Unsteady non-darcianCouette flow in porous medium with heat transfer subject to uniform suction or injection

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Abstract

The unsteady non-DarcianCouette flow through a porous medium of a viscous incompressible fluid bounded by two parallel porous plates is studied with heat transfer. A non-Darcy model that obeys the Forchheimer extension is assumed for the characteristics of the porous medium. A uniform suction and injection are applied perpendicular to the plates while the fluid motion is subjected to a constant pressure gradient. The two plates are kept at different but constant temperatures while the viscous dissipation is included in the energy equation. The effects of the porosity of the medium, inertial effects and the uniform suction and injection velocity on both the velocity and temperature distributions are investigated.

Keywords : Non-Darcian flow; Couette flow; parallel plates; Forchheimer equation; Finite Difference; Numerical solution.

1 Introduction

The flow between two parallel plates is a classical problem that has many applications in accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil, fluid droplets and sprays, magnetohydrodynamic (MHD) power generators and MHD pumps [1]. A lot of research work concerning the flow between two parallel plates has been obtained under different physical effects [1–10].

Fluid flow in porous media is now one of the most important topics due to its wide applications in both science and engineering [11, 12]. In most of the previous work, the Darcy model was

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adopted when studying porous flows. The Darcy law is sufficient in studying small rate flows where the Reynolds number is very small. For larger Reynolds numbers the Darcy law is insufficient and a variety of models have been implemented in studying flows in porous media. The DarcyForchheimer (DF) model is probably the most popular modification to Darcianflows utilized in simulating inertial effects [13–16]. It has been used extensively in chemical engineering analysis and also in materials processing simulations.

In the present study, the unsteady non-DarcianCouette flow and heat transfer in a porous medium of an incompressible viscous fluid between two infinite horizontal porous plates is studied and the DF model is used for the characteristics of the porous medium. The fluid is acted upon by a constant pressure gradient and a uniform suction and injection perpendicular to the plates. The upper plate is moving with a constant velocity while the lower plate is kept stationary. The non-Darcy flow in the porous medium deals with the analysis in which the partial differential equations governing the fluid motion are based on the non-Darcy law (Darcy -Forchheimer flow model) that accounts for the drag exerted by the porous medium [17–19] in addition to the inertial effect [16, 20–25]. The two plates are maintained at two different but constant temperatures. This configuration is a good approximation of some practical situations such as heat exchangers, flow meters, and pipes that connect system components. The cooling of these devices can be achieved by utilizing a porous surface through which a coolant, either a liquid or gas, is forced. Therefore, the results obtained here are important for the design of the wall and the cooling arrangements of these devices. The governing equations are solved numerically taking the viscous dissipation into consideration. The inclusion of the porosity effect, inertial effects as well as the velocity of suction or injection leads to some interesting effects, on both the velocity and temperature distributions to be investigated.

2 Description of the Problem

The two non-conducting plates are located at the $y = \pm h$ planes and extend from $x = -\infty$ to ∞ and $z = -\infty$ to ∞ embedded in a DF porous medium where a high Reynolds number is assumed [13–16]. The lower and upper plates are kept at the two constant temperatures T_1 and T_2, respectively, where $T_2 > T_1$, as shown in Fig (1). The fluid flows between the two plates under the influence of a constant pressure gradient $\frac{dP}{dX}$ in the x-direction, and a uniform suction from above and injection from below which are applied at $t = 0$ with velocity v_0 in the positive y-direction. The upper plate is moving with a constant velocity U_0 while the lower plate is kept stationary.

