Generalization of equation of continuity and electro-magnetic scalar radiation

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ABSTRACT

Steady current circuits are not closed in all cases if we continue to define current in its usual sense. In high frequency current oscillations current changes along its path even if there is no net charge oscillations. To bring these into mathematical framework, Maxwell's equations of electrodynamics are generalized extending the equation of continuity. This generalization led to prediction of electro-magnetic scalar radiations to be emitted along with the usual electromagnetic radiations. Calculation for these radiations was undertaken for dipole antennas under no assumed charge distributions. Substantial amount of electro-magnetic scalar radiations are shown to be emitted at high frequency current oscillations revealing directional property like electromagnetic radiation with maximum intensity not along the perpendicular bisector of the antenna but along the direction of its alignment.

I. Introduction

The science of electricity began with study of interactions between static charge distributions and forces between steady current circuits. In all these circuits the flow stream of charges are taken to be closed on itself. But in certain generators and some media forming parts of such circuits , the cross section of the flow stream of charges is not well defined. The cathode of vacuum tube devices receives conduction electrons in one side, which then get merged into the sea of valence electrons. On the other side, electrons come out of the surface of cathode through thermionic emission to maintain the current in the circuit. In npn transistor some of the incoming electrons to the p-type material of the base get trapped by the holes at the entry point. Close to the exit point some covalent bonds are ruptured to generate electrons and holes of which the electrons flow out of the base to maintain the base current. The behavior of such media can be viewed as reducing current to zero value at the entry point without any charge deposit and giving rise current at the exit point with no depletion of any charge deposit. This aspect along with certain other aspects to be discussed in the following section can be included in the mathematical framework of classical electrodynamics through generalization of equation of continuity of charge flow given as

$$\vec{\nabla}.\vec{J}+\frac{\partial\rho}{\partial t}=0 \qquad (1.1)$$

Where $\vec{J}$ and ρ are current density and charge density respectively. This can be done through generalization of Maxwell's equations of electrodynamics.

II. Generalization of Maxwell's equations

The field around charged matter is visualized by drawing a set of diverging electric field lines starting from every positively charged particle and a set of converging electric field lines ending at every negatively charged particle. Amount of charge within a region of space is determined through the electric flux over the surface enclosing the region. If the flux over the region varies in time changing between outward and inward flux periodically, we infer that charge distribution in the region is changing its sign with the same period.

Concentric circular magnetic lines of force can be imagined to exist around a straight conductor carrying a steady current. If the current changes in time alternating in sign the magnetic field lines change accordingly to generate circular electric field lines. These field lines in turn, because of its time variation generate magnetic field lines. The coupled electric and magnetic field lines go out into space leaving behind the oscillating current and are called electromagnetic radiation. In a similar manner if a charge oscillation exists at a point without being translated to a current or, a time varying current undergoes a change along its path failing to bring a net charge oscillation at the point of change, alternately converging and diverging electric field lines are generated to propagate outwards into space. This is manifested as a coupled radial electric field and a magnetic scalar field to be named as electro-magnetic scalar radiation.

Electromagnetic radiation was predicted when Maxwell included eq. (1.1) into the then existing equations of electrodynamics to deduce a set of four field equations known as Maxwell's equations. The relation does not allow visualization of converging or diverging electric field lines in space unless they surround charged matter. One case may be examined where it fails to reveal the physical realities. From Maxwell's equations it follows that electric and magnetic fields are generated owing to rapid variations of current and charge distributions in some region of space. The current in metallic conductors is established due to drift speed of valence electrons, a combined effect of electric field in the material of the conductor owing to the dc or ac sources and charges developed on the surface of the conductor [1] , and collision of the accelerating electrons against the thermal vibrations of the metal . For the current to pile up charges along its flow stream, the valence electrons are to travel 10 to 100 atomic distances or more before the applied electric field changes its direction. In the microwave frequency range this condition is rarely satisfied. For example, with source frequency of 10 GHz, electric field of rms value 1000 volt/m, relaxation time of 2×10^{-14} s the distance travelled is 3.5×10^{-10}m, about three atomic diameters. Thus in these high frequency range of oscillation of applied electric field, the relation (1.1) is rarely satisfied. Hence Maxwell's equations need be generalized to include such behavior of charged materials at high frequencies of current oscillation. Such generalization can conveniently be brought about if studies on electrodynamics would begin with the inhomogeneous potential wave equations given below.

