Modal Analysis of Composite Beam Reinforced by Aluminium-Synthetic Fibers with and without Multiple Cracks Using ANSYS

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Abstract

The composite materials consist of two or more different materials that form regions large enough to be regarded as continua and which are usually firmly bonded together at the interface. Many natural and Synthetic materials are of this nature, such as: reinforced rubber, filled polymers, GFRP (Glass Fiber Reinforcement Plastic), Nylon, aligned and chopped fiber composites, polycrystalline aggregates (metals), etc. It is widely used in high speed machinery, aircraft and light weight structures. Crack is a main cause of damage occurring upon dynamic loading and may cause serious failure of structure. The influence of cracks on dynamic characteristics like natural frequencies, modes of vibration of structures has been investigated.

The paper presents the Computational modal analysis of a composite beam with and without cracks. In this work, the mechanical properties of aluminum and fiber (Nylon and Glass fiber reinforcement plastic) are measured a universal testing machine. The three-dimensional finite element models of composite beam with and without cracks are constructed and then computational modal analysis on ANSYS-14 is then performed to generate natural frequencies and mode shapes. The location of cracks will vary from 10 to 90 % of beam length. The finite element model agrees well with the analytical values.

Keywords: GFRP, Nylon, Aluminium, Natural Frequency, Mode Shapes

1 Introduction

Raciti and Kapania (1989) collected a report of developments in the vibration analysis of laminated composite beams. Classical laminate plate theory and first order shear deformation theory are used for analysis. The assumption of displacements as linear functions of the coordinate in the thickness direction has proved to be inadequate for predicting the response of thick laminates [1]. Yuan and Miller (1990) derived a new finite element model for laminated composite beams. The model includes sufficient degrees of freedom to allow the cross-sections of each lamina to deform into a shape which includes up through cubic terms in thickness co-ordinate. The element consequently admits shear deformation up through quadratic terms for each lamina but not interfacial slip or delamination [2].Maiti & Sinha (1994) used higher order shear deformation theory for the analysis of composite beams. Nine nodes iso parametric elements are used in the analysis. Natural frequencies of composite beam are compared for different stacking sequences, different (l/h) ratios and different boundary conditions. They had shown that natural frequency decreases with an increase in ply angle and a decrease in (l/h) ratio [3].Teboub and Hajela (1995) approved the symbolic computation technique to analyze the free vibration of generally layered composite beam on the basis of a first-order shear deformation theory. The model used considering the effect of poisson effect, coupled extensional, bending and torsional deformations as well as rotary inertia[4]. Banerjee (1999) has investigated the free vibration of axially laminated composite Timoshenko beams using dynamic stiffness matrix method.

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This is accomplished by developing an exact dynamic stiffness matrix of a composite beam with the effects of axial force, shear deformation and rotatory inertia taken into account. The effects of axial force, shear deformation and rotator inertia on the natural frequencies are demonstrated. The theory developed has applications to composite wings and helicopter blades [5]. Bassiouni (1999) proposed a finite element model to investigate the natural frequencies and mode shapes of the laminated composite beams. The FE model needed all lamina had the same lateral displacement at a typical cross-section, but allowed each lamina to rotate to a different amount from the other. The transverse shear deformations were included [6]. Kisa (2003) the effects of the location and depth of the cracks, and the volume fraction and orientation of the fibers on the natural frequencies and mode shapes of the beam with transverse non-propagating open cracks, were explored. The results of the study leaded to conclusions that, presented method was adequate for the vibration analysis of cracked cantilever composite beams, and by using the drop in the natural frequencies and the change in the mode shapes, the presence and nature of cracks in a structure can be detected [7].
Jafari and Ahmadian (2007) had done free vibration analysis of a cross-ply laminated composite beam on Pasternak Foundation. The model is designed in such a way that it can be used for single-stepped cross-section. For the first time to-date, the same analysis was conducted for a single-stepped LCB on Pasternak foundation. Stiffness and mass matrices of a cross-ply LCB on Pasternak foundation using the energy method are computed [9]. Ramanamurthy (2008) the cracks can be present in structures due to their limited fatigue strengths or due to the manufacturing processes. These cracks open for a part of the cycle and close when the vibration reverses its direction. These cracks will grow over time, as the load reversals continue, and may reach a point where they pose a threat to the integrity of the structure. As a result, all such structures must be carefully maintained and more generally, SHM denotes a reliable system with the ability to detect and interpret adverse "change" in a structure due to damage or normal operation. [10]. Lu and Law (2009) the finite beam element was formulated using the composite element method with a one-member–one-element configuration with cracks where the interaction effect between cracks in the same element was automatically included. The accuracy and convergence speed of the proposed model in computation were compared with existing models and experimental results. [11].Gaith (2011) the effects of crack depth and location, fiber orientation, and fiber volume fraction on the flexibility and consequently on natural frequency and mode shapes for cracked fiber-reinforced composite beams are investigated [12].

