Numerical Solution for Hybrid Fuzzy System by Adams fourth order predictor-corrector method

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Abstract

In this paper three numerical methods to solve for hybrid fuzzy differential equations are discussed. These methods are Adams-Bashforth, Adams-Moulton and Predictor-Correctormethod is obtained by combining Adams-Bashforth and Adams-Moulton methods. Convergence and stability of the proposed methods are also proved in detail. In addition, these methods are illustrated by solving two Cauchy problems.

Keywords: Hybrid systems; Fuzzy differential equations; Adams fourth order predictor-corrector method.

1. Introduction

Hybrid systems are devoted to modelling, design and validation of interactive systems of computer programs and continuous systems. That is, control systems that are capable of controlling complex systems which have discrete event dynamics as well as continuous time dynamics can be modelled by hybrid systems. The differential systems containing fuzzy valued functions and interaction with a discrete time controller are named as hybrid fuzzy differential systems. For analytical results on stability properties and comparision theorems we refer to [18, 19, 24].

In the last few years, many works have been performed by several authors in numerical solutions of fuzzy differential equations [1−−3, 20, 21]. Pederson and Sambandham [21, 24] have investigated the numerical solution of hybrid fuzzy differential equations by using Runge-Kutta and Euler methods. Recently, the numerical solutions of fuzzy differential equations by predictor-corrector method has been studied in [21, 24]. Prakash and Kalaiselvi [23] have studied the numerical solution

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of hybrid fuzzy differential equations by predictor-corrector three step method. Kanagarajan and sambath[17] studied the numerical solution of fuzzy differential equations by improved prdictor-corrector method.

The structure of this paper organized as. In Section 2. we bring definitions to fuzzy valued functions. In Section 3 we define hybrid fuzzy differential systems. In Sections 4, 5 and 6 we apply the Adams-Basforth, Adams-Moulton and Adams fourth order predictor-corrector methods for solving hybrid fuzzy differential equations. In Section 7, we give converge and stability results. The proposed algorithm is illustrated by solving some examples in Section 8.

2. Preliminaries

Let $P_K(\mathcal{R}^n)$ denote the family of all non-empty, compact, convex subsets of $\mathcal{R}^n$. If $\alpha, \beta \in \mathcal{R}$ and $A, B \in P_K(\mathcal{R}^n)$, then $\alpha(A+B) = \alpha A + \alpha B,\ \alpha(\beta A) = (\alpha\beta)A, 1A = A$ and if $\alpha, \beta \geq 0$, then $(\alpha+\beta)A = \alpha A + \beta A$.

Denote by E^n the set of $u : \mathcal{R}^n \to [0,1]$ such that u satiesfies $(i) - (iv)$ mentioned below:

(i) u is normal, that is, there exists an $x_0 \in \mathcal{R}^n$ such that $u(x_0) = 1$,

(ii) u is fuzzy convex, that is, for $x, y \in \mathcal{R}^n$ and $0 \leq \lambda \leq 1$.

$$u(\lambda x + (1-\lambda)y) \geq min[u(x), u(y)],$$

(iii) u is upper semicontinuous,

(iv) $[u]^0 =$ the closure of $[x \in \mathcal{R}^n : u(x) > 0]$ is compact.

For $0 < \alpha \leq 1$, we denote $[u]^\alpha = [x \in \mathcal{R}^n : u(x) \geq \alpha]$. Then from (i) to (iv), it follows that $\alpha-$level sets $[u]^\alpha \in P_k(\mathcal{R}^n)$ for $0 \leq \alpha \leq 1$. An example of a $u \in E^1$ is given by

$$u(x) = \begin{cases} 4x - 3 \quad if \quad x \in (0.75, 1], \\ -2x + 3 \quad if \quad x \in (1, 1.5), \\ 0 \quad if \quad x \notin (0.75, 1.5), \end{cases} \tag{1}$$

The $\alpha-$level sets are given by

$$[u]^\alpha = [0.75 + 0.25\alpha, 1.5 - 0.5\alpha].$$

Let I be a real interval. A mapping $y : I \to E$ is called a fuzzy process and its α-level set is denoted by $[y(t)]^\alpha = [\underline{y}^\alpha(t), \overline{y}^\alpha(t)], \quad t \in I, \quad \alpha \in (0,1]$.

Triangular fuzzy numbers are those fuzzy sets in E which are characterized by an ordered triple $(x^l, x^c, x^r) \in \mathcal{R}^3$ with $x^l \leq x^c \leq x^r$ such that $[U]^0 = [x^l, x^r]$ and $[U]^1 = [x^c]$, then

$$[U]^\alpha = [x^c - (1-\alpha)(x^c - x^l), \quad x^c + (1-\alpha)(x^r - x^c)] \tag{2}$$

for any $\alpha \in I$.

Definition 2.1. An m-step method for solving the initial-value problem is one whose difference equation for finding the approximation $y(t_{i+1})$ at the mesh point t_{i+1} can be represented by the following equation :

$$\begin{aligned} y(t_{i+1}) = a_{m-1}y(t_i) + a_{m-2}y(t_{i-1}) + \cdots + a_0 y(t_{i+1-m}) \\ +h\{b_m f(t_{i+1}, y_{i+1}) + b_{m-1}f(t_i, y_i) + \cdots + b_0 f(t_{i+1-m}, y_{i+1-m})\} \end{aligned} \tag{3}$$

for $i = m-1, m, \cdots, N-1$, such that $a = t_0 \leq t_1 \leq \cdots \leq t_N = b, h = \frac{(b-a)}{N} = t_{i+1} - t_i$ and $a_0, a_1, \cdots, a_{m-1}, b_0, b_1, \cdots, b_m$ are constants with the starting values

$$y_0 = \alpha_0, \qquad y_1 = \alpha_1, \cdots \qquad y_{m-1} = \alpha_{m-1}.$$

When $b_m = 0$, the method is known as explicit, since equation (3) gives y_{i+1} explicit in terms of previously determined values. When $b_m \neq 0$, the method is known as implicit, since y_{i+1} occurs on both sides of Equation (3) and is specfied only implicitly.
With consideration of Definition 2.1, the multi-step method as follows.