$$\nabla^2\vec{A} - \varepsilon\mu\frac{\partial^2\vec{A}}{\partial t^2} = -\mu\vec{J} \qquad (2.1)$$

$$\nabla^2\Phi - \varepsilon\mu\frac{\partial^2\Phi}{\partial t^2} = -\frac{\rho}{\varepsilon} \qquad (2.2)$$

where $\vec{A}\,(\vec{r}\,,\mathrm{t})$ is the vector potential and $\Phi\,(\vec{r}\,,\mathrm{t})$ is the scalar potential . ε and μ are respectively the electrical permittivity and magnetic permeability of the medium in which the potential functions prevail.

The solutions of eq. (2.1) and eq. (2.2) are [2] given as below.

$$\vec{A}(\vec{r},t) = \frac{\mu}{4\pi}\int\frac{\vec{J}\left(\vec{\mathrm{r}}',\mathrm{t}-\frac{|\vec{\mathrm{r}}-\vec{\mathrm{r}}'|}{\mathrm{c}}\right)}{|\vec{r}-\vec{r}'|}\mathrm{d}\tau' \qquad (2.3)$$

$$\Phi(\vec{r},t) = \frac{1}{4\pi\varepsilon}\int\frac{\rho\left(\vec{\mathrm{r}}',\mathrm{t}-\frac{|\vec{\mathrm{r}}-\vec{\mathrm{r}}'|}{\mathrm{c}}\right)}{|\vec{r}-\vec{r}'|}\mathrm{d}\tau' \qquad (2.4)$$

Three physical fields - electric field, $\vec{E}(\vec{r},t)$,magnetic field,$\vec{B}(\vec{r},t)$ and magnetic scalar field $S(\vec{r},t)$ can be obtained from the potential functions as given in the following.

$$\vec{E} = -\left(\vec{\nabla}\Phi + \frac{\partial\vec{A}}{\partial t}\right) \qquad (2.5)$$

$$\vec{B} = \vec{\nabla}\times\vec{A} \qquad (2.6)$$

$$S = -\left(\vec{\nabla}.\vec{A} + \varepsilon\mu\frac{\partial\Phi}{\partial \mathrm{t}}\right) \qquad (2.7)$$

These fields, following eq. (2.1) and eq.(2.2)can be found to satisfy the following equations, the first four of which shall be called generalized Maxwell's equations.

$$\vec{\nabla}.\vec{E} = \frac{1}{\varepsilon}\left(\rho + \varepsilon\frac{\partial S}{\partial t}\right) \qquad (2.8)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad (2.9)$$

$$\vec{\nabla}.\vec{B} = 0 \qquad (2.10)$$

$$\vec{\nabla}\times\vec{B} - \varepsilon\mu\frac{\partial\vec{E}}{\partial t} = \mu\left(\vec{J} - \frac{\nabla S}{\mu}\right) \qquad (2.11)$$

$$\nabla^2\vec{E} - \varepsilon\mu\frac{\partial^2\vec{E}}{\partial t^2} = \mu\left(\frac{\partial\vec{J}}{\partial t} + \frac{1}{\varepsilon\mu}\vec{\nabla}\rho\right) \qquad (2.12)$$

$$\nabla^2\vec{B} - \varepsilon\mu\frac{\partial^2\vec{B}}{\partial t^2} = -\mu(\vec{\nabla}\times\vec{J}) \qquad (2.13)$$

$$\nabla^2 S - \varepsilon\mu \frac{\partial^2 S}{\partial t^2} = \mu\left(\vec{\nabla}.\vec{J} + \frac{\partial \rho}{\partial t}\right) \quad (2.14)$$