2 Mathematical model

The model chosen is a cantilever composite beam of uniform cross-section having dimension 500x30x6 mm. This is a three layer sandwich composite beam. The middle layer is of Synthetic fiber (Nylon or GFRP) and the upper and lower layer is of Aluminium. In this model the location of cracks will vary from $L_1/L=0.1$ to $L_9/L=0.9$. The ANSYS model of the used beam is shown in Fig.1.

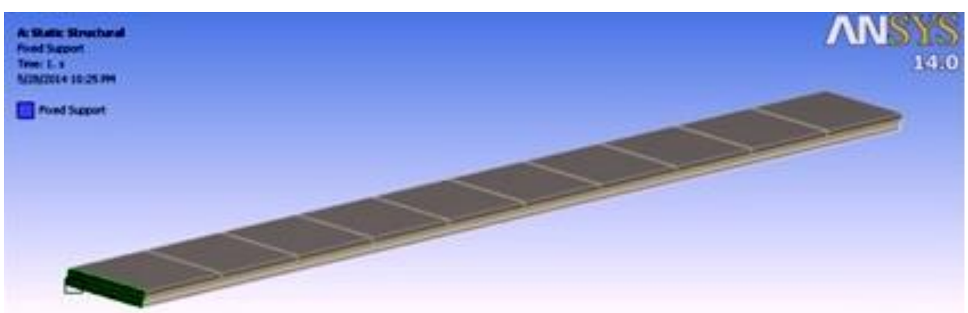


Figure 1 Mathematical Model of Composite Sandwich beam

3 Governing equation

3.1 Deflection of cantilever beam [16]

The bending moment at position x is given by $M=-w\frac{x^2}{2}$, substituting this value in equation

$$M= EI\frac{d^2y}{dx^2} \quad (1)$$

$$EI\frac{d^2y}{dx^2}= -w\frac{x^2}{2} \quad (2)$$

$$EI\frac{dy}{dx}= -w\frac{x^3}{6} + A \quad (3)$$

$$EIy = -w\frac{x^4}{24} + Ax + B \quad (4)$$

Applying boundary condition:

At x=L, y=0 and at x=L, $\frac{dy}{dx}$=0 in equation 4 giving:

$$EIy = -w\frac{x^4}{24} + w\frac{L^3}{6}x - w\frac{L^4}{8} \quad (5)$$

Deflection at free end at x=0

$$Y= -\frac{wL^4}{8EI} \quad (6)$$

3.2 Modal analysis

3.2.1 Damping matrices

Damping may be introduced into a transient, harmonic, or damped modal analysis as well as a response spectrum. The type of damping allowed depends on the analysis as described in the subsequent sections.

3.2.2 Transient (full or reduced) analysis and damped modal analysis

The damping matrix, [C], may be used in transient and damped modal analyses as well as substructure generation. In its most general form, the damping matrix is composed of the following components.

$$[C]=\alpha[M]+(\beta+\frac{2}{\Omega}g)[K]+\sum_{i=1}^{N_{ma}}\alpha_i{}^m[M_i]+\sum_{j=1}^{N_m}[(\beta_j{}^m+\frac{2}{\Omega}g_j+\frac{1}{\Omega}g_j{}^E)[K_j]]+\sum_{k=1}^{N_e}[C_k]+\sum_{m=1}^{N_v}\frac{1}{\Omega}[C_m]+\sum_{l=1}^{N_g}[G_l] \quad (7)$$

Where:

[C] = structure damping matrix , α = mass matrix multiplier, [M] = structure mass matrix, β = stiffness matrix multiplier, [K] = structure stiffness matrix, Nma = number of materials, αim = mass matrix multiplier for material I, [Mi] = portion of structure mass matrix based on material I , Nmb = number of materials, β_j^m = stiffness matrix multiplier for material j, [Kj] = portion of structure stiffness matrix based on material j, Ne = number of elements with specified damping, [Ck] = element damping matrix , Ng = number of elements with Coriolis or gyroscopic damping, [Gl] = element Coriolis or gyroscopic damping matrix.