Adams-Bashforth four-Step method:

$$y_0 = \alpha_0, \qquad y_1 = \alpha_1, \qquad y_2 = \alpha_2, \qquad y_3 = \alpha_3,$$

$$y_{i+1} = y_i + \tfrac{h}{24}[55f(t_i, y_i) - 59f(t_{i-1}, y_{i-1}) + 37f(t_{i-2}, y_{i-2} - 9f(t_{i-3}, y_{i-3})],$$

$$where \;\; i = 3, 4, \ldots, N-1.$$

Adams-Moulton three-step method :

$$\begin{aligned} y_1 = \alpha_1, \qquad y_2 = \alpha_2, \qquad y_3 = \alpha_3, \\ y_{i+1} = y_i + \frac{h}{24}[9f(t_{i+1}, y_{i+1}) + 19f(t_i, y_i) - 5f(t_{i-1}, y_{i-1}) + f(t_{i-2}, y_{i-2})]. \end{aligned}$$

where $i = 3, 4, \cdots, N-1$.

Definition 2.2. Assosisted with the difference equation

$$\begin{aligned} y_{i+1} = a_{m-1}y_i + a_{m-2}y_{i-1} + \cdots + a_0 y_{i+1-m} + hF(t_i, h, y_{i+1}, y_i, \cdots, y_{i+1-m}), \\ y_0 = \alpha_0, \qquad y_1 = \alpha_1, \qquad y_{m-1} = \alpha_{m-1}, \end{aligned} \tag{4}$$

the characteristic polynomial of the method is defined by

$$p(\lambda) = \lambda^m - a_{m-1}\lambda^{m-1} - a_{m-2}\lambda^{m-2} - \cdots - a_1\lambda - a_0.$$

If $|\lambda_i| \leq 1$ for each $i = 1, 2, \cdots, m$, and all roots with absolute value 1 are simple roots, then the difference method is said to satisfy the root condition.

Theorem 2.1. [13] A multistep method of the form (4) is stable if and only if it satisfies the roots condition.

Notations used in this paper are as follows:

A tilde is placed over a symbol to denote a fuzzy set so $\tilde{\alpha_1}, \tilde{f}(t), \cdots$

An arbitrary fuzzy number with an ordered pair of functions $(\underline{u}(\alpha), \overline{u}(\alpha))$. $0 \leq \alpha \leq 1$, which satisfy the following requirements is represented.

1. $\underline{u}(\alpha)$ is a bounded left continuous non-decreasing function over $[0, 1]$,
2. $\overline{u}(\alpha)$ is a bounded right continuous non-increasing function over $[0, 1]$,
3. $\underline{u}(\alpha) \leq \overline{u} \ (\alpha), \quad 0 \leq \alpha \leq 1$.

Definition 2.3. The supremum metric d_∞ on E is defined by

$$d_\infty(U, V) = sup \left\{ d_H([U]^\alpha, [V]^\alpha) : \alpha \in I \right\},$$

and (E, d_∞) is a complete metric space.

Definition 2.4. A mapping $F : T \to E$ is a Hukuhara differentiable at $t_0 \in T \subseteq R$ if for some $h_0 > 0$ the Hukuhara differences $F(t_0 + \Delta t) \sim_h F(t_0), \quad F(t_0) \sim_h F(t_0 - \Delta t)$, exist in E for all $0 < \Delta t < h_0$ and if there exist an $F'(t_0) \in E$ such that

$$\lim_{\Delta t \to 0^+} d_\infty \left(\frac{F(t_0 + \Delta t) -_h F(t_0)}{\Delta t, F'(t_0)} \right) = 0, \text{and} \lim_{\Delta t \to 0^+} d_\infty \left(\frac{F(t_0) -_h F(t_0 - \Delta t)}{\Delta t, F'(t_0)} \right) = 0$$

the fuzzy set $F'(t_0)$ is called the Hukuhara derivative of F at t_0.

Recall that $U \sim_h V = W \in E$ are defined on level sets, where $[U]^\alpha \sim_h [V]^\alpha = [W]^\alpha$ for all $\alpha \in I$. By consideration of definition of the metric d_∞, all the level set mappings $[F(.)]^\alpha$ are Hukuhara differentiable at t_0 with Hukuhara derivatives $[F'(t_0)]^\alpha$ for each $\alpha \in I$ when $F : T \to E$ is Hukuhara differentiable at t_0 with Hukuhara derivative $F'(t_0)$.

Definition 2.5. The fuzzy integral $\int_a^b y(t)dt, \quad 0 \leq a \leq b \leq 1$, is defined by

$$\left[\int_a^b y(t)dt \right]^\alpha = \left[\int_a^b \underline{y}^\alpha(t)dt, \int_a^b \overline{y}^\alpha(t)dt \right],$$

provided the Lebesgue integrals on the right exist.

Definition 2.6. If $F : T \to E$ is Hukuhara differentialble and its Hukuhara derivative F' is integrable over [0,1], then

$$F(t) = F(t_0) + \int_{t_0}^t F'(s)ds,$$

for all values of t_0, t where $0 \leq t_0 \leq t \leq 1$.

Definition 2.7. A mapping $y : I \to E$ is called a fuzzy process. We denote

$$\big[y(t)\big]^{\alpha} = \big[\underline{y}^{\alpha}(t), \overline{y}^{\alpha}(t)\big], \qquad t \in I, \qquad \alpha \in (0,1].$$

The Seikkala's derivative $y'(t)$ of a fuzzy process y is defined by $\big[y(t)\big]^{\alpha} = \big[\underline{y}^{\alpha}(t), \overline{y}^{\alpha}(t)\big]$, $t \in I,\ \alpha \in (0,1]$ provided the equation defines a fuzzy number $y'(t) \in E$.

Theorem 2.2. [14] Let $(t_i, \widetilde{u}_i), i = 0, 1, 2, \cdots, n$ be the observed data and suppose that each of the $\widetilde{u}_i = (u_i^l, u_i^c, u_i^r)$ is an element of E. Then for each $t \in [t_0, t_n]$,
$\tilde{f}(t) = (f^l(t), f^c(t), f^r(t)) \in E$.

$$f^l(t) = \sum_{l_i(t) \geq 0} l_i(t) u_i^l + \sum_{l_i(t) < 0} l_i(t) u_i^r,$$

$$f^c(t) = \sum_{i=0} l_i(t) u_i^c,$$

$$f^r(t) = \sum_{l_i(t) \geq 0} l_i(t) u_i^r + \sum_{l_i(t) < 0} l_i(t) u_i^l,$$

such that $l_i(t) = \prod_{j \neq i}^{n} \dfrac{(t - t_j)}{(t_i - t_j)}$.

