The gravitational phenomena without the curved spacetime

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Abstract: In this paper was presented a description of the gravitational phenomena in the new medium, different than the curved spacetime, which merges the Minkowski spacetime and the mass density into the single idea. The new description of the gravitational phenomena is kept in the spirit of Mach, in opposite to the Newtonian gravity and GR. All phenomena of gravity are expressed in terms of the relationship between a bodies and not between the body and surrounding the spacetime.

keywords: *general relativity; modified gravity; gravitational waves*

1 Introduction

General Relativity (GR) is a theory, which a since about 100 years describes the gravitational phenomena as a geometric properties of the four-dimensional spacetime. Although GR is widely accepted as a fundamental theory of gravitation for the many physicists still this is not a perfect theory.

In GR the spacetime plays a very important role as the medium. The spacetime continuum is the mathematical model, that merges the three-dimensional space and one dimension time into a single idea - the four-dimensional spacetime. Under influence outer gravitational field the four-dimensional spacetime is curved. Although the GR survived so far all tests observation, direct detection of the gravitational waves still remains missing part of the puzzle.

In this paper was presented a description of the gravitational phenomena in the new *medium*[1], different than the curved spacetime, which merges the Minkowski spacetime and the mass density into the single idea. The physical consequences of a such replacement are a very interesting and far-reaching.

2 The alternative description of the gravitational phenomena

The arena, where gravitational phenomena take place is the continuous medium, immersed in the Minkowski four-dimensional spacetime, which is the only the background. The medium is infinite collection of material bodies filling the whole spacetime about a certain mass density ρ, which has the capacity to propagate the gravitational interactions.

Let us assume that in the absence of the outer gravitational field, the medium becomes *the bare medium*. This medium, with *the bare mass density* ρ^{bare}, is homogeneous, isotropic, independent of the time and is defined as follows

$$\rho_{\mu\nu}^{bare} \overset{def}{=} \rho^{bare} \cdot \eta_{\mu\nu} = \mathrm{diag}(-\rho^{bare}, \rho^{bare}, \rho^{bare}, \rho^{bare}) \tag{1}$$

where: $\rho_{\mu\nu}^{bare}$ is *the bare mass density tensor*, $\eta_{\mu\nu}$ is *the Minkowski tensor*, μ, ν = 0, 1, 2, 3.

This mathematical model merges the Minkowski spacetime and the mass density into the single idea.

[1] This is way out beyond the well-known scheme.

Note that $\rho_{\mu\nu}^{bare}$ never reaches zero, although it may be very close. The bare medium is never empty, in the contrast to the vacuum. For example, the bare mass density tensor in the spherical coordinates has form

$$\rho_{\mu\nu}(r) = \rho^{bare} \cdot \mathrm{diag}(-1,\ 1,\ r^2,\ r^2 \cdot \sin^2\theta)$$

components: ρ_{00} and ρ_{rr} has dimension [kg m^{-3}], but components $\rho_{\phi\phi}$ and $\rho_{\theta\theta}$ [kg m^{-1}].

Under influence outer gravitational field the bare medium with the bare mass density ρ^{bare} is stressed and deformed and becomes *the effective medium* with *the effective mass density tensor* $\rho_{\mu\nu}$.

2.1 The metric

The metric in the effective medium is defined as

$$ds^2(\rho_{\mu\nu}) \overset{def}{=} \frac{\rho_{\mu\nu}(x)}{\rho^{bare}} dx^\mu dx^\nu \tag{2}$$

where: $\rho_{\mu\nu}(x)$ is a symmetric, position dependent, the effective mass density tensor. Tensor $\rho_{\mu\nu}(x)$ describes the mathematical relationship between the effective medium and the bare medium under influence gravitation and, in a some sense, is equivalent to *the metric tensor* $g_{\mu\nu}(x)$.

The tensor $\rho_{\mu\nu}(x)$ describes all the properties of the effective medium and establishes metric, which is described by the equation (2). The gravitational field change $\rho_{\mu\nu}(x)$ and also metric (2). In absence of the gravitational field, the metric (2) becomes *the Minkowski metric*.

Let us consider the spacetime with metric

$$ds^2(g_{\mu\nu}) = g_{\mu\nu}(x) dx^\mu dx^\nu$$

and metric of the effective medium (2).

2.2 Postulate

All gravitational laws in curved spacetime and in the effective medium are the same.

This is an extension of the *Principle of Relativity*, which means, that the mathematical structures describing all gravitational phenomena in two different media are the same. We expect that they must lead to the same empirical data in the both different media (we assume also, that $\rho^{bare} = 1$), therefore

$$ds^2(\rho_{\mu\nu}) = ds^2(g_{\mu\nu})$$

and finally we get the mathematical bridge between these the two different media in form

$$\rho_{\mu\nu} = g_{\mu\nu}\,. \tag{3}$$

This mathematical relationship will be helpful in finding the field equations in the effective medium.