Writing

$$-\frac{\vec{\nabla}\mathrm{S}}{\mu} = \vec{J}_F \quad (2.15)$$

and $$\varepsilon\frac{\partial S}{\partial t} = \rho_F \quad (2.16)$$

eq.(2.14) can be cast in the following form.

$$\vec{\nabla}.\left(\vec{J} + \vec{J}_F\right) + \frac{\partial(\rho+\rho_F)}{\partial t} = 0 \quad (2.17)$$

$\vec{J}_F$ and ρ_F can be called dynamic current density and dynamic charge density respectively and eq.(2.17) may be taken as a statement of generalized equation of continuity. One can call the statement as generalized charge conservation relation. The generalized Maxwell's equations, eq. (2.8) to eq. (2.11) are Lorentz invariant and $\vec{J}_F$,ρ_F define a current four vector. S is a Lorentz scalar.

The generalized Maxwell's equations have been obtained and discussed earlier [3] *and* [4] . They have derived an equation for conservation of energy of electrodynamic fields, which is stated as below.

$$\vec{J}.\vec{E} + \frac{1}{\varepsilon\mu}\rho S = -\frac{\partial\left(\frac{\varepsilon E^2}{2} + \frac{B^2}{2\mu} + \frac{S^2}{2\mu}\right)}{\partial t} - \vec{\nabla}.\left(\vec{E} \times \frac{\vec{B}}{\mu} - \vec{E}\frac{S}{\mu}\right) \quad (2.18)$$

The generalized Poynting vector $\vec{\mathcal{P}} = \vec{E} \times \frac{\vec{B}}{\mu} - \vec{E}\frac{S}{\mu}$ will now comprise of two distinct types of radiations-electromagnetic and electro-magnetic scalar radiations. In the following sections a few simple cases are considered to show how eq.(2.17)leads to electro-magnetic scalar radiation apart from the usual electromagnetic radiation.

III.Radiations from dipole antenna

Centre-fed and multiple-fed dipole antennas are now in wide use and they appear in text books[5] ,[6].Radiations from these antennas have been calculated in several works. Current distribution in them have been experimentally determined and charge distribution along their length have been calculated in conformity with eq.(1.1).The radiations from the antennas are all shown to be electromagnetic in nature. A large number of experiments have conformed to the theoretical findings. In what follows I consider the antennas with the same current distributions but without assuming any net charge oscillations as required by eq.(1.1).The details are shown for a half wave dipole antenna.

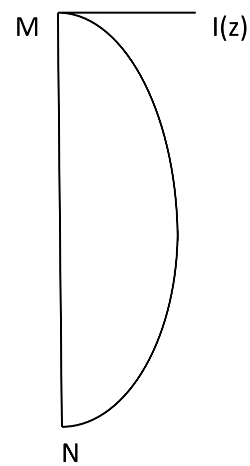


Fig.(2)

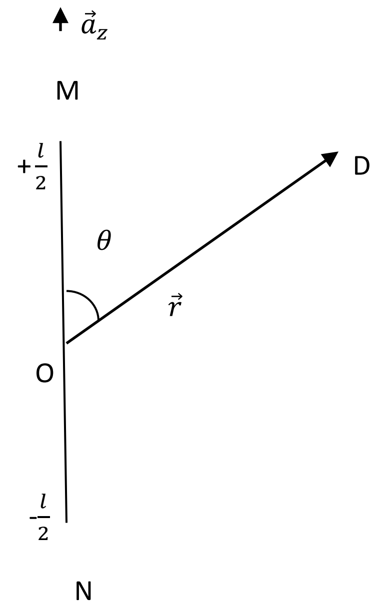


Fig.(1)