3. The Hybrid Fuzzy Differential Systems

Consider the hybrid fuzzy differential systems

$$\begin{cases} x'(t) = f(t, x(t), \lambda_k(x_k)), & t \in [t_k, t_{k+1}], \\ x(t_k) = x_k, \end{cases} \tag{5}$$

where $0 \leq t_0 < t_1 < \cdots < t_k < \cdots, t_k \to \infty$,as $k \to \infty$ $f \in C[R_+ \times E_1 \times E_1, E_1], \lambda_k \in C[E_1, E_1]$. Here we assume the existence and uniqueness of the hybrid system hold on each $[t_k, t_{k+1}]$. To be specfic the system would look like:

$$x'(t) = \begin{cases} x_0'(t) = f(t, x_0(t), \lambda_0(x_0)), & x_0(t_0) = x_0, & t_0 \leq t \leq t_1, \\ x_1'(t) = f(t, x_1(t), \lambda_1(x_1)), & x_1(t_1) = x_1, & t_1 \leq t \leq t_2, \\ \vdots \\ x_k'(t) = f(t, x_k(t), \lambda_k(x_k)), & x_k(t_k) = x_k, & t_k \leq t \leq t_{k+1}. \\ \vdots \end{cases}$$

By the solution of equation (5) we mean the following function;

$$x(t) = x(t, t_0, x_0) = \begin{cases} x_0(t_0) = x_0, & t_0 \le t \le t_1, \\ x_1(t_1) = x_1, & t_1 \le t \le t_2, \\ \vdots & \\ x_k(t_k) = x_k, & t_k \le t \le t_{k+1}, \\ \vdots & \end{cases}$$

We note that the solution of (5) are piecewise differentiable in each interval for $t \in [t_k, t_{k+1}]$ for a fixed $x_k \in E^1$ and $k = 0, 1, 2, \ldots$.

4. Adams-Bashforth methods

In this section, we develop the Adams-Bashforth method for hybrid fuzzy differential equation (5), via an application of the Adams-Bashforth method for fuzzy differential equations in [4] when f and λ_k in equation (5) can be obtained via the Zadeh extension principle from $f \in C[R_+ \times R \times R, R]$ and $\lambda_k \in C[R; R]$, we assume that the existence and uniqueness of solutions of equation (5) hold for each $[t_k, t_{k+1}]$.

For fixed r, we replace each interval $[t_k, t_{k+1}]$ by a set of $N_k + 1$ discrete equally spaced grid points, $t_k = t_{k,0} < t_{k,1} < \cdots < t_{k,N} = t_{k+1}$(including the endpoints) at which the exact solution $\tilde{x}(t)$ is approximated by some $\tilde{y}_k(t)$.

Fix $k \in Z^+$. The fuzzy initial value problem

$$\left.\begin{array}{ll} x'_k(t) = f(t, x_k(t), \lambda_k(x_k)), & t_k \le t \le t_{k+1}, \\[2ex] \tilde{x}(t_k) = \tilde{x}_k, & \end{array}\right\} \tag{6}$$

can be solved by Adams-Bashforth four-step method. Let the fuzzy initial values be $\widetilde{y}(t_{i-1}), \widetilde{y}(t_i), \widetilde{y}(t_{i+1})$, $\widetilde{y}(t_{i+2})$, that is $\widetilde{f}(t_{i-1}, y(t_{i-1}))$, $\widetilde{f}(t_i, y(t_i))$, $\widetilde{f}(t_{i+1}, y(t_{i+1}))$, $\widetilde{f}(t_{i+2}, y(t_{i+2}))$, which are triangular fuzzy numbers and are shown by

$$\left\{f^l(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)), f^c(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)), f^r(t_{i-1}, y(t_{i-1}), \lambda_k(y_k))\right\},$$

$$\left\{f^l(t_i, y(t_i), \lambda_k(y_k)), f^c(t_i, y(t_i), \lambda_k(y_k)), f^r(t_i, y(t_i)), \lambda_k(y_k)\right\},$$

$$\left\{f^l(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)), f^c(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)), f^r(t_{i+1}, y(t_{i+1}), \lambda_k(y_k))\right\}.$$

Also integrating equation (6) from t_{i+2} to t_{i+3} we get

$$\widetilde{y}(t_{i+3}) = \widetilde{y}(t_{i+2}) + \int_{t_{i+2}}^{t_{i+3}} \widetilde{f}(t, y_k(t), \lambda_k(y_k))dt. \tag{7}$$

By fuzzy linear spline interpolation for $\widetilde{f}(t_{i-1},(t_{i-1}),\lambda_k(y_k))$, $\widetilde{f}(t_i,(t_i),\lambda_k(y_k))$, $\widetilde{f}(t_{i+1},(t_{i+1}),\lambda_k(y_k))$, $\widetilde{f}(t_{i+2},(t_{i+2}),\lambda_k(y_k))$ we have;

$$\begin{aligned}
f^l(t,y_k(t),\lambda_k(y_k)) &= \sum_{j=i-1,l_j(t)\geq 0}^{i+2} l_j(t)f^l(t_{k,j},y(t_{k,j}),\lambda_k(y_k)) \\
&+ \sum_{j=i-1,l_i(t)<0}^{i+2} l_j(t)f^r(t_{k,j},y(t_{k,j}),\lambda_k(y_k)), \\
f^c(t,y_k(t),\lambda_k(y_k)) &= \sum_{j=i-1}^{i+2} l_j(t)f^c(t_{k,j},y(t_{k,j}),\lambda_k(y_k)), \\
f^r(t,y_k(t),\lambda_k(y_k)) &= \sum_{j=i-1,l_j(t)\geq 0}^{i+2} l_j(t)f^r(t_{k,j},y(t_{k,j}),\lambda_k(y_k)) \\
&+ \sum_{j=i-1,l_j(t)<0}^{i+2} l_j(t)f^l(t_{k,j},y(t_{k,j}),\lambda_k(y_k)),
\end{aligned}$$

for $t_{i+2}\leq t\leq t_{i+3}$:

