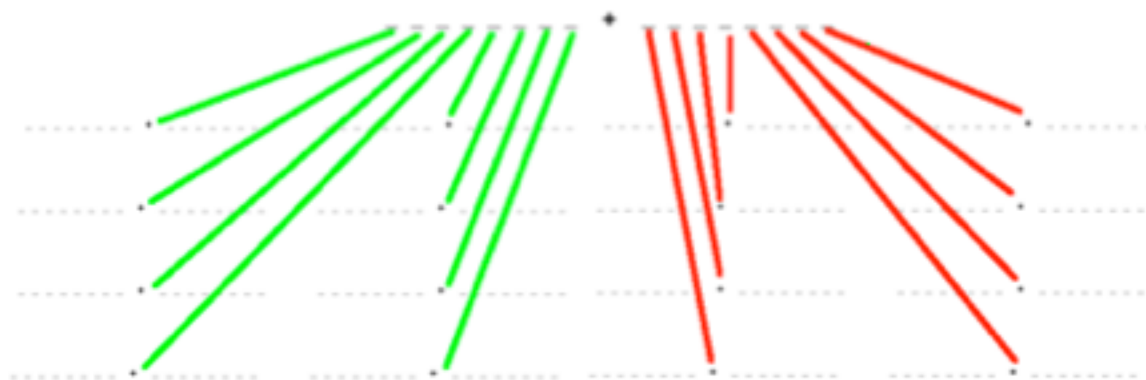
Consider, following basic ideas of Geoffrey Dixon related to his characterization of 64-dimensional spinor spaces as C x H x O (C = complex, H = quaternion, O = ocrtonion),
64-dim 64s++ = 8s+ x 8s+ of Cl(8) x Cl(8) = Cl(16)
and
64-dim 64s+- = 8s+ x 8s- of Cl(8) x Cl(8) = Cl(16)
so that
64s++ + 64s+- = 128s+ are +half-spinors of Cl(16) which is in E8

Then Cl(16) contains
128-dim +half-spinor space 64s++ + 64s+- of Cl(16) in E8 = Fermion Generation
and
128-dim -half-spinor space 64s-+ + 64s-- of Cl(16) not in E8 = Fermion AntiGeneration

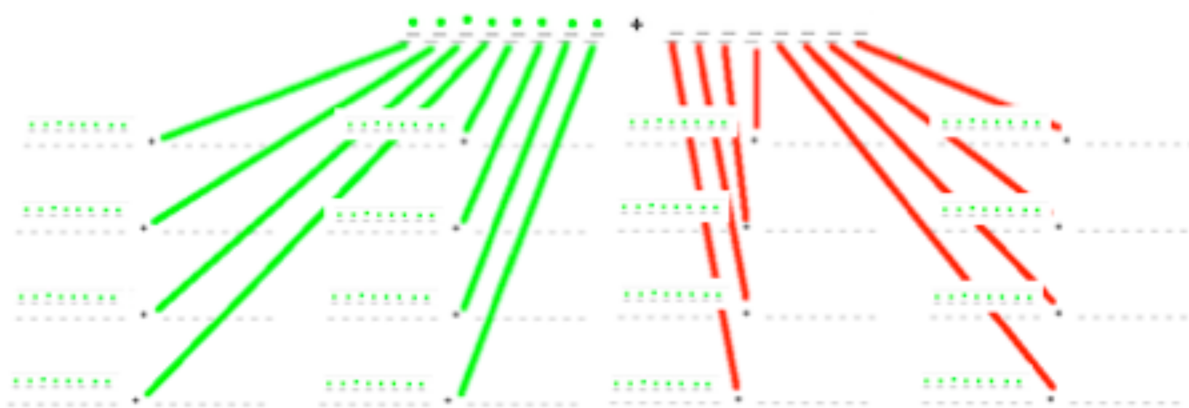
Since E8 contains only the 128 +half-spinors and none of the 128 -half-spinors of Cl(16)
and
since, due to their +half-spinor property with respect to the first Cl(8),
the 128s+ = 64s++ + 64s+- have only Effective Paths of QF Energy that go to the Fermion Particle slots that are also of type +
that is, to the 8 Fermion Particle Representation slots

Next, consider **the first Unfolding step of Octonionic Inflation**. It is based on all 16 = 8 Fermion Particle slots + 8 Fermion Antiparticle Representation slots whether or not they have been filled by QF Energy.

7 of the 8 Fermion Particle slots correspond to the 7 Imaginary Octonions and
therefore to the 7 Independent E8 Integral Domain Lattices and
therefore to 7 New Cl(16) E8 Local Lagrangian Regions.
The 8th Fermion Particle slot corresponds to the 1 Real Octonion and
therefore to the 8th E8 Integral Domain Lattice (not independent - see Kirmse's mistake) and
therefore to the 8th New Cl(16) E8 Local Lagrangian Region.
Similarly, the 8 Fermion Antiparticle slots Unfold into 8 more New New Cl(16) E8 Local Lagrangian Regions, so that one Unfolding Step is a 16-fold multiplication of Cl(16) E8 Local Lagrangian Regions:



If the QF Energy is sufficient, the Fermion Particle content after the first Unfolding is



so it is clear that **the Octonionic Inflation Unfolding Process creates Fermion Particles with no Antiparticles, thus explaining the dominance of Matter over AntiMatter in Our Universe**.

Each Unfolding has duration of the Planck Time Tplanck
and none of the components of the Unfolding Process Components are simultaneous,
so that **the total duration of N Unfoldings is 2^N Tplanck**.
Paola Zizzi in gr-qc/0007006 said: "... **during inflation,**
the universe can be described as a superposed state of quantum ... [qubits].
the self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... **the decoherence time ... [Tdecoh = 10^9 Tplanck = 10^(-34) sec]** ...
and **corresponds to a superposed state of ... [10^19 = 2^64 qubits]**. ...".

Why decoherence at 64 Unfoldings = 2^64 qubits ?

2^64 qubits corresponds to the Clifford algebra Cl(64) = Cl(8x8).
By the periodicity-8 theorem of Real Clifford algebras, Cl(64) is the smallest Real Clifford algebra for which we can reflexively identify each component Cl(8) with a vector in the Cl(8) vector space. This reflexive identification/reduction causes our universe to decohere at N = 2^64 = 10^19
which is roughly the number of Quantum Consciousness Tubulins in the Human Brain.
The Real Clifford Algebra Cl(8) is the basic building block of Real Clifford Algebras due to 8-Periodicity whereby Cl(8N) = Cl(8) x ...(N times tensor product)... x Cl(8)
An Octonionic basis for the Cl(8) 8-dim vector space is {1,i,j,k,E,I,J,K}
NonAssociativity, NonUnitarity, and Reflexivity of Octonions is exemplified by the 1-1 correspondence between Octonion Basis Elements and E8 Integral Domains

1 <=> 0E8
i <=> 1E8
j <=> 2E8
k <=> 3E8
E <=> 4E8
I <=> 5E8
J <=> 6E8
K <=> 7E8

where 1E8,2E8,3E8,4E8,5E8,6E8,7E8 are 7 independent Integral Domain E8 Lattices and 0E8 is an 8th E8 Lattice (Kirmse's mistake) not closed as an Integral Domain.
Using that correspondence expands the basis {1,i,j,k,E,I,J,K} to

{0E8,1E8,2E8,3E8,4E8,5E8,6E8,7E8}

Each of the E8 Lattices has 240 nearest neighbor vectors so the total dimension of the Expanded Space is 240 x 240 x 240 x 240 x 240 x 240 x 240 x 240
Everything in the Expanded Space comes directly from the original Cl(8) 8-dim space so all Quantum States in the Expanded Space can be held in Coherent Superposition.
However,
if further expansion is attempted, there is no direct connection to original Cl(8) space and any Quantum Superposition undergoes Decoherence.
If each 240 is embedded reflexively into the 256 elements of Cl(8) the total dimension is

256 x 256 x 256 x 256 x 256 x 256 x 256 x 256 = 256^8 = 2^(8x8) = 2^64 =
= Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) = Cl(8x8) = Cl(64)

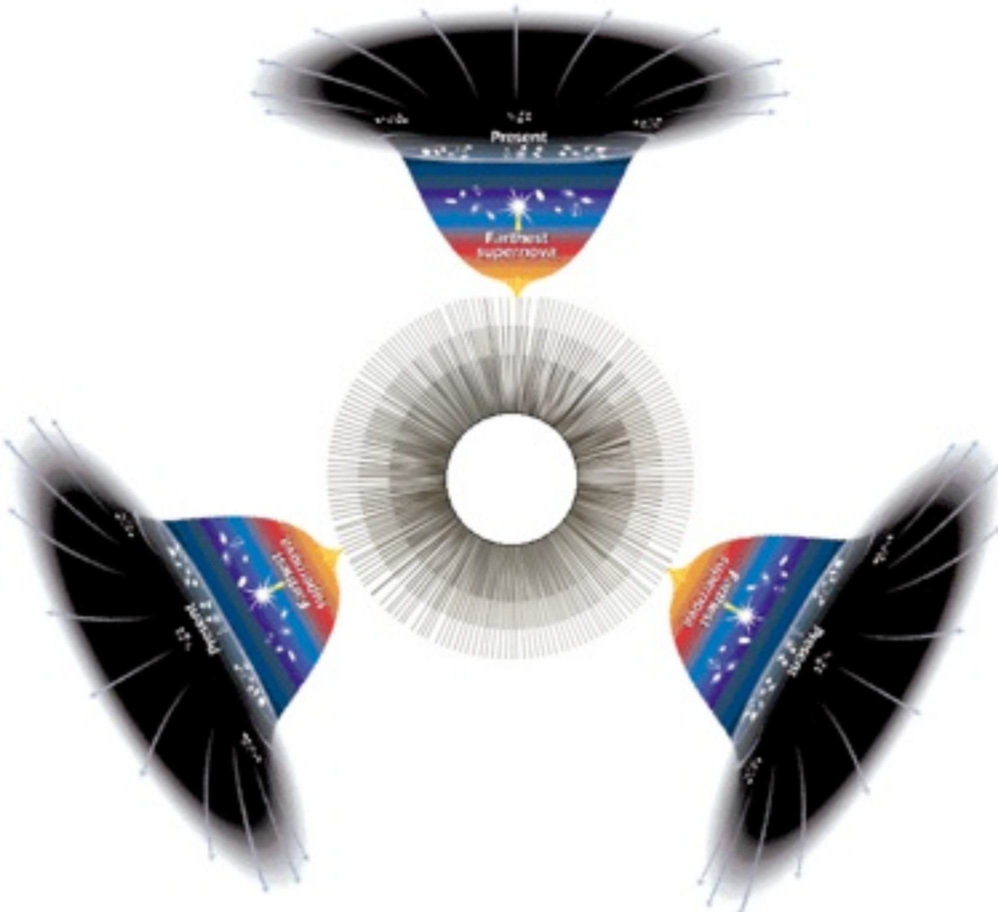
so the largest Clifford Algebra that can maintain Coherent Superposition is Cl(64) which is why Zizzi Quantum Inflation ends at the Cl(64) level.

At the end of 64 Unfoldings, Non-Unitary Octonionic Inflation ended having produced about (1/2) 16^64 = (1/2) (2^4)^64 = 2^255 = 6 x 10^76 Fermion Particles

The End of Inflation time was at about 10^(-34) sec = 2^64 Tplanck
and
the size of our Universe was then about 10^(-24) cm
which is **about the size of a Fermion Schwinger Source Kerr-Newman Cloud**.

End of Inflation and Low Initial Entropy in Our Universe:

Roger Penrose in his book The Emperor's New Mind (Oxford 1989, pages 316-317) said:
"... in our universe ... Entropy ... increases ... Something forced the entropy to be low in the past. ... **the low-entropy states in the past are a puzzle**. ...".
The key to solving Penrose's Puzzle is given by Paola Zizzi in gr-qc/0007006:
"... **The self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... the decoherence time ... [Tdecoh = 10^9 Tplanck = 10(-34) sec] ... and corresponds to a superposed state of ... [10^19 = 2^64 qubits]**. ...
... **This is also the number of superposed tubulins-qubits in our brain ... leading to a conscious event**. ...".
The Zizzi Inflation phase of our universe ends with decoherence "collapse" of the 2^64 Superposition Inflated Universe into Many Worlds of Quantum Theory,



only one of which Worlds is our World. The central white circle is the Inflation Era in which everything is in Superposition; the boundary of the central circle marks the decoherence/collapse at the End of Inflation; and each line radiating from the central circle corrresponds to one decohered/collapsed Universe World (of course, there are many more lines than actually shown), only three of which are explicitly indicated in the image, and only one of which is Our Universe World.

Since our World is only a tiny fraction of all the Worlds, it carries only a tiny fraction of the entropy of the 2^64 Superposition Inflated Universe, thus solving Penrose's Puzzle.

End of Inflation and Quaternionic Structure

In Cl(16)-E8 Physics (vixra 1405.0030) Octonionic symmetry of 8-dim spacetime is broken at the End of Non-Unitary Octonionic Inflation to Quaternionic symmetry of (4+4)-dim Kaluza-Klein M4 x CP2 physical spacetime x internal symmetry space.

Here are some details about that transition:

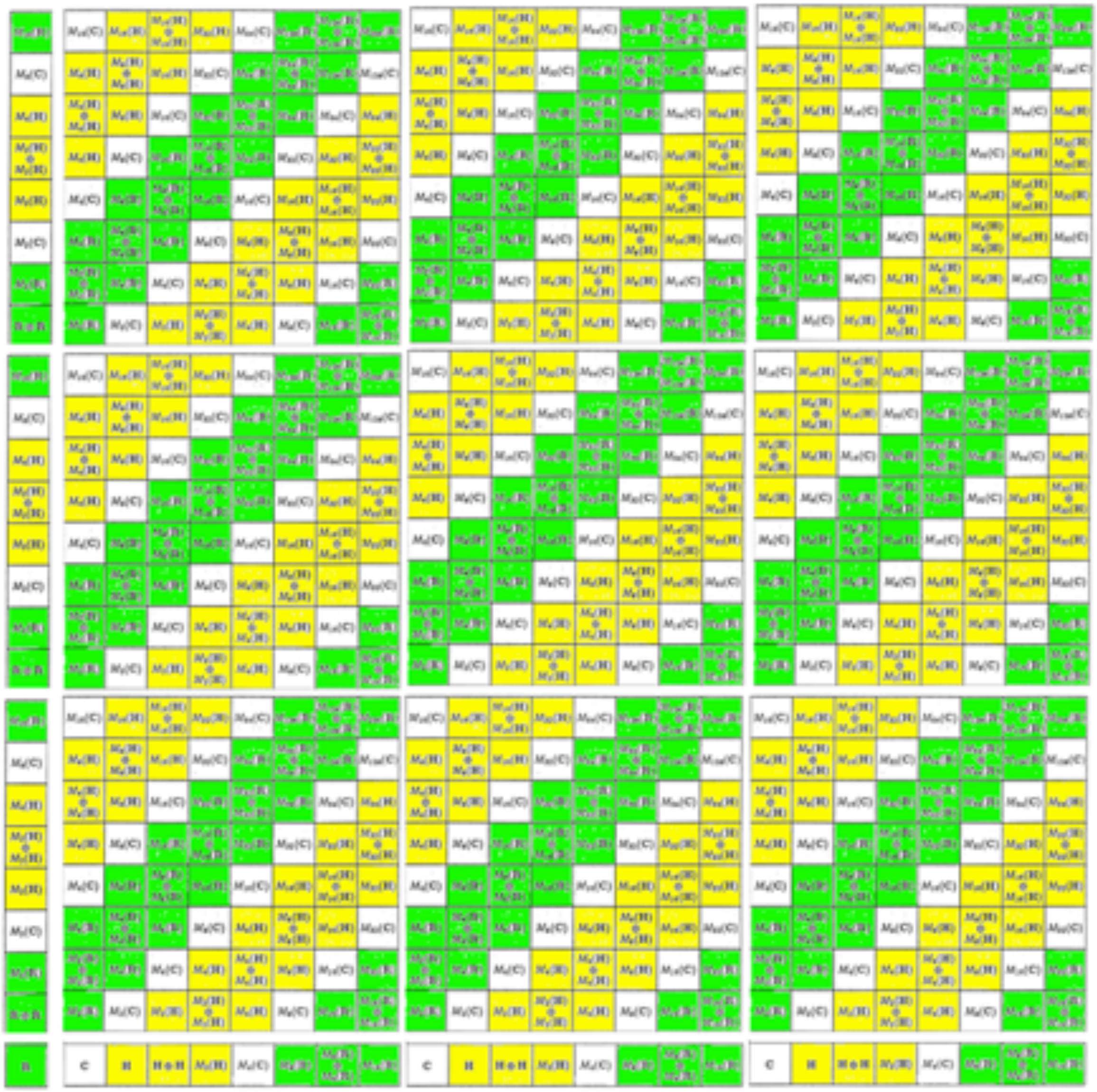
The basic local entity of Cl(16)-E8 Physics is
Cl(0,16) = Cl(1,15) = Cl(4,12) = Cl(5,11) = Cl(8,8) = M(R,256) = 256x256 Real Matrices
which contains E8 with 8-dim Octonionic spacetime
and is the tensor product Cl(0,8) x Cl(0,8) = Cl(1,7) x Cl(1,7)
where Cl(0,8) = Cl(1,7) = M(R,16) is the Clifford Algebra of the 8-dim spacetime.