In Fig.(1) MN is the dipole antenna of length l. It has its midpoint at O .This point is taken as the origin of co-ordinate axes with z-axis along the dipole. Point D of position vector $\vec{r}$ is observation point of the fields. The current distribution along the antenna, I(z,t) is taken as

$$I(z,t)\vec{a}_z = I_0 \cos\left(\frac{\omega z'}{c}\right)\cos\omega t\,\vec{a}_z = I(z)\cos\omega t\,\vec{a}_z \qquad (3.1)$$

The variation of current amplitude along the antenna is given in Fig.(2). The length of antenna is $\frac{\lambda}{2}$ where λ is free space wavelength corresponding to the angular frequency ω of current oscillation. The current is driven by external time varying source fed at the centre of the dipole. The vector and scalar potential functions for the aforesaid current distribution and no charge distribution are given using eq.(2.3) and eq.(2.4)to be

$$\Phi=0 \qquad (3.2)$$

$$\vec{A}(\vec{r},t) = \frac{\mu I_0}{4\pi}\,\vec{a}_z \int_{-\frac{l}{2}}^{+\frac{l}{2}} \frac{\cos\left(\frac{\omega z'}{c}\right)\cos\omega\left(t-\frac{|\vec{r}-\vec{a}_z z'|}{c}\right)}{|\vec{r}-\vec{a}_z z'|}\,dz' \qquad (3.3)$$

For observation point far away from the dipole such that

$$l \ll r \qquad (3.4)$$

$$|\vec{r}-\vec{a}_z z'|=\sqrt{r^2+z'^2-2rz'\cos\theta} \cong r - z'\cos\theta \qquad (3.5)$$

$$\frac{1}{|\vec{r}-\vec{a}_z z'|} \cong \frac{1}{r}\left(1+\frac{z'\cos\theta}{r}\right) \qquad (3.6)$$

Using eq.{3.5) and eq.(3.6)eq.(3.3) simplifies to

$\vec{A}(\vec{r},t) = \frac{\mu I_0}{4\pi}\frac{\cos w}{r} f_{1(\theta)(\cos\theta\vec{a}_r - \sin\theta\vec{a}_\theta)}$ (3.7)

Where $w = \omega\left(t - \frac{r}{c}\right)$ (3.8)

$f_{1(\theta)=}2\int_0^{\frac{l}{2}} \cos\left(\frac{\omega z\prime}{c}\right)\cos\left(\frac{\omega z\prime\cos\theta}{c}\right)dz'$ (3.9)

and $\vec{a}_z = (\cos\theta\vec{a}_r - \sin\theta\vec{a}_\theta)$ (3.10)

Following definitions in eq.(2.5),(2.6) and (2.7), the three fields are obtained from the expression of vector potential in eq.(3.7) as given below.

$\vec{E}(\vec{r},t) = \frac{\mu\omega I_0}{4\pi}\frac{\sin w}{r} f_{1(\theta)(\cos\theta\vec{a}_r - \sin\theta\vec{a}_\theta)}$ (3.11)

$$\vec{B}(\vec{r},t) = \frac{\mu I_0}{4\pi}\left\{f_1(\theta)\sin\theta\left(-\frac{\omega}{cr}\sin w + \frac{\cos w}{r^2}\right) - \frac{\cos w}{r^2}\cos\theta\frac{\partial f_1}{\partial\theta}\right\}\vec{a}_\varphi$$

(3.12)

$S(\vec{r},t) = -\frac{\mu I_0}{4\pi}\left\{f_1(\theta)\cos\theta\left(\frac{\sin w}{cr} + \frac{\cos w}{r^2}\right) - \left(2\cos\theta\, f_1(\theta) + \sin\theta\frac{\partial f_1}{\partial\theta}\right)\frac{\cos w}{r^2}\right\}$

(3.13)

The terms in the above expressions falling as $\frac{1}{r}$ *with distance* are called far field terms and they only contribute to radiation. These terms are given below.