$$\begin{aligned}
l_{i-1}(t) &= \frac{(t-t_i)(t-t_{i+1})(t-t_{i+2})}{(t_{i-1}-t_i)(t_{i-1}-t_{i+1})(t_{i-1}-t_{i+2})}\leq 0, \\
l_i(t) &= \frac{(t-t_{i-1})(t-t_{i+1})(t-t_{i+2})}{(t_i-t_{i-1})(t_i-t_{i+1})(t_i-t_{i+2})}\geq 0, \\
l_{i+1}(t) &= \frac{(t-t_{i-1})(t-t_i)(t-t_{i+2})}{(t_{i+1}-t_{i-1})(t_{i+1}-t_i)(t_{i+1}-t_{i+2})}\leq 0, \\
l_{i+2}(t) &= \frac{(t-t_{i-1})(t-t_i)(t-t_{i+1})}{(t_{i+2}-t_{i-1})(t_{i+2}-t_i)(t_{i+2}-t_{i+1})}\geq 0,
\end{aligned}$$

therefore the following results will be obtained:

$$\begin{aligned}
f^l(t,y(t),\lambda_k(y_k)) &= l_{i-1}(t)f^r(t_{i-1},y(t_{i-1}),\lambda_k(y_k))+l_i(t)f^l(t_i,y(t_i),\lambda_k(y_k)) \\
&+l_{i+1}(t)f^r(t_{i+1},y(t_{i+1}),\lambda_k(y_k))+l_{i+2}(t)f^l(t_{i+2},y(t_{i+2}),\lambda_k(y_k)), \quad (8) \\
f^c(t,y(t),\lambda_k(y_k)) &= l_{i-1}(t)f^c(t_{i-1},y(t_{i-1}),\lambda_k(y_k))+l_i(t)f^c(t_i,y(t_i),\lambda_k(y_k)) \\
&+l_{i+1}(t)f^c(t_{i+1},y(t_{i+1}),\lambda_k(y_k))+l_{i+2}(t)f^c(t_{i+2},y(t_{i+2}),\lambda_k(y_k))), \quad (9) \\
f^r(t,y(t),\lambda_k(y_k)) &= l_{i-1}(t)f^l(t_{i-1},y(t_{i-1}),\lambda_k(y_k))+l_i(t)f^r(t_i,y(t_i),\lambda_k(y_k)) \\
&+l_{i+1}(t)f^l(t_{i+1},y(t_{i+1}),\lambda_k(y_k))+l_{i+2}(t)f^r(t_{i+2},y(t_{i+2},\lambda_k(y_k))). \quad (10)
\end{aligned}$$

From (2) and (7) it follows that:

$$\widetilde{y}^{\alpha}(t_{i+3})=[\underline{y}^{\alpha}(t_{i+3}),\overline{y}^{\alpha}(t_{i+3})],$$

where

$$\underline{y}^{\alpha}(t_{i+3}) = \underline{y}^{\alpha}(t_{i+2}) + \int_{t_{i+2}}^{t_{i+3}} \left\{\alpha f^{c}(t, y(t), \lambda_k(y_k)) + (1-\alpha) f^{l}(t, y(t), \lambda_k(y_k))\right\} dt \tag{11}$$

and

$$\overline{y}^{\alpha}(t_{i+3}) = \overline{y}^{\alpha}(t_{i+2}) + \int_{t_{i+2}}^{t_{i+3}} \left\{\alpha f^{c}(t, y(t), \lambda_k(y_k)) + (1-\alpha) f^{r}(t, y(t), \lambda_k(y_k))\right\} dt, \tag{12}$$

If (8) and (9) are used in (11) and (9),(10) in (12)

$$\begin{aligned}
\underline{y}^{\alpha}(t_{i+3}) &= \underline{y}^{\alpha}(t_{i+2}) + \int_{t_{i+2}}^{t_{i+3}} \{\alpha(l_{i-1}(t) f^{c}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)) + l_i(t) f^{c}(t_i, y(t_i), \lambda_k(y_k)) \\
&\quad + l_{i+1}(t) f^{c}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) + l_{i+2}(t) f^{c}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k)) \\
&\quad + (1-\alpha) l_{i-1}(t) f^{r}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)) + l_i(t) f^{l}(t_i, y(t_i), \lambda_k(y_k)) \\
&\quad + l_{i+1}(t) f^{r}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) + l_{i+2}(t) f^{l}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k))\} dt,
\end{aligned}$$

and

$$\begin{aligned}
\overline{y}^{\alpha}(t_{i+3}) &= \overline{y}^{\alpha}(t_{i+2}) + \int_{t_{i+2}}^{t_{i+3}} \{\alpha(l_{i-1}(t) f^{c}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)) + l_i(t) f^{c}(t_i, y(t_i), \lambda_k(y_k)) \\
&\quad + l_{i+1}(t) f^{c}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) + l_{i+2}(t) f^{c}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k)) \\
&\quad + (1-\alpha) l_{i-1}(t) f^{l}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)) + l_i(t) f^{r}(t_i, y(t_i), \lambda_k(y_k)) \\
&\quad + l_{i+1}(t) f^{l}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) + l_{i+2}(t) f^{r}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k))\} dt.
\end{aligned}$$

The following results will be obtained by integration:

$$\begin{aligned}
\underline{y}^{\alpha}(t_{i+3}) &= \underline{y}^{\alpha}(t_{i+2}) + \frac{55h}{24}[\alpha f^{c}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k)) + (1-\alpha) f^{l}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k))] \\
&\quad - \frac{59h}{24}[\alpha f^{c}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) + (1-\alpha) f^{r}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k))] \\
&\quad + \frac{37h}{24}[\alpha f^{c}(t_i, y(t_i), \lambda_k(y_k)) + (1-\alpha) f^{l}(t_i, y(t_i), \lambda_k(y_k))] \\
&\quad - \frac{9h}{24}[\alpha f^{c}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)) + (1-\alpha) f^{r}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k))],
\end{aligned}$$

and

$$\begin{aligned}
\overline{y}^{\alpha}(t_{i+3}) &= \overline{y}^{\alpha}(t_{i+2}) + \frac{55h}{24}[\alpha f^{c}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k)) + (1-\alpha) f^{r}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k))] \\
&\quad - \frac{59h}{24}[\alpha f^{c}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) + (1-\alpha) f^{l}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k))] \\
&\quad + \frac{37h}{24}[\alpha f^{c}(t_i, y(t_i), \lambda_k(y_k)) + (1-\alpha) f^{r}(t_i, y(t_i))] \\
&\quad - \frac{9h}{24}[\alpha f^{c}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)) + (1-\alpha) f^{l}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k))].
\end{aligned}$$