3.2.3 Harmonic (full or reduced) analysis

The damping matrix ([C]) used in harmonic analyses is composed of the following components.

The input exciting frequency, Ω, is defined in the range between Ω_B and Ω_E via

$\Omega_B = 2\pi f_B$, rad/s

$\Omega_E = 2\pi f_E$, rad/s

f_B = beginning frequency, Hz

f_E = end frequency, Hz

Substituting equation (7) into the harmonic response equation of motion and rearranging terms yields

$$[[K]+i\left\{2g[K]+\sum(2g_j+g_j^{E})[K_j]+\sum[C_m]\right\}+i\Omega[\alpha[M]+\sum\alpha_i^{m}[M_i]+\beta[K]+ \sum\beta_j^{m}[K_j]+\sum[C_k]+\sum[G_j]]](u_1+iu_2)=F_1-\Omega^2[M] \quad (8)$$

The complex stiffness matrix in the first row of the equation consists of the normal stiffness matrix augmented by the structural damping terms given by g, g_i, g_i^E, and $[C_m]$ which produce an imaginary contribution. Structural damping is independent of the forcing frequency, Ω, and produces a damping force proportional to displacement (or strain). The terms g, g_i, and g_i^E are damping ratios (i.e., the ratio between actual damping and critical damping, not to be confused with modal damping).

The second row consists of the usual viscous damping terms and is linearly dependent on the forcing frequency, Ω, and produces forces proportional to velocity.

3.3 Mode-superposition analysis

The damping matrix is not explicitly computed, but rather the damping is defined directly in terms of a damping ratio ξd. The damping ratio is the ratio between actual damping and critical damping. The damping ratio ζ_i^d for mode i is the combination of

$$\zeta_i^d = \zeta + \zeta_i^m + \frac{\alpha}{2\omega_i} + \frac{\beta}{2}\omega_i \quad (9)$$

where

ξ = constant modal damping ratio

ζ_i^m = modal damping ratio for mode shape i (see below)

ω_i = circular natural frequency associated with mode shape i = $2\pi f_i$

f_i = natural frequency associated with mode shape i

α = mass matrix multiplier

The modal damping ratio ζ_i^m can be defined for each mode directly (undamped modal analysis only). Alternatively, for the case where multiple materials are present whose damping ratios are different, an effective mode-dependent damping ratio ζ_i^m can be defined in the modal analysis if material-dependent damping is defined and the element results are calculated. This effective damping ratio is computed from the ratio of the strain energy in each material in each mode using

$$\zeta_i^m = \frac{\sum_{j=1}^{N_m}\beta^m{}_j E_j^s}{\sum_{j=1}^{N_m} E_j^s} \quad (10)$$

where:

N_m = number of materials

β_j^m = damping ratio for material j

$$E_j^s = \frac{1}{2}(\varphi_i)^T[K_j](\varphi)$$

Strain Energy contained in mode i for material j

$\{\varphi_i\}$ = displacement vector for mode i

$[K_j]$ = stiffness matrix of part of structure of material j

These mode-dependent (and material-dependent) ratios, ζ_i^m , will be carried over into the subsequent mode-superposition or spectrum analysis.

4 Results and Discussions

In order to check the natural frequency of cantilever composite beam, first we calculate the mechanical properties of synthetic fiber (Nylon and GFRP) and Aluminium, like ultimate tensile strength, yielding strength and Poisson ratio.These mechanical properties are fed into ANSYS-14 to calculate the deflection and natural frequency for composite cantilever beam.

4.1 Deflection of beams on various loads

Table 1:- Deflection of Nylon Beam with and without cracks

0 crack		
S.No	**Force (N)**	**Deflection (mm)**
1	10	1.9217
2	20	3.8434
3	30	5.7651
4	40	7.6868
5	50	9.6085

9 cracks		
S.No	**Force (N)**	**Deflection (mm)**
1	10	2.0091
2	20	4.0181
3	30	6.0272
4	40	8.0362
5	50	10.045

Table 2:- Deflection of Composite Nylon Beam with and without cracks

0 Crack		
S.No	**Force (N)**	**Deflection (mm)**
1	10	1.2136
2	20	2.4273
3	30	3.6409
4	40	4.8545
5	50	6.0682

9 Crack		
S.No	**Force (N)**	**Deflection (mm)**
1	10	1.2749
2	20	2.5497
3	30	3.8246
4	40	5.0995
5	50	6.3743

As the load on the cantilever beam increases, the deflection also increases. The deflection in all the cases the deflection is minimum at 10N load and is maximum at 50N load. This shows that defection is directly proportional to the applied load on the beam.As the cracks appear on the beam (nylon and composite) the deflection increases, minimum at beam with no cracks and maximum at beam with 9 cracks.
It can also be seen that when the Nylon is sandwiched between the layers of aluminium, its deflection decreases in both the cases i.e with and without cracks. It shows that deflection of pure nylon decreases when it is used as composite with aluminium.