Non-Unitary Octonionic Inflation is based on Octonionic spacetime structure with superposition of E8 integral domain lattices. At the End of Inflation the superposition ends and Octonionic 8-dim structure is replaced by Quaternionic (4+4)-dim structure.

Since M(R,16) = M(Q,2) x M(Q,2) and M(Q,2) = Cl(1,3) = Cl(0,4)
Cl(0,8) = Cl(1,7) can be represented as Cl(1,3) x Cl(0,4)
where
Cl(1,3) is the Clifford Algebra for M4 physical spacetime
and
Cl(0,4) is the Clifford Algebra for CP2 = SU(3) / U(2) internal symmetry space
thus
making explicit the Quaternionic structure of (4+4)-dim M4 x CP2 Kaluza-Klein.

Quaternionic structure similar to that of Cl(1,3) = Cl(0,4) = M(Q,2) is seen in
Cl(2,4) = M(Q,4) = 4x4 Quaternion matrices with grading based on 4x4 = 1 4 6 4 1

1	2	1				
	4	8	4			
		6	12	6		
			4	8	4	
				1	2	1
1	6	15	20	15	6	1



Conformal Gravity Spin(2,4) = SU(2,2) of Cl(2,4) = M(Q,4) 4x4 Quaternionic Matrices

Appendix - Grothendieck Universe Quantum Theory and Code

The First Grothendieck Universe is the Empty Set.

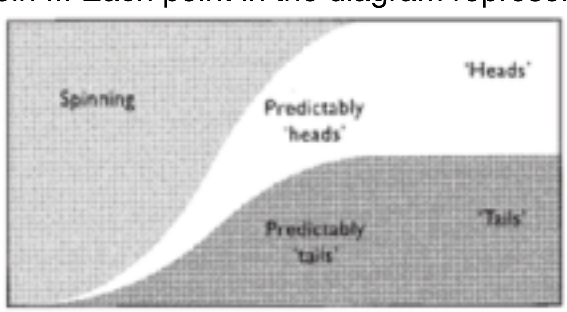
The Second Grothendieck Universe is Hereditarily Finite Sets such as a Generalized Feynman Checkerboard Quantum Theory based on E8 Lattices and Discrete Cl(16) Clifford Algebra.
(viXra 1501.0078)

The Third Grothendieck Universe is the Completion of Union of all tensor products of Cl(16) Real Clifford algebra

Since the Cl(16)-E8 Lagrangian is Local and Classical,
it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global Cl(16)-E8 Algebraic Quantum Field Theory (AQFT).

The usual Hyperfinite II1 von Neumann factor for creation and annihilation operators on Fermionic Fock Space over C^(2n) is constructed by completion of the union of all tensor products of 2x2 Complex Clifford algebra matrices, which have Periodicity 2, so
for the Cl16)-E8 model based on Real Clifford Algebras with Periodicity 8,
whereby any Real Clifford Algebra, no matter how large,
can be embedded in a tensor product of factors of Cl(8) and of Cl(8)xCl(8) = Cl(16),
the completion of the union of all tensor products of Cl(16) = Cl(8)xCl(8)
produces a generalized Hyperfinite II1 von Neumann factor
that gives the Cl(16)-E8 model a natural Algebraic Quantum Field Theory.

The overall structure of Cl(160-E8 AQFT is similar to the Many-Worlds picture described by David Deutsch in his 1997 book "The Fabric of Reality" said (pages 276-283):
"… there is no fundamental demarcation between snapshots of other times and snapshots of other universes ... Other times are just special cases of other universes ... Suppose ... we toss a coin ... Each point in the diagram represents one snapshot



... in the multiverse there are far too many snapshots for clock readings alone to locate a snapshot relative to the others. To do that, we need to consider the intricate detail of which snapshots determine which others. …
in some regions of the multiverse, and in some places in space,

the snapshots of some physical objects do fall, for a period, into chains,
each of whose members determines all the others to a good approximation …".

The Real Clifford Algebra Cl(16) containing E8 for the Local Lagrangian of a Region is equivalent to a " snapshot" of the Deutsch "multiverse".
The completion of the union of all tensor products of all Cl(16)-E8 Local Lagrangian Regions forms a generalized hyperfinite II1 von Neumann factor AQFT and emergently self-assembles into a structure = Deutsch multiverse.

For the Cl(16)-E8 model AQFT to be realistic, it must be consistent with **EPR entanglement relations**. Joy Christian in arXiv 0904.4259 said:
"... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings ... The alleged non-localities ... result from misidentified [geometries] of the EPR elements of reality. ... The correlations are ... the classical correlations [such as those] among the points of a 3 or 7-sphere ... S3 and S7 ... are ... parallelizable ... The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...".
Since E8 is a Lie Group and therefore parallelizable and lives in Clifford Algebra Cl(16), **the Cl(16)-E8 model is consistent with EPR**.

The Creation-Annihilation Operator structure of Cl(16)-E8 AQFT is given by the

Maximal Contraction of E8 = semidirect product A7 x h92

where h92 = 92+1+92 = 185-dim Heisenberg algebra and A7 = 63-dim SL(8)
The Maximal E8 Contraction A7 x h92 can be written as a 5-Graded Lie Algebra

28 + 64 + (SL(8,R) + 1) + 64 + 28

Central Even Grade 0 = SL(8,R) + 1

The 1 is a scalar and SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices,
so SL(8,R) represents a local 8-dim SpaceTime in Polar Coordinates.

Odd Grades -1 and +1 = 64 + 64

Each = 64 = 8x8 = Creation/Annihilation Operators for 8 components of 8 Fundamental Fermions.

Even Grades -2 and +2 = 28 + 28

Each = Creation/Annihilation Operators for 28 Gauge Bosons of Gravity + Standard Model.

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

$\int$ Gauge Gravity + Standard Model + Fermion Particle-AntiParticle
8-dim SpaceTime .

The Cl(16)-E8 generalized Hyperfinite II1 von Neumann factor Algebraic Quantum Field Theory is based on the Completion of the Union of all Tensor Products of the form

Cl(16) x ...(N times tensor product)... x Cl(16) = Cl(16N)

For N = 2^8 = 256 the copies of Cl(16) are on the 256 vertices of the 8-dim HyperCube

For N = 2^16 = 65,536 = 4^8 the copies of Cl(16) fill in the 8-dim HyperCube
as described by William Gilbert's web page: "... **The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube**. ...".

The vertices of the Hilbert curve are at the centers of the 2^8 sub-8-HyperCubes whose edge lengths are 1/2 of the edge lengths of the original 8-dim HyperCube

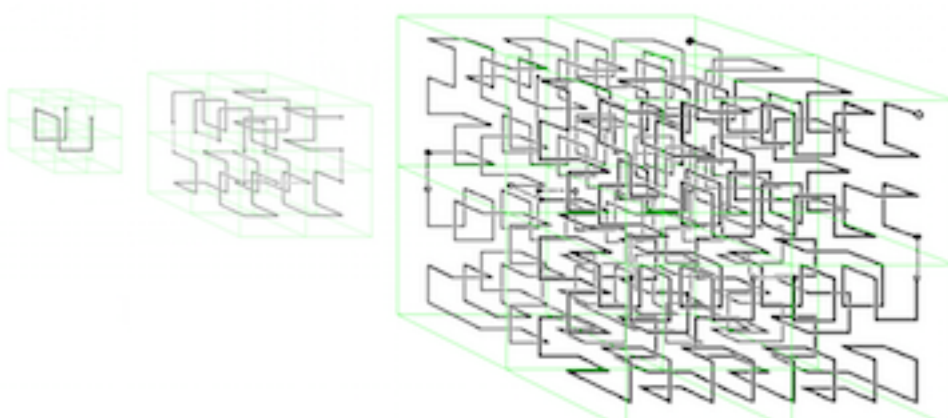
As N grows, the copies of Cl(16) continue to fill the 8-dim HyperCube of E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes.

If edges of sub-HyperCubes, equal to the distance between adjacent copies of Cl(16), remain constantly at the Planck Length, then the

full 8-dim HyperCube of our Universe expands as N grows to 2^16 and beyond

similarly to the way shown by this 3-HyperCube example for N = 2^3, 4^3, 8^3

from Wiliam Gilbert's web page:



The Union of all Cl(16) tensor products is
the Union of all subdivided 8-HyperCubes
and
their Completion is a huge superposition of 8-HyperCube Continuous Volumes
which Completion belongs to the Third Grothendieck Universe.

AQFT Quantum Code

Cerf and Adami in quantum-ph/9512022 describe virtualqubit-anti-qubit pairs (they call them ebit-anti-ebitpairs) that are related to negative conditional entropies for quantum entangled systems and are similar to fermion particle-antiparticle pairs. Therefore quantum information processes can be described by particle-antiparticle diagrams much like particle physics diagrams and **the Algebraic Quantum Field Theory of the E8 - Cl(16) = Cl(8)xCl(8) Physics Model should be equivalent to a Quantum Code Information System.**

Quantum Reed-Muller code [[256 , 0 , 24]]
corresponds to
Real Clifford Algebra Cl(8)

Tensor Product Quantum Reed-Muller code [[256 , 0 , 24]] x [[256 , 0 , 24]]
corresponds to
Real Clifford Algebra Cl(8) x Cl(8) = Cl(16) containing E8

Completion of the Union of All Tensor Products of [[256 , 0 , 24]] x [[256 , 0 , 24]]
corresponds to
AQFT (Algebraic Quantum Field Theory) hyperfinite von Neumann factor algebra that is Completion of the Union of All Tensor Products of Cl(16)

Quantum Reed-Muller code [[256 , 0 , 24]] is described in quantum-ph/9608026 by Steane as mapping a quantum state space of 256 qubits into 256 qubits, correcting [(24-1)/2] = 11 errors, and detecting 24/2 = 12 errors.
Let C(n,t) = n! / t! (n-t)!
Then

[[256, 0, 24]] is of the form

```
[[ 2^n,   2^n - C(n,t) - 2 SUM(0 k t-1) C(n,k),   2^t + 2^(t-1) ]]
[[ 2^8,   2^8 - C(8,4) - 2 SUM(0 k 3) C(8,k),     2^4 + 2^(4-1) ]]
[[ 2^8,   2^8 - 70 - (1+8+28+56) -  (1+8+28+56),  16 + 8 ]]
[[ 256,   256 - (1+8+28+56+70+56+28+8+1),         16 + 8 ]]
[[ 256,   16x16 - SUM(0 k 8) 8/\8/\..(k)../\8,    16 + 8 ]]
```

The quantum code [[256, 0, 24]] can be constructed from the classical Reed-Muller code (256, 93, 32) of the form

```
(  2^8,   2^8 - SUM(0 k t) C(n,k),             2^(t+1) )
(  2^8,   2^8 - SUM(0 k 4) C(n,k),             2^5 )
(  2^8,   2^8 - (70+56+28+8+1),                32 )
(  2^8,   1+8+28+56,                           32 )
```

To construct the quantum code [[256, 0, 24]] :

First, form a quantum code generator matrix
from the 128x256 generator matrix G of the classical code (256, 93, 32) :

```
|         |         |
|    G    |    0    |
|         |         |
|    0    |    G    |
|         |         |
```

Second, form the generator matrix of a quantum code of distance 16
by adding to the quantum generator matrix a matrix Dx such that
G and Dx together generate the classical Reed-Muller code (256, 163, 16) :

```
(  2^8,   1+8+28+56+70,                           16 ) :
```

```
|         |         |
|    G    |    0    |
|         |         |
|    0    |    G    |
|         |         |
|    Dx   |    0    |
|         |         |
```

This quantum code has been made by combining the classical codes
(256, 93, 32) and (256, 163, 16), so that it is of the form

```
[[ 256, 93 + 163 - 256, min(32,16) ]]  =  [[ 256, 0, 16 ]].
```

It is close to what we want, but has distance 16.
For the third and final step, increase the distance to 16+8 = 24
by adding Dz to the quantum generator matrix:

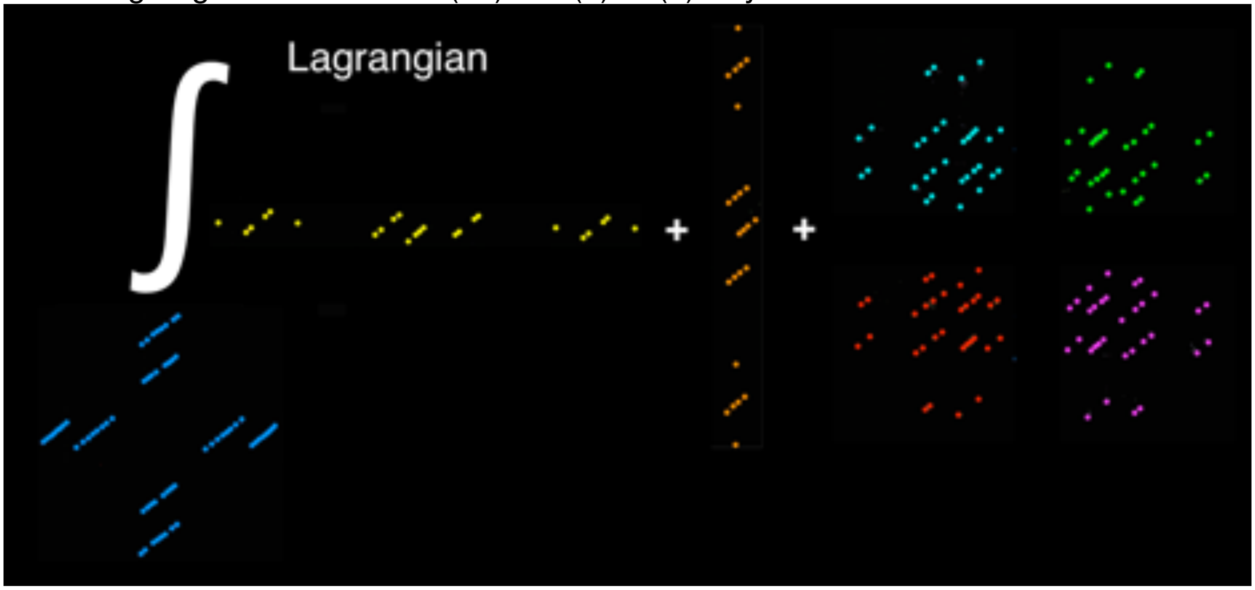
```
|         |         |
|    G    |    0    |
|         |         |
|    0    |    G    |
|         |         |
|    Dx   |    Dz   |
|         |         |
```

This is the generator matrix of the quantum code [[256, 0, 24]]
as constructed by Steane.

