Explanation why “when one takes money he becomes a slave” using automata theory and Van der Pol Equation

"Rely on the teaching, not on the person;
Rely on the meaning, not on the words;
Rely on the definitive meaning, not on the provisional;
Rely on your wisdom mind, not on your ordinary mind."
Buddha

1. Introduction

We study the possibility for a Finite State Automaton (FSA) to find a signal masked by the noise (“white noise”).
Such problems have applications in marine engineering where such automata are used to risque underwater crew.
We show that the only possible automaton that leads to such rescue mission success is automaton built on the up-and-down principle. FSA built on other principles “gets stacked” and not able to fulfill their “duties”.
Similar problems were considered in [4], [5], [6],[7].

2. Main result

Let us recall a definition of FSA. More precisely, a definition of a transducer.

Definition
A finite state transducer is a sextuple $(\Sigma, \Gamma, S, s_0, \delta, \omega)$, where:

- Σ is the input alphabet (a finite non-empty set of symbols).
- Γ is the output alphabet (a finite, non-empty set of symbols).
- S is a finite, non-empty set of states.
- s_0 is the initial state, an element of S.
- δ is the state-transition function: $\delta : S \times \Sigma \to S$.
- ω is the output function $\omega : S \times \Sigma \to \Gamma$.

Let consider the case when $\Sigma = \Gamma = \{0, 1\}$. Other cases can be considered similarly.

Definition
A signal is a sequence $\alpha = (\alpha_1, ..., \alpha_n) \in \{0, 1\}^n$ of 0 *or* 1 of length n.

Definition
A signal $\beta = (\beta_1, ..., \beta_n)$ is called masked by the noise if there exists a random vector $\xi = (\xi_1, ..., \xi_n)$ with values in $\{0, 1\}^n$ such that

$$\beta = \alpha \oplus \xi, E(\xi_i) = 1/2 \, for \; i = 1, ..., n \; and \, for \; all \; n \qquad (1)$$

where $\oplus$ is a boolean addition and E is a expected value of ξ.
For simplicity, we let $\xi_i = \xi \, for \; all \; i$ where ξ is some random variable.

Let there be a transducer $A = (\Sigma, \Gamma, S, s_0, \delta, \omega)$ and let β be its input signal. The transducer output signal is denoted by γ.

Definition
We call a transducer $A = (\Sigma, \Gamma, S, s_0, \delta, \omega)$ capable of recognizing a signal α if

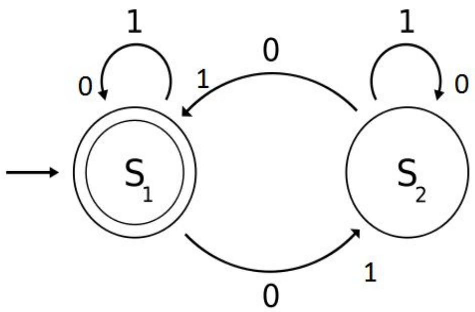
$$D(\gamma_n - \beta_n) \rightarrow 0 \qquad (2)$$

when $n \rightarrow \infty$, where D is a variance.

Definition
Up-and-down automaton is an automaton for which the output signal is determined by the rules
if input is 1 and previous input is 0, then the output is 1,
if input is 0 and previous input is 1, then the output is 0,
if input is 0 and previous input is 0, then the output is 1,
if input is 1 and previous input is 1, then the output is 0,
if no previous input, then the output is BLANK. `

As one can see Up-and-down automaton is the automaton that follows the input by taking the previous input and inverting it, and it reminds a well-known method from statistics theory called up-and-down method since it is based on the same principle - “up-and-down” [7].

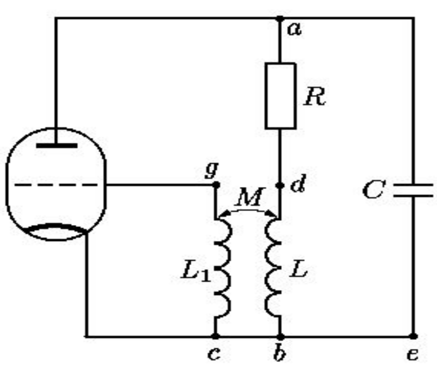
Example of such up-and-down automaton is given on Pic. 1.



Pic. 1 Example Up-and-down automaton with initial state s_1.

Theorem.
Up-and-down automaton is capable to recognize any signal α.
Proof.
Self-evident. It follows from (2).
QED

Theorem.
Any transducer that is not equivalent to Up-and-down automaton is not capable to recognize a signal α (there exists a signal α that (2) is not fulfilled).
Proof.
If such transducer exists, then it will be complex enough. The work of such transducers can be described through electronic elements. It is seen that this transducer will contain enough complex elements such as depicted on the Pic. 2.

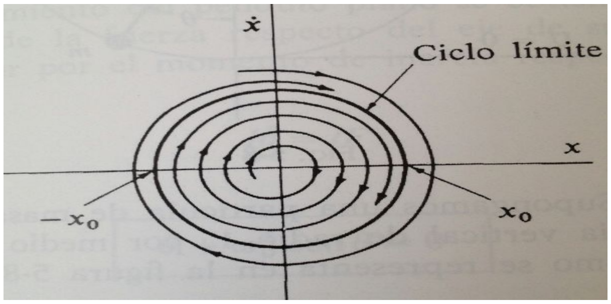


Pic. 2 Typical electronic schema

The signal inside schema depicted on Pic. 2 is described by Van Der Pol equation

$$x'' + \lambda(x^2 - 1)x' + x = 0 \qquad (3).$$

See [1] for further discussions. It is known that (3) posses solutions of type depicted on Pic. 3.



Pic.3 The Phase portrait of Van Der Pol equation (3)

If $\lambda = 0$, then the solution is harmonic oscillation.
As one can see such output signals can not predict any input signal except of the same type because they are too predictable themselves.
QED

3. Conclusion

The above results can be generalized for the case of “white noise” and bigger input-output alphabets Σ, Γ.
We do not use methods of probabilistic theory such as in [3] but rather methods developed by deterministic approaches that are described by differential equations [1], [2].
Such methods give another way in understanding the “reality”.

As one can see the above result explains why **“when one takes money he becomes a slave”** (**this formula is universal** and there is **no one who can escape this law**):
when one takes money he inevitably makes more complexity into his life, by this his way of thinking becomes a cycle (he does not see the reality anymore), and therefore he has no really new ideas never, he falls into complete illusion that he achieves something.

This article describes the meaning of the following Buddha’s story:
Prince Gautama who had become Buddha saw one of his followers meditating under a tree at the edge of the Ganges river. Upon inquiring why he was meditating, his follower stated he was attempting to become so enlightened he could cross the river unaided. Buddha gave him a few pennies and said: "Why don't you seek passage with that boatman. It is much easier."

As one can see **the modern civilization result is actually “three pennies”** because this civilization is in the cycle for several thousands years.

References

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