The flow is through a porous medium and affected by another inertial effects where the non-Darcy law (Darcy -Forchheimer flow model) is assumed [16, 20–25]. From the geometry of the problem and due to the infinite dimensions in the x and z-directions, it is evident that $\frac{\partial}{\partial x} = \frac{\partial}{\partial z}$ for all quantities, i.e. they are independent in the x and z-coordinates, apart from the pressure gradient $\frac{dp}{dx}$ which is assumed constant, thus the velocity vector of the fluid is $\overrightarrow{v}(y,t) = u(y,t)\overrightarrow{i} + v_0\overrightarrow{j}$, with the initial and boundary conditions ($u = 0$ at $t0$), and ($u = 0$ at $y = -h$, and $u = U_o$ at $y = h$ for $t > 0$). The temperature $T(y,t)$ at any point in the fluid satisfies both the initial and boundary conditions $T = T_1$ at $t \leq 0, T = T_2$ at $y = +h$, and $T = T_1$ at $y = -h$ for $t > 0$.

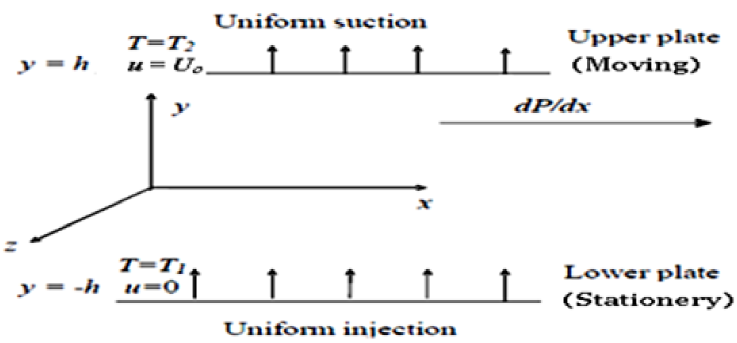

Figure 1: The geometry of the proplem

The fluid flow is governed by the momentum equation see [26].

$$\rho\frac{\partial u}{\partial t}+\rho v_0\frac{\partial u}{\partial y}=-\frac{p}{x}+\mu\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{k}u-\rho\frac{\lambda}{k}u^2. \tag{1}$$

Where, ρ and μ are, respectively, the density of the fluid and the coefficient of viscosity,K is the Darcy permeability [17–19] and λ is the inertial coefficient (i.e. the non-Darcian Forchheimer geometrical constant which is related to the geometry of the porous medium [16]). The last two terms in the right side of Eq.(1) represent the non-Darcy porosity forces. To find the temperature distribution inside the fluid we use the energy equation see [26].

$$\rho c\frac{\partial T}{\partial t}+\rho c v_0\frac{\partial T}{\partial y}=k\frac{\partial^2 T}{\partial y^2}+(\frac{\partial u}{\partial y})^2 \tag{2}$$

Where, c and k are, respectively, the specific heat capacity and the thermal conductivity of the fluid. The second term on the right-hand side represents the viscous dissipation.

The problem is simplified by writing the equations (1) and (2) in the non-dimensional form. We define the following non-dimensional quantities.

$$\hat{x}=\frac{x}{h},\quad \hat{y}=\frac{y}{h},\quad \hat{z}=\frac{z}{h},\quad \hat{p}=\frac{p}{\rho U_0^2},\quad \hat{u}=\frac{u}{U_0},\quad \hat{t}=\frac{tU_0}{h},\quad \hat{T}=\frac{T-T_1}{T_2-T_1}$$

where, $=\frac{v_0}{U_0}$ (the suction parameter), $pr=\mu\frac{c}{k}$, (the Prandt l number), $Ec=\frac{U_0^2}{c(T_2-T_1)}$, (the Eckert number), $Re=\frac{\rho U_0 h}{\mu}$, (the Reynolds number), $\beta=h^2k$, (the porosity parameter), $\gamma=\frac{\lambda h}{k}$, (the dimensionless non-Darcy parameter).