Thus

$$\begin{aligned} \underline{y}^{\alpha}(t_{i+3}) &= \underline{y}^{\alpha}(t_{i+2}) + \frac{h}{24}[55\underline{f}^{\alpha}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k)) - 59\overline{f}^{\alpha}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) \\ &\quad +37\underline{f}^{\alpha}(t_i, y(t_i), \lambda_k(y_k)) - 9\overline{f}^{\alpha}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k))], \end{aligned} \tag{13}$$

$$\begin{aligned} \overline{y}^{\alpha}(t_{i+3}) &= \overline{y}^{\alpha}(t_{i+2}) + \frac{h}{24}[55\overline{f}^{\alpha}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k)) - 59\underline{f}^{\alpha}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) \\ &\quad +37\overline{f}^{\alpha}(t_i, y(t_i), \lambda_k(y_k)) - 9\underline{f}^{\alpha}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k))]. \end{aligned} \tag{14}$$

Therefore Adams-Basforth four-step method is obtained as follows:

$$\begin{cases} \underline{y}^{\alpha}(t_{i+3}) = \underline{y}^{\alpha}(t_{i+2}) + \frac{h}{24}[55\underline{f}^{\alpha}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k)) - 59\overline{f}^{\alpha}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) \\ \qquad +37\underline{f}^{\alpha}(t_i, y(t_i), \lambda_k(y_k)) - 9\overline{f}^{\alpha}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)), \\ \overline{y}^{\alpha}(t_{i+3}) = \overline{y}^{\alpha}(t_{i+2}) + \frac{h}{24}[55\overline{f}^{\alpha}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k)) - 59\underline{f}^{\alpha}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) \\ \qquad +37\overline{f}^{\alpha}(t_i, y(t_i), \lambda_k(y_k)) - 9\overline{f}^{\alpha}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)), \\ \underline{y}^{\alpha}(t_{i-1}) = \alpha_0,\ \underline{y}^{\alpha}(t_i) = \alpha_1,\ \underline{y}^{\alpha}(t_{i+1}) = \alpha_2,\ \underline{y}^{\alpha}(t_{i+2}) = \alpha_3, \\ \overline{y}^{\alpha}(t_{i-1}) = \alpha_4,\ \overline{y}^{\alpha}(t_i) = \alpha_5,\ \overline{y}^{\alpha}(t_{i+1}) = \alpha_6\ , \overline{y}^{\alpha}(t_{i+2}) = \alpha_7. \end{cases} \tag{15}$$

5. Adams-Moulton methods

Fix $k \in Z^+$. The fuzzy initial value problem (6) can be solved by Adams-Moulton three-step method. The Adams-Moulton three step method is obtained as follows:

$$\begin{cases} \underline{y}^{\alpha}(t_{i+2}) = \underline{y}^{\alpha}(t_{i+1}) + \frac{h}{24}[9\underline{f}^{\alpha}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k)) + 19\underline{f}^{\alpha}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) \\ \qquad -5\overline{f}^{\alpha}(t_i, y(t_i), \lambda_k(y_k)) + \underline{f}^{\alpha}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)) \\ \overline{y}^{\alpha}(t_{i+2}) = \overline{y}^{\alpha}(t_{i+1}) + \frac{h}{24}[9\overline{f}^{\alpha}(t_{i+2}, y(t_{i+2}), \lambda_k(y_k)) + 19\overline{f}^{\alpha}(t_{i+1}, y(t_{i+1}), \lambda_k(y_k)) \\ \qquad -5\underline{f}^{\alpha}(t_i, y(t_i), \lambda_k(y_k)) + 9\overline{f}^{\alpha}(t_{i-1}, y(t_{i-1}), \lambda_k(y_k)) \\ \underline{y}^{\alpha}(t_{i-1}) = \alpha_0,\ \underline{y}^{\alpha}(t_i) = \alpha_1,\ \underline{y}^{\alpha}(t_{i+1}) = \alpha_2 \\ \overline{y}^{\alpha}(t_{i-1}) = \alpha_3,\ \overline{y}^{\alpha}(t_i) = \alpha_4\ , \overline{y}^{\alpha}(t_{i+1}) = \alpha_5 \end{cases} \tag{16}$$

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6. Adams fourth order predictor-corrector method.

The following algorithm is based on Adams-Bashforth four-step method as a predictor and also an iteration of Adams-Moulton three-step method as a corrector.

ALGORITHM:

Fix $k \in Z^+$. To approximate the solution of following fuzzy initial value problem.

$$x'_k(t) = f(t_{k,i}, y(t_{k,i}), \lambda_k(y_k))$$

$$\underline{y}^\alpha(t_{k,i-1}) = \underline{\alpha}_0,\ \underline{y}^\alpha(t_{k,i}) = \underline{\alpha}_1,\ \underline{y}^\alpha(t_{k,i+1}) = \underline{\alpha}_2,\ \underline{y}^\alpha(t_{k,i+2}) = \underline{\alpha}_3.$$

$$\overline{y}^\alpha(t_{k,i-1}) = \overline{\alpha}_0,\ \overline{y}^\alpha(t_{k,i}) = \overline{\alpha}_1,\ \overline{y}^\alpha(t_{k,i+1}) = \overline{\alpha}_2,\ \overline{y}^\alpha(t_{k,i+2}) = \overline{\alpha}_3.$$

positive integer N_k is chosen.