The two classical Reed-Muller codes used to build [[256, 0, 24]]
are (256, 163, 32) and (256, 93, 16),
classical Reed-Muller codes of orders 4 and 3, which are dual to each other.
Due to the nested structure of Reed-Muller codes,
they contain the Reed-Muller codes of orders 2, 1, and 0 :

Classical Reed-Muller Codes of Length 2^8 = 256			Order
(256,	1+8+28+56+70+56+28+8+1,	1)	8
(256,	1+8+28+56+70+56+28+8,	2)	7
(256,	1+8+28+56+70+56+28,	4)	6
(256,	1+8+28+56+70+56,	8)	5
(256,	1+8+28+56+70,	16)	4
(256,	1+8+28+56,	32)	3
(256,	1+8+28,	64)	2
(256,	1+8,	128)	1
(256,	1,	256)	0

In the Lagrangian of the E8 - Cl(16) = Cl(8)xCl(8) Physics Model

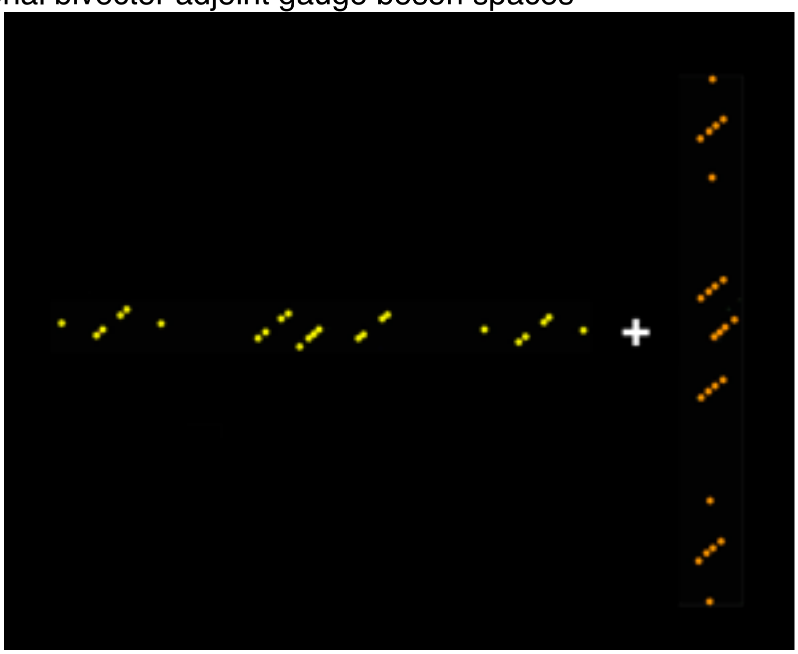


the Higgs scalar prior to dimensional reduction corresponds to the
0th order classical Reed-Muller code (256, 1, 256), the classical repetition code;

the 8-dimensional vector spacetime

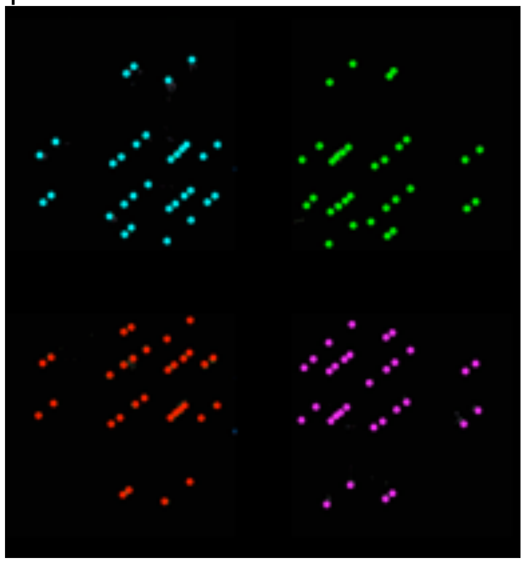
prior to dimensional reduction corresponds to non-0th-order part of the 1st order classical Reed-Muller code (256, 9, 128),
which is dual to the 6th order classical Reed-Muller code (256, 247, 4),
which is the extended Hamming code,
extended from the binary Hamming code (255, 247, 3),
which is dual to the simplex code (255, 8, 128) ;

the 28-dimensional bivector adjoint gauge boson spaces



prior to dimensional reduction correspond to the non-1st-order part of the 2nd order classical Reed-Muller code (256, 37, 64) .

The 8 first generation fermion particles and 8 first generation fermion antiparticles
of the 16-dim full spinor representation of the 256-dimensional Cl(0,8) Clifford algebra



correspond to the distance of the classical Reed-Muller code (256, 93, 16),
and to the 16-dimensional Barnes-Wall lattice Λ16,
which lattice comes from the (16,5,8) Reed-Muller code.
Each Λ16 vertex has 4320 nearest neighbors.

The other 8 of the 16+8 = 24 distance of the quantum Reed-Muller code [[256, 0, 24]]
corresponds to the 8-dimensional vector spacetime, and to the 8-dimensional E8 lattice
which comes from the (8,4,4) Hamming code, with weight distribution 0(1) 4(14) 8(1).
It can also be constructed from the repetition code (8,1,1).
The dual of (8,1,1) is (8,7,2), a zero-sum even weight code,
containing all binary vectors with an even number of 1s.
Each E8 lattice vertex has 240 nearest neighbors. In Euclidean R8,
there is only one way to arrange 240 spheres so that they all touch one sphere,
and only one way to arrange 56 spheres so that they all touch
a set of two spheres in contact with each other,
and so forth, giving the following classical spherical codes:
(8,240,1/2), (7,56,1/3), (6,27,1/4), (5,16,1/5), (4,10,1/6), and (3,6,1/7).

(If you use an Octonion Integral Domain instead of Euclidean R8 without multiplication
then there are 7 algebraically independent ways to arrange the 240 spheres.)

The total 24 distance of the quantum Reed-Muller code [[256, 0, 24]]
corresponds to the 24-dimensional Leech lattice,
and to the classical extended Golay code (24, 12, 8)
in which lattice each vertex has 196,560 nearest neighbors. In Euclidean R24,
there is only one way to arrange 196,560 spheres so that they all touch one sphere,
and only one way to arrange 4600 spheres so that they all touch a set of two spheres in
contact with each other, and so forth, giving the following classical spherical codes:
(24,196560,1/2), (23,4600,1/3), (22,891,1/4), (21,336,1/5), (20,170,1/6), ...

Appendix - Details of World-Line String Bohm Quantum Theory

A physically realistic Lattice Bosonic String Theory with Strings = World-Lines and
Monster Group Symmetry
containing gravity and the Standard Model
can be constructed consistently with the E8 physics model
248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8
= (28 + 28 + 64) + (64 + 64)

Joseph Polchinski, in his books String Theory vols. I and II(Cambridge 1998), says: "... the **closed ... unoriented ... bosonic string ... theory** has the maximal 26-dimensional Poincare invariance ... It is possible to have a consistent theory ...[with]... the **dilaton** ... the [**string-**]**graviton** ...[and]... the **tachyon** ...[whose]... negative mass-squared means that the no-string 'vacuum' is actually unstable ... ".
The **dilaton** of E8 Physics sets the Planck scale as the scale for
the 16 dimensions that are orbifolded fermion particles and anti-particles
and the 4 dimensions of the CP2 Internal Symmetry Space of M4xCP2 spacetime.
The remaining 26-16-4 = 6 dimensions are the Conformal Physical Spacetime with Spin(2,4) = SU(2,2) symmetry that produces M4 Physical Spacetime.
The **string-graviton** of E8 Physics is a spin-2 interaction among strings.
If Strings = World Lines and World Lines are past and future histories of particles, then string-graviton interactions determine a Cramer Transaction Quantum Theory discussed in quantum-ph/0408109. Roger Penrose in "Road to Reality" (Knopf 2004) says: "... **quantum** mechanics ... alternates between ... **unitary** evolution **U** ... and state reduction R ... quantum state **reduction** ... is ... **objective** ... **OR** ...
it is always a gravitational phenomenon ... [A] conscious event ... would be ... orchestrated OR ... of ... large-scale quantum coherence ... of ... microtubules ...".
String-Gravity produces Sarfatti-Bohm Quantum Potential with Back-Reaction.
It is distinct from the MacDowell-Mansouri Gravity of stars and planets.
The **tachyon** produces the instability of a truly empty vacuum state with no strings.
It is natural, because if our Universe were ever to be in a state with no strings, then tachyons would create strings = World Lines thus filling our Universe with the particles and World-Lines = strings that we see. Something like this is necessary for particle creation in the Inflationary Era of non-unitary Octonionic processes.
Our construction of a 26D String Theory consistent with E8 Physics uses a structure that is not well-known, so I will mention it here before we start:

There are 7 independent E8 lattices, each corresponding to one of the 7 imaginary octionions denoted by iE8, jE8, kE8, EE8, IE8, JE8, and KE8 and related to both D8 adjoint and half-spinor parts of E8 and with 240 first-shell vertices. An 8th E8 lattice 1E8 with 240 first-shell vertices related to the D8 adjoint part of E8 is related to the 7 octonion imaginary lattices (viXra 1301.0150v2) .
It can act as an effectively independent lattice as part of the basis subsets {1E8,EE8} or {1E8,iE8,jE8,kE8}.

With that in mind, here is the construction:

Step 1:

Consider the 26 Dimensions of Bosonic String Theory as the 26-dimensional traceless part J3(O)o

a	O+	Ov
O+*	b	O-
Ov*	O-*	-a-b

(where Ov, O+, and O- are in Octonion space with basis {1,i,j,k,E,I,J,K} and a and b are real numbers with basis {1}) of the 27-dimensional Jordan algebra J3(O) of 3x3 Hermitian Octonion matrices.

Step 2:

Take a D3 brane to correspond to the Imaginary Quaternionic associative subspace spanned by {i,j,k} in the 8-dimenisonal Octonionic Ov space.

Step 3:

Compactify the 4-dimensional co-associative subspace spanned by {E,I,J,K} in the Octonionic Ov space as a CP2 = SU(3)/U(2), with its 4 world-brane scalars corresponding to the 4 covariant components of a Higgs scalar.

Add this subspace to D3, to get D7.

Step 4:

Orbifold the 1-dimensional Real subspace spanned by {1} in the Octonionic Ov space by the discrete multiplicative group Z2 = {-1,+1}, with its fixed points {-1,+1} corresponding to past and future time. This discretizes time steps and gets rid of the world-brane scalar corresponding to the subspace spanned by {1} in Ov. It also gives our brane a 2-level timelike structure, so that its past can connect to the future of a preceding brane and its future can connect to the past of a succeeding brane.

Add this subspace to D7, to get D8.

D8, our basic Brane, looks like two layers (past and future) of D7s.

Beyond D8 our String Theory has 26 - 8 = 18 dimensions, of which 25 - 8 have corresponding world-brane scalars:

- 8 world-brane scalars for Octonionic O+ space;
- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 5:

To get rid of the world-brane scalars corresponding to the Octonionic O+ space, orbifold it by the 16-element discrete multiplicative group Oct16 = {+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K} to reduce O+ to 16 singular points {-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K}.

- Let the 8 O+ singular points {-1,-i,-j,-k,-E,-I,-J,-K} correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the past D7 layer of D8.
- Let the 8 O+ singular points {+1,+i,+j,+k,+E,+I,+J,+K} correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the future D7 layer of D8.

The 8 components of the 8 fundamental first-generation fermion particles = 8x8 = 64 correspond to the 64 of the 128-dim half-spinor D8 part of E8.

This gets rid of the 8 world-brane scalars corresponding to O+, and leaves:

- 8 world-brane scalars for Octonionic O- space;
- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 6:

To get rid of the world-brane scalars corresponding to the Octonionic O- space, orbifold it by the 16-element discrete multiplicative group Oct16 = {+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K} to reduce O- to 16 singular points {-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K}.

- Let the 8 O- singular points {-1,-i,-j,-k,-E,-I,-J,-K} correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the past D7 layer of D8.
- Let the 8 O- singular points {+1,+i,+j,+k,+E,+I,+J,+K} correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the future D7 layer of D8.

The 8 components of the 8 fundamental first-generation fermion anti-particles = 8x8 = 64 correspond to the 64 of the 128-dim half-spinor D8 part of E8.

This gets rid of the 8 world-brane scalars corresponding to O-, and leaves:

- 1 world-brane scalars for real a space; and
- 1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 7:

Let the 1 world-brane scalar for real a space correspond to a Bohm-type Quantum Potential acting on strings in the stack of D8 branes.

Interpret strings as world-lines in the Many-Worlds, short strings representing virtual particles and loops.

Step 8:

Fundamentally, physics is described on HyperDiamond Lattice structures.

There are 7 independent E8 lattices, each corresponding to one of the 7 imaginary octionions. denoted by iE8, jE8, kE8, EE8, IE8, JE8, and KE8 and related to both D8 adjoint and half-spinor parts of E8 and with 240 first-shell vertices.

An 8th 8-dim lattice 1E8 with 240 first-shell vertices related to the E8 adjoint part of E8 is related to the 7 octonion imaginary lattices.

Give each D8 brane structure based on Planck-scale E8 lattices so that each D8 brane is a superposition/intersection/coincidence of the eight E8 lattices.
(see viXra 1301.0150)

Step 9:

Since Polchinski says "... If r D-branes coincide ... there are r^2 vectors, forming the adjoint of a U(r) gauge group ...", make the following assignments:

- a gauge boson emanating from D8 from its 1E8 and EE8 lattices is a U(2) ElectroWeak boson thus accounting for the photon and W+, W- and Z0 bosons.
- a gauge boson emanating from D8 from its IE8, JE8, and KE8 lattices is a U(3) Color Gluon boson thus accounting for the 8 Color Force Gluon bosons.

The 4+8 = 12 bosons of the Standard Model Electroweak and Color forces correspond to 12 of the 28 dimensions of 28-dim Spin(8) that corresponds to the 28 of the 120-dim adjoint D8 part of E8.

- a gauge boson emanating from D8 from its 1E8, iE8, jE8, and kE8 lattices is a U(2,2) boson for conformal U(2,2) = Spin(2,4)xU(1) MacDowell-Mansouri gravity plus conformal structures consistent with the Higgs mechanism and with observed Dark Energy, Dark Matter, and Ordinary matter.

The 16-dim U(2,2) is a subgroup of 28-dim Spin(2,6) that corresponds to the 28 of the 120-dim adjoint D8 part of E8.

Step 10:

Since Polchinski says "... there will also be r^2 massless scalars from the components normal to the D-brane. ... the collectives coordinates ... X^u ... for the embedding of n D-branes in spacetime are now enlarged to nxn matrices. This 'noncummutative geometry' ...[may be]... an important hint about the nature of spacetime. ...", make the following assignment:

The 8x8 matrices for the collective coordinates linking a D8 brane to the next D8 brane in the stack are needed to connect the eight E8 lattices of the D8 brane to the eight E8 lattices of the next D8 brane in the stack.

The 8x8 = 64 correspond to the 64 of the 120 adjoint D8 part of E8.

We have now accounted for all the scalars

and

have shown that the model has the physics content of the realistic E8 Physics model

with Lagrangian structure based on E8 = (28 + 28 + 64) + (64 + 64)

and AQFT structure based on Cl(16) with real Clifford Algebra periodicity and generalized Hyperfinite II1 von Neumann factor algebra.

A Single Cell of E8 26-dimensional Bosonic String Theory,
in which Strings are physically interpreted as World-Lines,
can be described by taking the quotient of its 24-dimensional O+, O-, Ov
subspace modulo the 24-dimensional Leech lattice.
Its automorphism group is the largest finite sporadic group, the Monster Group,
whose order is
8080, 17424, 79451, 28758, 86459, 90496, 17107, 57005, 75436, 80000, 00000
=
2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71
or about 8 x 10^53.

A Leech lattice construction is described by Robert A. Wilson in his 2009 paper "Octonions and the Leech lattice":
"... The (real) octonion algebra is an 8-dimensional (non-division) algebra with an orthonomal basis { 1=ioo , i0 , i1 , i2 , i3 , i4 , i5 , i6 } labeled
by the projective line PL(7) = { oo } u F7
...
The E8 root system embeds in this algebra ... take the 240 roots to be ...
112 octonions ... +/- it +/- iu for any distinct t,u
... and ...
128 octonions (1/2)(+/- 1 +/- i0 +/- ... +/- i6) which have an odd number of minus signs.
Denote by L the lattice spanned by these 240 octonions
...
Let s = (1/2)(- 1 + i0 + ... + i6) so s is in L ... write R for Lbar ...
...
(1/2) (1 + i0) L = (1/2) R (1 + i0) is closed under multiplication ... Denote this ...by A
... Writing B = (1/2) (1 + i0) A (1 + i0) ...from ... Moufang laws ... we have
L R = 2 B , and ... B L = L and R B = R ...[also]... 2 B = L sbar
...
the roots of B are
[16 octonions]... +/- it for t in PL(7)
... together with
[112 octonions]... (1/2) (+/- 1 +/- it +/- i(t+1) +/- i(t+3)) ...for t in F7
... and ...
[112 octonions]... (1/2) (+/- i(t+2) +/- i(t+4) +/- i(t+5) +/- i(t+6)) ...for t in F7
...
the octonionic Leech lattice ... contains the following 196560 vectors of norm 4 ,
where M is a root of L and j,k are in J = { +/- it l t in PL(7) },
and all permutations of the three coordinates are allowed:

(2 M, 0 , 0) **Number: 3x240 = 720**
(M sbar, +/- (M sbar) j , 0) **Number: 3x240 x 16 = 11520**
((M s) j , +/- M k , +/- (M j) k) **Number: 3x240 x 16 x 16 = 184320**
...