Table 3:-Deflection of GFRP Beam with and without cracks

0 Crack		
S.No	**Force (N)**	**Deflection (mm)**
1	10	1.0316
2	20	2.0633
3	30	3.0949
4	40	4.1266
5	50	5.1582

9 Crack		
S.No	**Force (N)**	**Deflection (mm)**
1	10	1.0816
2	20	2.1633
3	30	3.2449
4	40	4.3266
5	50	5.4082

Table 4:- Deflection of Composite GFRP Beam with and without cracks

0 Crack		
S.No	**Force (N)**	**Deflection (mm)**
1	10	1.1891
2	20	2.3783
3	30	3.5674
4	40	4.7565
5	50	5.9457

9 Crack		
S.No	**Force (N)**	**Deflection (mm)**
1	10	1.244
2	20	2.4893
3	30	3.734
4	40	4.9787
5	50	6.2233

As the cracks appear on the beam (GFRP and composite) the deflection increases, minimum at beam with no cracks and maximum at beam with 9 cracks.
It can also be seen that when GFRP is sandwiched between the layers of aluminium, its deflection increases in both the cases i.e with and without cracks. It shows that deflection of pure GFRP increases when it is used as composite with aluminium.

4.2 Natural frequency of composite beam with and without cracks

Table 5:- Natural Frequency of Aluminium beam with and without cracks in different Modes Shapes

Material	Mode Shapes	Natural Frequency (Hz) 0 Cracks	Natural Frequency (Hz) 9 Cracks
Aluminium	1	0.28664	0.2806
	2	1.4253	1.4132
	3	1.795	1.7571
	4	5.0215	4.9152
	5	8.3529	8.2711

Table 6:- Natural Frequency of GFRP beam with and without cracks in different Modes Shapes

Material	Mode Shapes	Natural Frequency (Hz) 0 Cracks	Natural Frequency (Hz) 9 Cracks
GFRP	1	0.37805	0.36993
	2	1.8816	1.8654
	3	2.3675	2.3165
	4	6.6229	6.4798
	5	11.255	11.144

Table 7:- Natural Frequency of Nylon beam with and without cracks in different Modes Shapes

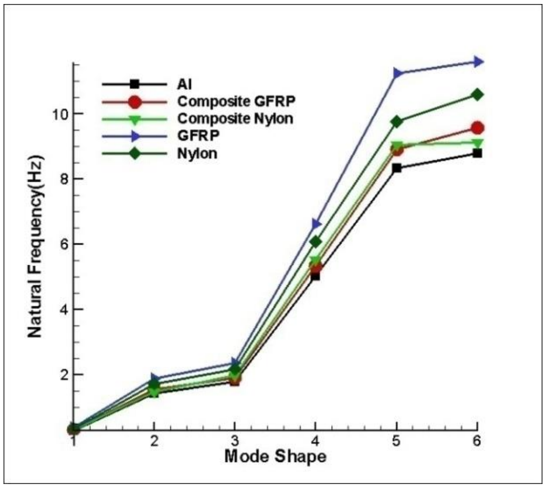
Material	Mode Shapes	Natural Frequency (Hz) 0 Cracks	Natural Frequency (Hz) 9 Cracks
Nylon	1	0.34655	0.3396
	2	1.72	1.7061
	3	2.1701	2.1265
	4	6.0716	5.949
	5	9.7726	9.6769

Table 8:- Natural Frequency of Composite GFRP beam with and without cracks in different Modes Shapes