In terms of the above non-dimensional variables and parameters, the basic Equations (1) and (2) are written as (the "hats" will be dropped for convenience):

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} - \frac{\beta}{Re}u - \gamma u^2. \tag{3}$$

$$\frac{\partial T}{\partial t} + \frac{\partial Y}{\partial y} = \frac{1}{RePr}\frac{\partial^2 T}{\partial y^2} + \frac{Ec}{Re}(\frac{\partial u}{\partial y})^2. \tag{4}$$

The initial and boundary conditions for the velocity become:

$$t \le 0 : u = 0 \quad and \quad t > 0 : T = 1, y = +1 \quad and \quad t > 0 : T = 0, y = -1. \tag{5}$$

and the initial and boundary conditions for the temperature are given by

$$t \le 0 : T = 0 \quad and \quad t > 0 : T = 1, y = +1 \quad and \quad t > 0, T = 0, y = -1. \tag{6}$$

3 Numerical Solution of the Governing Equations

Equations (3) and (4) are solved numerically using finite differences [27] under the initial and boundary conditions (5) and (6) to determine the velocity and temperature distributions for different values of the parameters , and S. The Crank-Nicolson implicit method [28] is applied. The finite difference equations are written at the mid-point of the computational cell and the different terms are replaced by their second-order central difference approximations in the y-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. Finally, the block tri-diagonal system is solved using Thomas algorithm. All calculations have been carried out for $\frac{dP}{dx} = -5, Re = 1, Pr = 1$ and $Ec = 0.2$.

4 Results and Discussion

Figures (2, 3, 4) show the time progression of the velocity profiles till the steady state for ($S = 1$) and various values of the porosity and non-Darcian parameters β and γ. It is observed that the velocity component u increases monotonically with time. The porosity parameter β and the non-Darcian parameter γ have a marked effect on the time development of u. It is obvious that increasing β decreases u and its steady state time as a result of increasing the resistive porosity force on u while, increasing γ for each value of β decreases more the velocity u and its steady state time which reflects the expected resistance because of the inertial effects. For $\gamma = 0$ in figures ($3-a$) and ($4-a$) we mean a flow without additional inertial effects and the Darcian case is obtained to provide an easier quick path for the fluid flow. Fig ($2-a$) represents the simpler linear Newtonian case with $\beta = \gamma = 0$ obtaining the highest velocity values.

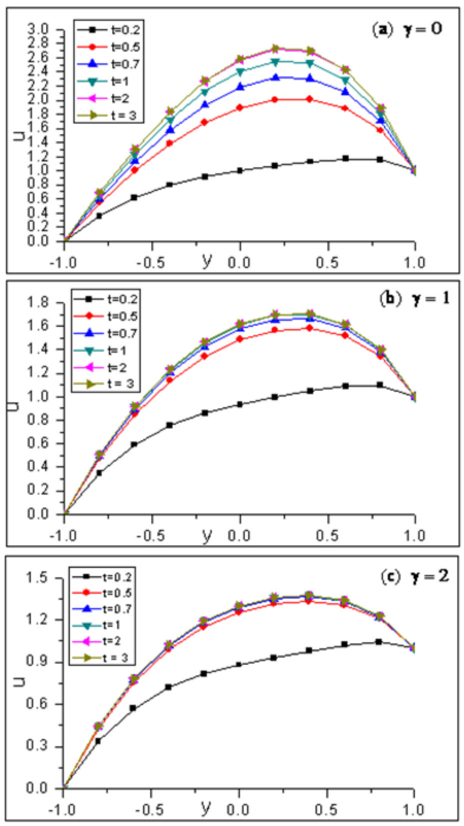

Figure 2: Time development in the velocity t for $\beta = 0$, $S = 1$ and various values of γ