The key to the simple proofs above is the observation that LR = 2B and BL = L: these remarkable facts appear not to have been noticed before ... some work ... by Geoffrey Dixon ...". Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting" using notation {e0,e1,e2,e3,e4,e5,e6,e7} for the Octonion basis elements that Robert A. Wilson denotes by {1=ioo,i0,i1,i2,i3,i4,i5 ,i6} and I often denote by {1,i,j,k,E,I,J,K}: "...

$$\begin{aligned} \Xi_0 &= \{\pm e_a\}, \\ \Xi_2 &= \{(\pm e_a \pm e_b \pm e_c \pm e_d)/2 : a,b,c,d \text{ distinct}, \\ &\quad e_a(e_b(e_c e_d)) = \pm 1\}, \end{aligned}$$

$$\begin{aligned} \Xi^{even} &= \Xi_0 \cup \Xi_2, \\ \mathcal{E}_8^{even} &= \text{span}\{\Xi^{even}\}, \end{aligned}$$

$$\begin{aligned} \Xi_1 &= \{(\pm e_a \pm e_b)/\sqrt{2} : a,b \text{ distinct}\}, \\ \Xi_3 &= \{(\textstyle\sum_{a=0}^{7} \pm e_a)/\sqrt{8} : \text{ even number of +'s}\}, \end{aligned}$$

$$\begin{aligned} \Xi^{odd} &= \Xi_1 \cup \Xi_3, \\ \mathcal{E}_8^{odd} &= \text{span}\{\Xi^{odd}\} \end{aligned}$$

(spans over integers) ...

Ξeven has 16+224 = 240 elements ... Ξodd has 112+128 = 240 elements ...
E8even does not close with respect to our given octonion multiplication ...[but]...
the set Ξeven[0-a], derived from Ξeven by replacing each occurrence of e0 ... with ea, and vice versa, is multiplicatively closed. ...".
Geoffrey Dixon's Ξeven corresponds to B
Geoffrey Dixon's **Ξeven[0-a] corresponds to the seven At**
Geoffrey Dixon's Ξodd corresponds to L
Ignoring factors like 2 , j , k , and +/-1 the Leech lattice structure is:

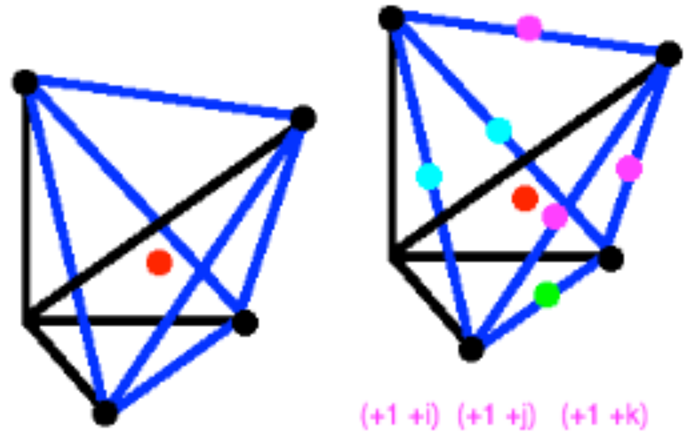
(L , 0 , 0)	Number: 3x240 = 720
(B , B , 0)	Number: 3x240 x 16 = 11520
(L s , L , L)	Number: 3x240 x 16 x 16 = 184320

(Ξodd , 0 , 0)	Number: 3x240 = 720
(Ξeven , Ξeven , 0)	Number: 3x240 x 16 = 11520
(Ξodd s , Ξodd , Ξodd)	Number: 3x240 x 16 x 16 = 184320

My view is that **the E8 domain B is fundamental**
and the E8 domains L and L s are derived from it.

That view is based on analogy with the 4-dimensional 24-cell
and its dual 24-cell. Using Quaternionic coordinates {1,i,j,k}
the 24-cell of 4-space has one Superposition Vertex for each 16-region of 4-space.

A Dual 24-cell gives a new Superposition Vertex at each edge of the region.



The Initial 24-cell Quantum Operators act with respect to 4-dim Physical Spacetime.
{1,i,j,k} represent time and 3 space coordinates.
(1/2)(+1+i+j+k) represents a fundamental first-generation Fermion particle/antiparticle (there is one for for each of the 16-regions).
The Dual 24-cell Quantum Operators act with respect to 4-dim CP2 Internal Symmetry Space. Since CP2 = SU(3)/SU(2)xU(1),
(+1 +i) (+1 +j) (+1 +k) are permuted by S3 to form the Weyl Group of Color Force SU(3),
(+i +j) (+i +k) are permuted by S2 to form the Weyl Group of Weak Force SU(2),
(+j +k) is permuted by S1 to form the Weyl Group of Electromagnetic Force U(1).
The B-type 24-cell is fundamental because it gives Fundamental Fermions.
The L-type dual 24-cell is derivative because it gives Standard Model Gauge Bosons.

Robert A.Wilson in "Octonions and the Leech lattice" also said
"... **B is not closed under multiplication** ... Kirmse's mistake
...[but]... as Coxeter ... pointed out ...
... there are seven non-associative rings At = (1/2) (1 + it) B (1 + it) , obtained from B by swapping 1 with it ... for t in F7 ...".
H. S. M. Coxeter in "Integral Cayley Numbers" (Duke Math. J. 13 (1946) 561-578) said
"... Kirmse ... defines ... an integral domain ... which he calls J1 [Wilson's B] ...[but]...
J1 itself is not closed under multiplication ... Bruck sent ... a revised description ...[of a]... domain J ... derived from J1 by transposing two of the i's [imaginary Octonions]...
It is closed under multiplication ... there are ... seven such domains, since the (7choose2) = 21 possible transpositions fall into 7 sets of 3, each set having the same effect. In each of the seven domains, one of the ... seven i's ... plays a special role, viz., that one which is not affected by any of the three transpositions. ...
J contains ... 240 units ... ". J is one of Wilson's seven At and, in Octonionic coordinates {1,i,j,k,e,ie,je,ke}, is shown below with physical interpretation color-coded as
8-dim Spacetime Coordinates x 8-dim Momentum Dirac Gammas
Gravity SU(2,2)=Spin(2,4) in a D4 + Standard Model SU(3)xU(2) in a D4
8 First-Generation Fermion Particles x 8 Coordinate Components
8 First-Generation Fermion AntiParticles x 8 Coordinate Components

```
112 = (16+48=64) + (24+24=48) Root Vectors corresponding to D8:

±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke,

(±1 ±i          ±e  ±ie           )/2
(±1     ±j      ±e       ±je      )/2
(±1         ±k  ±e           ±ke  )/2

(       ±j  ±k           ±je ±ke  )/2
(   ±i      ±k      ±ie      ±ke  )/2
(   ±i  ±j          ±ie  ±je      )/2

128 = 64 + 64 Root Vectors corresponding to half-spinor of D8:

(±1                 ±ie  ±je ±ke  )/2
(±1     ±j  ±k      ±ie           )/2
(±1 ±i      ±k           ±je      )/2
(±1 ±i  ±j                   ±ke  )/2

(   ±i  ±j  ±k  ±e                )/2
(   ±i          ±e       ±je ±ke  )/2
(       ±j      ±e  ±ie      ±ke  )/2
(           ±k  ±e  ±ie  ±je      )/2
```

The above Coxeter-Bruck J is, in the notation I usually use, denoted 7E8 .
It is one of Coxeter's seven domains (Wilson's seven {A0,A1,A2,A3,A4,A5,A6})
that I usually denote as { 1E8 , 2E8 , 3E8 , 4E8 , 5E8 , 6E8, 7E8 } .

Since the Leech lattice structure is

(L , 0 , 0)	**Number: 3x240 = 720**
(B , B , 0)	**Number: 3x240 x 16 = 11520**
(L s , L , L)	**Number: 3x240 x 16 x 16 = 184320**

if you replace the structural B with 7E8 and **the Leech lattice structure** becomes

(L , 0 , 0)	**Number: 3x240 = 720**
(7E8 , 7E8 , 0)	**Number: 3x240 x 16 = 11520**
(L s , L , L)	**Number: 3x240 x 16 x 16 = 184320**

and **the Leech lattice of E8 26-dim String Theory is the Superposition of 8 Leech lattices based on each of { B , 1E8 , 2E8 , 3E8 , 4E8 , 5E8 , 6E8, 7E8 }** just as the D8 branes of E8 26-dim String Theory are each the Superposition of the 8 domains { B , 1E8 , 2E8 , 3E8 , 4E8 , 5E8 , 6E8, 7E8 } .

What happens to a Fundamental Fermion Particle whose World-Line string intersects a Single Cell ?

The Fundamental Fermion Particle does not remain a single Planck-scale entity. **Tachyons create clouds of particles/antiparticles** as described by Bert Schroer in hep-th/9908021: "... any compactly localized operator applied to the vacuum generates clouds of pairs of particle/antiparticles ... More specifically it leads to the impossibility of having a local generation of pure one-particle vectors unless the system is interaction-free ...".

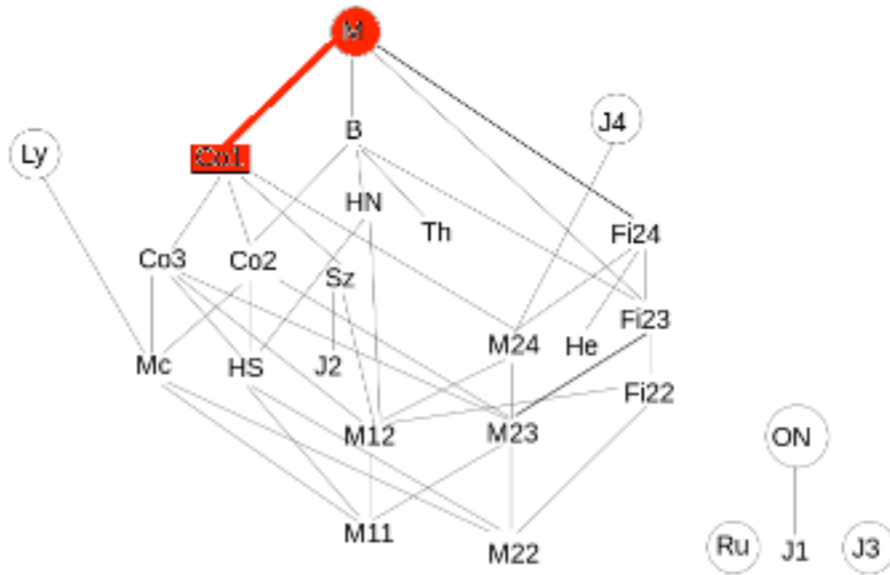
What is the structural form of the Fundamental Fermion Cloud ?

In "**Kerr-Newman [Black Hole]** solution as a Dirac particle", hep-th/0210103, H. I. Arcos and J. G. Pereira say: "... For m^2 < a^2 + q^2 , with m, a, and q respectively the source mass, angular momentum per unit mass, and electric charge, the Kerr-Newman (KN) solution of Einstein's equation reduces to a naked singularity of circular shape, enclosing a disk across which the metric components fail to be smooth ... due to its topological structure, the extended KN spacetime does admit states with half-integral angular momentum. ... The state vector ... evolution is ... governed by the Dirac equation. ... for symmetry reasons, the electric dipole moment of the KN solution vanishes identically, a result that is within the limits of experimental data ... a and m are thought of as parameters of the KN solution, which only asymptotically correspond respectively to angular momentum per unit mass and mass. Near the singularity, a represents the radius of the singular ring ... With ... renormalization ... for the usual scattering energies, the resulting radius is below the experimental limit for the extendedness of the electron ...".

What is the size of the Fundamental Fermion Kerr-Newman Cloud ?

The FFKN Cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively neutral cloud of particle/antiparticle pairs. The symmetry of the cloud is governed by the 24-dimensional Leech lattice by which the Single Cell was formed.

Here (adapted from Wikipedia) is a chart of the Monster M and its relation to other Sporadic Finite Groups and some basic facts and commentary:



The largest such subgroups of M are B, Fi24, and Co1.

B, the Baby Monster, is sort of like a downsized version of M,
as B contains Co2 and Fi23 while M contains Co1 and Fi24.

Fi24 (more conventionally denoted Fi24') is of order 1255205709190661721292800 = 1.2 x 10^24 It is the centralizer of an element of order 3 in the monster group M and is a triple cover of a 3-transposition group. It may be that Fi24' symmetry has its origin in the Triality of E8 26-dim String Theory.

The order of Co1 is 2^21.3^9.5^4.7^2.11.13.23 or about 4 x 10^18.
Aut(Leech Lattice) = double cover of Co1.
The order of the double cover 2.Co1 is 2^22.3^9.5^4.7^2.11.13.23 or about 0.8 x 10^19.
Taking into account the non-sporadic part of the Leech Lattice symmetry
according to the ATLAS at brauer.maths.qmul.ac.uk/Atlas/v3/spor/M/
the maximal subgroup of M involving Co1 is 2^(1+24).Co1 of order
139511839126336328171520000 = 1.4 x 10^26
As **2.Co1 is the Automorphism group of the Leech Lattice modulo to which the Single Cell was formed**, and as
the E8 26-dim String Theory Leech Lattice is a superposition of 8 Leech Lattices, 8 x 2^(1+24).Co1 describes the structure of the FFKN Cloud. Therefore,
the volume of the FFKN Cloud should be on the order of 10^27 x Planck scale, and
the FFKN Cloud should contain on the order of 10^27 particle/antiparticle pairs and **its size should be** somewhat larger than, but roughly similar to,
10^(27/3) x 1.6 x 10^(-33) cm = roughly **10^(-24) cm**.

The full 26-dimensional Lattice Bosonic String Theory can be regarded as an infinite-dimensional Affinization of the Theory of a Single Cell.

James Lepowsky said in math.QA/0706.4072:
"... **the Fischer-Griess Monster M ... was constructed by Griess as a symmetry group (of order about 10^54) of a remarkable new commutative but very, very highly nonassociative, seemingly ad-hoc, algebra B of dimension 196,883**. The "structure constants" of the Griess algebra B were "forced" by expected properties of the conjectured-to-exist Monster. It was proved by J. Tits that M is actually the full symmetry group of B. ...

There should exist a (natural) infinite-dimensional Z-graded module for M (i.e., representation of M)

V = DIRSUM(n=-1,0,1,2,3,...) V_n ...

such that
... the graded dimension of the graded vector space V ... = ... SUM(n=-1,0,1,2,3,...) (dim V_n) q^n
where
J(q) = q^(-1) + 0 + 196884q + higher-order terms,
the classical modular function with its constant term set to 0. J(q) is the suitably normalized generator of the field of SL(2, Z)-modular invariant functions on the upper half-plane, with q = exp(2 pi i tau) , tau in the upper half-plane ...

Conway and Norton conjectured ... for every g in M (not just g = 1), the the generating function

... the graded trace of the action of g on the graded space V ... = ... SUM(n=-1,0,1,2,3,...) (tr g I V_n) q^n

should be the analogous "Hauptmodul" for a suitable discrete subgroup of SL(2,R), a subgroup having a fundamental "genus-zero property," so that its associated field of modular-invariant functions has a single generator (a Hauptmodul) ... (... the graded dimension is of course the graded trace of the identity element g = 1.) The Conway-Norton conjecture subsumed a remarkable coincidence that had been noticed earlier

- that **the 15 primes giving rise to the genus-zero property ... are precisely the primes dividing the order of the ... Monster** ...

the McKay-Thompson conjecture ... that there should exist a natural ... infinite-dimensional Z-graded M-module V whose graded dimension is J(q) ... was (constructively) proved .... The graded traces of some, but not all, of the elements of the Monster - the elements of an important subgroup of M, namely, a certain involution centralizer involving the largest Conway sporadic group Co1 - were consequences of the construction, and these graded traces were indeed (suitably) modular functions ... We called this V **"the moonshine module V[flat]" ...**
The construction ... needed ... a natural infinite-dimensional "affinization" of the Griess algebra B acting on V[flat]

This "affinization," which was part of the new algebra of vertex operators, is analogous to, but more subtle than, the notion of affine Lie algebra .... More precisely, the vertex operators were needed for a "commutative affinization" of a certain natural 196884-dimensional enlargement B' of B, with an identity element (rather than a "zero" element) adjoined to B. This enlargement B' naturally incorporated the Virasoro algebra - the central extension of the Lie algebra of formal vector fields on the circle - acting on V[flat] ...

The vertex operators were also needed for a natural "lifting" of Griess's action of M from the finite-dimensional space B to the infinite-dimensional structure V[flat], including its algebra of vertex operators and its copy of the affinization of B'.

Thus the Monster was now realized as the symmetry group of a certain explicit "algebra of vertex operators" based on an infinite-dimensional Z-graded structure whose graded dimension is the modular function J(q).

Griess's construction of B and of M acting on B was a crucial guide for us, although we did not start by using his construction; rather, we recovered it, as a finite-dimensional "slice" of a new infinite-dimensional construction using vertex operator considerations. ...

The initally strange-seeming finite-dimensional Griess algebra was now embedded in a natural new infinite-dimensional space on which a certain algebra of vertex operators acts ... At the same time, the Monster, a finite group, took on a new appearance by now being understood in terms of a natural infinite-dimensional structure. ... the largest sporadic finite simple group, the Monster, was "really" infinite-dimensional ...