Step 1. Let $h = \dfrac{t_{k+1} - t_k}{N_k}$,

$$\underline{w}^\alpha(t_{k,0}) = \underline{\alpha}_0, \underline{w}^\alpha(t_{k,1}) = \underline{\alpha}_1, \underline{w}^\alpha(t_{k,2}) = \underline{\alpha}_2, \underline{w}^\alpha(t_{k,3}) = \underline{\alpha}_3,$$

$$\overline{w}^\alpha(t_{k,0}) = \overline{\alpha}_0, \overline{w}^\alpha(t_{k,1}) = \overline{\alpha}_1, \overline{w}^\alpha(t_{k,2}) = \overline{\alpha}_2, \overline{w}^\alpha(t_{k,3}) = \overline{\alpha}_3,$$

Step 2. Let $i = 1$,

Step 3. Let

$$\begin{cases} \underline{w}^{(0)\alpha}(t_{i+3}) = \underline{w}^\alpha(t_{i+2}) + \frac{h}{24}[55\underline{f}^\alpha(t_{i+2}, w(t_{i+2}), \lambda_k(w_k)) - 59\overline{f}^\alpha(t_{i+1}, w(t_{i+1}), \lambda_k(w_k)) \\ \qquad +37\underline{f}^\alpha(t_i, w(t_i), \lambda_k(w_k)) - 9\overline{f}^\alpha(t_{i-1}, w(t_{i-1}), \lambda_k(w_k))], \\ \overline{w}^{(0)\alpha}(t_{i+3}) = \overline{w}^\alpha(t_{i+2}) + \frac{h}{24}[55\underline{f}^\alpha(t_{i+2}, w(t_{i+2}), \lambda_k(w_k)) - 59\overline{f}^\alpha(t_{i+1}, w(t_{i+i}), \lambda_k(w_k)) \\ \qquad +37\overline{f}^\alpha(t_i, w(t_i), \lambda_k(w_k)) - 9\underline{f}^\alpha(t_{i-1}, w(t_{i-1}), \lambda_k(w_k))]. \end{cases}$$

Step 4. Let $t_{i+3} = t_0 + (i+3)h$.

Step 5. Let

$$\begin{cases} \underline{w}^\alpha(t_{i+2}) = \underline{y}^\alpha(t_{i+1}) + \frac{h}{24}[9\underline{f}^\alpha(t_{i+2}, w(t_{i+2}), \lambda_k(w_k)) + 19\underline{f}^\alpha(t_{i+1}, w(t_{i+1}), \lambda_k(w_k))], \\ \qquad -5\overline{f}^\alpha(t_i, w(t_i), \lambda_k(w_k)) + \underline{f}^\alpha(t_{i-1}, w(t_{i-1}), \lambda_k(w_k)), \\ \overline{w}^\alpha(t_{i+2}) = \overline{w}^\alpha(t_{i+1}) + \frac{h}{24}[9\overline{f}^\alpha(t_{i+2}, w(t_{i+2}), \lambda_k(w_k)) + 19\overline{f}^\alpha(t_{i+1}, w(t_{i+1}), \lambda_k(w_k)) \\ \qquad -5\underline{f}^\alpha(t_i, y(t_i), \lambda(w_k)) + \overline{f}^\alpha(t_{i-1}, w(t_{i-1}), \lambda_k(w_k))], \end{cases}$$

Step 6. $i = i + 1$.
Step 7. If $i \leq N - 2$ go to step 3.
Step 8. Algorithm will be completed and $(\underline{w}^{\alpha}(t_{k+1}), \overline{w}^{\alpha}(t_{k+1})$ approximates real value of $(\underline{x}^{\alpha}(t_{k+1}), \overline{x}^{\alpha}(t_{k+1})$.

7. Convergence and Stability

For a fixed α, to integrate the system (15) in $[t_0, t_1], [t_1, t_2], \cdots, [t_k, t_{k+1}], \cdots$, we replace each interval by a set of discrete equally spaced grid points $t_1 < t_2 < \ldots < t_N = T$ (including the end points) at which the exact solution $(\underline{x}(t,\alpha),\ \overline{x}(t,\alpha))$ is approximated by some $(\underline{y}(t,\alpha),\ \overline{y}(t,\alpha))$. The gird points which the solution is calculated are at $[t_k, t_{k+1}]$ at $t_{k,n} = t_k + nh_k, h_k = (t_{k+1} - t_k)/N_k, 0 \leq n \leq N_k$, the exact and approximate solutions are denoted by $x_{k,n}(\alpha) = [\underline{x}_{k,n}(\alpha), \overline{x}_{k,n}(\alpha)]$ and $y_{k,n}(\alpha) = [\underline{y}_{k,n}(\alpha), \overline{y}_{k,n}(\alpha)]$, respectively. However, we will use

$$\underline{y}_{0,0}(\alpha) = \underline{\alpha}_0,\ \ \underline{y}_{0,1}(\alpha) = \underline{\alpha}_1,\ \ \underline{y}_{0,2}(\alpha) = \underline{\alpha}_2,\ \ \underline{y}_{0,3}(\alpha) = \underline{\alpha}_3,$$

$$\overline{y}_{0,0}(\alpha) = \overline{\alpha}_0,\ \ \overline{y}_{0,1}(\alpha) = \overline{\alpha}_1,\ \ \overline{y}_{0,2}(\alpha) = \overline{\alpha}_2,\ \ \overline{y}_{0,3}(\alpha) = \overline{\alpha}_3,$$

and

$$\underline{y}_{k,0}(\alpha) = \underline{y}_{k-1,N_{k-1}-1}(\alpha),\ \underline{y}_{k,1}(\alpha) = \underline{y}_{k-1,N_{k-1}}(\alpha),\ \underline{y}_{k,2}(\alpha) = \underline{y}_{k-1,N_{k-1}+1}(\alpha),$$

$$\overline{y}_{k,0}(\alpha) = \overline{y}_{k-1,N_{k-1}-1}(\alpha),\ \overline{y}_{k,1}(\alpha) = \overline{y}_{k-1,N_{k-1}}(\alpha),\ \overline{y}_{k,2}(\alpha) = \overline{y}_{k-1,N_{k-1}+1}(\alpha).$$

Theorem 7.1. For arbitrary fixed $\alpha : 0 \leq \alpha \leq 1$ and $k \in Z^+$, the Adams-Moulton three-step approximates equation (16) converge to the exact solutions $\underline{x}(t_{k+1}, \alpha),\ \ \overline{x}(t_{k+1}, \alpha)$.

Proof. It is sufficient to show

$$\lim_{h_0,\cdots,h_k} \underline{y}_{k,N_k}(\alpha) = \underline{x}(t_{k+1}, \alpha), \qquad \lim_{h_0,\cdots,h_k} \overline{y}_{k,N_k}(\alpha) = \overline{x}(t_{k+1}, \alpha)$$

Remark 7.1. The convergence order of Adams-Moulton three-step method is $O(h^4)$.
Theorem 7.2. For arbitrary fixed $\alpha : 0 \leq \alpha \leq 1$ and $k \in Z^+$, the Adams-Bashforth four-step approximates equation (15) converge to the exact solutions $\underline{x}(t_{k+1}, \alpha),\ \ \overline{x}(t_{k+1}, \alpha)$.
Remark 7.2. The convergence order of Adams-Bashforth four-step method is $O(h^4)$.
Theorem 7.3. Fix $k \in Z^+$, the Adams-Bashforth three and four-step methods are stable.
Proof. Fix $k \in Z^+$. For Adams-Bashforth three-step method, there exist only one characteristic polynomial $p(\lambda) = \lambda^2 - \lambda$ and it is clear that satisfies the root condition by Theorem 2.1; then the method is stable.