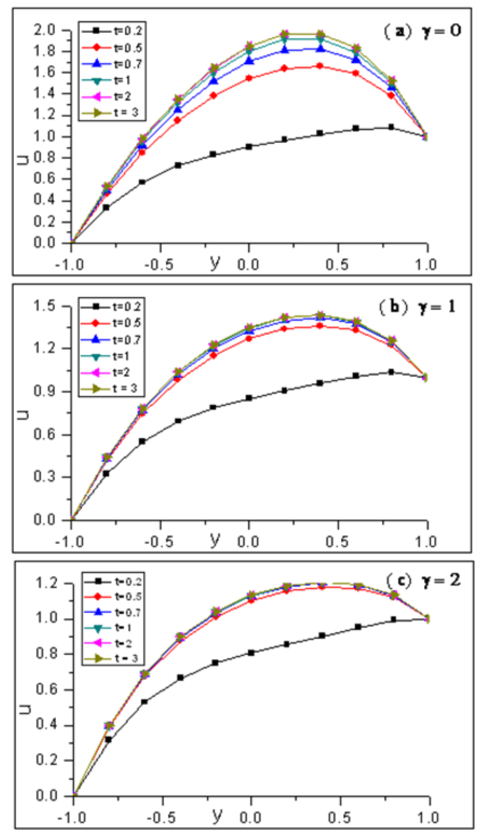

Figure 3: Time development in the velocity t for $\beta = 1$, $S = 1$ and various values of γ

Figures ($5-a$, 6,7) show the time development of the temperature profiles for ($S = 1$) and various values of β and γ. It is observed that the temperature T increases monotonically with t.The parameters β and γ affect the time progression of the temperature T; increasing β and γ decreases T and its steady state time, as increasing β and γ decreases u which, in turn, decreases the viscous dissipation which decreases T. Increasing γ for each value of β decreases more the temperature and its steady time because of the additional resistive inertial effects on u. In figures ($6-a$) and ($7-a$) where ($\gamma = 0$) we obtain the linear Darcian case with higher temperature values while, figure ($5-a$) shows the linear Newtonian case ($\beta = \gamma = 0$) in which the highest temperature values are reached. It is observed that the velocity component u and temperature T reach the steady state monotonically and that u reaches the steady state faster than T. This is expected, since u acts as the source of temperature.

Figures (8) and (9) indicate the effect of suction and injection on the time progression of both the velocity u and the temperature T at the center of the channel respectively for various values of β and γ. It is observed that increasing the suction parameter S decreases the velocity and its steady state time at the center of the channel due to the convection of the fluid from regions in the lower half to the center which has higher fluid speed. On the other hand, increasing the suction parameter S decreases the temperature T at the center of the channel which is influenced more by the convection term, which pushes the fluid more from the cold lower half towards the center.

Figures ($8-a$) and ($9-a$) indicate the linear Newtonian case where the plates are non-porous ($\beta = 0$) and there is no inertial effects ($\gamma = 0$) to obtain the highest velocity and temperature distributions. Figures ($8-b$) and ($9-b$) show the Darcian case in which the velocity and temperature decrease more because of the porosity of the medium ($= 1$). Also, figures ($8-c$) and ($9-c$) represent the non-Darcy flow in porous medium ($\beta = \gamma = 1$) which shows an obvious resistive effect in decreasing u and T where a noticeable close in the velocity profiles for the different values of the suction parameter S is achieved. Also, it can be seen from Figure (9) that T may exceed the value 1 which is the temperature of the hot plate and this is due to the viscous dissipation.

Tables (1) and (2) summarize the variation of the steady state values of both the velocity u and the temperature T at the center of the channel ($y = 0$) respectively for various values of β and γ and different values of the suction parameter ($= 0, 1, 2$). The results assure the inversely proportionality between both the parameters β and γ with both the velocity$u$ and the temperature T reaching the steady state of both because the increase in the porosity resistance and the inertial effects reduce u and hence T. The results also show that increasing the suction parameter S decreases the velocity and the temperature and that the higher velocity and temperature values for various β and γ are obtained at $S = 0$.