The very-highly-nonassociative Griess algebra, or rather, from our viewpoint, the natural modification of the Griess algebra, with an identity element adjoined, coming from a "forced" copy the Virasoro algebra, became simply the conformalweight-two subspace of an algebra of vertex operators of a certain "shape." ...

the constant term of J(q) is zero, and this choice of constant term, which is not uniquely determined by number-theoretic principles, is not traditional in number theory. It turned out that the vanishing of the constant term ... was canonically "forced" by the requirement that the Monster should act naturally on V[flat] and on an associated algebra of vertex operators.

This vanishing of the degree-zero subspace of V[flat] is actually analogous in a certain strong sense to the absence of vectors in the Leech lattice of square-length two; the Leech lattice is a distinguished rank-24 even unimodular (self-dual) lattice with no vectors of square-length two.

In addition, this vanishing of the degree-zero subspace of V[flat] and the absence of square-length-two elements of the Leech lattice are in turn analogous to the absence of code-words of weight 4 in the Golay error-correcting code, a distinguished selfdual binary linear code on a 24-element set, with the lengths of all code-words divisible by 4. In fact, the Golay code was used in the original construction of the Leech lattice, and the Leech lattice was used in the construction of V[flat]

This was actually to be expected ... because it was well known that the automorphism groups of both the Golay code and the Leech lattice are (essentially) sporadic finite simple groups; the automorphism group of the Golay code is the Mathieu group M24 and the automorphism group of the Leech lattice is a double cover of the Conway group Co1 mentioned above, and both of these sporadic groups were well known to be involved in the Monster ... in a fundamental way....

The Golay code is actually **unique** subject to its distinguishing properties mentioned above ... and t**he Leech lattice is unique** subject to its distinguishing properties mentioned above ... I**s V[flat] unique? If so, unique subject to what? ... this uniqueness is an unsolved problem** ...

V[flat] came to be viewed in retrospect by string theorists as an inherently stringtheoretic structure: the "chiral algebra" underlying the Z2-orbifold conformal field theory based on the Leech lattice.

The string-theoretic geometry is this: One takes the torus that is the quotient of 24-dimensional Euclidean space modulo the Leech lattice, and then one takes the quotient of this manifold by the "negation" involution x -> -x, giving rise to an orbit space called an "orbifold"&emdash;a manifold with, in this case, a "conical" singularity. Then one takes the "conformal field theory" (presuming that it exists mathematically) based on this orbifold, and from this one forms a "string theory" in two-dimensional space-time by compactifying a 26-dimensional "bosonic string" on this 24-dimensional orbifold. The string vibrates in a 26-dimensional space, 24 dimensions of which are curled into this 24-dimensional orbifold ...

Borcherds used ... ideas, including his results on generalized Kac-Moody algebras, also called Borcherds algebras, together with certain ideas from string theory, including the "physical space" of a bosonic string along with the "no-ghost theorem" ... to prove the remaining Conway-Norton conjectures for the structure V[flat] ... What had remained to prove was ... that ... the conjugacy classes outside the involution centralizer - were indeed the desired Hauptmoduls ... He accomplished this by constructing a copy of his "Monster Lie algebra" from the "physical space" associated with V[flat] enlarged to a central-charge-26 vertex algebra closely related to the 26-dimensional bosonic-string structure mentioned above. He transported the known action of the Monster from V[flat] to this copy of the Monster Lie algebra, and ... he proved certain recursion formulas ... ... he succeeded in concluding that all the graded traces for V[flat] must coincide with the formal series for the Hauptmoduls ...

this vertex operator algebra V[flat] has the following three simply-stated properties ...

• (1) V[flat], which is an irreducible module for itself ... , is its only irreducible module, up to equivalence ... every module for the vertex operator algebra V [flat] is completely reducible and is in particular a direct sum of copies of itself. Thus the vertex operator algebra V[flat] has no more representation theory than does a field! (I mean a field in the sense of mathematics, not physics. Given a field, every one of its modules - called vector spaces, of course - is completely reducible and is a direct sum of copies of itself.)

• (2) dim V[flat]_0 = 0. This corresponds to the zero constant term of J(q); while the constant term of the classical modular function is essentially arbitrary, and is chosen to have certain values for certain classical numbertheoretic purposes, the constant term must be chosen to be zero for the purposes of moonshine and the moonshine module vertex operator algebra.

• (3) The central charge of the canonical Virasoro algebra in V[flat] is 24. "24" is the "same 24" so basic in number theory, modular function theory, etc. As mentioned above, this occurrence of 24 is also natural from the point of view of string theory.

These three properties are actually "smallness" properties in the sense of conformal field theory and string theory. These properties allow one to say that V[flat] essentially defines the smallest possible nontrivial string theory ... (These "smallness" properties essentially amount to: "no nontrivial representation theory," "no nontrivial gauge group," i.e., "no continuous symmetry," and "no nontrivial monodromy"; this last condition actually refers to both the first and third "smallness" properties.)

Conversely, conjecturally ... V[flat] is the unique vertex operator algebra with these three "smallness" properties (up to isomorphism). This conjecture ... remains unproved. It would be the conformal-field-theoretic analogue of the uniqueness of the Leech lattice in sphere-packing theory and of the uniqueness of the Golay code in error-correcting code theory ...

Proving this uniqueness conjecture can be thought of as the "zeroth step" in the program of classification of (reasonable classes of) conformal field theories. M. Tuite has related this conjecture to the genus-zero property in the formulation of monstrous moonshine.

Up to this conjecture, then, we have the following remarkable characterization of the largest sporadic finite simple group: **The Monster is the automorphism group of the smallest nontrival string theory** that nature allows ... **Bosonic 26-dimensional space-time ... "compactified" on 24 dimensions, using the orbifold construction V[flat]** ... or more precisely, the automorphism group of the vertex operator algebra with the canonical "smallness" properties. ...

This definition of the Monster in terms of "smallness" properties of a vertex operator algebra provides a remarkable motivation for the definition of the precise notion of vertex (operator) algebra. The discovery of string theory (as a mathematical, even if not necessarily physical) structure sooner or later must lead naturally to the question of whether this "smallest" possible nontrivial vertex operator algebra V . exists, and the question of what its symmetry group (which turns out to be the largest sproradic finite simple group) is.

And on the other hand, the classification of the the finite simple groups - a mathematical problem of the absolutely purest possible sort - leads naturally to the question of what natural structure the largest sporadic group is the symmetry group of; the answer entails the development of string theory and vertex operator algebra theory (and involves modular function theory and monstrous moonshine as well).

The Monster, a singularly exceptional structure - in the same spirit that the Lie algebra E8 is "exceptional," though M is far more "exceptional" than E8 - helped lead to, and helps shape, the very general theory of vertex operator algebras. (The exceptional nature of structures such as E8, the Golay code and the Leech lattice in fact played crucial roles in the construction of V[flat] ...

V[flat] is defined over the field of real numbers, and in fact over the field of rational numbers, in such a way that the Monster preserves the real and in fact rational structure, and that the Monster preserves a rational-valued positive-definite symmetric bilinear form on this rational structure. ...

t**he "orbifold" construction of V[flat] ...[has been]... interpreted in terms of algebraic quantum field theory, specifically, in terms of local conformal nets of von Neumann algebras on the circle** ...

the notion of vertex operator algebra is actually the "one-complex-dimensional analogue" of the notion of Lie algebra. But at the same time that it is the "onecomplex-dimensional analogue" of the notion of Lie algebra, the notion of vertex operator algebra is also the "one-complex- dimensional analogue" of the notion of commutative associative algebra (which again is the corresponding "one-realdimensional"
notion). ... This analogy with the notion of commutative associative algebra comes from the "commutativity" and "associativity" properties of the vertex operators ... in a vertex operator algebra ...

The remarkable and paradoxical-sounding fact that the notion of vertex operator algebra can be, and is, the "one-complex-dimensional analogue" of BOTH the notion of Lie algebra AND the notion of commutative associative algebra lies behind much of the richness of the whole theory, and of string theory and conformal field theory.

When mathematicians realized a long time ago that complex analysis was qualitatively entirely different from real analysis (because of the uniqueness of analytic continuation, etc., etc.), a whole new point of view became possible. In vertex operator algebra theory and string theory, there is again a fundamental passage from "real" to "complex," this time leading from the concepts of both Lie algebra and commutative associative algebra to the concept of vertex operator algebra and to its theory, and also leading from point particle theory to string theory. ...

While a string sweeps out a two-dimensional (or, as we've been mentioning, one-complex-dimensional) "worldsheet" in space-time, **a point particle of course sweeps out a one-real-dimensional "world-line" in space-time**, with time playing the role of the "one real dimension," and this "one real dimension" is related in spirit to the "one real dimension" of the classical operads that I've briefly referred to - the classical operads "mediating" the notion of associative algebra and also the notion of Lie algebra (and indeed, any "classical" algebraic notion), and in addition "mediating" the classical notion of braided tensor category. The "sequence of operations performed one after the other" is related (not perfectly, but at least in spirit) to the ordering ("time-ordering") of the real line.

But as we have emphasized, the "algebra" of vertex operator algebra theory and also of its representation theory (vertex tensor categories, etc.) is "mediated" by an (essentially) one-complex-dimensional (analytic partial) operad (or more precisely, as we have mentioned, the infinite-dimensional analytic structure built on this). When one needs to compose vertex operators, or more generally, intertwining operators, after the formal variables are specialized to complex variables, one must choose not merely a (time-)ordered sequencing of them, but instead, a suitable complex number, or more generally, an analytic local coordinate as well, for each of the vertex operators.

This process, very familiar in string theory and conformal field theory, is a reflection of how the one-complex-dimensional operadic structure "mediates" the algebraic operations in vertex operator algebra theory.

Correspondingly, "algebraic" operations in this theory are not instrinsically "timeordered";
they are instead controlled intrinsically by the one-complex-dimensional operadic structure. The "algebra" becomes intrinsically geometric.

"Time," or more precisely, as we discussed above, the one-real-dimensional world-line, is being replaced by a one-complex-dimensional world-sheet.

This is the case, too, for the vertex tensor category structure on suitable module categories. In vertex operator algebra theory, "algebra" is more concerned with one-complex-
dimensional geometry than with one-real-dimensional time. ...".

Appendix - World-Line String Bohm Quantum Consciousness

"... **Bohm's Quantum Potential can be viewed as an internal energy** of a quantum system ..." according to Dennis, de Gosson, and Hiley (arXiv 1412.5133) and Peter R. Holland says in "The Quantum Theory of Motion" (Cambridge 1993): "... **the total force ... from the quantum potential ... does not ... fall off with distance** ... because ... the quantum potential ... depends on the form of ...[the quantum state]... rather than ... its ... magnitude ...".

Penrose-Hameroff-type Quantum Consciousness is due
to Resonant Quantum Potential Connections among Quantum State Forms.
The Quantum State Form of a Conscious Brain is determined by
the configuration of a subset of its 10^18 to 10^19 Tubulin Dimers
with math description in terms of a large Real Clifford Algebra:

Resonance is discussed by Carver Mead in "Collective Electrodynamics" (MIT 2000):
"... we can build ... a resonator from ... electric dipole ... configuration[s] ...

[such as   Tubulin Dimers]

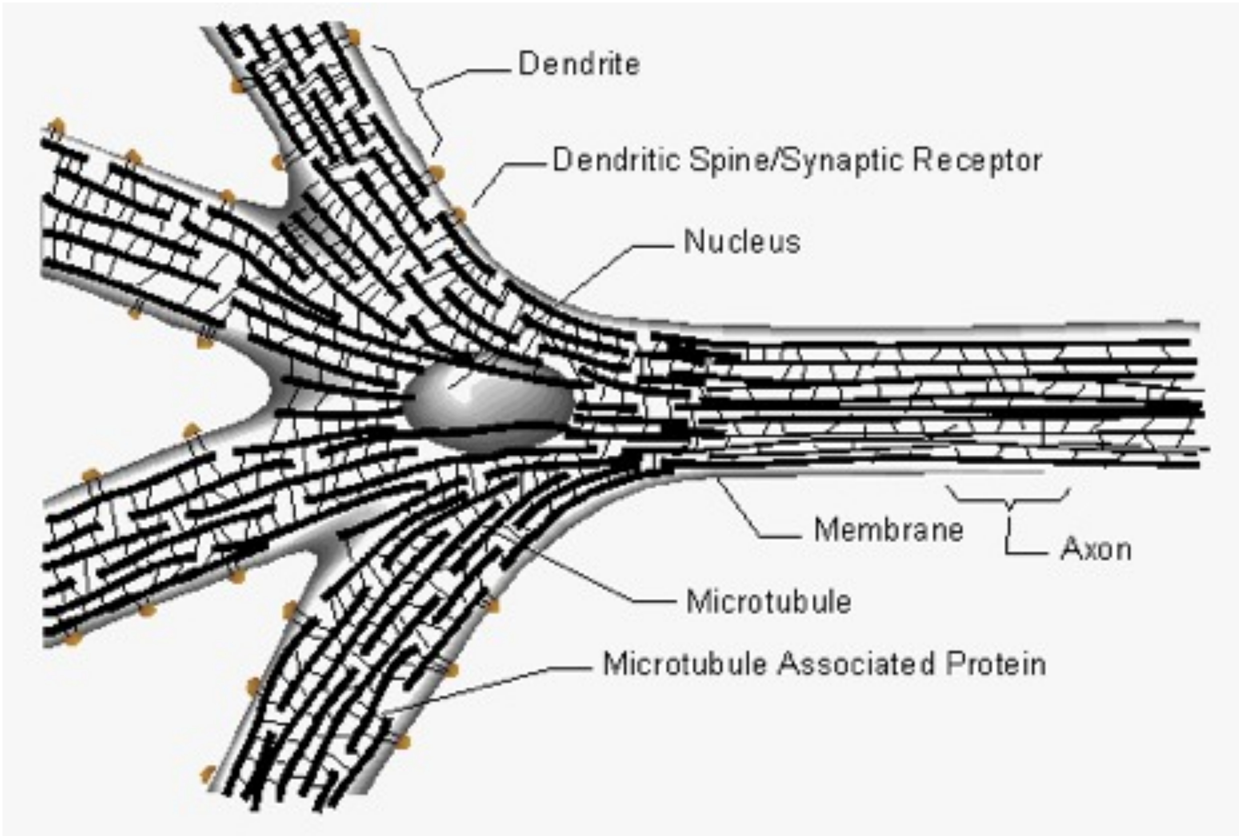
Because there are charges at the two ends of the dipole, we can have a contribution to the electric coupling from the scalar potential ... as well [as] from the magnetic coupling ... from the vector potential ... electric dipole coupling is stronger than magnetic dipole coupling ... the coupling of ... two ... configurations ... is the same, whether retarded or advanced potentials are used. Any ... configuration ... couples to any other on its light cone, whether past or future. ... The total phase accumulation in a ... configuration ... is the sum of that due to its own current, and that due to currents in other ... configurations ... far away ...
The energy in a single resonator alternates between the kinetic energy of the electrons (inductance), and the potential energy of the electrons (capacitance). With the two resonators coupled, the energy shifts back and forth between the two resonators in such a way that the total energy is constant ... The conservation of energy holds despite an arbitrary separation between the resonators ... Instead of scaling linearly with the number of charges that take part in the motion, the momentum of a collective system scales as the square of the number of charges! ... The inertia of a collective system, however, is a manifestation of the interaction, and cannot be assigned to the elements separately. ... Thus, it is clear that collective quantum systems do not have a classical correspondence limit. ...".

For **the 10^18 Tubulin Dimers of the human brain**,
the resonant frequencies are the same and exchanges of energy among them
act to keep them **locked in a Quantum Protectorate collective coherent state**.

