Relativity and Quantum Mechanics - the route towards unification

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The notes explicitly describe unification of Special Relativity with Quantum Mechanics based on fundamental similarities between interval and Heisenberg's relation. Outline for unification of General Relativity with Quantum Mechanics is presented.

KEYWORDS: relativity, quantum mechanics, unification, Heisenberg's.

1. Introduction

Remarkably successful approach of making Quantum Mechanics compatible with the predictions of Special Relativity [1] has been, on the other hand, quite fruitless in unification of either of Relativities with Quantum Mechanics. These notes explicitly describe unification of Special Relativity with Quantum Mechanics based on fundamental similarities between interval and Heisenberg's relation. Outline for unification of General Relativity with Quantum Mechanics is presented.

2. Unification

For the sake of simplicity let's temporarily use a following notation:

x (more correctly: $|\Delta x| \geq 0$) – x-component of the spatial displacement of the material object between two events (*e.g.* between two measurements); (n1)

t (more correctly: $|\Delta t| \geq 0$) – temporal displacement of the material object between two events; (n2)

p (more conventionally: p_x) – x-component of momentum of the material object (e.g. between two measurements); (n3)

m (more conventionally: m_0) – rest mass of the object. (n4)

One can show that a following relation is valid [2]:

$$\frac{m\,x}{s/c} = p \qquad (1)$$

where

c - speed of light; (n5)

s (more conventionally: s_x) – x-component of the interval between two events on the world line of the material object. (n6)

Interval is invariant and defined in Special Relativity as [2]:

$$s^2 = c^2 t^2 - x^2 \qquad (2)$$

It follows from eq.(1):

$$\left\langle p^2 \right\rangle = \left(\frac{mc}{s}\right)^2 \left\langle x^2 \right\rangle \qquad (3)$$

$$\left\langle p \right\rangle^2 = \left(\frac{mc}{s}\right)^2 \left\langle x \right\rangle^2 \qquad (4)$$

where $\langle q \rangle$ stands for mean value of quantity q (n7)

Subtraction of eq.(4) from eq.(3) gives:

$$\left\langle p^2 \right\rangle - \left\langle p \right\rangle^2 = \left(\frac{mc}{s}\right)^2 \left\{ \left\langle x^2 \right\rangle - \left\langle x \right\rangle^2 \right\} \qquad (5)$$

Using:

$$\left\langle (\Delta q)^2 \right\rangle = \left\langle q^2 \right\rangle - \left\langle q \right\rangle^2 \qquad [3] \qquad (6)$$

one can obtain from eq.(5):

$$\left\langle (\Delta p)^2 \right\rangle = \left(\frac{mc}{s}\right)^2 \left\langle (\Delta x)^2 \right\rangle \tag{7}$$

It should be mentioned here that simplification of notation (n1) (*i.e.* usage of x instead of Δx) can be omitted with respect to eq.(7) because increment Δ does not have an absolute value. For this reason $\Delta(\Delta x)$ can legally be replaced by Δx.

It follows from eq.(7):

$$\left\langle (\Delta p)^2 \right\rangle \left\langle (\Delta x)^2 \right\rangle = \left(\frac{mc}{s}\right)^2 \left\langle (\Delta x)^2 \right\rangle^2 \tag{8}$$

On the other hand, according to Heisenberg's relation [3]:

$$\left\langle (\Delta p)^2 \right\rangle \left\langle (\Delta x)^2 \right\rangle \geq \frac{\hbar^2}{4} \tag{9}$$

where $\hbar$ – reduced Planck's constant (n8)

Thus, it follows from eqs.(8),(9):

$$\left(\frac{mc}{s}\right)^2 \left\langle (\Delta x)^2 \right\rangle^2 \geq \frac{\hbar^2}{4} \tag{10}$$

or

$$s \leq 2\left(\frac{mc}{\hbar}\right) \left\langle (\Delta x)^2 \right\rangle \tag{11}$$

More conventionally (please recall (n6)):

$$s_x \leq 2\left(\frac{mc}{\hbar}\right) \left\langle (\Delta x)^2 \right\rangle \tag{12}$$

One might anticipate that in general form eq.(12) should look like:

$$s_r \leq 2\left(\frac{mc}{\hbar}\right) \left\langle (\Delta r)^2 \right\rangle \tag{13}$$

where $$\left(\Delta r\right)^2=\left(\Delta x\right)^2+\left(\Delta y\right)^2+\left(\Delta z\right)^2 \qquad \text{(n9) (14)}$$

Metric in Special Relativity can be written as [2]:

$$\left(ds_r\right)^2=c^2\left(dt\right)^2-\left(dr\right)^2 \qquad (15)$$

Thus, eq.(13) for the metric of the material object in Special Relativity should look like:

$$\left(ds_r\right)^2\le 4\left(\frac{mc}{\hbar}\right)^2\left\langle\left(dr\right)^2\right\rangle^2 \qquad (16)$$

Eq.(16) allows one to obtain a relation for geodesic of the material object in Special Relativity:

$$\left(ds_r\right)^2_{\max}=4\left(\frac{mc}{\hbar}\right)^2\left\langle\left(dr\right)^2\right\rangle^2 \qquad (17)$$

3. Concluding Remarks

(i) It is interesting to note that derivation of the relation for geodesic (eq.(17)) was only possible because of the *inequality* of Heisenberg's relation.

(ii) Eqs.(16),(17) are applicable for both, macro- and micro-objects. Indeed, it is hardly possible to define *trajectory* for micro object. Thus, mean value of quadratic deviation of the displacement should be used in this case. For macro object, on the other hand, mean values become indistinguishable from the values itself.

(iii) Because of the presence of Planck's constant along with the rest mass of the material object in eqs.(16),(17), one would reasonably expect that eqs.(16),(17) put some limitations on possible values of rest masses of material objects (*e.g.* elementary particles). That is why such predictions, once explicitly expressed, could serve a reliable way of verification of the theory.

(iv) One must agree that unification of General Relativity with Quantum Mechanics should logically become the next important step.

(v) Indeed, with the help of the Principle of Equivalence, the method described here should be applied to the metric, written in a general form (*i.e.* metric should allow for cross-term interactions). Newtonian method for extremum searching should lead to a relation for geodesic, while Heisenberg's principle used as a boundary condition will make the equation for geodesic explicit.

(vi) However, as one can see, the route of further unification (see (v)) looks quite straightforward and very incremental intellectually with respect to the method already described here. For these reasons actual derivations are left for the passionate fans of laborious math.

(vii) Nevertheless, predictions of unified General Relativity-Quantum Mechanics theory should open new ways for testing principles of General Relativity.

References

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