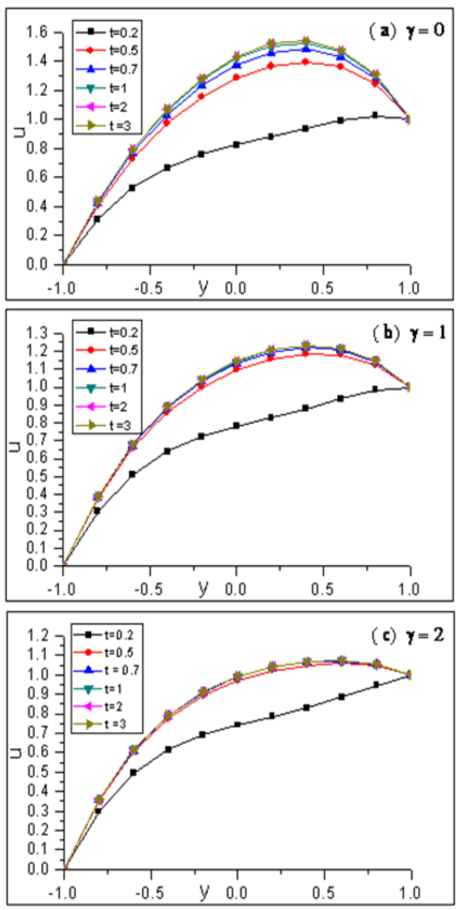

Figure 4: Time development in the velocity t for $\beta = 2$, $S = 1$ and various values of γ

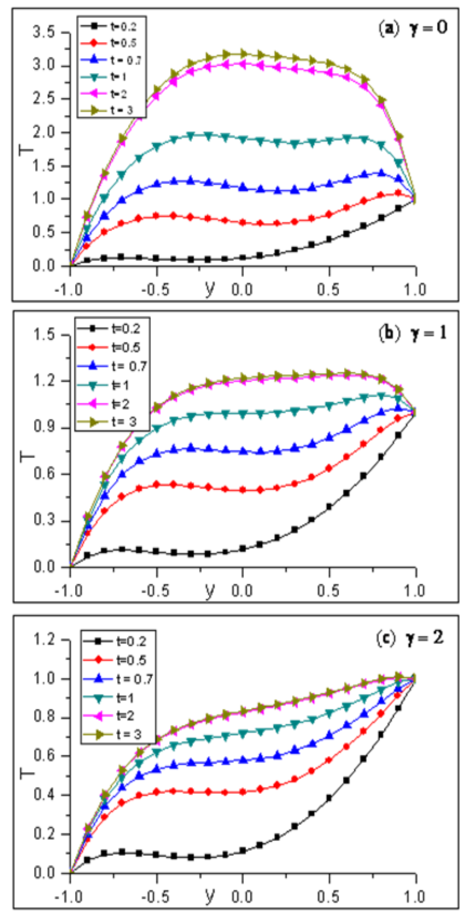

Figure 5: Time development in the velocity t for $\beta = 0$, $S = 1$ and various values of γ

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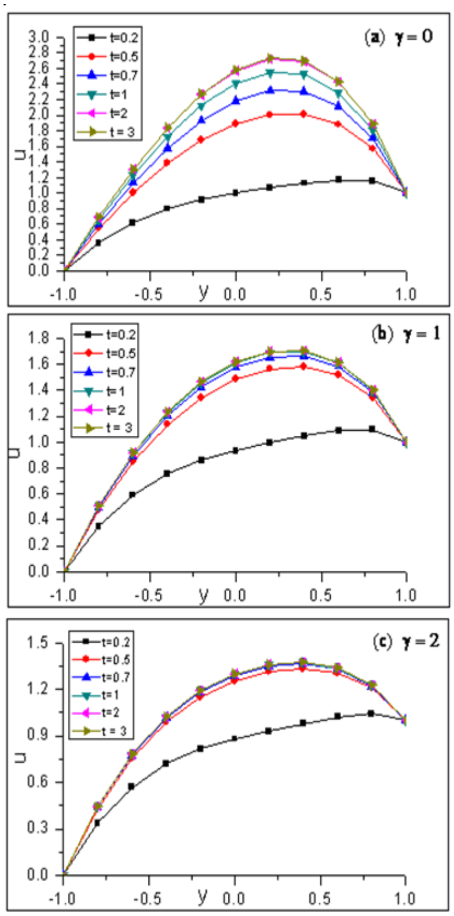


Figure 6: Time development in the velocity t for $\beta = 1$, $S = 1$ and various values of γ