2.3 Building a bridge between different media

Einstein's field equation takes the form

$$R_{\mu\nu}(g_{\mu\nu}) - \frac{1}{2} g_{\mu\nu} \cdot R(g_{\mu\nu}) = \frac{8\pi G}{c^4} T_{\mu\nu}(g_{\mu\nu}) \tag{4}$$

where physical quantities: $R_{\mu\nu}(g_{\mu\nu})$ *the Ricci tensor* and $R(g_{\mu\nu})$ *the Ricci scalar* are describing the curved spacetime, both are the function of the metric tensor $g_{\mu\nu}$, tensor $T_{\mu\nu}(g_{\mu\nu})$ is *the stress–energy tensor*, *G* is the gravitational constant, *c* is the speed of light.

The left side of the equation (4) describe the Riemann geometry and has the physical dimension [m^{-2}], while the right side of the equation (4) describe the material source, and without coefficient, has the physical dimension [N m^{-2}]. The coefficient $\frac{8\pi G}{c^4}$ has the physical dimension [N^{-1}] and joins two different worlds, the world of the Riemann geometry with the physical world of the energy, momentum and pressure.

When the stress–energy tensor disappears, i.e. $T_{\mu\nu}(g_{\mu\nu}) = 0$, then the world of the Riemann geometry closes in itself, what can lead to the singularity.

According to *Postulate*, when the curved spacetime we will replace with the effective medium then, **the general form of the field equation (4) will not change**

$$\sigma_{\mu\nu}(\rho_{\mu\nu}) - \frac{1}{2} \rho_{\mu\nu} \cdot \sigma(\rho_{\mu\nu}) \sim T_{\mu\nu}(\rho_{\mu\nu}) \tag{4a}$$

The new physical quantities: $\sigma_{\mu\nu}(\rho_{\mu\nu})$ *the stress tensor of the effective medium* and $\sigma(\rho_{\mu\nu})$ *the stress scalar of the effective medium* are describing the effective medium during the stress. The stress scalar of the effective medium can express as $\sigma(\rho_{\mu\nu}) \equiv \rho^{\mu\nu} \cdot \sigma_{\mu\nu}(\rho_{\mu\nu})$. All physical quantities are now the function of the effective mass density tensor $\rho_{\mu\nu}$.

Both sides of the equation (4a) have now the same physical dimension [N m^{-2}] and describe the same physical world.

The gravitational field interacts with the bare medium, causing that this medium is stress and becomes the effective medium. The stress–energy tensor $T_{\mu\nu}(\rho_{\mu\nu})$ describes the source of these stress. The stress tensor of the effective medium $\sigma_{\mu\nu}(\rho_{\mu\nu})$ describes the magnitude of these stress. Under the influence of these stress the effective medium is strain. We suppose also, that $\sigma_{\mu\nu}(\rho_{\mu\nu}) \sim R_{\mu\nu}(g_{\mu\nu})$ and $\sigma(\rho_{\mu\nu}) \sim R(g_{\mu\nu})$.

This mathematical considerations will help us to find the equation of the field.

2.4 The field equation

We search for the field equation of the second order in the field variables $\rho_{\mu\nu}$, which, to be consistent with the observations, and in the weak field and slow motion limit (Newtonian limit), must reduce to the classical Poisson equation for the classical gravitation.

The Lagrangian density L_{eff} for the effective medium should contain the field variables $\rho_{\mu\nu}$ and their first derivatives $\frac{\partial \rho_{\mu\nu}}{\partial x^{\gamma}}$ only. The field variables are describing the effective medium. In agreement with these requirements, we assume that

$$L_{eff} = \frac{1}{16\pi}\sqrt{-\rho}\cdot\sigma$$

where: $\rho = \det(\rho_{\mu\nu})$ is the determinant of the effective mass density tensor. We write the total action over an arbitrary the effective medium region Ω as

$$S = \int_{\Omega}\left(L_{eff} + L_m\right)\sqrt{-\rho}\cdot d^4x$$

where: L_m is the Lagrangian density for the matter and fields. The term $\sqrt{-\rho}\cdot d^4x$ describes the volume change in the effective medium, while $d^4x = dx^0 \cdot dx^1 \cdot dx^2 \cdot dx^3$ describes the volume in the bare medium.