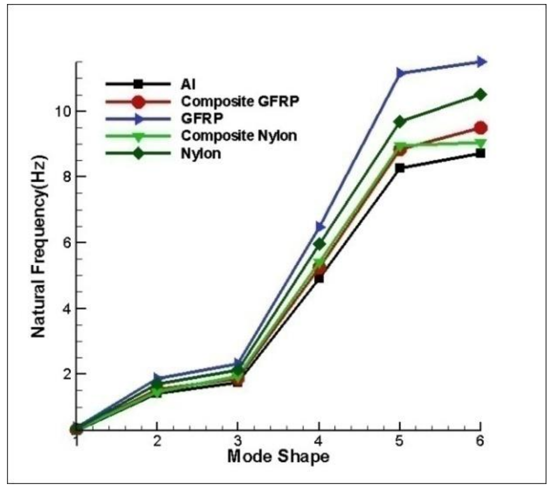
Material	Mode Shapes	Natural Frequency (Hz) 0 Cracks	Natural Frequency (Hz) 9 Cracks
Composite GFRP	1	0.30496	0.29871
	2	1.5524	1.5404
	3	1.9097	1.8705
	4	5.3429	5.2326
	5	8.9061	8.8231

Table 9:- Natural Frequency of Composite Nylon beam with and without cracks in different Modes Shapes

Material	Mode Shapes	Natural Frequency (Hz) 0 Cracks	Natural Frequency (Hz) 9 Cracks
Composite Nylon	1	0.31648	0.3095
	2	1.4824	1.4676
	3	1.9814	1.9378
	4	5.5406	5.4192
	5	9.0497	8.9615



(a)



(b)

Figure 2Variation between Natural frequency and Mode Shape (a) 0 Crack (b) 9 Cracks

The natural frequencies in both the cases either with or without the cracks are compared for the different materials and their composites and it had been found that the natural frequency of aluminium is minimum and that of GFRP is highest while the natural frequencies of nylon, GFRP composite and Nylon composite lie in between in all mode shapes.It has also been observed that in case of polymers used here like GFRP and nylon, the natural frequencies of GFRP and nylon is much higher. But when they are bonded with aluminium to form composites, their natural frequencies decreases. The Natural frequency is decreases when we increase the crack on the upper surface of the beam.

4.3 Natural frequencies for various mode shapes

The first mode of vibration is a bending mode. In this mode shape, the frequency is 0.3095 Hz. The beam is tending to bend about the root section's minimum moment of inertia .The analysis shows that the parameters that affect root stiffness have a large impact on the first mode frequency. The first mode frequency is also affected by parameters that affect tip mass.

The second modes of vibration is also bending mode with one node formation about the root, the frequency is more than the first mode. The deflection was in the vertical direction. The frequency is correspondingly higher due to the increased stiffness in that direction i.e. 1.9378 Hz. The third mode of vibration is also bending mode with 2 Node formations. The frequency of the third mode shape is 5.4192 Hz. The fourth mode of vibration is twisting about the root, the frequency is affected by tip rotational moment of inertia. The frequency of the fifth mode is 8.9615Hz. The fifth mode of vibration is also bending mode in horizontal direction. The frequency of sixth mode shape is 9.0433 Hz, which is the highest frequency in all the fifth mode shapes.

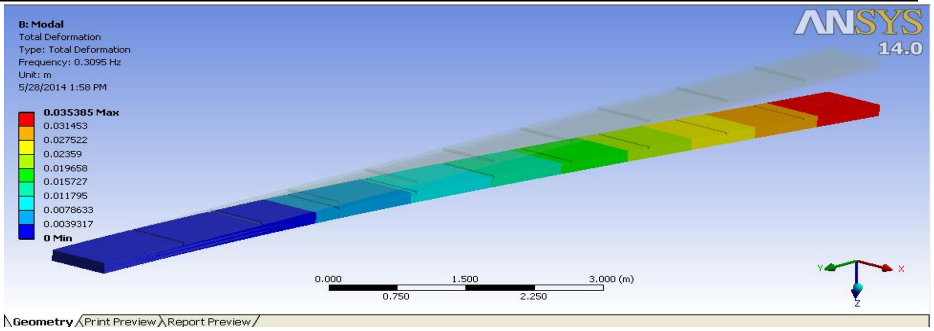


Figure 3 First mode shape of composite Nylon beam with natural frequency 0.3095Hz

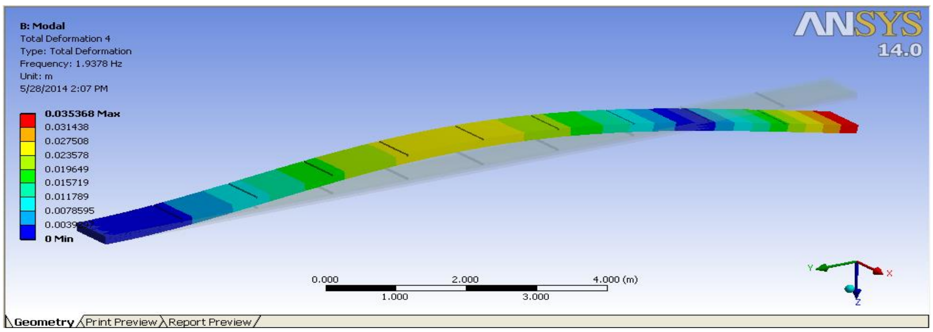


Figure 4 Second mode shape of composite Nylon beam with natural frequency 1.9378Hz

