Pi Formulas

Part 5: Formula for the constant Pi

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abstract

In this note we give one formula for the constant Pi

Una Representación Para La Constante Pi : $\pi = 3.141592...$

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Resumen. Se muestra una fórmula general para la constante Pi:

$$\pi = 4\sum_{n=0}^{\infty}(-1)^n(2n+1)^{-1}$$

1. Fórmula

Para $n \in \mathbb{N} = \{1,2,3,...\}, 0 < a < 1$, se tiene:

$$\pi = 6\sum_{k=0}^{\infty}(-1)^k \int_0^v \left(nax^{n-1}+1-a\right)\left(ax^n+(1-a)x\right)^{2k} dx \qquad (1)$$

donde $v \equiv v(n,a) \in (0,1)$, satisface la ecuación:

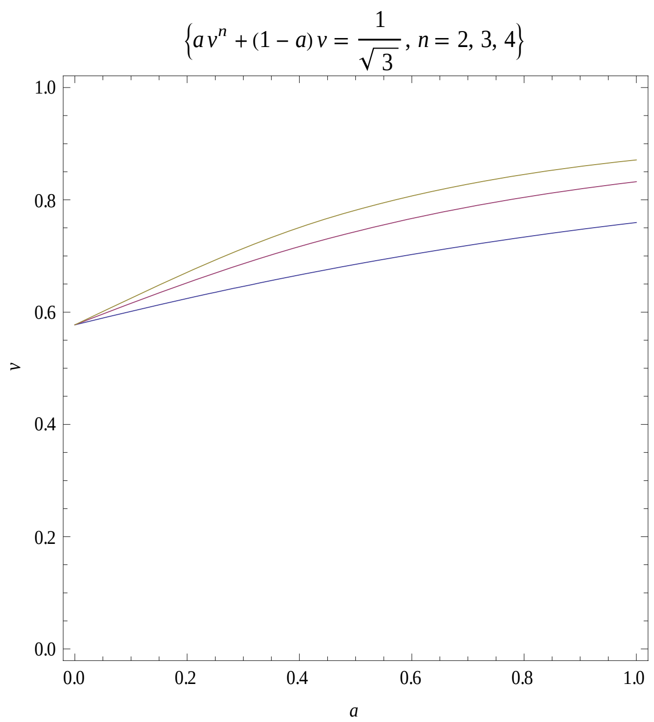
$$av^n+(1-a)v = \frac{1}{\sqrt{3}} \quad , n \in \mathbb{N} \qquad (2)$$

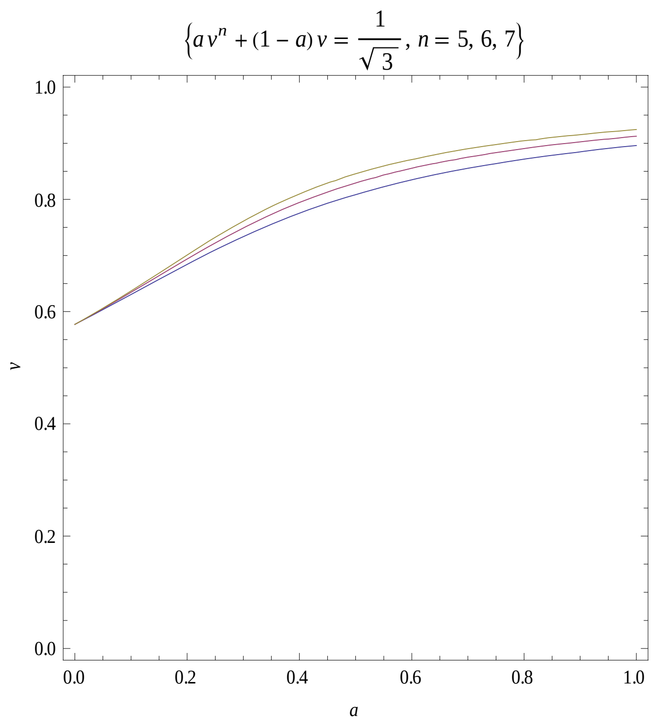
2. Representación Gráfica de la Función: $v(n,a), n = 2,3,4,...$

Para $n = 2,3,4,...$, y $s = \dfrac{n}{n-1}$, se tiene:

$$u = \sqrt[s]{a\sqrt{3}+(1-a)\sqrt{3}\sqrt[s]{a\sqrt{3}+(1-a)\sqrt{3}\sqrt[s]{a\sqrt{3}+...}}} \qquad (3)$$

$$v = \left(\frac{1}{u}\right)^{\frac{1}{n-1}} \qquad (4)$$


$\left\{a\,v^n + (1-a)\,v = \frac{1}{\sqrt{3}},\; n = 2, 3, 4\right\}$
v
a
0.0
0.2
0.4
0.6
0.8
1.0


$\left\{a\,v^n + (1-a)\,v = \frac{1}{\sqrt{3}},\; n = 5, 6, 7\right\}$
v
a
0.0
0.2
0.4
0.6
0.8
1.0

3. Una Integral

$$\int_0^v \left(nax^{n-1}+1-a\right)\left(ax^n+\left(1-a\right)x\right)^{2k}dx = \\ =\sum_{m=0}^{2k}\binom{2k}{m}a^{2k-m}\left(1-a\right)^m\left(\frac{nav^{(2k-m+1)n+m}}{\left(2k-m+1\right)n+m}+\frac{\left(1-a\right)v^{(2k-m)n+m+1}}{\left(2k-m\right)n+m+1}\right) \quad (5)$$

4. Referencias

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