After substituting, the total action has form

$$S = \int\left(\frac{1}{16\pi}\sigma + L_m\right)\sqrt{-\rho}\cdot d^4x$$

After the mathematical calculations well-known from GR, according to the equation (3), the search of the field equation will get the correct form

$$\sigma_{\mu\nu}(\rho_{\mu\nu}) - \frac{1}{2}\rho_{\mu\nu}\cdot\sigma(\rho_{\mu\nu}) = 8\pi\cdot T_{\mu\nu}(\rho_{\mu\nu}) \tag{5}$$

where

$$T_{\mu\nu}(\rho_{\mu\nu}) \equiv -2\frac{\partial L_m}{\partial \rho^{\mu\nu}} + L_m \rho_{\mu\nu}\,.$$

The sources and fields in the equation (5) are the same and not differ from each other (see also the equation (7)). Were obtained a new understanding of phenomena of gravity.

The distribution of the mass density in the Universe $T_{\mu\nu}(\rho_{\mu\nu})$ determines the stress of the effective medium in the whole Universe. Please note, that $T_{\mu\nu}(\rho_{\mu\nu})$ never reach zero.

Paraphrasing John Archibald Wheeler's words, we can say that: *matter tells the effective medium how to stress, the effective medium under the stress tells matter how to move.*

The field equation (5) has exactly the same form as the Einstein's field equation (4), if we will use the following relations

$$\begin{aligned}
\rho_{\mu\nu}(x) &= g_{\mu\nu}(x) \\
\sigma_{\mu\nu}(\rho_{\mu\nu}) &= \frac{c^4}{G}\cdot R_{\mu\nu}(g_{\mu\nu}) \\
\sigma(\rho_{\mu\nu}) &= \frac{c^4}{G}\cdot R(g_{\mu\nu})
\end{aligned} \tag{6}$$

The coefficient $\frac{c^4}{G} = 1.21\cdot 10^{44}$ N and is a probably the greatest force in the Universe. It tells us about the scale of the transition between these two different worlds.

2.5 Correspondence with Newtonian theory of the gravitation

In the Newtonian approximation we can decompose $\rho_{\mu\nu}$ to following simple form

$$\rho_{\mu\nu}(x) = \rho_{\mu\nu}^{bare} + \rho_{\mu\nu}^{*}(x),$$

where $\rho_{\mu\nu}^{*}(x) << 1$ is a very small perturbation in the effective mass density tensor, then the field equation (5) takes the form

$$\nabla^2 \rho_{00}^{*}(x) \approx -\frac{8\pi G}{c^2}\rho(x) \tag{7}$$

The distribution of the mass density $\rho(x)$ (the source) determines the distribution of the effective mass density $\rho_{00}^{*}(x)$ (the field) around the source.

The equation (7) is a special case, for $\rho_{00}^{*}(x) \approx \frac{2V(x)}{c^2}$, the well-known Poisson equation for the classical gravity

$$\nabla^2 V(x) \approx -4\pi G\rho(x) \tag{8}$$

where $V(x)$ is the gravitational potential.

2.6 The equations of motion

The Lagrangian function for the body with the effective mass density tensor $\rho_{\mu\nu}(x)$ has form

$$L = \frac{1}{2}\rho_{\mu\nu}(x)\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$$

The equations of motion have the form

$$\frac{dp_\gamma(x)}{d\tau}-\frac{1}{2}\frac{\partial\rho_{\mu\nu}(x)}{\partial x^\gamma}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}=0 \tag{9}$$

where: $p_\gamma(x)=\rho_{\gamma\nu}(x)\frac{dx^\nu}{d\tau}$ is the effective density of the four-momentum, τ is the proper time.

The effective density of the four-momentum is an external physical quantity, which describes the relationship the tested object **with respect to the all surrounding bodies**, not respect to the spacetime.

For a change of the effective density of the four-momentum at proper time τ, i.e. $\frac{dp_\gamma(x)}{d\tau}$, in the equation (9), is responsible the term $-\frac{1}{2}\frac{\partial\rho_{\mu\nu}(x)}{\partial x^\gamma}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}$, which describes the distribution and motion of the all surrounding masses.

The body motion does not depend on the inner properties of the spacetime, but the from presence and the motion of the all surrounding bodies and their distribution.

The new quality of the understanding, kept in Mach's spirit, has been reached[2].

When the all surrounding bodies consist only with the bare masses, i.e. when $\frac{\partial\rho_{\mu\nu}(x)}{\partial x^\gamma}=0$, then $\frac{dp_\gamma}{d\tau}=0$ and the effective density of the four-momentum p_γ is constant along the world line.