Philip W. Anderson in cond-mat/0007287 and cond-mat/007185 said:
"... Laughlin and Pines have introduced the term "Quantum protectorate" as a general descriptor of the fact that certain states of quantum many-body systems exhibit properties which are unaffected by imperfections, impurities and thermal fluctuations. They instance ... flux quantization in superconductors, equivalent to the Josephson frequency relation which again has mensuration accuracy and is independent of imperfections and scattering. ...
... the source of quantum protection is a collective state of the quantum field involved such that the individual particles are sufficiently tightly coupled that elementary excitations no longer involve a few particles but are collective excitations of the whole system, and therefore, macroscopic behavior is mostly determined by overall conservation laws ... **a "quantum protectorate" ...[is]... a state in which the many-body correlations are so strong that the dynamics can no longer be described in terms of individual particles, and therefore perturbations which scatter individual particles are not effective** ...".
Mershin, Sanabria, Miller, Nawarathna, Skoulakis, Mavromatos, Kolomenskii, Scheussler, Ludena, and Nanopoulos in physics/0505080 "Towards Experimental Tests of Quantum Effects in Cytoskeletal Proteins" said:

Classically, the various dimers can only be in the ...[]... conformations. Each dimer is influenced by the neighboring dimers resulting in the possibility of a transition. This is the basis for classical information processing, which constitutes the picture of a (classical) cellular automaton.
If we assume ... that each dimer can find itself in a QM superposition of ...[those]... states, a quantum nature results. Tubulin can then be viewed as a typical two-state quantum mechanical system, where the dimers couple to conformational changes with 10^(-9) - 10^(-11) sec transitions, corresponding to an angular frequency ~ 10^10 - 10^12 Hz. In this approximation, the upper bound of this frequency range is assumed to represent (in order of magnitude) the characteristic frequency of the dimers, viewed as a two-state quantum-mechanical system ...[
The Energy Gap of our Universe as superconductor condensate spacetime is from 3 x 10^(-18) Hz (radius of universe) to 3 x 10^43 Hz (Planck length). Its RMS amplitude is 10^13 Hz = 10 THz = energy of neutrino masses = critical temperature Tc of BSCCO superconducting crystal Josephson Junctions]...
large-scale quantum coherence ...[has been observed]... at temperatures within a factor of three of biological temperatures. MRI magnets contain hundreds of miles of superconducting wire and routinely carry a persistent current. There is no distance limit - the macroscopic wave function of the superfluid condensate of electron pairs, or Cooper pairs, in a sufficiently long cable could maintain its quantum phase coherence for many thousands of miles ... there is no limit to the total mass of the electrons participating in the superfluid state. The condensate is "protected" from thermal fluctuations by the BCS energy gap at the Fermi surface ... The term "quantum protectorate" ... describe[s] this and related many-body systems ...".

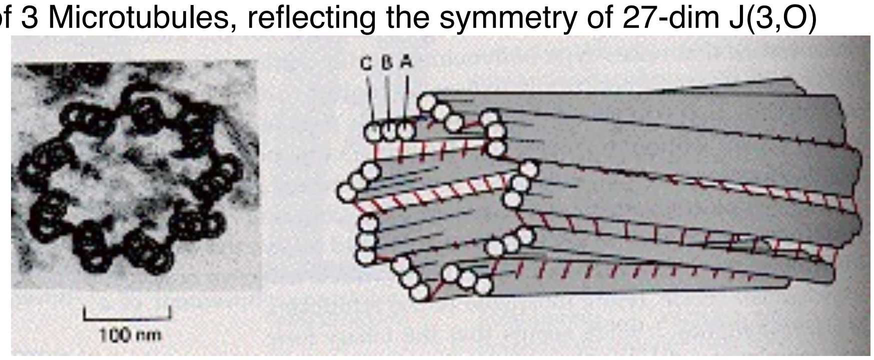
The Human Brain has about 10^11 Neuron cells, each about 1,000 nm in size.
The cytoskeleton of cells, including neurons of the brain, is made up of Microtubules



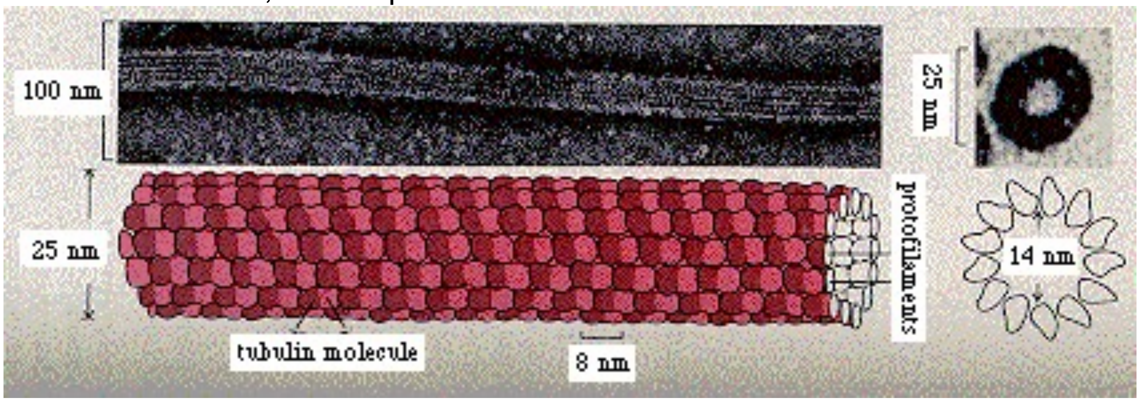
(image from "Orchestrated Objective Reduction of Quantum Coherence in Brain Microtubules: The "Orch OR" Model for Consciousness" by Penrose and Hameroff)

Each Neuron contains about 10^9 Tubulin Dimers, organized into Microtubules some of which are organized by a Centrosome. Centrosomes contain a pair of Centrioles.

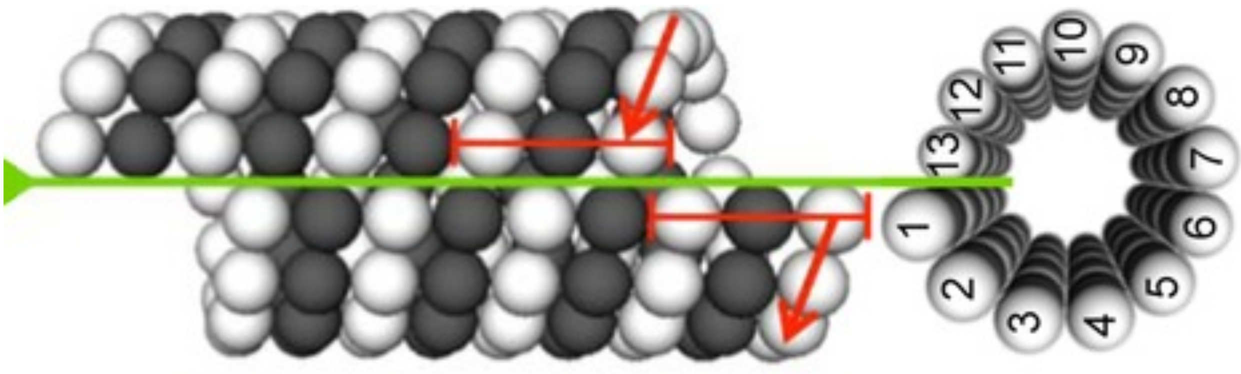
A Centriole is about 200 nm wide and 400 nm long. Its wall is made up of 9 groups of 3 Microtubules, reflecting the symmetry of 27-dim J(3,O)



Each Microtubule is a hollow cylindrical tube with about 25 nm outside diameter and 14 nm inside diameter, made up of 13 columns of Tubulin Dimers



(illustrations and information about cells, microtubules, and centrioles are from Molecular Biology of the Cell, 2nd ed, by Alberts, Bray, Lewis, Raff, Roberts, and Watson (Garland 1989))



(image from Wikipedia on Microtubule)

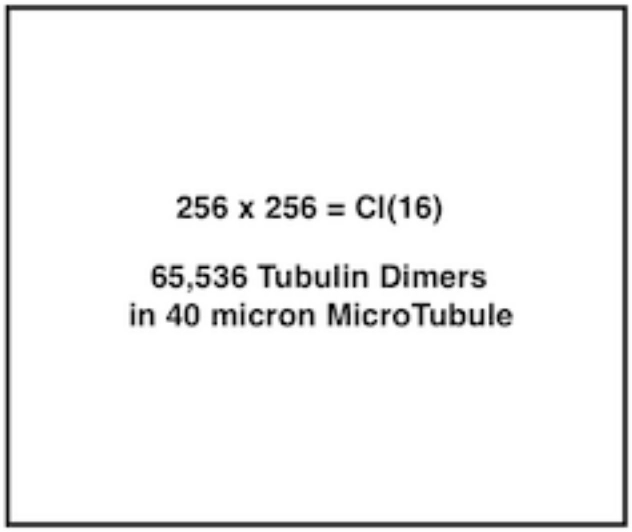
Each Tubulin Dimer is about 8 nm x 4 nm x 4 nm, consists of two parts, alpha-tubulin and beta-tubulin (each made up of about 450 Amino Acids, each containing roughly 20 Atoms)
A Microtubule 40 microns = 40,000 nm long contains 13 x 40,000 / 8 = 65,000 Dimers

(images adapted from nonlocal.com/hbar/microtubules.html by Rhett Savage)

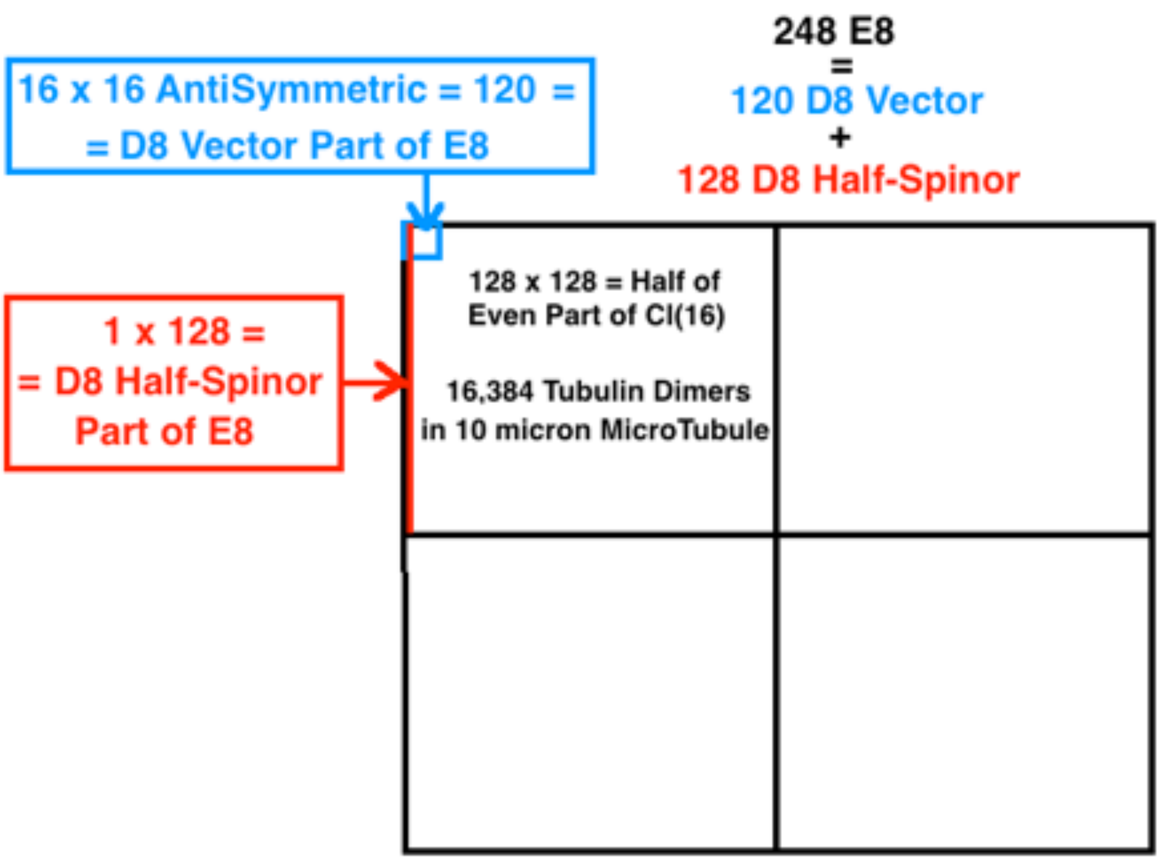
The black dots indicate the position of the Conformation Electrons.
There are two energetically distinct configurations for the Tubulin Dimers:
Conformation Electrons Similarly Aligned (left image) - State 0
Conformation Electrons Maximally Separated (right image) - State 1

The two structures - State 0 ground state and State 1 higher energy state - make Tubulin Dimers the basis for a Microtubule binary math / code system.

Microtubule binary math / code system corresponds to Clifford Algebras Cl(8) and Cl(8)xCl(8) = Cl(16) containing E8

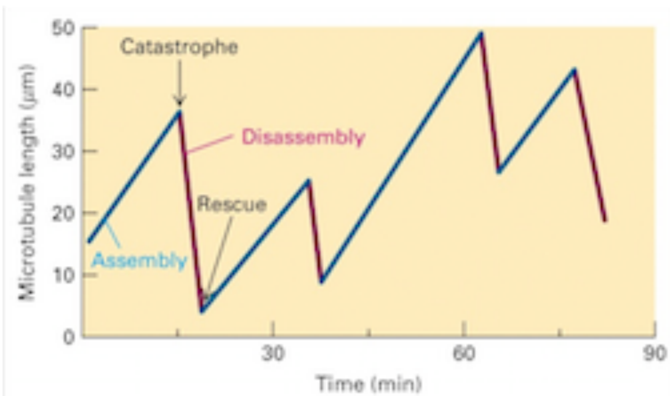


A 40 micron Microtubule contains Dimers representing the 65,536 elements of Cl(16) which contains the 248 elements of Lie Algebra E8 that defines E8 Physics Lagrangian.



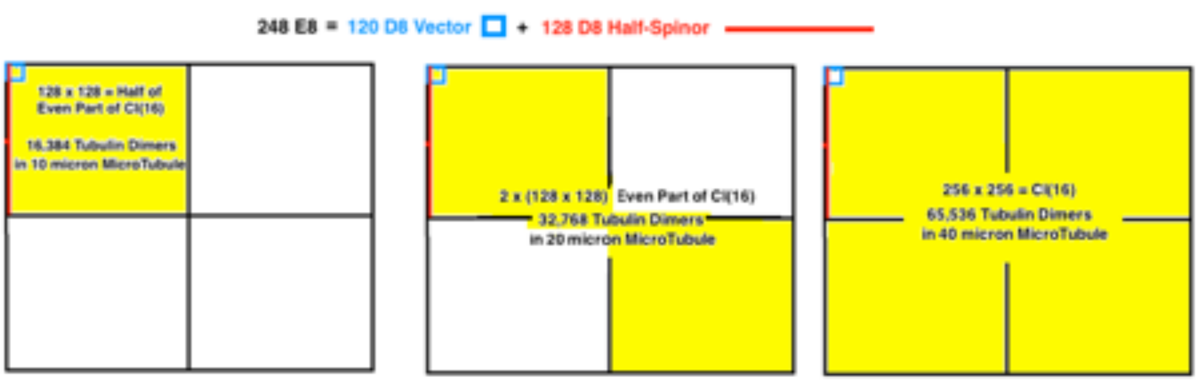
E8 lives in only half of the block diagonal Even Part half of Cl(16) so that E8 of E8 Physics can be represented by the 16,384 Dimers of a 10 micron Microtubule.

According to 12biophys.blogspot.com Lecture 11 Microtubule structure is dynamic:
“... One end of the microtubule is composed of stable (GTP) monomers
while the rest of the tubule is made up of unstable (GDP) monomers.
The GTP end comprises a cap of stable monomers.
Random fluctuations either increase or decrease the size of the cap.
This results in 2 different dynamic states for the microtubule.
Growing: cap is present   Shrinking: cap is gone ...



...”.

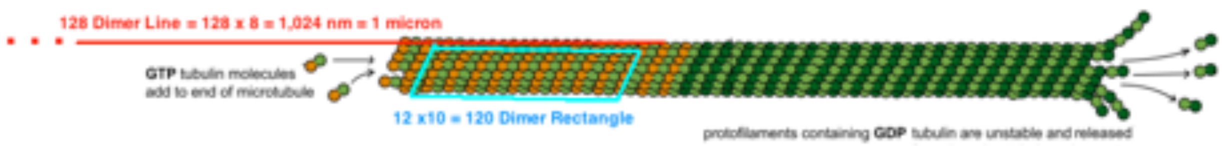
Microtubules spend most of their lives between 10 microns and 40 microns,
sizes that can represent E8 as half of the Even Part (half) of Cl(16) (10 microns)



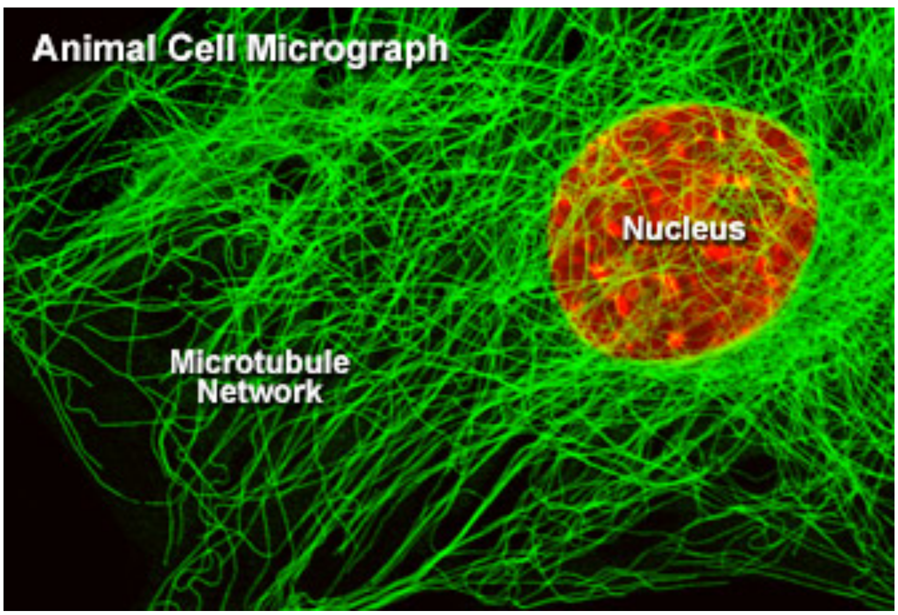
or as the Even Part (half) of Cl(16) (20 microns) or as full Cl(16) (40 microns).