Also, for Adams-Bashforth four-step method, there exist only one characteristic polynomial $p(\lambda) = \lambda^3 - \lambda^2$ and it satisfies the root condition, therefore it is a stable.
Theorem 7.4. Fix $k \in Z^+$, the Adams-Moulton three and four-step methods are stable.
Proof. Similar to Theorem 7.3.

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8. Numerical Examples

Consider the fuzzy initial value problem,

$$\begin{aligned} \tilde{x}'(t) &= -\tilde{x}(t), \\ \tilde{x}(0) &= [0.75, 1, 1.125]. \end{aligned} \tag{17}$$

We translate the problem (17) into the following system of equations

$$\begin{array}{l} x^{l'}(t) = -x^r(t), \\ x^{c'}(t) = -x^c(t), \\ x^{r'}(t) = -x^l(t), \\ x^l(0) = 0.75,\ x^c(t) = 1,\ x^r(t) = 1.125. \end{array} \tag{18}$$

and its solution is $x^l(t) = -0.1875e^t + 0.9375e^{-t},\ x^c(t) = e^{-t},\ x^r(t) = 0.1875e^t + 0.9375e^{-t}$. By [15], the exact solution of (17) is

$$x(t) = [-0.1875e^t + 0.9375e^{-t},\ e^{-t},\ 0.1875e^t + 0.9375e^{-t}]$$

which compares well with the predictor-corrector method.

Example 8.1. Consider the fuzzy initial value problem,

$$\begin{cases} \tilde{x}'(t) & = -\tilde{x}(t) + m(t)\lambda_k(x(t_k)), \quad t \in [t_k, t_{k+1}], \quad t_k, t_{k+1}], \quad t_k = k, \quad k = 0, 1, 2, ..., \\ \tilde{x}(0) & = [0.75, \quad 1, \quad 1.125], \\ \tilde{x}(0.1) & = [-0.1875e^{0.1} + 0.9375e^{-0.1}, \quad e^{0.1}, \quad 0.1875e^{0.1} + 0.9375e^{-0.1}], \\ \tilde{x}(0.2) & = [-0.1875e^{0.2} + 0.9375e^{-0.2}, \quad e^{0.2}, \quad 0.1875e^{0.2} + 0.9375e^{-0.2}], \\ \tilde{x}(0.3) & = [-0.1875e^{0.3} + 0.9375e^{-0.3}, \quad e^{0.3}, \quad 0.1875e^{0.3} + 0.9375e^{-0.3}], \end{cases} \tag{19}$$

where

$$m(t) = |sin(\pi t)|, \qquad k = 0, 1, 2, \ldots \tag{20}$$

$$\lambda(t) = \begin{cases} 0, & if \quad k = 0 \\ \mu, & k \in \{1, 2, \ldots, \} \end{cases} \tag{21}$$

The hybrid fuzzy initial value problem (19) is equivalent to the following system of fuzzy initial value problems:

$$\begin{cases} \tilde{x}_0^{l'}(t) & = -\tilde{x}_0^r(t), \\ \tilde{x}_0^{c'}(t) & = -\tilde{x}_0^c(t), \\ \tilde{x}_0^{r'}(t) & = -\tilde{x}_0^l(t), \quad t \in [0,1] \\ \tilde{x}(0) & = [0.75, 1, 1.125], \\ \tilde{x}(0.1) & = [-0.1875e^{0.1} + 0.9375e^{-0.1}, \quad e^{-0.1}, \quad 0.1875e^{0.1} + 0.9375e^{-0.1}], \\ \tilde{x}(0.2) & = [-0.1875e^{0.2} + 0.9375e^{-0.2}, \quad e^{-0.2}, \quad 0.1875e^{0.2} + 0.9375e^{-0.2}], \\ \tilde{x}(0.3) & = [-0.1875e^{0.3} + 0.9375e^{-0.3}, \quad e^{-0.3}, \quad 0.1875e^{0.3} + 0.9375e^{-0.3}], \\ \tilde{x}^{l'}(t_i) & = -\tilde{x}_i{}^r(t_i) + m(t)x_i^l(t_i), \\ \tilde{x}^{c'}(t_i) & = -\tilde{x}_i{}^c(t_i) + m(t)x_i^c(t_i) \\ \tilde{x}^{r'}(t_i) & = -\tilde{x}_i{}^l(t_i) + m(t)x_i^r(t_i) \quad t \in [t_i, t_{i+1}], \\ x_i(t_i) & = x_{i-1}(t_i), x_i(t_i-1) = x_{i-1}(t_i-1), x_i(t_i-2) = x_{i-1}(t_i-2), \quad i = 1, 2 \ldots. \end{cases} \tag{22}$$

For $t \in [0,1]$, the exact solution of equation (17) satisfies

$$x(t) = [-0.1875e^t + 0.9375e^{-t}, \quad e^{-t}, \quad 0.1875e^t + 0.9375e^{-t}]. \tag{23}$$

For $t \in [1,2]$, the exact solution of equation (19) satisfies,

$$x(t)^T = \begin{pmatrix} -0.1875\left[e^t + \dfrac{1}{1+\pi^2}(e(sin(\pi t) + \pi cos(\pi t)) + \pi e^t)\right] \\ +0.9375\left[e^{-t} - \dfrac{1}{1+\pi^2}(e^{-1}(sin(\pi t) - \pi cos(\pi t)) - \pi e^{-t})\right] \\ e^{-t} - \dfrac{1}{1+\pi^2}(e^{-1}(sin(\pi t) - \pi cos(\pi t)) - \pi e^{-t}) \\ 0.1875\left[e^t + \dfrac{1}{1+\pi^2}(e(sin(\pi t) + \pi cos(\pi t)) + \pi e^t)\right] \\ +0.9375\left[e^{-t} - \dfrac{1}{1+\pi^2}(e^{-1}(sin(\pi t) - \pi cos(\pi t)) - \pi e^{-t})\right] \end{pmatrix} \tag{24}$$

The results of Example 8.1 on [0,2] are shown in Figure 1.