The left side of the equation (9) has a different, equivalent form

$$\frac{dp_\gamma(x)}{d\tau}=\frac{d}{d\tau}\left(\rho_{\gamma\nu}(x)\frac{dx^\nu}{d\tau}\right)=\frac{d\rho_{\gamma\nu}(x)}{d\tau}\frac{dx^\nu}{d\tau}+\rho_{\gamma\nu}(x)\frac{d^2x^\nu}{d\tau^2}=0$$

In case, when $\rho_{\gamma\nu}(x)$ does not depends *explicitly* on τ, the equation (9) has form

$$\rho_{\gamma\nu}(x)\frac{d^2x^\nu}{d\tau^2}+\Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}(x))\cdot\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}=0 \tag{10}$$

where:

$$\Gamma_{\gamma\mu\nu}(\rho_{\mu\nu}(x))\overset{def}{=}\frac{1}{2}\left(\frac{\partial\rho_{\gamma\mu}(x)}{\partial x^\nu}+\frac{\partial\rho_{\gamma\nu}(x)}{\partial x^\mu}-\frac{\partial\rho_{\mu\nu}(x)}{\partial x^\gamma}\right)$$

Inertial forces appears, when the body changes its state of motion ***only*** *with respect to the all surrounding bodies.*

[2] *This is the reason why E. Mach was led to make the attempt to eliminate space as an active cause in the system of mechanics. According to him, a material particle does not move in not accelerated motion relatively to space, but relatively to the centre of all the other masses in the Universe; in this way the series of causes of mechanical phenomena was closed, in contrast to the mechanics of Newton and Galileo* [1].

2.7 The rotation bucket with water problem

When the bucket of water put at the pole then, due the Earth's rotation, the water in a bucket will change his state of motion relative to the all surrounding bodies and will appear the centrifugal force. The water surface will adopt a parabolic shape. If a bucket of water will be turned with the same angular velocity as the rotating Earth, but in the opposite direction, then surface should be flat, because surface of water will be at rest respect to all the surrounding bodies.

What will happen if, instead bucket of water, we put the Sagnac interferometer? The experiment can confirm (or not) Berkeley and Mach point of view, that **only rotation with respect to the all surrounding masses, have physical sense**.

The rotation of the body with respect to the spacetime loses its *raison d'être* and the spacetime ceases to be the frame of reference.

2.8 The bare medium as the reference frame

In particular case, when the all surrounding masses are the bare masses, then the inertial forces will disappear

$$\rho^{bare}\frac{d^2x^\nu}{d\tau^2}=0\,. \tag{11}$$

The body with the bare mass density ρ^{bare} is in the rest or moves in a straight line with the constant speed in respect to the all surrounding bare masses. The equation (11) determines the new reference frame – *the bare medium reference frame.*

2.9 The equation of motion in a weak field

Suppose that, the surrounding masses and their distribution generates a weak gravitational field. The equation of motion (10) takes now the form (i = 1, 2, 3)

$$\left(\rho^{bare}+\frac{1}{2}\overset{*}{\rho}_{00}(x)\right)\cdot\frac{d^2x^i}{dt^2}\approx-\frac{1}{2}c^2\frac{\partial\overset{*}{\rho}_{00}(x)}{\partial x^i} \tag{12}$$

The equation (12) is different than Newton's equation of the motion for the gravity. It what currently we consider to be the inertial mass density, really is the sum of the bare mass density ρ^{bare} and the effective mass density $\overset{*}{\rho}_{00}(x)$. The effective mass density $\overset{*}{\rho}_{00}(x)$ of the body is a results of interactions between the body and the all surrounding bodies, during the change state of motion.

Note that in the equation (12) gravitational mass does not appear explicitly, so *the Equivalence Principle*, underlying the GR, lost *raison d'être.*

3 Which medium is correct?

The idea of gravitational waves is a very attractive to many researchers. Although the GR survived so far all tests observation, direct detection of the gravitational waves still remains missing part of the puzzle. It is believed that the gravitational waves can convey important information about the early

Universe. However, despite strong circumstantial evidence for the existence of these waves, so far, they have not been detected directly. This is contrary to expectations many theorists therefore the very important question: *Why is there no gravitational waves?* is still open.

The left side of the equations (4) and (5) describes the two different media, respectively: the curved space-time medium and the effective medium, in which the gravitational waves are propagated. Thus, there are two different ways description of the propagation of these waves, as: a fluctuations in the curvature of the spacetime or a fluctuations in the effective mass density [2]? Which one is correct?

4 Conclusion

To describe the gravitational phenomena applied a new mathematical model, which merges the Minkowski spacetime and the mass density into the single idea. This description is kept in the spirit of Mach, in opposite to the Newtonian gravity and GR. All phenomena of gravity are expressed in terms of the relationship between bodies and not between the body and surrounding the spacetime.

Interaction, which appears during the change state of motion, and which propagates immediately, wakens our hope.

References

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