In a given Microtubule
the 128 D8 Half-Spinor part is represented by a line of 128 Dimers in its stable GTP region
and

the 120 D8 Vector part by a 12 x 10 block of Dimers in its stable GTP region
(image adapted from 12biophys.blogspot.com Lecture 11)



The image immediately above does not show how thin is the Microtubule.
The following image (from micro.magnet.fsu.edu) shows overall Microtubule shape



How do the Microtubules communicate with each other ?

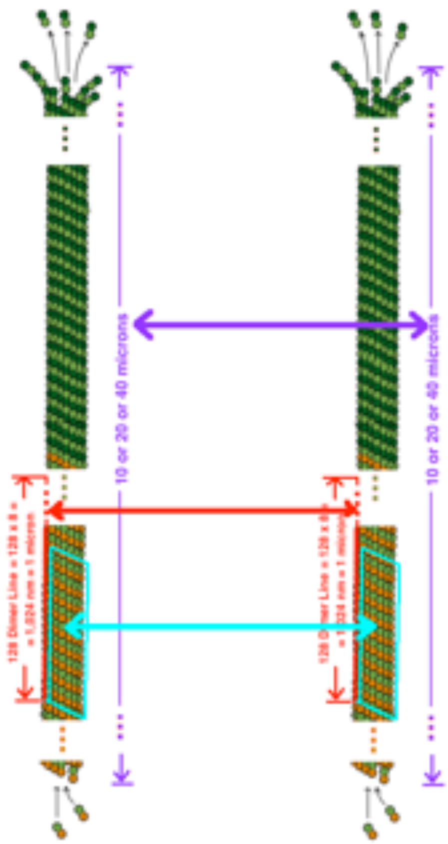
Consider the Superposition of States State 0 and State 1 involving one Tubulin Dimer with Conformation Electron mass m and State1 / State 0 position separation a .

The Superposition Separation Energy Difference is the internal energy

$$E\_ssediff = G m^2 / a$$

that can be seen as either the **energy of 26D String Theory spin two gravitons** or the **Bohm Quantum Potential internal energy**, equivalently.

Communication between two Microtubules is by the Bohm Quantum Potential
between their respective corresponding Dimers (purple arrow)
with the correspondence being based on connection between respective E8 subsets,
the 128 D8 Half-Spinors (red arrow) and the 120 D8 BiVectors (cyan arrow)



How is information encoded in the Microtubules ?

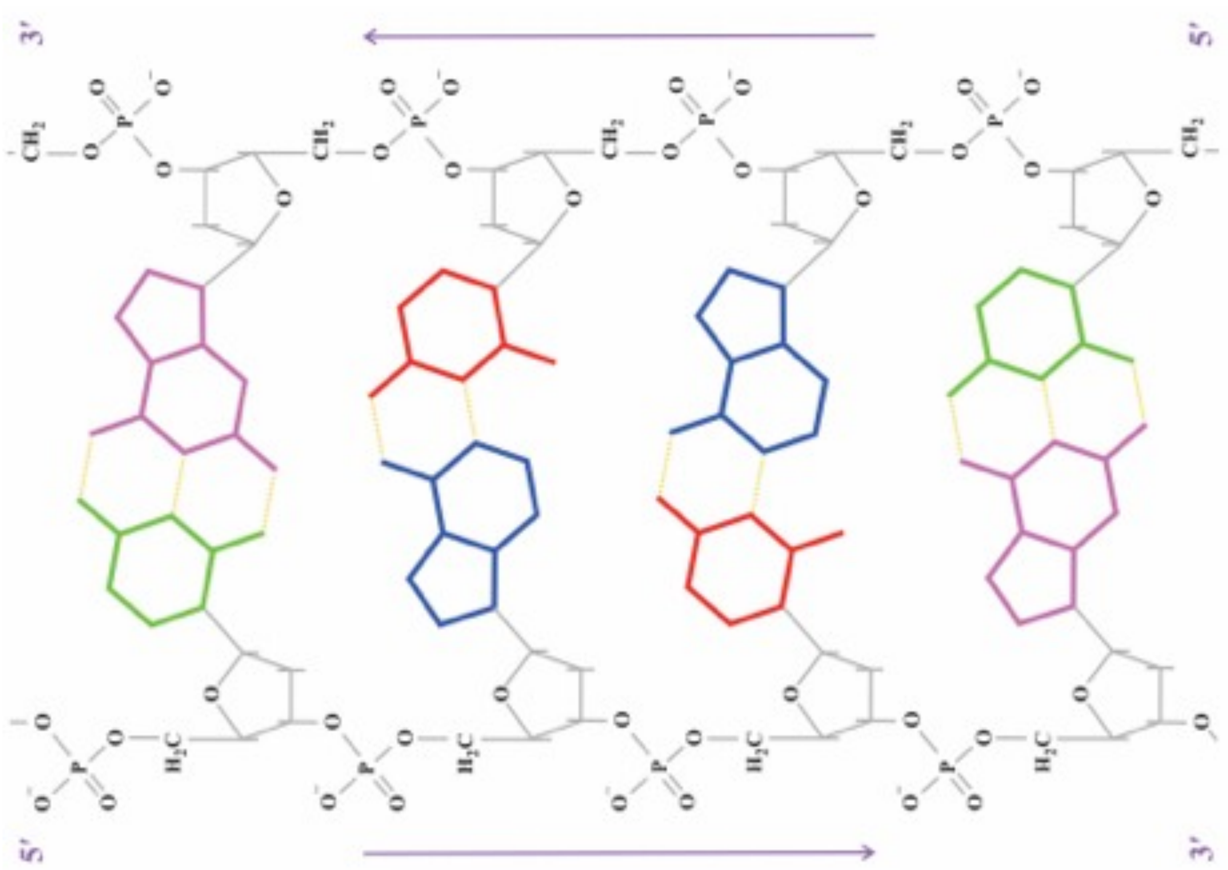
Each Microtubule contains E8, allowing Microtubules to be corrrelated with each other.
The parts of the Microtubule beyond E8 are in Cl(16) for 40 micron Microtubules,
or the Even Subalgebra of Cl(16) for 20 micron Microtubules,
or half of the Even Subalgebra of Cl(16) for 10 micron Microtubules
so since by 8-Periodicity of Real Clifford Algebras Cl(16) = Cl(8) x Cl(8) and
since Cl(8) information is described by the Quantum Reed-Muller code [[256 , 0 , 24]]
the information content of Cl(16) and its Subalgebras **is described by the Tensor Product Quantum Reed-Muller code [[256 , 0 , 24]] x [[256 , 0 , 24]]**

For a 40-micron Microtubule there are, outside the 248-E8 part, about 65,000 TD Qubits
available to describe one Quantum Thought State among about 2^65,000 possibilities,
analagous to the Book of Genesis of (22+5)^78,064 Hebrew Letter/Final possibilities.

65,536-dimensional Cl(16) not only contains the E8 of E8 Physics
and the information content of Microtubules
but also contains **the information content of DNA chromosome condensation**
and **the information content of mRNA triple - amino acid transformations**.

In “Living Matter: Algebra of Molecules” (CRC Press 2016) Valery V. Stcherbic and Leonid P. Buchatsky say: “... DNA structure contains four nucleotides:

adenine A, guanine G, cytosine C and thymine T. ...



... The Sugar-phosphate group consists of 2-deoxyribose and phosphoric acid residues. DNA chain orientation is identified by carbon atoms of 2-deoxyribose: (5′)CH2 and (3′)COH. The biological function of DNA and storage and transfer of genetic information to daughter cells is based on specific, complimentary pairing of nucleotides:

A is paired with T, and G with C.

...

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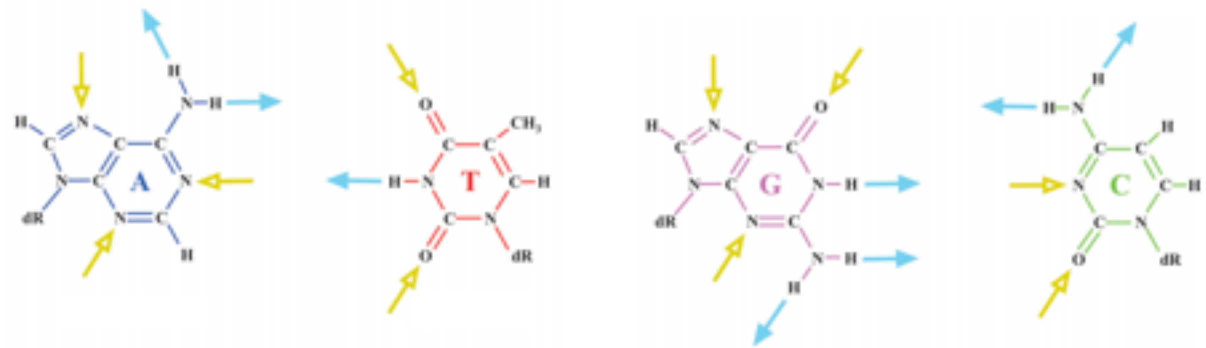


Figure 1.4 Potential vectors of hydrogen bond of DNA nucleotides.
Yellow arrows—acceptors, blue arrows—donors of hydrogen.

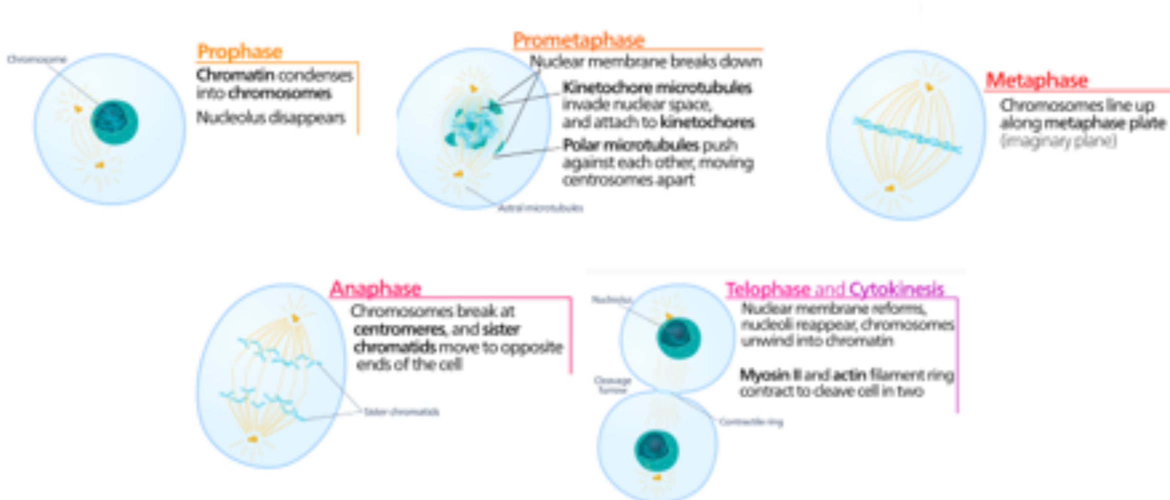
The space of DNA nucleotide states contains T2^3 ⊗ C2^4 ⊗ A2^5 ⊗ G2^6 = 2^18 elements of Clifford algebras. This space reduction to four nucleotides means compression of DNA information by a factor of 2^18 / 4 = 65536.
Reduction of the nucleotide state space leads
to DNA compactization and chromosome condensation. ...”.

In “Chromosome Condensation and Cohesion” (eLS December 2010) Laura Angelica Diaz-Martinez and Hongtau Yu say: “... The diploid human genome consists of 46 chromosomes, which collectively contain about 2 m of deoxyribonucleic acid (DNA). During mitosis, the genome is packaged into 46 pairs of sister chromatids,
each less than 10 μm long. ...”.

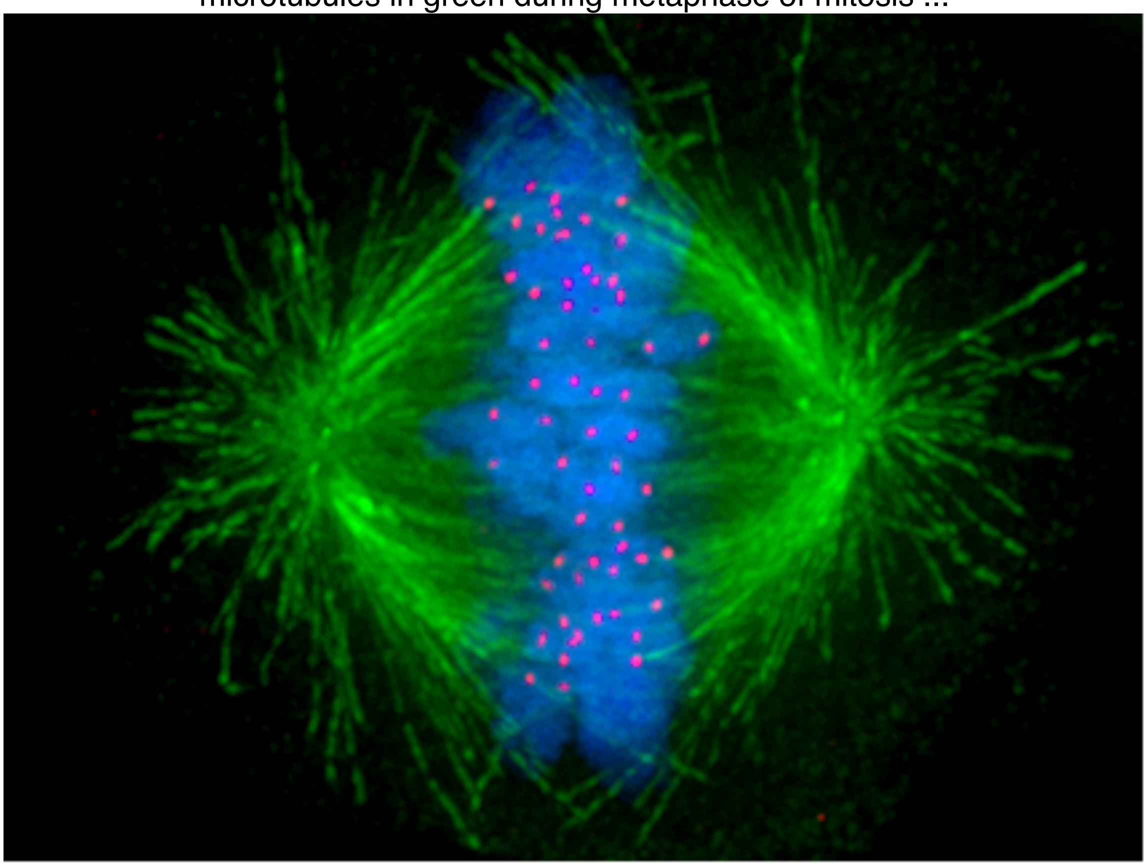
The DNA information condensation factor of 65,536 is the dimension of Cl(16)
which is
the Real Clifford Algebra containing 248-dim E8 of E8 Physics
as 120-dim bivector D8 plus 128-dim D8 half-spinor
and is also
the Clifford Algebra of Microtubule information in Quantum Consciousness.

Microtubule information = 65,536 = Cl(16) = DNA condensation information

Wikipedia describes interaction of Microtubules with DNA in mitosis condensation: “...



... Micrograph showing condensed chromosomes in blue, kinetochores in pink, and microtubules in green during metaphase of mitosis ...