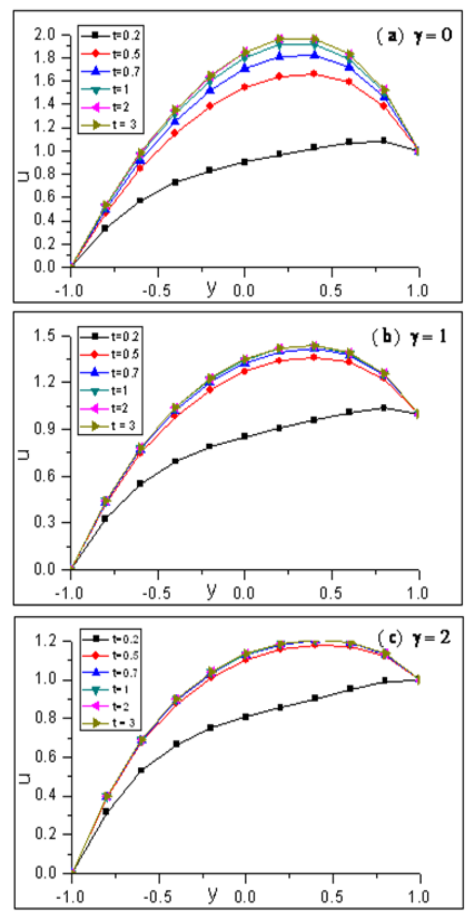
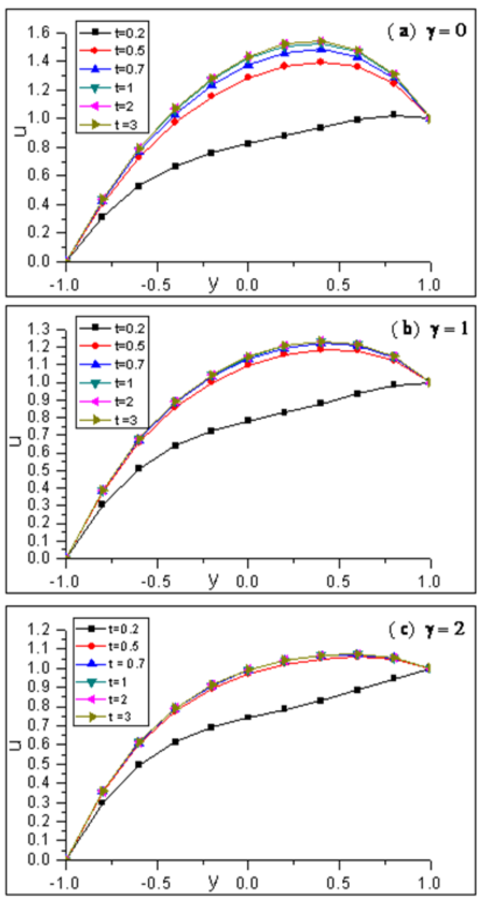


Figure 7: Time development in the velocity t for $\beta = 2$, $S = 1$ and various values of γ



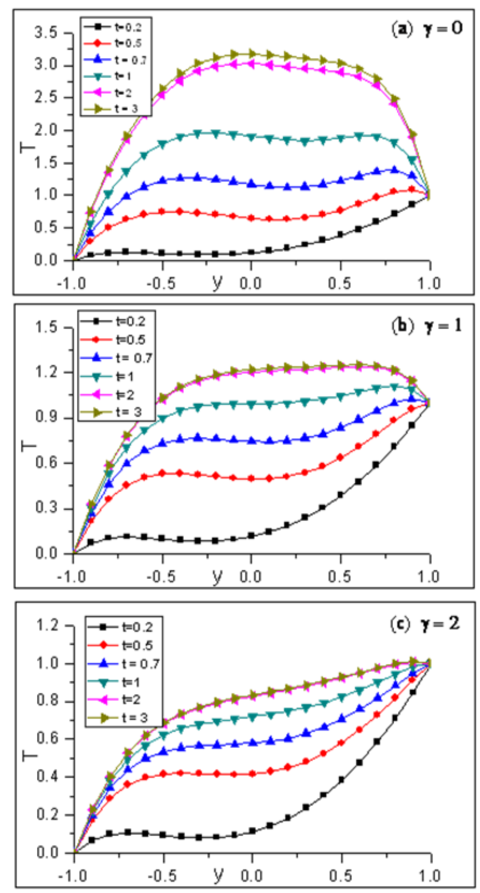


Figure 8: Effect to the suction parameter S on the time development on the velocity u at the center of the channel $(y = 0)$ for various values of the parameters β and γ