$\vec{E}(\vec{r},t) = \frac{\mu\omega I_0}{4\pi}\frac{\sin w}{r} f_{1(\theta)(\cos\theta\vec{a}_r - \sin\theta\vec{a}_\theta)}$ (3.14)

$\vec{B}(\vec{r},t) = \frac{\mu I_0}{4\pi}\left\{f_1(\theta)\sin\theta\left(-\frac{\omega}{cr}\sin w\right)\right\}\vec{a}_\varphi$ (3.15)

$S(\vec{r},t) = -\frac{\mu I_0}{4\pi}\left\{f_1(\theta)\cos\theta\left(\frac{\sin w}{cr}\right)\right\}$ (3.16)

$\vec{\mathcal{P}} = \vec{E}\times\frac{\vec{B}}{\mu} - \vec{E}\frac{S}{\mu} = \vec{\mathcal{P}}_{em} + \vec{\mathcal{P}}_{es}$ (3.17)

Where, $\vec{\mathcal{P}}_{em} = \frac{\mu I_0^2}{16\pi^2 c}\omega^2\frac{sin^2 w}{r^2}sin^2\theta f_1{}^2(\theta)\vec{a}_r$ (3.18)

$\vec{\mathcal{P}}_{es} = \frac{\mu I_0^2}{16\pi^2 c}\omega^2\frac{sin^2 w}{r^2}cos^2\theta f_1{}^2(\theta)\vec{a}_r$ (3.19)

For half wave antenna

$$f_{1(\theta)=}2\int_0^{\frac{\lambda}{4}} \cos\left(\frac{\omega z'}{c}\right)\cos\left(\frac{\omega z'\cos\theta}{c}\right)dz'$$

$=\frac{2c}{\omega sin^2\theta}\cos\left(\frac{\pi}{2}\cos\theta\right)$ (3.20)

Average power radiation from the antenna is obtained by integrating the Poynting vector over a sphere of radius r centered at the midpoint, o of the antenna.

$$P = \int_0^\pi (\vec{\mathcal{P}}_{em} + \vec{\mathcal{P}}_{es}).\vec{a}_r \frac{1}{2r^2} r^2 \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= \pi \int_0^\pi (\mathcal{P}_{em} + \mathcal{P}_{es}) \sin\theta \, d\theta \tag{3.21}$$

If electromagnetic and electro-magnetic scalar radiations are noted by P_{em} *and* P_{es} respectively, then they can be obtained as below.

$P_{em} = \pi \int_0^\pi \mathcal{P}_{em} \sin\theta \, d\theta$ (3.22)

$P_{es} = \pi \int_0^\pi \mathcal{P}_{es} \sin\theta \, d\theta$ (3.23)

Using eq.(3.18) in eq.(3.22) and eq.(3.19) in eq.(3.23) and solving integrals the following results are obtained.

$$P_{em} = \frac{\eta I_0^2}{4\pi}\frac{1}{2}\int_0^{2\pi} \frac{1-\cos v}{v} dv$$

$$= \frac{\eta I_0^2}{4\pi}\frac{1}{2}\left(\gamma + \ln(2\pi) - C_i(2\pi)\right)$$

$$= \frac{\eta I_0^2}{4\pi}(1.219) \tag{3.24}$$

$$P_{es} = \frac{\eta I_0^2}{4\pi}\frac{\pi}{4}\int_0^{2\pi} \frac{1-\cos v}{v^2} dv$$

$$-\frac{\eta I_0^2}{4\pi}\frac{1}{4}\int_0^{2\pi} \frac{1-\cos v}{v} dv$$

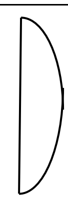
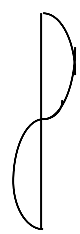
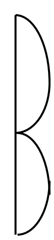
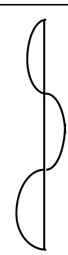
$$=\frac{\eta I_0^2}{4\pi}\left\{\frac{\pi}{4} S_i(2\pi) - \frac{1}{4}\left(\gamma + \ln 2\pi - C_i(2\pi)\right)\right\}$$

$$= \frac{\eta I_0^2}{4\pi}(0.504) \tag{3.25}$$

In the above expressions γ is Euler-Mascheroni constant, $C_i(2\pi)$ is the cosine integral, $S_i(2\pi)$ is the sine integral and $\eta = \sqrt{\frac{\mu}{\varepsilon}}$,is the impedance of the medium where the fields prevail. For material-free space, $\eta = 120\pi\ \Omega$. It is to be pointed out here that the value of P_{em} is the same as would have obtained by restricting to eq.(1.1).