...”. **Information lost by condensing DNA is stored in Microtubules through Anaphase after which it has been restored to the new Duplicated DNA.**

Stcherbic and Buchatsky also say: “... Ribonucleic acid (RNA) can also store genetic information. A single RNA helix is seldom used as a carrier of genetic information (only in some viruses); its main role is storing DNA sites as copies of individual proteincoding genes (mRNA) or in formation of large structural complexes, e.g., ribosomes and spliceosomes. At self-splicing, RNA may perform the function of an enzyme. RNA also performs an important role during DNA replication. So called RNA-primers are necessary to synthesize DNA complementary chains, although this fact is not obvious. RNA contains sugar, ribose, which hydroxyl groups make more reactive than DNA. Besides, RNA contains uracil U, which is somewhat lighter than thymine.
...
At translation of mRNA triplets into genetic code amino acids,
the dynamics of triplets to amino acids transformation should be taken into account.
...
At transition ... functional volume is equal to 3^5 = 243.
To this volume there should be added the volume of auxiliary spaces,
equal to 13 = 5 + 4 + 3 + 1.
Accordingly, we get
256 functions of mRNA triplet transformation into amino acids of the genetic code.
Reverse transition ... from amino acids ... to triplet ... needs 5^3 + 3^1 = 128 functions.
In addition, 128 triplets of mRNA-tRNA pairing should be added to this number. ...”.

The 256 of mRNA triplet to amino acids is represented by Cl(8) Clifford algebra
and
the 128+128 = 256 of amino acids to mRNA triplets is representd by another Cl(8)
so
that the mRNA triple - amino acid connection is represented by the tensor product Cl(8) x Cl(8) which by 8-Periodicity of Real Clifford Algebras is the Real Clifford Algebra Cl(16)
which also contains 248-dim E8 of viXra 1508.0157 E8 Physics
and is also the Clifford Algebra
of Microtubule information in viXra 1512.0300 Quantum Consciousness.

What about information in a large number of Microtubules ?

Since the information in one Microtubule is based on Cl(16)
and
since by 8-Periodicity Cl(16) x...(8N times tensor product)...x Cl(16) = Cl(16 N)
information of a large number of Microtubules is described by

the large-number Tensor Products of
Tensor Product [[256 , 0 , 24]] x [[256 , 0 , 24]] Quantum Reed-Muller codes

How does all this give rise to Penrose-Hameroff Quantum Consciousness ?

Consider the Superposition of States State 0 and State 1 involving one Tubulin Dimer with Conformation Electron mass m and State1 / State 0 position separation a .
The Superposition Separation Energy Difference is the internal energy

E_ssediff = G m^2 / a

that can be seen as either the **energy of 26D String Theory spin two gravitons** or the **Bohm Quantum Potential internal energy**, equivalently.

For a given Tubulin Dimer a = 1 nanometer = 10^(-7) cm so that
T = h / E_electron = (Compton / Schwarzschild) (a / c) = 10^26 sec = 10^19 years

Now consider the case of N Tubulin Dimers in Coherent Superposition
connected by the Bohm Quantum Potential Force that does not fall off with distance.
Jack Sarfatti defines coherence length L by L^3 = N a^3 so that
the Superposition Energy E_N of N superposed Conformation Electrons is

E_N = G M^2 / L = N^(5/3) E_ssediff

The decoherence time for the system of N Tubulin Electrons is

T_N = h / E_N = h / N^(5/3) E_ssediff = N^(-5/3) 10^26 sec

so we have the following rough approximate Decoherence Times T_N

Number of Involved Tubulin Dimers	Time T_N	
10^(11+9) = 10^20	10^(-33 + 26) = 10^(-7) sec	10^11 neurons x 10^9 TD / neuron 10^20 Tubuin Dimers in Human Brain
10^16	10^(-27 + 26) = 10^(-1) sec - 10 Hz	

Human Alpha EEG is 8 to 13 Hz
Fundamental Schumann Resonance is 7.8 Hz
Time of Traverse by a String World-Line Quantum Bohmion of a Quantum Consciousness Hamiltonian Circuit of 10^16 TD separated from nearest neighbors by 10 nm is 10^16 x 10 nm / c = (10^16 x 10^(-6)) cm / c = 10^10 cm / c = 0.3 sec

Appendix - Adinkra and Pyramid

According to The Oxford Encyclopedia of African Thought, Vol. 1, by Irele and Jeyifo:

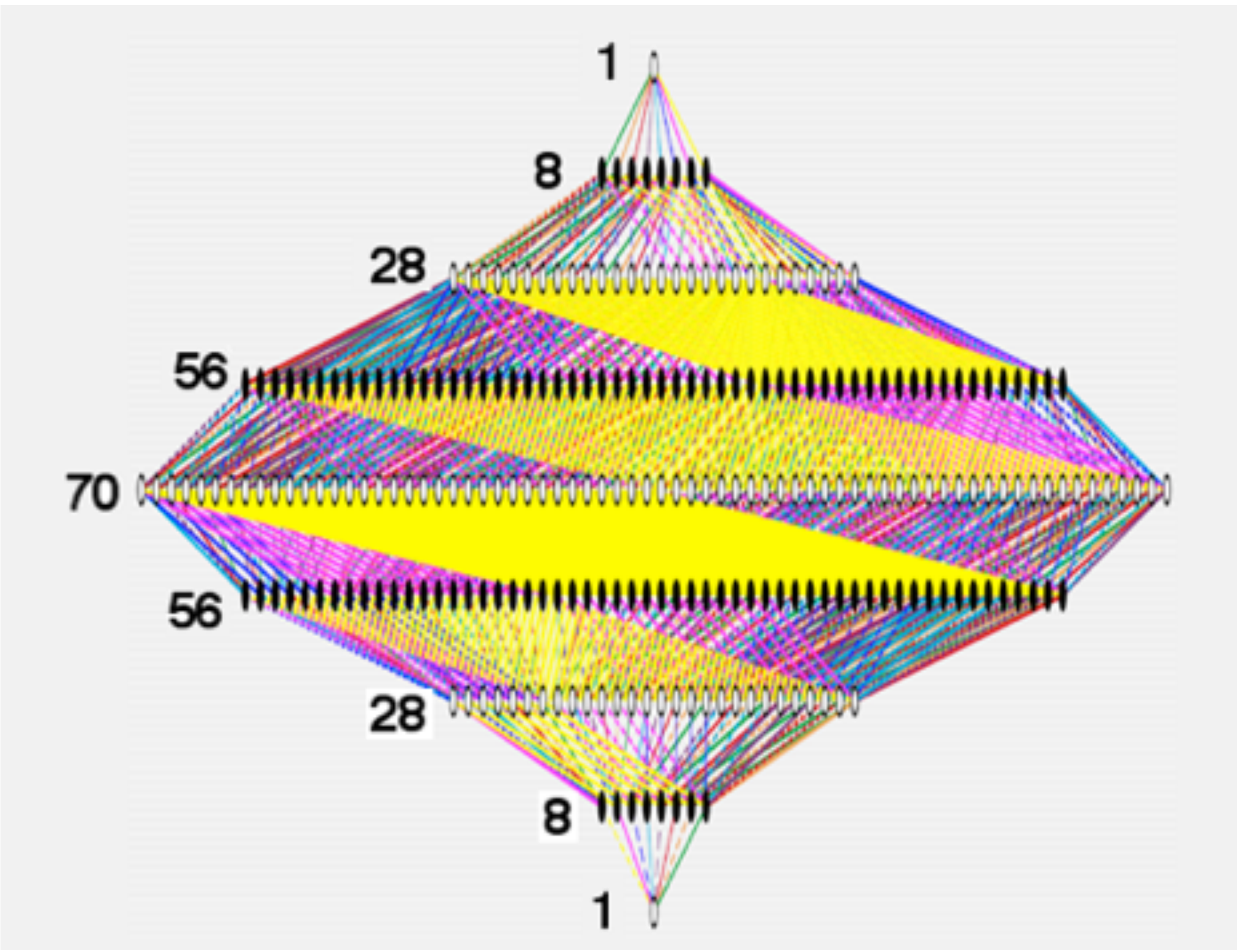
"... Adinkra are visual forms that ... integrate striking aesthetic power, evocative mathematical structures, and philosophical conceptions ...:.

Gates, Doran, Faux, Hubsch, Iga, Landweber, and Miller (arxiv 0811.3410) said:

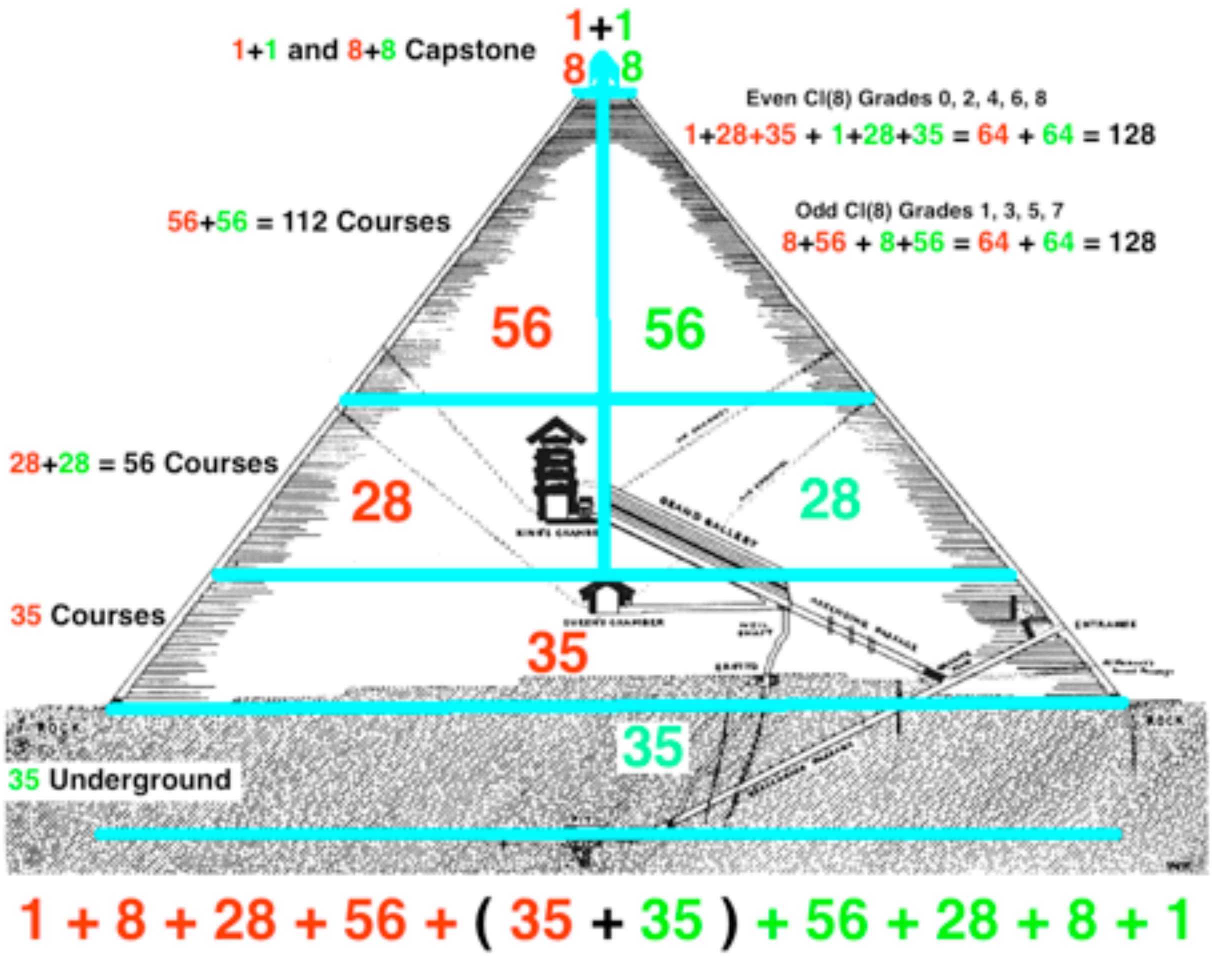
"... we **relate Adinkras to Clifford algebras** ...".

G. D. Landweber's 2006 program Adinkramat at http://www.cohomology.com/
produces Adinkra graphs of MI^Ncubic, such as

N = 8 of real Clifford Algebra Cl(8) with 28-dim grade 2 = Spin(8)
and graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
with 2^8 = 256 elements corresponding to the 256 Odu and
sqrt(256) = 16-dim spinors = 8-dim +half-spinors and 8-dim -half-spinors

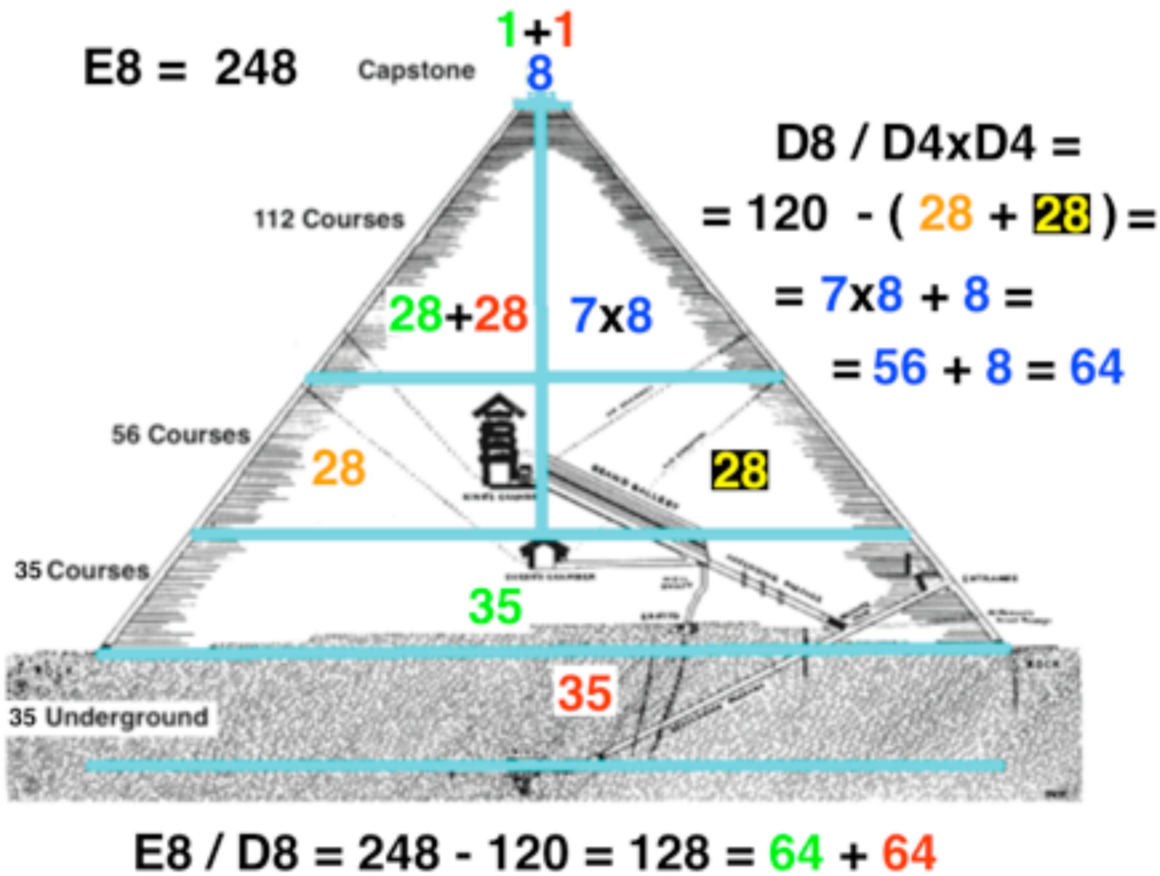


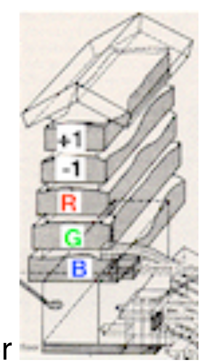
Clifford Algebras were not known to European mathematicians until Clifford in the 19th century and not known to European physicists until Dirac in the 20th century but it seems to me that their structure was known to Africans in ancient times. For example, the courses of the Great Pyramid of Giza correspond to the graded structure of Cl(8):



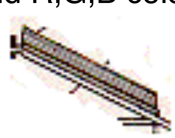
(image adapted from David Davidson image - for larger size see tony5m17h.net/GreatPyrCl8.png)

248-dim E8 (like 256-dim Cl(8)) can also be seen in terms of the Great Pyramid
(the 8-dim difference is related to the Cl(8) Primitive Idempotent and the Higgs).



The 28 is in the area of the Upper Chamber  which has 5 slabs

that represent the 5 charges (+1,-1 electric and R,G,B color) of the Standard Model.

The 28 is in the area of the Grand Gallery  which rises at a
slope of about 26 degrees, or about half of the Golden Ratio slope of the Great Pyramid
which is arccosine(1 / ((1 + sqrt(5))/2)) = 51.8 degrees.
The Grand Gallery could represent a segment of a space-time path (World-Line)
in the context of Conformal Gravity.