Figure 9: Effect to the suction parameter S on the time development on the temperature T at the center of the channel $(y = 0$ for various values of the parameters β and $\gamma)$

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Table 1: Variation of the steady state velocity u at the center of the channel $(y = 0)$ for various values of β and γ.

(a) S=0	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	2.999989	1.744921	1.377762
$\beta = 1$	2.085598	1.452757	1.198116
$\beta = 2$	1.583725	1.226317	1.048155
(b) S=1	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	2.580994	1.618268	1.30164
$\beta = 1$	1.850517	1.351063	1.132651
$\beta = 2$	1.432185	1.144794	0.9923267
(c) S=2	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	1.90749	1.435628	1.193
$\beta = 1$	1.544194	1.212631	1.044423
$\beta = 2$	1.240852	1.039225	0.9203554

Table 2: Variation of the steady state temperature T at the center of the channel $(y = 0)$ for various values of and .

(a) $S = 0$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	4.049962	1.426117	1.008442
$\beta = 1$	1.980979	1.090037	0.8639441
$\beta = 2$	1.249388	0.8911143	0.7681971
(b) $S = 1$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	3.193807	1.221758	0.8334498
$\beta = 1$	1.626091	0.8984181	0.6823296
$\beta = 2$	1.013675	0.7007263	0.5787295
(c) $S = 2$	$\gamma = 0$	$\gamma = 1$	$\gamma = 2$
$\beta = 0$	1.844336	0.8935586	0.6171848
$\beta = 1$	1.080732	0.6489245	0.4905686
$\beta = 2$	0.6980676	0.4943847	0.4017469

5 Conclusions

The unsteady non-DarcianCouette flow through a porous medium of a viscous incompressible fluid has been studied in the presence of uniform suction and injection. The effect of the porosity of the medium, inertial effects and the suction and injection velocity on the velocity and temperature distributions has been investigated. It is found that the porosity, inertial effects and suction or injection velocity has a marked effect on both the velocity and temperature distributions in an inverse proportionality manner. Various cases were monitored passing through the Newtonian fluid flow in non-porous medium, the Darcian flow model and the non-Darcian flow in porous medium which showed the greatest flow resistance resulting in lower velocity and temperature values.

References

[1] Hartmann, J., Lazarus, F.: Kgl. DanskeVidenskab. Selskab. Mat.-Fys. Medd. 15, 6–7 (1937).

[2] Tao, I.N.: Magnetohydrodynamic Effects on the Formation of Couette Flow. J. of Aerospace Sci. 27, 334–347 (1960)

[3] Alpher, R.A.: Heat transfer in magnetohydrodynamic flow between parallel plates. Int. J. Heat and Mass Transfer. 3, 108 (1961)

[4] Sutton, G.W., Sherman, A.: Engineering Magnetohydrodynamics. McGraw-Hill Book Co., New York (1965)

[5] Cramer, K., Pai, S.-I.:Magnetofluid dynamics for engineers and applied physicists. McGraw-Hill Book Co., New York (1973)

[6] Nigam, S.D., Singh, S.N.:Heat transfer by laminar flow between parallel plates under the action of transverse magnetic field. Quart. J. Mech. Appl. Math. 13, 85–97 (1960)

[7] Tani, I.:Steady motion of conducting fluids in channels under transverse magnetic fields with consideration of Hall effect. J. of Aerospace Sci. 29, 287 (1962)

[8] Soundalgekar, V.M., Vighnesam, N.V., Takhar,H.S.:Hall and Ion-slip effects in MHD Couettee flow with heat transfer. IEEE Trans. Plasma Sci.PS-7(3), 178(1979)