Example 8.2. Consider the fuzzy initial value problem,

$$\begin{cases} \tilde{x}'(t) & = \tilde{x}(t) + m(t)\lambda_k(x(t_k)), \quad t \in [t_k, t_{k+1}], \quad t_k, t_{k+1}], \quad t_k = k, \quad k = 0, 1, 2, ..., \\ \tilde{x}(0) & = [0.75, \quad 1, \quad 1.125], \\ \tilde{x}(0.1) & = [0.75e^{0.1}, \quad e^{0.1}, \quad 1.125e^{0.1}], \\ \tilde{x}(0.2) & = [0.75e^{0.2}, \quad e^{0.2}, \quad 1.125e^{0.2}], \\ \tilde{x}(0.3) & = [0.75e^{0.3}, \quad e^{0.3}, \quad 1.125e^{0.3}], \end{cases} \tag{25}$$

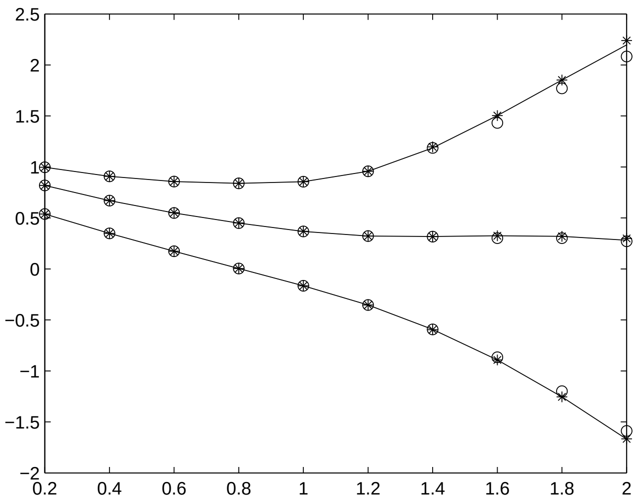

Figure 1: h=0.1
- Exact, *-Predictor corrector order 4, o-Predictor corrector order 3

where

$$m(t) = \begin{cases} 2(t(mod1)), & if \quad t(mod1) \le 0.5 \\ 2(1 - t(mod1)), & if \quad t(mod1) > 0.5, \end{cases} \tag{26}$$

$$\lambda(t) = \begin{cases} 0, & if \quad k = 0 \\ \mu, & k \in \{1, 2, \dots, \}. \end{cases} \tag{27}$$

The hybrid fuzzy initial value problem (25) is equivalent to the following system of fuzzy initial value problems:

$$\begin{cases} \tilde{x}_0'(t) & = \tilde{x}_0(t), \quad t \in [0,1] \\ \tilde{x}(0) & = [0.75 + 0.25\alpha, \quad 1.125 - 0.125\alpha], \\ \tilde{x}(0.1) & = [(0.75 + 0.25\alpha)e^{0.1}, \quad (1.125 - 0.125\alpha)^{0.1}], \\ \tilde{x}(0.2) & = [(0.75 + 0.25\alpha)e^{0.2}, \quad (1.125 - 0.125\alpha)^{0.2}], \\ \tilde{x}(0.3) & = [(0.75 + 0.25\alpha)e^{0.3}, \quad (1.125 - 0.125\alpha)^{0.3}], \quad 0 \le \alpha \le 1. \\ \tilde{x}'(t_i) & = \tilde{x}_i(t_i) + m(t)x_i(t_i)), \quad t \in [t_i, t_{i+1}], \\ x_i(t_i) & = x_{i-1}(t_i), x_i(t_i - 1) = x_{i-1}(t_i - 1), x_i(t_i - 2) = x_{i-1}(t_i - 2), \quad i = 1, 2 \dots. \end{cases} \tag{28}$$

In equation(25), $x(t) + m(t)\lambda_k(x((t_k)))$ is a continuous function of t, x and $\lambda_k(x(t_k))$. Therefore, by

Example 6.1 0f Kaleva [15], for each $k = 0, 1, 2, \ldots$, fuzzy initial value problem

$$\begin{cases} \tilde{x}'(t_i) & = \tilde{x}_i(t_i) + m(t)x_i(t_i)), \quad t \in [t_i, t_{i+1}], \\ x_i(t_i) & = x_{i-1}(t_k), \\ x(t_k - 1) & = x_{t_k} - 1, \\ x(t_k - 2) & = x_{t_k} - 2, \quad i = 1, 2 \ldots. \end{cases} \tag{29}$$

has a unique solution $[t_k, t_{k+1}]$.

For $\in [0, 1]$, the exact solution of equation (25) satisfies

$$x(t) = [0.75e^t, \quad e^t, \quad 1.125e^t]. \tag{30}$$

For $t \in [1, 2]$, the exact solution of equation (25) satisfies,

$$x(t) = x(1)(2t - 2 + e^{t-1.5}(3\sqrt{e} - 4)). \tag{31}$$

The results of Example 8.2 [1,2] are shown in Figure 2.

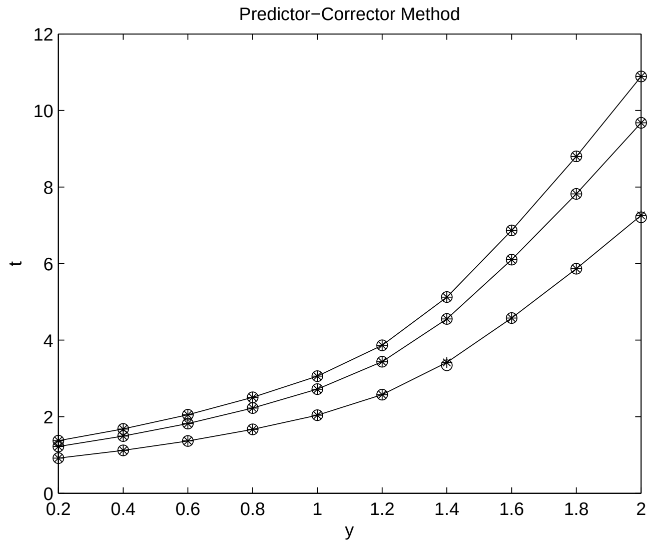

Figure 2: h=0.1
- Exact, o-Predictor corrector order 4, *-Predictor corrector order 3

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