Similar calculations are made for a few other antennas. The results are presented in the table-I.

TABLE-I

Sl. No.	Antenna type	Current distribution curve	$P_{em}/\frac{\eta I_0^2}{4\pi}$	$P_{es}/\frac{\eta I_0^2}{4\pi}$
1	Half wave antenna, centre-fed		1.219	0.504
2	Full wave antenna,offcentre-fed, Out of phase		1.557	1.9
3	Full wave antenna, dual-fed, Inphase,symmetric		3.32	0.451
4	3/2 wave antenna, centre-fed, Out of phase		1.758	2.6976
5	3/2 wave antenna, multiple-fed Inphase,symmetric		5.28	0.472

IV. Discussion

I. In the calculations I have assumed the average distance between two neighbouring conducting electrons is more than the distance travelled by an electron during one period of oscillation of the source drifting the electrons. This prevents piling of conducting electrons. $\vec{J}$ has different amplitudes along the length of antenna due to similar amplitude variation of electric field of the driving source. Since $\vec{J} = \rho\vec{u}$, periodic alteration of $\vec{J}$ can be due to ρ remaining constant at ρ_0 while $\vec{u}$ alternates with the same period or, equivalently $\vec{u}$ remains constant at some small value $\vec{u}_0$ and ρ oscillates about ρ_0 with time period remaining the same. The latter variation mostly generates electro-magnetic scalar radiation. Keeping this in mind the calculation has been undertaken. The electric field now had radial component besides transverse component and there emerged a new S-field . The transverse component of electric field together with magnetic field gave rise to electromagnetic radiation same as that would have been obtained imposing relation (1.1).The radial components of electric field and the magnetic scalar field gave rise to electro-magnetic scalar radiation, quite independent of electromagnetic radiation. The two radiations have directional behavior-the electromagnetic radiation is maximum along perpendicular bisector of the antenna while the other radiation is maximum along the direction of alignment of the dipole. Further, absorption of energy is maximum from the electromagnetic radiation when the receiving antenna is held perpendicular to the direction of propagation of energy but in case of the other radiation it is maximum when antenna is aligned in the direction of energy propagation. Calculations for different current distributions along any given antenna show that electro-magnetic scalar radiation increases when electromagnetic radiation falls for a current distribution.

In regions far away from the charge and current distributions, eq. (2.8) takes the form,

$$\vec{\nabla}.\vec{E} = \varepsilon\frac{\partial S}{\partial t}$$

as if there exist oscillatory charges of density , $\varepsilon\frac{\partial S}{\partial t}$ giving rise to alternating converging and diverging Coulomb lines of force around the point. These electric field configurations propagate in space at speed, c same as that of light. The term, $\frac{1}{\varepsilon\mu}\rho S$ in the generalized power equation (2.18)suggests that there is exchange of energy between S-field and oscillating charge distribution at locations where the latter is present or absent – no charge aggregation even if $\vec{\nabla}.\vec{J} = 0$.

In steady current cases there exists a definite gradient of magnetic scalar field, S in the media discussed in section-I so as to satisfy $\vec{\nabla}.(\vec{J} + \vec{J}_F) = 0$ with $\vec{J}_F = -\frac{\vec{\nabla}S}{\mu}$.

V. Conclusion

Electro-magnetic scalar radiations are predicted by the generalized Maxwell's equations. The electromagnetic radiations are independent of charge distributions as long as the current distribution is held fixed while the electro-magnetic scalar radiations depend on the charge distributions.

Experimental studies need be taken up to measure the radiations from millimeter antennas under appropriate alternating high frequency driving source.

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