[9] Soundalgekar, V.M. ,Uplekar, A.G.:Hall effects in MHD Couette flow with heat transfer. IEEE Trans. Plasma Sci. PS-14(5), 579–584 (1986)

[10] Attia, H.A.:Transient MHD flow and heat transfer between two parallel plates with temperature dependent viscosity. Mech. Res. Comm. 26, no. 1, 115 (1999)

[11] Scheidegger, A.E.: The physics of flow through porous media. University of Toronto (1974)

[12] Kaviany, M.: Principles of heat transfer in porous media. Springer (1995)

[13] Lee, S.L., Yang, J.H.:Modelling of DarcyForchheimer drag for fluid flow across a bank of circular cylinders. Int. J. Heat Mass Transfer. 40, 3149–3155 (1997)

[14] Andrade, Jr. J.S., Costa, U.M.S., Almeida, M.P.,Makse, H.A., Stanley,H.E.:Inertial effects on fluid flow through disordered porous media.Phys Rev Lett. 82, 5249–8252 (1999)

[15] Jeong, N., Choi, D.H., Lin,C.L.:Prediction of DarcyForchheimer drag for micro-porous structures of complex geometry using the lattice Boltzmann method. J. MicromechMicroeng. 16, 2240–2250 (2006)

[16] Khani, F., Farmany, A.,AhmadzadehRaji, M.,Addul Aziz, Samadi,F.: Analytic solution for heat transfer of a third grade viscoelastic fluid in non-Darcy porous media with thermophysical effects.Commun. Nonlinear Sci. Numer. Simulat. 14, 3867–3878 (2009)

[17] Joseph, D.D.,Nield, D.A., Papanicolaou, G.:Nonlinear equation governing flow in a staturated porous media. Water Resources Research.18, no. 4, 1049–1052 (1982)

[18] Ingham, D.B., Pop,I.: Transport phenomena in porous media. Pergamon, Oxford (2002)

[19] Khaled, A.R.A., Vafai, K.: The role of porous media in modeling flow and heat transfer in biological tissues. Int. J. Heat Mass Transf. 46, 4989–5003 (2003)

[20] Anwar Bg, O., Zueco, J.,Takhar, H.S.:Laminar free convection from a continuously-moving vertical surface in thermally-stratified non-Darcian high-porosity medium: Network numerical study. International Communications in Heat and Mass Transfer. 35, 810–816 (2008)

[21] Singh, A.K., Agnihotri, P.,Singh, N.P., Singh,A.K.:Transient and non-Darcian effects on natural convection flow in a vertical channelpartially filled with porous medium: Analysis with ForchheimerBrinkman extended Darcy model. International Journal of Heat and Mass Transfer. 54, 1111–1120 (2011)

[22] Chen, Y.:Non-Darcy flow stability of mixed convection in a vertical channel filled with a porous medium. International Journal of Heat and Mass Transfer. 47, 1257–1266 (2004)

[23] Dulal Pal:Magnetohydrodynamic non-Darcy mixed convection heat transfer from a vertical heated plate embedded in a porous medium with variable porosity.Commun. Nonlinear Sci. Numer. Simulat. 15, 3974–3987 (2010)

[24] Afify, A.A.:Effects of variable viscosity on non-Darcy MHD free convection along a non-isothermal vertical surface in a thermally stratified porous medium. Applied Mathematical Modelling. 31, 1621–1634 (2007)

[25] Dulal Pal, Mondal, H.:MHD non-Darcy mixed convective diffusion of species over a stretching sheet embedded in a porous medium with non-uniform heat source/sink, variable viscosity and Soret effect.Commun. Nonlinear Sci. Numer. Simulat. 17, 672–684 (2012)

[26] Schlichting, H.: Boundary layer theory. McGraw-Hill Book Co. (1968)

[27] Ames,W.F.: Numerical solutions of partial differential equations. Academic Press, 2nd ed., New York (1977)

[28] Mitchell, A.R., Griffiths, D.F.: The Finite Difference Method in Partial Differential Equations. John Wiley, New York (1980)