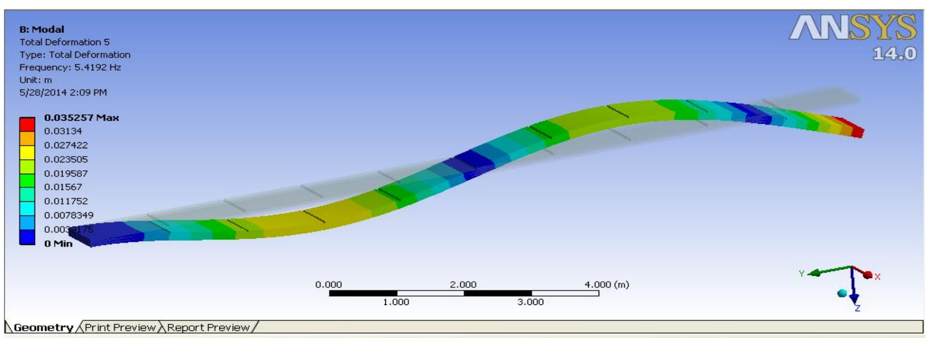


Figure 5 Third mode shape of composite Nylon beam with natural frequency 5.4192Hz

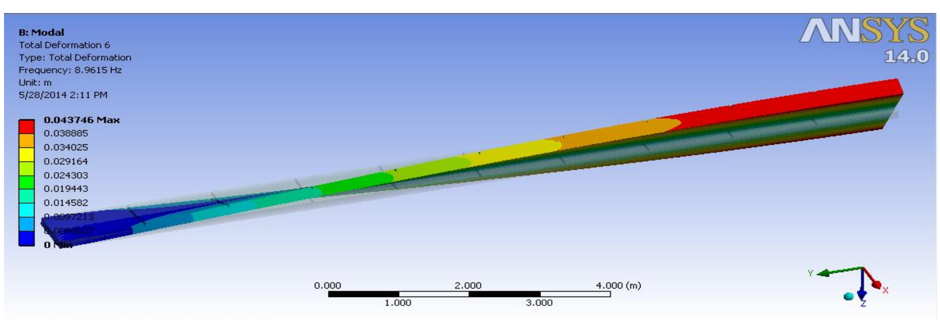


Figure 6 Fourth mode shape of composite Nylon beam with natural frequency 8.9615Hz

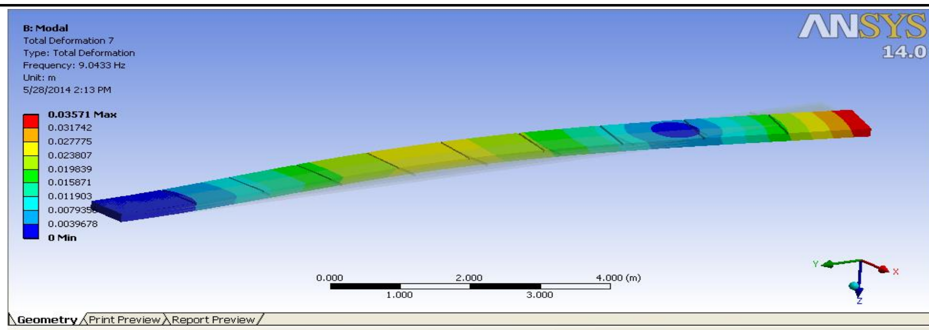


Figure 7 Fifth mode shape of composite Nylon beam with natural frequency 9.0433Hz

Conclusions

The following conclusions can be made from this research paper:

- The deflection of composite beams is less than that of pure material beams if nylon is taken as synthetic fiber with Al, but if GFRP is taken then its deflection is found to be increased when compared to pure GFRP. So, nylon suits good to make composite beam with Al as compared to other synthetic fibers like GFRP.
- As the number of cracks increases the deflection in beam increases.
- The natural frequencies of pure materials (GFRP & Nylon) are larger than those of composite beams made by them
- As the number of cracks on beams increases, the natural frequencies decrease.
- The natural frequency found higher in the fifth mode shape for all composite and pure materials.

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